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Understanding Physics

JEE Main & Advanced

ELECTRICITY AND MAGNETISM

DC PANDEY

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卐 **ADMINISTRATIVE & PRODUCTION OFFICES**

Regd. Office
'Ramchhaya' 4577/15, Agarwal Road, Darya Ganj, New Delhi -110002
Tele: 011- 47630600, 43518550; Fax: 011- 23280316

Head Office
Kalindi, TP Nagar, Meerut (UP) - 250002
Tel: 0121-2401479, 2512970, 4004199; Fax: 0121-2401648

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PREFACE

The overwhelming response to the previous editions of this book gives me an immense feeling of satisfaction and I take this an opportunity to thank all the teachers and the whole student community who have found this book really beneficial.

In the present scenario of ever-changing syllabus and the test pattern of JEE Main & Advanced, the NEW EDITION of this book is an effort to cater all the difficulties being faced by the students during their preparation of JEE Main & Advanced. The exercises in this book have been divided into two sections viz., JEE Main & Advanced. Almost all types and levels of questions are included in this book. My aim is to present the students a fully comprehensive textbook which will help and guide them for all types of examinations. An attempt has been made to remove all the printing errors that had crept in the previous editions. I am extremely thankful to (Dr.) Mrs. Sarita Pandey, Mr. Anoop Dhyani, Nisar Ahmad for their endless efforts during the project.

Comments and criticism from readers will be highly appreciated and incorporated in the subsequent editions.

DC Pandey

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23

Current Electricity

Chapter Contents

23.1 Introduction

An electrical circuit consists of some active and passive elements. The active elements such as a battery or a cell, supply electric energy to the circuit. On the contrary, passive elements consume or store the electric energy. The basic passive elements are resistor, capacitor and inductor.

A resistor opposes the flow of current through it and if some current is passed by maintaining a potential difference across it, some energy is dissipated in the form of heat. A capacitor is a device which stores energy in the form of electric potential energy. It opposes the variations in voltage. An inductor opposes the variations in current. It does not oppose the steady current through it. Fundamentally, electric circuits are a means for conveying energy from one place to another. As charged particles move within a circuit, electric potential energy is transferred from a source (such as a battery or a cell) to a device in which that energy is either stored or converted to another form, like sound in a stereo system or heat and light in a toaster or light bulb. Electric circuits are useful because they allow energy to be transported without any moving parts (other than the moving charged particles themselves).

In this chapter, we will study the basic properties of electric currents. We'll study the properties of batteries and how they cause current and energy transfer in a circuit. In this analysis, we will use the concepts of current, potential difference, resistance and electromotive force.

23.2 Electric Current

Flow of charge is called electric current. The direction of electric current is in the direction of flow of positive charge or in the opposite direction of flow of negative charge.

Current is defined quantitatively in terms of the rate at which net charge passes through a cross-section area of the conductor.

Thus, $$\boxed{I = \frac{dq}{dt} \quad \text{or} \quad i = \frac{dq}{dt}}$$

We can have the following two concepts of current, as in the case of velocity, **instantaneous current** and **average current.**

Instantaneous current $= \dfrac{dq}{dt}$ = current at any point of time and

Average current $= \dfrac{q}{t}$

Hence-forth unless otherwise referred to, current would signify instantaneous current. By convention, the direction of the current is assumed to be that in which positive charge moves. In the SI system, the unit of current is ampere (A).

$$\boxed{1\ \text{A} = 1\ \text{C/s}}$$

Household currents are of the order of few amperes.

Flow of Charge

If current is passing through a wire then it implies that a charge is flowing through that wire. Further,

$$i = \frac{dq}{dt} \quad \Rightarrow \quad dq = i\,dt \qquad \text{...(i)}$$

Now, three cases are possible :

Case 1 If given current is constant, then from Eq. (i) we can see that flow of charge can be obtained directly by multiplying that constant current with the given time interval. Or,

$$\boxed{\Delta q = i \times \Delta t}$$

Case 2 If given current is a function of time, then charge flow can be obtained by integration. Or,

$$\boxed{\Delta q = \int_{t_i}^{t_f} i\,dt}$$

Case 3 If current *versus* time is given, then flow of charge can be obtained by the area under the graph.

$$\boxed{\Delta q = \text{area under } i\text{-}t \text{ graph}}$$

Extra Points to Remember

- The current is the same for all cross-sections of a conductor of non-uniform cross-section. Similar to the water flow, charge flows faster where the conductor is smaller in cross-section and slower where the conductor is larger in cross-section, so that charge rate remains unchanged.
- Electric current is very similar to water current, consider a water tank kept at some height and a pipe is connected to the water tank. The rate of flow of water through the pipe depends on the height of the tank. As the level of water in the tank falls, the rate of flow of water through the pipe also gets reduced. Just as the flow of water depends on the height of the tank or the level of water in the tank, the flow of current through a wire depends on the potential difference between the end points of the wire. As the potential difference is changed, the current will change. For example, during the discharging of a capacitor potential difference and hence, the current in the circuit decreases with time. To maintain a constant current in a circuit a constant potential difference will have to be maintained and for this a battery is used which maintains a constant potential difference in a circuit.
- Though conventionally a direction is associated with current (opposite to the motion of electrons), it is not a vector as the direction merely represents the sense of charge flow and not a true direction. Further, current does not obey the law of parallelogram of vectors, i.e. if two currents i_1 and i_2 reach a point we always have $i = i_1 + i_2$ whatever be the angle between i_1 and i_2.

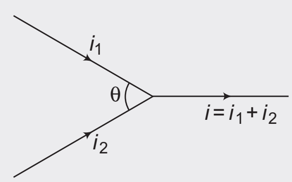


Fig. 23.1

- According to its magnitude and direction, current is usually divided into two types :
  (i) **Direct current (DC)** If the magnitude and direction of current does not vary with time, it is said to be direct current (DC). Cell, battery or DC dynamo are its sources.
  (ii) **Alternating current (AC)** If a current is periodic (with constant amplitude) and has half cycle positive and half negative, it is said to be alternating current (AC). AC dynamo is the source of it.
- If a charge q revolves in a circle with frequency f, the equivalent current,

$$\boxed{i = qf}$$

- In a conductor, normally current flow or charge flow is due to flow of free electrons.
- Charge is quantised. The quantum of charge is e. The charge on any body will be some integral multiple of e, i.e.

$$\boxed{q = \pm ne}$$

where, $n = 1, 2, 3\ldots$

Example 23.1 *In a given time of* 10 s, 40 *electrons pass from right to left. In the same interval of time* 40 *protons also pass from left to right. Is the average current zero? If not, then find the value of average current.*

Solution No, the average current is not zero. Direction of current is the direction of motion of positive charge or in the opposite direction of motion of negative charge. So, both currents are from left to right and both currents will be added.

$$\therefore \quad I_{av} = I_{electron} + I_{proton}$$

$$= \frac{q_1}{t_1} + \frac{q_2}{t_2}$$

$$= \frac{40\,e}{10} + \frac{40\,e}{10} \qquad (q = ne)$$

$$= 8\,e$$

$$= 8 \times 1.6 \times 10^{-19}\ \text{A}$$

$$= 1.28 \times 10^{-18}\ \text{A}$$ **Ans.**

Example 23.2 *A constant current of* 4 *A passes through a wire for* 8 s. *Find total charge flowing through that wire in the given time interval.*

Solution Since, $i =$ constant

$$\therefore \quad \Delta q = i \times \Delta t$$

$$= 4 \times 8$$

$$= 32\ \text{C}$$

Example 23.3 *A wire carries a current of* 2.0 *A. What is the charge that has flowed through its cross-section in* 1.0 *s ? How many electrons does this correspond to?*

Solution $\because \quad i = \frac{q}{t}$

$$\therefore \quad q = it = (2.0\ \text{A})(1.0\ \text{s}) = 2.0\ \text{C}$$ **Ans.**

$$q = ne$$

$$\therefore \quad n = \frac{q}{e} = \frac{2.0}{1.6 \times 10^{-19}}$$

$$= 1.25 \times 10^{19}$$ **Ans.**

Example 23.4 *The current in a wire varies with time according to the relation*

$$i = (3.0\ A) + (2.0\ A/s)\,t$$

(a) *How many coulombs of charge pass a cross-section of the wire in the time interval between* $t = 0$ *and* $t = 4.0$ *s?*

(b) *What constant current would transport the same charge in the same time interval?*

Solution (a)

$$i = \frac{dq}{dt}$$

$$\therefore \quad \int_0^q dq = \int_0^4 i\,dt$$

$$\therefore \quad q = \int_0^4 (3 + 2t)\,dt$$

$$= [3t + t^2]_0^4 = [12 + 16]$$

$$= 28\text{ C}$$ **Ans.**

(b) $i = \dfrac{q}{t} = \dfrac{28}{4} = 7\text{ A}$ **Ans.**

Example 23.5 *Current passing through a wire decreases linearly from* 10 *A to* 0 *in* 4 *s. Find total charge flowing through the wire in the given time interval.*

Solution Current *versus* time graph is as shown in figure.

Area under this graph will give us net charge flow.

Hence,

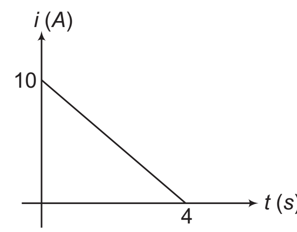


Fig. 23.2

$$\Delta q = \text{Area}$$

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 4 \times 10$$

$$= 20\text{ C}$$ **Ans.**

INTRODUCTORY EXERCISE 23.1

1. How many electrons per second pass through a section of wire carrying a current of 0.7 A?
2. A current of 3.6 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in 3.0 h?
3. A current of 7.5 A is maintained in wire for 45 s. In this time,
   (a) how much charge and
   (b) how many electrons flow through the wire?
4. In the Bohr model, the electron of a hydrogen atom moves in a circular orbit of radius 5.3×10^{-11} m with a speed of 2.2×10^6 m/s. Determine its frequency f and the current I in the orbit.
5. The current through a wire depends on time as, $i = (10 + 4t)$

   Here, i is in ampere and t in seconds. Find the charge crossed through a section in time interval between $t = 0$ to $t = 10$ s.
6. In an electrolyte, the positive ions move from left to right and the negative ions from right to left. Is there a net current? If yes, in what direction?

23.3 Electric Currents in Conductors

Conductors are those materials which can conduct electricity. Conductors can be broadly classified into two groups :

(i) Solid conductors

(ii) Electrolyte conductors

Normally in atoms and molecules, the negatively charged electrons and the positively charged nuclei are bound to each other and are thus not free to move. In solid conductors (notably metals), some of the electrons (called free electrons) are free to move within the bulk materials. In these conductors, current flow takes place due to these free electrons. Positive ions in these conductors are almost fixed. They do not move. So, they do not contribute in the current. In electrolyte solutions however, both positive and negative ions can move.

In our following discussions, we will focus only on solid conductors so that the current is carried by the negatively charged free electrons in the background of fixed positive ions.

Theory of Current Flow through Solid Conductors

At room temperature, the free electrons in a conductor move randomly with speeds of the order of 10^5 m/s. Since, the motion of the electrons is random, there is no net charge flow in any direction. For any imaginary plane passing through the conductor, the number of electrons crossing the plane in one direction is equal to the number crossing it in the other direction. Therefore, net current is zero from any section.

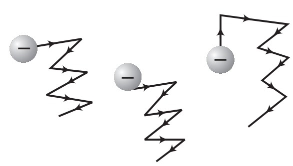


Fig. 23.3

When a constant potential difference V is applied between the ends of the conductor as shown in Fig. 23.4, an electric field **E** is produced inside the conductor. The conduction electrons within the conductor are then subjected to a force $-e\mathbf{E}$ and move overall in the direction of increasing potential.

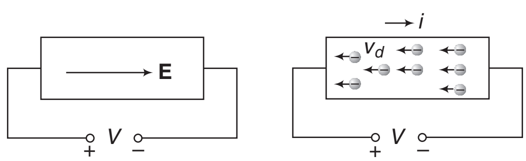


Fig. 23.4

However, this force does not cause the electrons to move faster and faster. Instead, a conduction electron accelerates through a very small distance (about 5×10^{-8} m) and then collides with fixed ions or atoms of the conductor. Each collision transfers some of the electron's kinetic energy to the ions (or atoms). Because of the collision, electron moves slowly along the conductor or we can say that it acquires a **drift velocity** $\boldsymbol{v_d}$ in the direction opposite to **E** (in addition to its random motion.)

The drift motion of free electrons produce an electric current in the opposite direction of this motion or in the direction of electric field (from higher potential to lower potential). It is interesting to note that the magnitude of the drift velocity is of the order of 10^{-4} m/s or about 10^9 times smaller than the average speed of the electrons of their random (or thermal) motion. The above discussion can be summarized as follows :

1. Free electrons inside a solid conductor can have two motions :
   (i) random or thermal motion (speed of the order of 10^5 m/s)
   (ii) drift motion (speed of the order of 10^{-4} m/s)
2. Net current due to random (or thermal motion) is zero from any section, whereas net current due to drift motion is non-zero.
3. In the absence of any electric field (or a potential difference across the conductor) free electrons have only random motion. Hence, net current from any section is zero.
4. In the presence of an electric field (or a potential difference across the conductor) free electrons have both motions (random and drift). Therefore, current is non-zero due to drift motion.
5. Drift motion of free electrons is opposite to the electric field. Therefore, direction of current is in the direction of electric field from higher potential to lower potential.

Example 23.6 *Electric field inside a conductor is always zero. Is this statement true or false?*

Solution False. Under electrostatic conditions when there is no charge flow (or no current) in the conductor, electric field is zero. If current is non-zero, then electric field is also non-zero. Because the drift motion (of free electrons) which produces a net current starts only due to electric force on them.

INTRODUCTORY EXERCISE 23.2

1. All points of a conductor are always at same potential. Is this statement true or false?

23.4 Drift Velocity and Relaxation Time

As discussed before, in the presence of electric field, the free electrons experience an electric force of magnitude.

$$F = qE \text{ or } eE \qquad (\text{as } q = e)$$

This will produce an acceleration of magnitude,

$$a = \frac{F}{m} = \frac{eE}{m} \qquad (m = \text{mass of electron})$$

Direction of force (and acceleration) is opposite to the direction of electric field.

After accelerating to some distance an electron will suffer collisions with the heavy fixed ions. The collisions of the electrons do not occur at regular intervals but at random times.

Relaxation time τ is the average time between two successive collisions. Its value is of the order of 10^{-14} second.

After every collision, let us assume that drift motion velocity of electron becomes zero. Then, it accelerates for a time interval τ, then again it collides and its drift motion velocity becomes zero and so on.

If v_d is the average constant velocity (called drift velocity) in the direction of drift motion, then relation between v_d and τ is given by

$$\boxed{v_d = \frac{eE\tau}{m}}$$

Extra Points to Remember

- In some standard books of Indian authors, the relation is given as

$$v_d = \frac{eE\tau}{2m}$$

Initially, I was also convinced with this expression. But later on after consulting many more literatures in this. I found that $v_d = \dfrac{eE\tau}{m}$ is correct. But at this stage it is very difficult for me to give its correct proof. Because the correct proof requires a knowledge of high level of physics which is difficult to understand for a class XII student.

Current and Drift Velocity

Consider a cylindrical conductor of cross-sectional area A in which an electric field E exists. Drift velocity of free electrons is v_d and n is number of free electrons per unit volume (called free electron density).

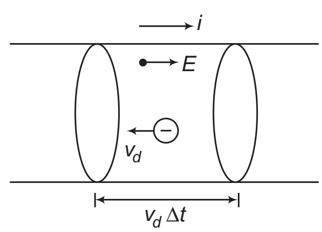


Fig. 23.5

Consider a length $v_d \Delta t$ of the conductor.

The volume of this portion is $Av_d \Delta t$.

Number of free electrons in this volume = (free electron density) × (volume)

$$= (n)\,(Av_d \Delta t)$$

$$= nAv_d \Delta t$$

All these electrons cross the area A in time Δt.

Thus, the charge crossing this area in time Δt is

$$\therefore \qquad \Delta q = (nAv_d \Delta t)\,(e)$$

or
$$i = \frac{\Delta q}{\Delta t} = neAv_d$$

or
$$\boxed{i = neAv_d}$$

Thus, this is the relation between current and drift velocity.

Current Density

Current per unit area (taken normal to the current), i/A is called current density and is denoted by j. The SI units of current density are A/m^2. Current density is a vector quantity $\mathbf{j}$ directed along $\mathbf{E}$.

$$\boxed{j = \frac{i}{A}}$$

But $i = neAv_d$, therefore

$$\boxed{j = nev_d}$$

Extra Points to Remember

- Drift velocity of electrons in a conductor is of the order of 10^{-4} m/s, then question arises in everybody's mind that why a bulb glows instantly when switched on? Reason is : when we close the circuit, electric field is set up in the entire closed circuit instantly (with the speed of light). Due to this electric field, the free electrons instantly get drift velocity in the entire circuit and a current is established in the circuit instantly. The current so set up does not wait for the electrons to flow from one end of the conductor to the other end.
- If a current i is flowing through a wire of non-uniform cross-section, then current will remain constant at all cross-sections. But drift velocity and current density are inversely proportional to the area of cross-section. This is because

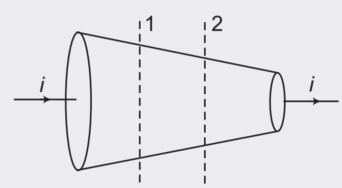


Fig. 23.6

$$i = neAv_d \quad \text{or} \quad v_d = \frac{i}{neA} \quad \text{or} \quad v_d \propto \frac{1}{A}$$

Further, $$j = \frac{i}{A} \quad \text{or} \quad j \propto \frac{1}{A}$$

So, in the figure $i_1 = i_2 = i$ but, $(v_d)_2 > (v_d)_1$ and $j_2 > j_1$ because $A_2 < A_1$

Note *Later we can also prove that electric field at 2 is also more than electric field at 1.*

Example 23.7 *An electron beam has an aperture of* $1.0\ mm^2$. *A total of* 6.0×10^{16} *electrons go through any perpendicular cross-section per second. Find (a) the current and (b) the current density in the beam.*

Solution (a) The current is given by

$$i = \frac{q}{t} = \frac{ne}{t}$$

Substituting the values we have,

$$i = \frac{(6.0 \times 10^{16})(1.6 \times 10^{-19})}{1}$$

$$= 9.6 \times 10^{-3}\ \text{A}$$ **Ans.**

(b) The current density is

$$j = \frac{i}{A} = \frac{9.6 \times 10^{-3}}{10^{-6}}$$

$$= 9.6 \times 10^{3}\ \text{A/m}^2$$ **Ans.**

Example 23.8 *Calculate the drift speed of the electrons when* $1\,A$ *of current exists in a copper wire of cross-section* $2\,mm^2$. *The number of free electrons in* $1\,cm^3$ *of copper is* 8.5×10^{22}.

Solution n = free electron density,

$$= 8.5 \times 10^{22} \text{ per cm}^3 \qquad \text{(Given)}$$
$$= (8.5 \times 10^{22})(10^6) \text{ per m}^3$$
$$= 8.5 \times 10^{28} \text{ per m}^3$$

From $i = neAv_d$, we get

$$v_d = \frac{i}{neA}$$

Substituting the values in SI units we have,

$$v_d = \frac{1}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(2 \times 10^{-6})}$$
$$= 3.6 \times 10^{-5} \text{ m/s}$$ **Answer**

INTRODUCTORY EXERCISE 23.3

1. When a wire carries a current of 1.20 A, the drift velocity is 1.20×10^{-4} m/s. What is the drift velocity when the current is 6.00 A?
2. Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section 1cm^2 and length 10 km. Free electron density of copper is $8.5 \times 10^{28}/\text{m}^3$. How long will it take the electric charge to travel from one end of the conductor to the other?

23.5 Resistance of a Wire

Resistance of a wire is always required between two points or two surfaces (say P and Q).

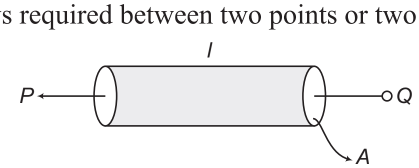


Fig. 23.7

Here, l = length of wire, A = area of cross-section

Now, $$R \propto l \qquad \ldots\text{(i)}$$

and $$R \propto \frac{1}{A} \qquad \ldots\text{(ii)}$$

Combining Eqs. (i) and (ii), we get

$$\boxed{R = \rho \frac{l}{A}} \qquad \ldots\text{(iii)}$$

Here, ρ is called resistivity of the material of the wire. This depends on number of free electrons present in the material. With increase in number of free electrons the value of ρ decreases.

Note *(i)* $\rho = \dfrac{1}{\sigma}$, *where* σ = *conductivity.*

(ii) SI units of resistivity are Ω-m (ohm-metre).

(iii) SI units of conductivity are $(\Omega\text{-m})^{-1}$.

(iv) In Eq. (iii), l is that dimension of conductor which is parallel to P and Q and A is that cross-sectional area, which is perpendicular to P and Q.

Example 23.9 *Two copper wires of the same length have got different diameters,*

(a) which wire has greater resistance?

(b) greater specific resistance?

Solution (a) For a given wire, $R = \rho \dfrac{l}{A}$, i.e. $R \propto \dfrac{1}{A}$

So, the thinner wire will have greater resistance.

(b) Specific resistance (ρ) is a material property. It does not depend on l or A.

So, both the wires will have same specific resistance.

Example 23.10 *A wire has a resistance R. What will be its resistance if it is stretched to double its length?*

Solution Let V be the volume of wire, then

$$V = Al$$

$$\therefore \qquad A = \frac{V}{l}$$

Substituting this in $R = \rho \dfrac{l}{A}$, we have $\qquad R = \rho \dfrac{l^2}{V}$

So, for given volume and material (i.e. V and ρ are constants)

$$R \propto l^2$$

When l is doubled, resistance will become four times, or the new resistance will be $4R$.

Example 23.11 *The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.*

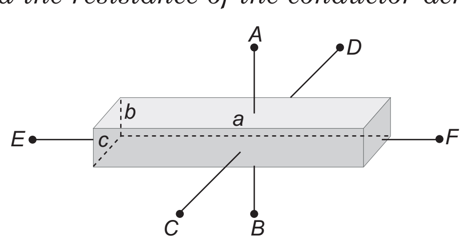


Fig. 23.8

Solution $R = \rho \dfrac{l}{A}$

Resistance across *AB, CD* and *EF* in tabular form is shown below.

Table 23.1

	l	*A*	*R*
AB	*c*	$a \times b$	$\rho \dfrac{c}{ab}$
CD	*b*	$a \times c$	$\rho \dfrac{b}{ac}$
EF	*a*	$b \times c$	$\rho \dfrac{a}{bc}$

Example 23.12 *A copper wire is stretched to make it 0.1% longer. What is the percentage change in its resistance?* **(JEE 1978)**

Solution
$$R = \rho \frac{l}{A} = \frac{\rho l}{V/l} \qquad (V = \text{volume of wire})$$
$$= \frac{\rho l^2}{V}$$
$$\therefore \qquad R \propto l^2 \qquad (\rho \text{ and } V = \text{constant})$$

For small percentage change

% change $R = 2$(% change in l) $= 2\,(0.1\%) = 0.2\%$

Since $R \propto l^2$, with increase in the value of *l*, resistance will also increase.

INTRODUCTORY EXERCISE 23.4

1. In household wiring, copper wire 2.05 mm in diameter is often used. Find the resistance of a 35.0 m long wire. Specific resistance of copper is $1.72 \times 10^{-8}\ \Omega$-m.
2. The product of resistivity and conductivity of a conductor is constant. Is this statement true or false?
3. You need to produce a set of cylindrical copper wires 3.50 m long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires? Specific resistance of copper $= 1.72 \times 10^{-8}\ \Omega$-m, density of copper $= 8.9 \times 10^3$ kg/m^3.
4. Consider a thin square sheet of side *L* and thickness *t*, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **(JEE 2010)**
   (a) directly proportional to *L*
   (b) directly proportional to *t*
   (c) independent of *L*
   (d) independent of *t*

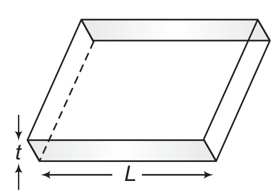


Fig. 23.9

23.6 Temperature Dependence of Resistance

If we increase the temperature of any material, the following two effects can be observed :

(i) Numbers of free electrons increase. Due to this effect conductivity of the material increases. So, resistivity or resistance decreases.

(ii) The ions of the material vibrate with greater amplitude and the collision between electrons and ions become more frequent. Due to this effect resistivity or resistance of the material increases.

In Conductors

There are already a large number of free electrons. So, with increase in temperature effect-(i) is not so dominant as effect-(ii). Hence, resistivity or resistance of conductors increase with increase in temperature.

Over a small temperature range (upto 100°C), the resistivity of a metal (or conductors) can be represented approximately by the equation,

$$\boxed{\rho(T) = \rho_0 \, [1 + \alpha \, (T - T_0)]} \qquad \text{...(i)}$$

where, ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0°C or 20°C) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0. The factor α is called the **temperature coefficient of resistivity.**

The resistance of a given conductor depends on its length and area of cross-section besides the resistivity. As temperature changes, the length and area also change. But these changes are quite small and the factor l/A may be treated as constant.

Then, $\quad R \propto \rho \quad$ and hence, $\quad \boxed{R(T) = R_0 \, [1 + \alpha \, (T - T_0)]} \qquad$ ...(ii)

In this equation, $R(T)$ is the resistance at temperature T and R_0 is the resistance at temperature T_0, often taken to be 0°C or 20°C. The temperature coefficient of resistance α is the same constant that appears in Eq. (i), if the dimensions l and A in equation $R = \rho \dfrac{l}{A}$ do not change with temperature.

In Semiconductors

At room temperature, numbers of free electrons in semiconductors (like silicon, germanium etc.) are very less. So, with increase in temperature, effect-(i) is very dominant. Hence, resistivity or resistance of semiconductors decreases with increase in temperature or we can say that temperature coefficient of resistivity α for semiconductors is negative.

Example 23.13 *The resistance of a thin silver wire is $1.0\ \Omega$ at 20°C. The wire is placed in a liquid bath and its resistance rises to $1.2\ \Omega$. What is the temperature of the bath? α for silver is 3.8×10^{-3} /°C.*

Solution $R(T) = R_0[1 + \alpha(T - T_0)]$

Here, $R(T) = 1.2\ \Omega$, $R_0 = 1.0\ \Omega$, $\alpha = 3.8 \times 10^{-3}/°\text{C}$ and $T_0 = 20°\text{C}$

Substituting the values, we have $\quad 1.2 = 1.0[1 + 3.8 \times 10^{-3}\ (T - 20)]$

or $\quad 3.8 \times 10^{-3}\ (T - 20) = 0.2$

Solving this, we get $\quad T = 72.6°\text{C}$ **Ans.**

Example 23.14 *Read the following statements carefully* (JEE 1993)

Y : The resistivity of semiconductor decreases with increase of temperature.

Z : In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature.

Select the correct statement (s) from the following

(a) Y is true but Z is false *(b) Y is false but Z is true*

(c) Both Y and Z are true *(d) Y is true and Z is the correct reason for Y*

Solution Resistivity of conductors increases with increase in temperature because rate of collisions between free electrons and ions increase with increase of temperature. However, the resistivity of semiconductors decreases with increase in temperature, because more and more covalent bonds are broken at higher temperatures and free electrons increase with increase in temperature. Therefore, the correct option is (c).

Example 23.15 *An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it. Its resistance at room temperature* $(27.0\,^\circ C)$ *is found to be* $75.3\ \Omega$. *When the toaster is connected to a* $230\ V$ *supply, the current settles, after a few seconds, to a steady value of* $2.68\ A$. *What is the steady temperature of the nichrome element? The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is* $1.70 \times 10^{-4}\ {}^\circ C^{-1}$.

Solution Given, $T_0 = 27^\circ\text{ C}$ and $R_0 = 75.3\ \Omega$

At temperature T, $R_T = \dfrac{V_T}{i_T}$ $\left(R = \dfrac{V}{i}\right)$

$$= \frac{230}{2.68} = 85.82\ \Omega$$

Using the equation, $R_T = R_0\,[1 + \alpha\,(T - T_0)]$

We have $85.82 = 75.3\,[1 + (1.70 \times 10^{-4})(T - 27)]$

Solving this equation, we get $T \approx 850\,^\circ\text{C}$ **Ans.**

Thus, the steady temperature of the nichrome element is $850\,^\circ\text{C}$.

INTRODUCTORY EXERCISE 23.5

1. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of (JEE 1988)
   (a) each of them increases
   (b) each of them decreases
   (c) copper increases and germanium decreases
   (d) copper decreases and germanium increases
2. The resistance of a copper wire and an iron wire at 20°C are 4.1 Ω and 3.9 Ω, respectively. Neglecting any thermal expansion, find the temperature at which resistances of both are equal.
   $\alpha_{\text{Cu}} = 4.0 \times 10^{-3}\ \text{K}^{-1}$ and $\alpha_{\text{Fe}} = 5.0 \times 10^{-3}\ \text{K}^{-1}$.

23.7 Ohm's Law

The equation, $V = iR$ is not Ohm's law. It is a mathematical relation between current passing through a resistance, value of resistance R and the potential difference V across it.

According to Ohm's law, there are some of the materials (like metals or conductors) or some circuits for which, current passing through them is proportional to the potential difference applied across them or

$$i \propto V \quad \text{or} \quad V \propto i$$

$$\Rightarrow \qquad V = iR \quad \text{or} \quad \frac{V}{i} = R = \text{constant}$$

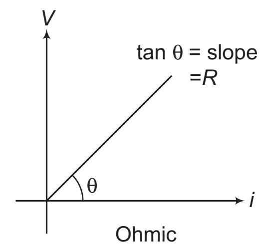


Fig. 23.10

or V - i graph for such materials and circuits is a straight line passing through origin. Slope of this graph is called its resistance. The materials or circuits which follow this law are called **ohmic.**

The materials or circuits which do not follow this law are called **non-ohmic**. V - i graph for non-ohmic circuits is not a straight line passing through origin. $\frac{V}{i}$ or R is not constant and i is not proportional to V.

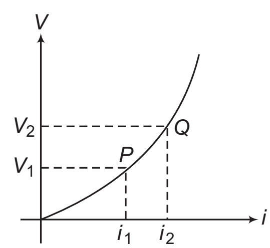


Fig. 23.11

Note *Equation $V = iR$ is applicable for even non-ohmic circuits also.*

For example, $\frac{V_1}{i_1} = R_1$ = *resistance at P.*

$\frac{V_2}{i_2} = R_2$ = *resistance at Q, but*

$R_1 \neq R_2$

Example 23.16 *The current-voltage graphs for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure. The temperature T_2 is greater than T_1. Is this statement true or false?* **(JEE 1985)**

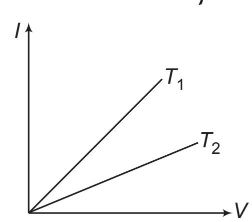


Fig. 23.12

Solution $\frac{I}{V}$ = Slope of given graph $=\frac{1}{R}$

or $\quad R=\frac{1}{\text{Slope}} \quad \Rightarrow \quad (\text{Slope})_{T_2} < (\text{Slope})_{T_1} \quad \Rightarrow \quad \frac{1}{(\text{Slope})_{T_2}} > \frac{1}{(\text{Slope})_{T_1}}$

Resistance of a metallic wire increases with increase in temperature.

or $\quad R_{T_2} > R_{T_1}$ or $T_2 > T_1$

Therefore, the statement is true.

23.8 The Battery and the Electromotive Force

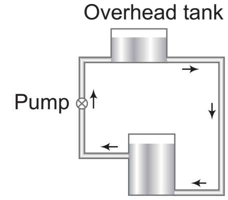


Fig. 23.13

Before studying the electromotive force (emf) of a cell let us take an example of a pump which is more easy to understand. Suppose we want to recycle water between a overhead tank and a ground water tank. Water flows from overhead tank to ground water tank by itself (by gravity). No external agent is required for this purpose. But to raise the water from ground water tank to overhead tank a pump is required or some external work has to be done. In an electric circuit, a battery or a cell plays the same role as the pump played in the above example.

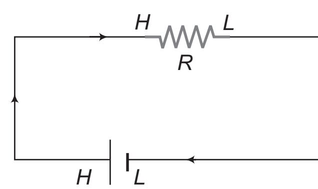


Fig. 23.14

Suppose a resistance (R) is connected across the terminals of a battery. A potential difference is developed across its ends. Current (or positive charge) flows from higher potential to lower potential across the resistance by itself. But inside the battery, work has to be done to bring the positive charge from lower potential to higher potential. The influence that makes current flow from lower to higher potential (inside the battery) is called **electromotive force** (abbreviated **emf**). If W work is done by the battery in taking a charge q from negative terminal to positive terminal, then work done by the battery per unit charge is called emf (E) of the battery.

Thus, $$\boxed{E=\frac{W}{q}}$$

The name electromotive force is misleading in the sense that emf is not a force it is work done per unit charge. The SI unit of emf is J/C or V ($1\text{ V}=1\text{ J/C}$).

23.9 Direct Current Circuits, Kirchhoff's Laws

Single current in a simple circuit (single loop) can be found by the relation,

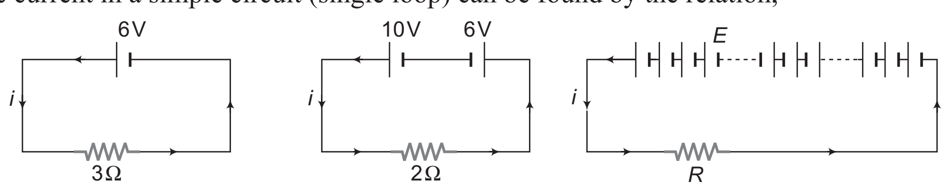


Fig. 23.15

$$\boxed{i = \frac{\text{net emf}}{\text{net resistance}} = \frac{E_{\text{net}}}{R_{\text{net}}}}$$

For example :

In Fig. (a) Net emf is 6 V and net resistance is 3 Ω. Therefore,

$$i = \frac{6}{3} = 2 \text{ A}$$

In Fig. (b) $\qquad$ Net emf $= (10 - 6)$ V $= 4$ V

and $\qquad$ Net resistance $= 2\ \Omega$

Therefore, $\qquad i = \frac{4}{2} = 2$ A

In Fig. (c) We have n cells each of emf E. Of these polarity of m cells (where $n > 2m$) is reversed. Then, net emf in the circuit is $(n - 2m)\,E$ and resistance of the circuit is R. Therefore,

$$\boxed{i = \frac{(n - 2m)\,E}{R}}$$

Resistors in Series and in Parallel

In series :

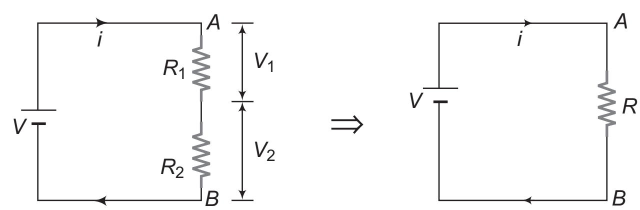


Fig. 23.16

Figure represents a circuit consisting of a source of emf and two resistors connected in series. We are interested in finding the resistance R of the network lying between A and B. That is, what single **equivalent resistor *R*** would have the same resistance as the two resistors linked together.

Because there is only one path for electric current to follow, i must have the same value everywhere in the circuit. The potential difference between A and B is V. This potential difference must somehow be divided into two parts V_1 and V_2 as shown,

∴ $\qquad V = V_1 + V_2 = iR_1 + iR_2$

or $\qquad V = i\,(R_1 + R_2) \qquad$ ...(i)

Let R be the equivalent resistance between A and B, then

$$V = iR \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\boxed{R = R_1 + R_2 \quad \text{for resistors in series}}$$

This result can be readily extended to a network consisting of n resistors in series.

∴ $\qquad R = R_1 + R_2 + \ldots\ldots + R_n$

In parallel :

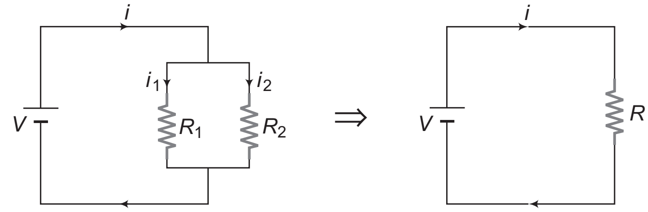


Fig. 23.17

In Fig. 23.17, the two resistors are connected in parallel. The voltage drop across each resistor is equal to the source voltage V. The current i, however, divides into two branches, which carry currents i_1 and i_2.

$$i = i_1 + i_2 \qquad \text{...(iii)}$$

If R be the equivalent resistance, then

$$i = \frac{V}{R}, \quad i_1 = \frac{V}{R_1} \quad \text{and} \quad i_2 = \frac{V}{R_2}$$

Substituting in Eq. (iii), we get

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{for resistors in parallel}}$$

This result can also be extended to a network consisting of n resistors in parallel. The result is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$$

Example 23.17 *Compute the equivalent resistance of the network shown in figure and find the current i drawn from the battery.*

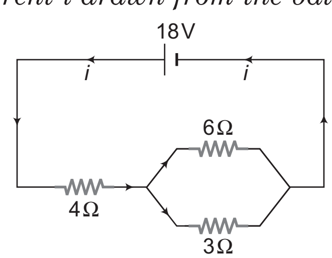


Fig. 23.18

Solution The 6 Ω and 3 Ω resistances are in parallel. Their equivalent resistance is

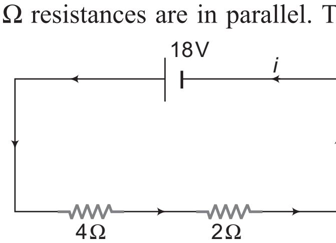


Fig. 23.19

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{3} \quad \text{or} \quad R = 2\,\Omega$$

Now, this 2 Ω and 4 Ω resistances are in series and their equivalent resistance is 4 + 2 = 6 Ω.

Therefore, equivalent resistance of the network = 6 Ω. **Ans.**

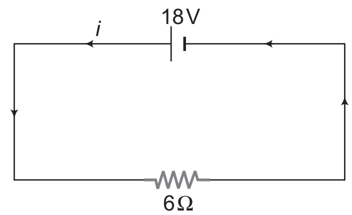


Fig. 23.20

Current drawn from the battery is

$$i = \frac{\text{net emf}}{\text{net resistance}} = \frac{18}{6}$$

$$= 3\text{ A}$$ **Ans.**

Kirchhoff's Laws

Many electric circuits cannot be reduced to simple series-parallel combinations. For example, two circuits that cannot be so broken down are shown in Fig. 23.21.

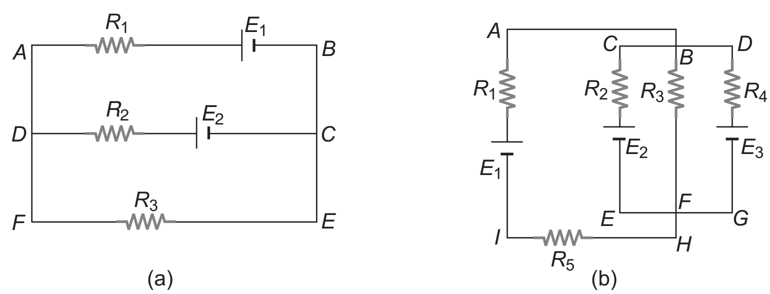


Fig. 23.21

However, it is always possible to analyze such circuits by applying two rules, devised by Kirchhoff in 1845 and 1846 when he was still a student.

First there are two terms that we will use often.

Junction

A junction in a circuit is a point where three or more conductors meet. Junctions are also called nodes or branch points.

For example, in Fig. (a) points *D* and *C* are junctions. Similarly, in Fig. (b) points *B* and *F* are junctions.

Loop

A loop is any closed conducting path. For example, in Fig. (a) *ABCDA*, *DCEFD* and *ABEFA* are loops. Similarly, in Fig. (b), *CBFEC*, *BDGFB* are loops.

Kirchhoff's rules consist of the following two statements :

Kirchhoff's Junction Rule

The algebraic sum of the currents into any junction is zero.

That is, $$\boxed{\sum_{\text{junction}} i = 0}$$

This law can also be written as, "the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point."

Thus, in figure $$i_1 + i_2 = i_3 + i_4$$

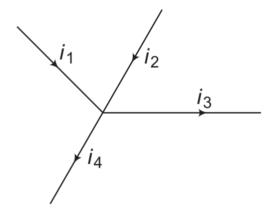


Fig. 23.22

The junction rule is based on **conservation of electric charge.** No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal to charge leaving per unit time. Charge per unit time is current, so if we consider the currents entering to be positive and those leaving to be negative, the algebraic sum of currents into a junction must be zero.

Kirchhoff's Loop Rule

The algebraic sum of the potential differences in any loop including those associated emf's and those of resistive elements, must equal zero.

That is, $$\boxed{\sum_{\text{closed loop}} \Delta V = 0}$$

Kirchhoff's second rule is based on the fact that the **electrostatic field is conservative** in nature. This result states that there is no net change in electric potential around a closed path. Kirchhoff's second rule applies only for circuits in which an electric potential is defined at each point. This criterion may not be satisfied if changing electromagnetic fields are present.

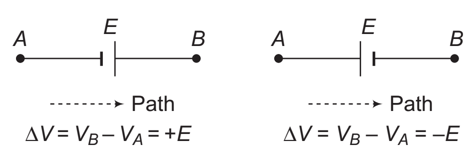


Fig. 23.23

In applying the loop rule, we need sign conventions. First assume a direction for the current in each branch of the circuit. Then starting at any point in the circuit, we imagine, travelling around a loop, adding emf's and iR terms as we come to them.

When we travel through a source in the direction from – to +, the emf is considered to be positive, when we travel from + to –, the emf is considered to be negative.

When we travel through a resistor in the same direction as the assumed current, the iR term is negative because the current goes in the direction of decreasing potential. When we travel through a resistor in the direction opposite to the assumed current, the iR term is positive because this represents a rise of potential.

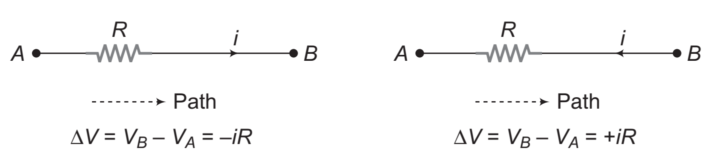


Fig. 23.24

Note *It is advised to write H (for higher potential) and L (for lower potential) across all the batteries and resistances of the loop under consideration while using the loop law. Then write – while moving from H to L and + for L to H. Across a battery write H on positive terminal and L on negative terminal. Across a resistance keep in mind the fact that current always flows from higher potential (H) to lower potential (L). For example, in the loop shown in figure we have marked H and L across all batteries and resistances. Now let us apply the second law in the loop ADCBA.*

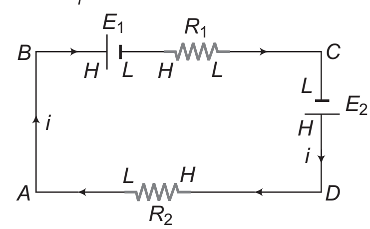


Fig. 23.25

The equation will be $$+iR_2 - E_2 + iR_1 + E_1 = 0$$

Example 23.18 *Find currents in different branches of the electric circuit shown in figure.*

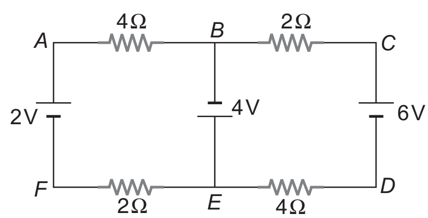


Fig. 23.26

HOW TO PROCEED *In this problem there are three wires EFAB, BE and BCDE. Therefore, we have three unknown currents i_1, i_2 and i_3. So, we require three equations. One equation will be obtained by applying Kirchhoff's junction law (either at B or at E) and the remaining two equations, we get from the second law (loop law). We can make three loops ABEFA, ACDFA and BCDEB. But we have to choose any two of them. Initially, we can choose any arbitrary directions of i_1, i_2 and i_3.*

Solution Applying Kirchhoff's first law (junction law) at junction B,

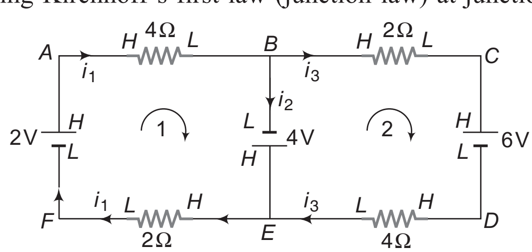


Fig. 23.27

$$i_1 = i_2 + i_3 \qquad \text{...(i)}$$

Applying Kirchhoff's second law in loop 1 ($ABEFA$),

$$-4i_1 + 4 - 2i_1 + 2 = 0 \qquad \text{...(ii)}$$

Applying Kirchhoff's second law in loop 2 ($BCDEB$),

$$-2i_3 - 6 - 4i_3 - 4 = 0 \qquad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1\,\text{A}$$

$$i_2 = \frac{8}{3}\,\text{A}$$

$$i_3 = -\frac{5}{3}\,\text{A}$$

Ans.

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

Example 23.19 *In example 23.18, find the potential difference between points F and C.*

HOW TO PROCEED *To find the potential difference between any two points of a circuit you have to reach from one point to the other via any path of the circuit. It is advisable to choose a path in which we come across the least number of resistors preferably a path which has no resistance.*

Solution Let us reach from F to C *via* A and B,

$$V_F + 2 - 4i_1 - 2i_3 = V_C$$

$$\therefore \qquad V_F - V_C = 4i_1 + 2i_3 - 2$$

Substituting, $i_1 = 1\,\text{A}$ and $i_3 = -\frac{5}{3}\,\text{A}$, we get

$$V_F - V_C = -\frac{4}{3}\ \text{volt}$$

Ans.

Here, negative sign implies that $V_F < V_C$.

Internal Resistance (r) and Potential Difference (V) across the Terminals of a Battery

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the **internal resistance** of the source, denoted by r. If this resistance behaves according to Ohm's law r is constant and independent of the current i. As the current moves through r, it experiences an associated drop in potential equal to ir. Thus, when a current is drawn through a source, the potential difference between the terminals of the source is

$$\boxed{V = E - ir}$$

This can also be shown as below.

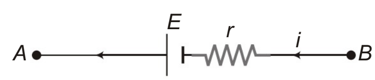


Fig. 23.28

$$V_A - E + ir = V_B$$

$$\text{or} \qquad V_A - V_B = E - ir$$

The following three special cases are possible :

(i) If the current flows in opposite direction (as in case of charging of a battery), then $V = E + ir$

(ii) $V = E$, if the current through the cell is zero.

(iii) $V = 0$, if the cell is short circuited.

This is because current in the circuit

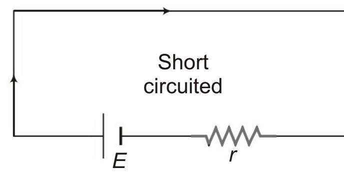


Fig. 23.29

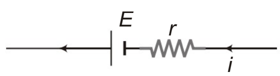
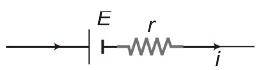
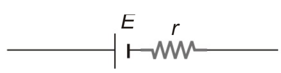
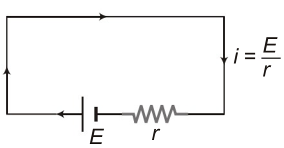
$$i = \frac{E}{r}$$

or $$E = ir$$

$\therefore$ $$E - ir = 0$$

or $$V = 0$$

Thus, we can summarise it as follows :

	$V = E - ir$	or	$V < E$
	$V = E + ir$	or	$V > E$
	$V = E$	if	$i = 0$
	$V = 0$	if short circuited	

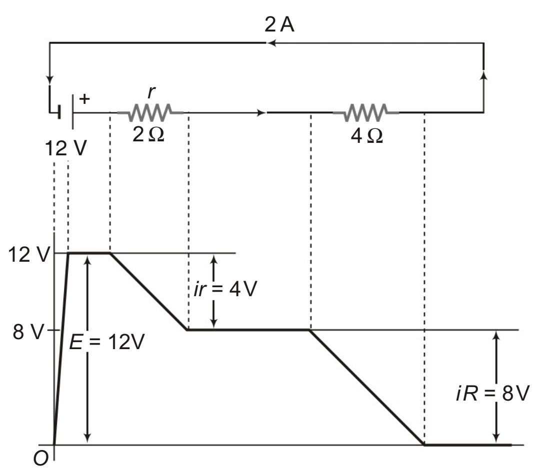


Fig. 23.30 Potential rise and fall in a circuit.

✓ *Extra Points to Remember*

- **In figure (a) :** There are eight wires and hence, will have eight currents or eight unknowns. The eight wires are *AB*, *BC*, *CE*, *EA*, *AD*, *BD*, *CD* and *ED*. Number of independent loops are four. Therefore, from the second law we can make only four equations. Total number of junctions are five (*A, B, C, D* and *E*). But by using the first law, we can make only four equations (one less). So, the total number of equations are eight.

  In figure (b) : Number of wires are six (*AB, BC, CDA, BE, AE* and *CE*). Number of independent loops are three so, three equations can be obtained from the second law. Number of junctions are four (*A, B, C* and *E*) so, we can make only three (one less) equations from the first law. But total number of equations are again six.

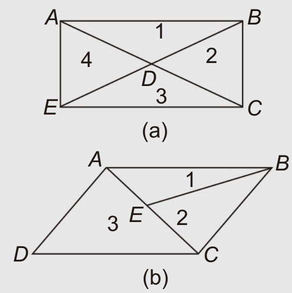


Fig. 23.31

- **Short circuiting :** Two points in an electric circuit directly connected by a conducting wire are called short circuited. Under such condition both points are at same potential.

  For example, resistance R_1 in the adjoining circuit is short circuited, i.e. potential difference across it is zero. Hence, no current will flow through R_1 and the current through R_2 is therefore, E/R_2.

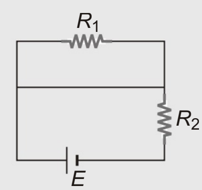


Fig. 23.32

- **Earthing :** If some point of a circuit is earthed, then its potential is taken to be zero.

  For example, in the adjoining figure,

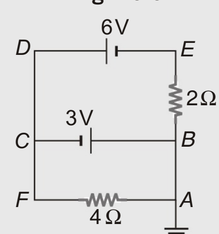


Fig. 23.33

$$V_A = V_B = 0$$
$$V_F = V_C = V_D = -3\text{ V}$$
$$V_E = -9\text{ V}$$

$\therefore$ $$V_B - V_E = 9\text{ V}$$

or current through 2 Ω resistance is

$$\frac{V_B - V_E}{2} \quad \text{or} \quad \frac{9}{2}\text{ A} \qquad \text{(from } B \text{ to } E\text{)}$$

Similarly, $$V_A - V_F = 3\text{ V}$$

and the current through 4 Ω resistance is $\dfrac{V_A - V_F}{4}$ or $\dfrac{3}{4}$ A (from *A* to *F*)

- For a current flow through a resistance there must be a potential difference across it but between any two points of a circuit the potential difference may be zero.

  For example, in the circuit,

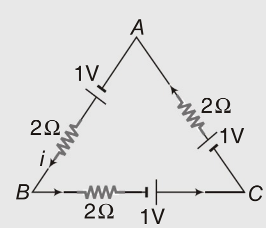


Fig. 23.34

$$\text{net emf } = 3\text{ V} \quad \text{and} \quad \text{net resistance } = 6\ \Omega$$

$\therefore$ current in the circuit, $i = \dfrac{3}{6} = \dfrac{1}{2}$ A

$\boldsymbol{V_A - V_B}$ $$V_A + 1 - 2 \times \frac{1}{2} = V_B \quad \text{or} \quad V_A - V_B = 0$$

or by symmetry, we can say that

$$V_A = V_B = V_C$$

So, the potential difference across any two vertices of the triangle is zero, while the current in the circuit is non-zero.

- **Distribution of current in parallel connections :** When more than one resistances are connected in parallel, the potential difference across them is equal and the current is distributed among them in inverse ratio of their resistance as

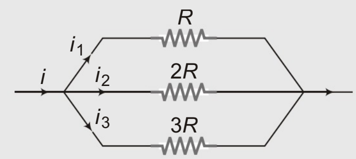


Fig. 23.35

$$i = \frac{V}{R}$$

or $$i \propto \frac{1}{R} \quad \text{for same value of } V$$

e.g. in the figure,

$$i_1 : i_2 : i_3 = \frac{1}{R} : \frac{1}{2R} : \frac{1}{3R} = 6 : 3 : 2$$

∴ $$i_1 = \left(\frac{6}{6+3+2}\right) i = \frac{6}{11} i$$

$$i_2 = \left(\frac{3}{6+3+2}\right) i = \frac{3}{11} i$$

and $$i_3 = \left(\frac{2}{6+3+2}\right) i = \frac{2}{11} i$$

Note *In case of only two resistances,* $\frac{i_1}{i_2} = \frac{R_2}{R_1}$

- **Distribution of potential in series connections :** When more than one resistances are connected in series, the current through them is same and the potential is distributed in the direct ratio of their resistance as

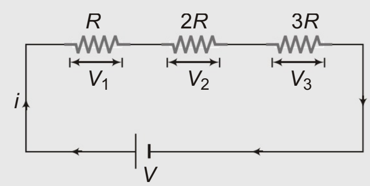


Fig. 23.36

$$V = iR \quad \text{or} \quad V \propto R \quad \text{for same value of } i.$$

For example in the figure,

$$V_1 : V_2 : V_3 = R : 2R : 3R = 1 : 2 : 3$$

∴ $$V_1 = \left(\frac{1}{1+2+3}\right) V = \frac{V}{6}$$

$$V_2 = \left(\frac{2}{1+2+3}\right) V = \frac{V}{3}$$

and $$V_3 = \left(\frac{3}{1+2+3}\right) V = \frac{V}{2}$$

Example 23.20 *In the circuit shown in figure,*

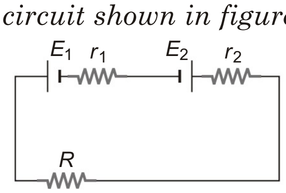


Fig. 23.37

$E_1 = 10\,V$, $E_2 = 4\,V$, $r_1 = r_2 = 1\,\Omega$ *and* $R = 2\,\Omega$.

Find the potential difference across battery 1 *and battery* 2.

Solution Net emf of the circuit $= E_1 - E_2 = 6$ V

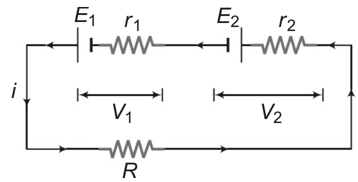


Fig. 23.38

Total resistance of the circuit $= R + r_1 + r_2 = 4\ \Omega$

$\therefore$ Current in the circuit, $i = \dfrac{\text{net emf}}{\text{total resistance}} = \dfrac{6}{4} = 1.5$ A

Now, $V_1 = E_1 - ir_1 = 10 - (1.5)(1)$

$= 8.5$ V **Ans.**

and $V_2 = E_2 + ir_2 = 4 + (1.5)(1)$

$= 5.5$ V **Ans.**

INTRODUCTORY EXERCISE 23.6

1. Find the current through 2 Ω and 4 Ω resistance.

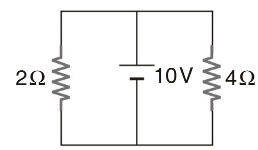


Fig. 23.39

2. In the circuit shown in figure, find the potentials of A, B, C and D and the current through 1 Ω and 2 Ω resistance.

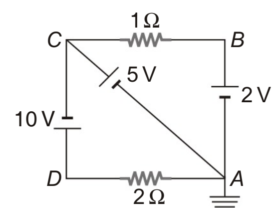


Fig. 23.40

3. For what value of E the potential of A is equal to the potential of B?

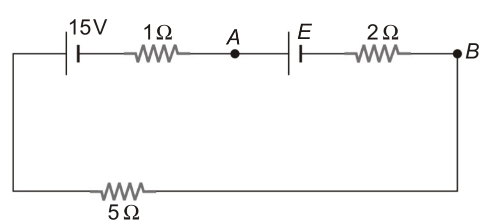


Fig. 23.41

4. Ten cells each of emf 1 V and internal resistance 1 Ω are connected in series. In this arrangement, polarity of two cells is reversed and the system is connected to an external resistance of 2 Ω. Find the current in the circuit.

5. In the circuit shown in figure, $R_1 = R_2 = R_3 = 10\ \Omega$. Find the currents through R_1 and R_2.

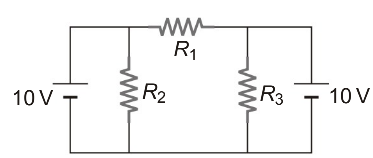


Fig. 23.42

23.10 Heating Effects of Current

An electric current through a resistor increases its thermal energy. Also, there are other situations in which an electric current can produce or absorb thermal energy.

Power Supplied or Power Absorbed by a Battery

When charges are transported across a source of emf, their potential energy changes. If a net charge Δq moves through a potential difference E in a time Δt, the change in electric potential energy of the charge is $E\,\Delta q$. Thus, the source of emf does work,

$$\Delta W = E\,\Delta q$$

Dividing both sides by Δt, then taking the limit as $\Delta t \to 0$, we find

$$\frac{dW}{dt} = E\,\frac{dq}{dt}$$

By definition, $\frac{dq}{dt} = i$, the current through the battery and $\frac{dW}{dt} = P$, the power output of (or input to) the battery. Hence,

$$\boxed{P = Ei}$$

The quantity P represents the rate at which energy is transferred from a discharging battery or to a charging battery.

In Fig. 23.43, energy is transferred from the source at a rate Ei

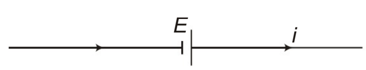


Fig. 23.43

In Fig. 23.44, energy is transferred to the source at a rate Ei

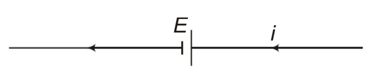


Fig. 23.44

Power dissipated across a resistance

Now, let's consider the power dissipated in a conducting element. Suppose it has a resistance R and the potential difference between its ends is V. In moving from higher to lower potential, a positive charge Δq loses energy $\Delta U = V\,\Delta q$. This electric energy is absorbed by the conductor through collisions between its atomic lattice and the charge carriers, causing its temperature to rise. This effect is commonly called **Joule heating.** Since, power is the rate at which energy is transferred, we have,

$$P = \frac{\Delta U}{\Delta t} = V \cdot \frac{\Delta q}{\Delta t} = V i$$

$$\therefore \qquad \boxed{P = V i}$$

which with the help of equation $V = iR$ can also be written in the forms,

$$\boxed{P = i^2 R} \quad \text{or} \quad \boxed{P = \frac{V^2}{R}}$$

Power is always dissipated in a resistance. With this rate, the heat produced in the resistor in time t is

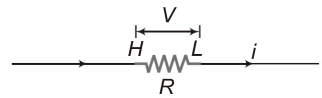


Fig. 23.45

$$H = Pt \quad \text{or} \quad \boxed{H = Vit = i^2Rt = \frac{V^2}{R}t}$$

Joule heating occurs whenever a current passes through an element that has resistance. To prevent the overheating of delicate electronic components, many electric devices like video cassette recorders, televisions and computer monitors have fans in their chassis to allow some of the heat produced to escape.

Extra Points to Remember

- We have seen above that power may be supplied or consumed by a battery. It depends on the direction of current.

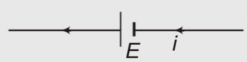


Fig. 23.46

In the above direction of current power is supplied by the battery (= Ei)

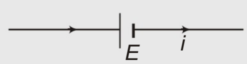


Fig. 23.47

In the opposite direction of current shown in Fig. 23.47, power is consumed by the battery. This normally happens during charging of a battery.

- A resistance always consumes power. It does not depend on the direction of current.

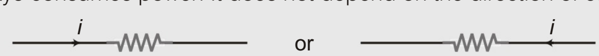


Fig. 23.48

In both cases shown in figure, power is only consumed and this power consumed is given by the formula.

$$P = i^2R = \frac{V^2}{R} = Vi$$

In the above equations V and i are the values across a resistance in which we wish to find the power consumed.

- In any electrical circuit, law of conservation of energy is followed.
  Net power supplied by all batteries of the circuit = net power consumed by all resistors in the circuit.

Example 23.21 *In the circuit shown in figure, find*

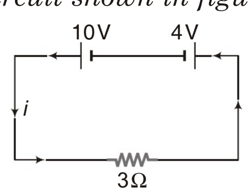


Fig. 23.49

(a) the power supplied by 10 V *battery*
(b) the power consumed by 4 V *battery and*
(c) the power dissipated in 3 Ω *resistance.*

Solution Net emf of the circuit = (10 − 4) V = 6 V

∴ Current in the circuit

$$i = \frac{\text{net emf}}{\text{total resistance}} = \frac{6}{3} = 2\text{ A}$$

(a) Power supplied by 10 V battery = $Ei = (10)(2) = 20$ W **Ans.**

(b) Power consumed by 4 V battery = $Ei = (4)(2) = 8$ W **Ans.**

(c) Power consumed by 3 Ω resistance = $i^2R = (2)^2(3) = 12$ W **Ans.**

Note *Here, we can see that total power supplied by 10 V battery (i.e. 20 W) = power consumed by 4 V battery and 3 Ω resistance. Which proves that conservation of energy holds good in electric circuits also.*

Example 23.22 *In the circuit shown in figure, find the heat developed across each resistance in 2 s.*

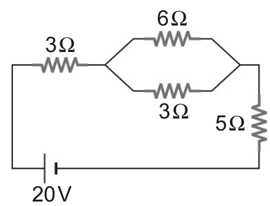


Fig. 23.50

Solution The 6 Ω and 3 Ω resistances are in parallel. So, their combined resistance is

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

or $$R = 2\,\Omega$$

The equivalent simple circuit can be drawn as shown.

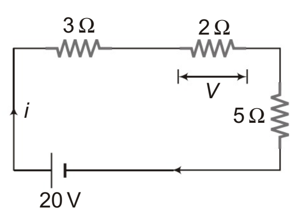


Fig. 23.51

Current in the circuit,

$$i = \frac{\text{net emf}}{\text{total resistance}} = \frac{20}{3+2+5} = 2\text{ A}$$

$$V = iR = (2)(2) = 4\text{ V}$$

i.e. Potential difference across 6 Ω and 3 Ω resistances are 4 V. Now,

$H_{3\Omega}$ (which is connected in series) = $i^2Rt = (2)^2(3)(2) = 24$ J

$$H_{6\Omega} = \frac{V^2}{R}t = \frac{(4)^2}{6}(2) = \frac{16}{3}\text{ J}$$

$$H_{3\,\Omega} \text{ (which is connected in parallel)} = \frac{V^2}{R}t = \frac{(4)^2\,(2)}{3} = \frac{32}{3}\text{ J}$$

and $$H_{5\,\Omega} = i^2Rt = (2)^2\,(5)\,(2) = 40\text{ J}$$ **Ans.**

INTRODUCTORY EXERCISE 23.7

1. In the circuit shown in figure, a 12 V battery with unknown internal resistance r is connected to another battery with unknown emf E and internal resistance $1\,\Omega$ and to a resistance of $3\,\Omega$ carrying a current of 2 A. The current through the rechargeable battery is 1 A in the direction shown. Find the unknown current i, internal resistance r and the emf E.

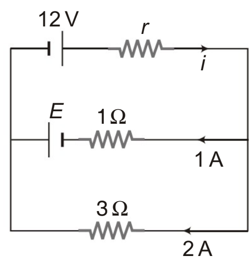


Fig. 23.52

2. In the above example, find the power delivered by the 12 V battery and the power dissipated in $3\,\Omega$ resistor.

23.11 Grouping of Cells

Cells are usually grouped in the following three ways :

Series Grouping

Suppose n cells each of emf E and internal resistance r are connected in series as shown in figure.

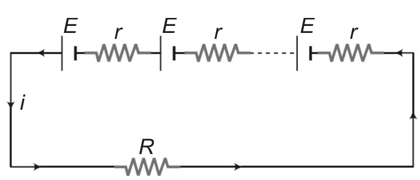


Fig. 23.53

Then, $$\text{Net emf} = nE$$

$$\text{Total resistance} = nr + R$$

$\therefore$ $$\text{Current in the circuit, } i = \frac{\text{net emf}}{\text{total resistance}} \quad \text{or} \quad \boxed{i = \frac{nE}{nr + R}}$$

Note *If polarity of m cells is reversed, then equivalent emf = (n − 2m) E, while total resistance is still nr + R*

$\therefore$ $$i = \frac{(n - 2m)\,E}{nr + R}$$

Parallel Grouping

Here, three cases are possible.

Case 1 *When E and r of each cell has same value and positive terminals of all cells are connected at one junction while negative at the other.*

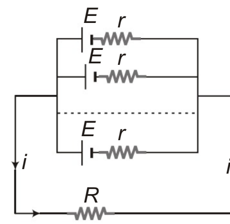


Fig. 23.54

In this situation, the net emf is E. The net internal resistance is $\frac{r}{n}$ as n resistances each of r are in parallel. Net external resistance is R. Therefore, total resistance is $\left(\frac{r}{n}+R\right)$ and so the current in the circuit will be,

$$i=\frac{\text{Net emf}}{\text{Total resistance}} \quad \text{or} \quad \boxed{i=\frac{E}{R+r/n}}$$

Note *A comparison of series and parallel grouping reveals that to get maximum current, cells must be connected in series if effective internal resistance is lesser than external and in parallel if effective internal resistance is greater than external.*

Case 2 *If E and r of each cell are different but still the positive terminals of all cells are connected at one junction while negative at the other.*

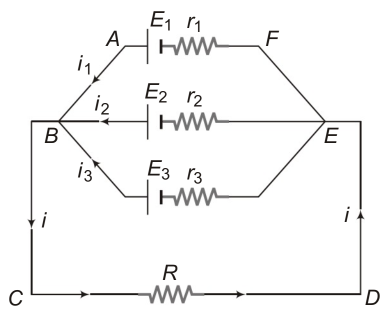


Fig. 23.55

Applying Kirchhoff's second law in loop $ABCDEFA$,

$$E_1-iR-i_1r_1=0 \quad \text{or} \quad i_1=-i\frac{R}{r_1}+\frac{E_1}{r_1} \qquad \ldots\text{(i)}$$

Similarly, we can write

$$i_2=-i\frac{R}{r_2}+\frac{E_2}{r_2} \qquad \ldots\text{(ii)}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

Adding all above equations, we have

$$(i_1+i_2+\ldots+i_n)=-iR\,\Sigma\left(\frac{1}{r}\right)+\Sigma\left(\frac{E}{r}\right)$$

But $\quad i_1+i_2+\ldots+i_n=i$

$\therefore \quad i=-iR\,\Sigma\left(\frac{1}{r}\right)+\Sigma\left(\frac{E}{r}\right)$

$$\therefore \qquad i = \frac{\Sigma\,(E/r)}{1 + R\,\Sigma\,(1/r)} = \frac{\Sigma\,(E/r)/\Sigma\,(1/r)}{\{1/\Sigma\,(1/r)\} + R} = \frac{E_{eq}}{R_{eq}}$$

where, $$\boxed{E_{eq} = \frac{\Sigma\,(E/r)}{\Sigma\,(1/r)} \quad \text{and} \quad R_{eq} = R + \frac{1}{\Sigma\,(1/r)}}$$

From the above expression, we can see that $i = \dfrac{E}{R + r/n}$ if n cells of same emf E and internal resistance r are connected in parallel. This is because,

$$\Sigma\,(E/r) = nE/r \quad \text{and} \quad \Sigma\,(1/r) = n/r$$

$$\therefore \qquad i = \frac{nE/r}{1 + nR/r}$$

Multiplying the numerator and denominator by r/n, we have

$$i = \frac{E}{R + r/n}$$

Exercise In parallel grouping (Case 2) prove that, $E_{eq} = E$ if $E_1 = E_2 = \ldots = E$ and $r_1 = r_2 = \ldots = r$

Case 3 This is the most general case of parallel grouping in which E and r of different cells are different and the positive terminals of few cells are connected to the negative terminals of the others as shown figure.

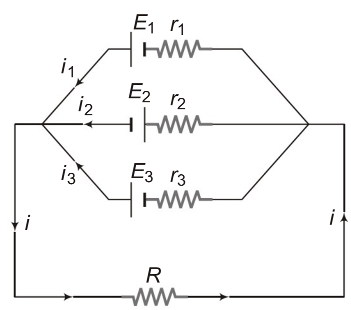


Fig. 23.56

Kirchhoff's second law in different loops gives the following equations :

$$E_1 - iR - i_1 r_1 = 0 \quad \text{or} \quad i_1 = \frac{E_1}{r_1} - \frac{iR}{r_1} \qquad \ldots\text{(i)}$$

$$-E_2 - iR - i_2 r_2 = 0 \quad \text{or} \quad i_2 = -\frac{E_2}{r_2} - \frac{iR}{r_2} \qquad \ldots\text{(ii)}$$

Similarly, $$i_3 = \frac{E_3}{r_3} - \frac{iR}{r_3} \qquad \ldots\text{(iii)}$$

Adding Eqs. (i), (ii) and (iii), we get

$$i_1 + i_2 + i_3 = (E_1/r_1) - (E_2/r_2) + (E_3/r_3) - iR\,(1/r_1 + 1/r_2 + 1/r_3)$$

or $$i\,[1 + R\,(1/r_1 + 1/r_2 + 1/r_3)] = (E_1/r_1) - (E_2/r_2) + (E_3/r_3)$$

$\therefore$
$$\boxed{i = \frac{(E_1/r_1) - (E_2/r_2) + (E_3/r_3)}{1 + R\,(1/r_1 + 1/r_2 + 1/r_3)}}$$

or
$$i = \frac{(E_1/r_1 - E_2/r_2 + E_3/r_3)\,/\,(1/r_1 + 1/r_2 + 1/r_3)}{R + 1/(1/r_1 + 1/r_2 + 1/r_3)}$$

Mixed Grouping

The situation is shown in figure.

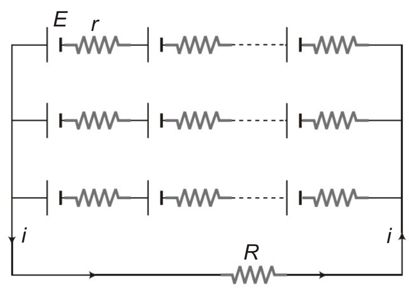


Fig. 23.57

There are n identical cells in a row and number of rows are m. Emf of each cell is E and internal resistance is r. Treating each row as a single cell of emf nE and internal resistance nr, we have

$$\text{Net emf} = nE$$

$$\text{Total internal resistance} = \frac{nr}{m}$$

$$\text{Total external resistance} = R$$

$\therefore$ Current through the external resistance R is

$$\boxed{i = \frac{nE}{R + \dfrac{nr}{m}}}$$

This expression after some rearrangements can also be written as

$$i = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnrR}}$$

If total number of cells are given then mn is fixed. E, r and R are also given, we have liberty to arrange the given number of cells in different rows. Then in the above expression the numerator nmE and in the denominator $2\sqrt{mnrR}$ all are fixed. Only the square term in the denominator is variable. Therefore,

i is maximum when,

$$\sqrt{mR} = \sqrt{nr} \quad \text{or} \quad R = \frac{nr}{m}$$

or total external resistance = total internal resistance

Thus, we can say that the current and hence power transferred to the load is maximum when load resistance is equal to internal resistance. This is known as **maximum power transfer theorem.**

Extra Points to Remember

- Regarding maximum current in the circuit or maximum power consumed by the external resistance R, there are three special cases. One we have discussed above, where we have to arrange the cells in such a manner that current and power in the circuit should be maximum. And this happens when we arrange the cells in such a manner that total internal resistance comes out to be equal to total external resistance. Rest two cases are discussed below.

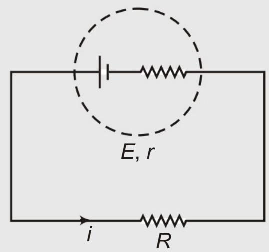


Fig. 23.58

In the figure shown,
$$i = \frac{E}{R + r}$$
$$P_R = i^2 R = \left(\frac{E}{R + r}\right)^2 R$$

(i) Now if r is variable. E and R are fixed, then i and P_R both are maximum when $r = 0$.

(ii) If R is variable. E and r are fixed. Then, current in the circuit is maximum when $R = 0$. But P_R will be maximum, when

$$R = r \quad \text{or} \quad \text{external resistance} = \text{internal resistance}$$

Example 23.23 *Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.*

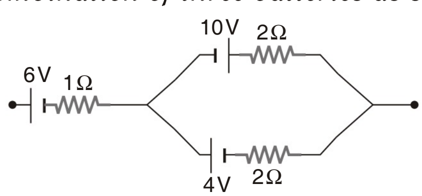


Fig. 23.59

Solution The given combination consists of two batteries in parallel and resultant of these two in series with the third one.

For parallel combination we can apply,

$$E_{eq} = \frac{\frac{E_1}{r_1} - \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{10}{2} - \frac{4}{2}}{\frac{1}{2} + \frac{1}{2}} = 3\text{ V}$$

Further,
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore$
$$r_{eq} = 1\,\Omega$$

Now this is in series with the third one, i.e.

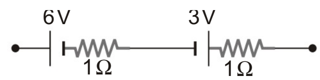


Fig. 23.60

The equivalent emf of these two is $(6-3)$ V or 3 V and the internal resistance will be $(1+1)$ or $2\,\Omega$

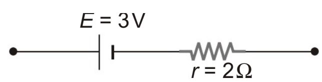


Fig. 23.61

Example 23.24 *In the circuit shown in figure $E_1 = 3V$, $E_2 = 2V$, $E_3 = 1$ V and $R = r_1 = r_2 = r_3 = 1\,\Omega$.* **(JEE 1981)**

(a) Find the potential difference between the points A and B and the currents through each branch.

(b) If r_2 is short-circuited and the point A is connected to point B, find the currents through E_1, E_2, E_3 and the resistor R.

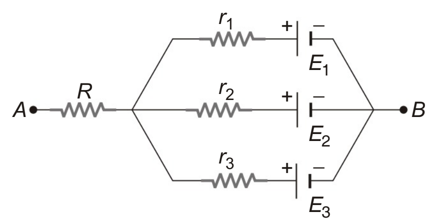


Fig. 23.62

Solution (a) Equivalent emf of three batteries would be

$$E_{eq} = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{(3/1+2/1+1/1)}{(1/1+1/1+1/1)} = 2\text{ V}$$

Further r_1, r_2 and r_3 each are of 1 Ω. Therefore, internal resistance of the equivalent battery will be $\frac{1}{3}\,\Omega$ as all three are in parallel.

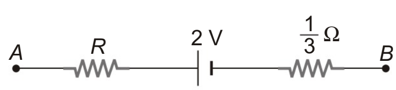


Fig. 23.63

The equivalent circuit is therefore shown in the figure.

Since, no current is taken from the battery.

$$V_{AB} = 2\text{ V} \qquad \text{(From } V = E - ir)$$

Further, $\quad V_A - V_B = E_1 - i_1 r_1$

$$\therefore \quad i_1 = \frac{V_B - V_A + E_1}{r_1} = \frac{-2+3}{1} = 1\text{ A}$$

Similarly, $$i_2 = \frac{V_B - V_A + E_2}{r_2} = \frac{-2+2}{1} = 0$$

and $$i_3 = \frac{V_B - V_A + E_3}{r_3} = \frac{-2+1}{1} = -1\text{ A}$$

(b) r_2 is short circuited means resistance of this branch becomes zero. Making a closed circuit with a battery and resistance R. Applying Kirchhoff's second law in three loops so formed.

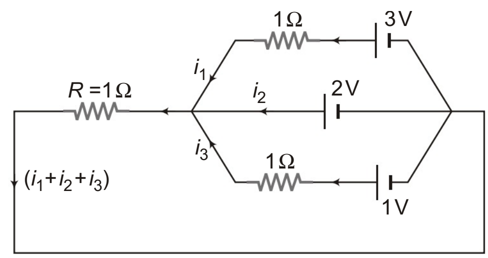


Fig. 23.64

$$3 - i_1 - (i_1 + i_2 + i_3) = 0 \quad \text{...(i)}$$

$$2 - (i_1 + i_2 + i_3) = 0 \quad \text{...(ii)}$$

$$1 - i_3 - (i_1 + i_2 + i_3) = 0 \quad \text{...(iii)}$$

From Eq. (ii) $\quad i_1 + i_2 + i_3 = 2\,\text{A}$

$\therefore$ Substituting in Eq. (i), we get $\quad i_1 = 1\,\text{A}$

Substituting in Eq. (iii), we get $\quad i_3 = -1\,\text{A}$

$\therefore \quad i_2 = 2\,\text{A}$

INTRODUCTORY EXERCISE 23.8

1. Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistances r_1 and r_2 respectively, with polarities as shown in figure

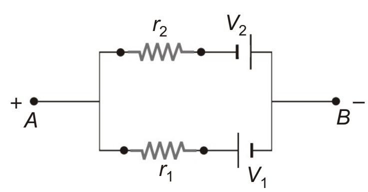


Fig. 23.65

2. Find the net emf of the three batteries shown in figure.

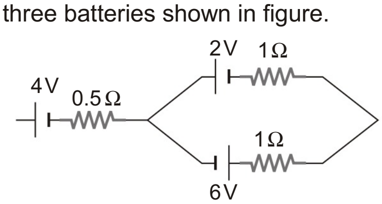


Fig. 23.66

3. Find the equivalent emf and internal resistance of the arrangement shown in figure.

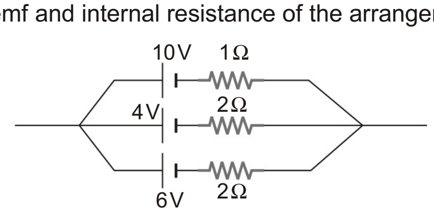


Fig. 23.67

23.12 Electrical Measuring Instruments

So far we have studied about current, resistance, potential difference and emf. Now, in this article we will study how these are measured. The basic measuring instrument is galvanometer, whose pointer shows a deflection when current passes through it. A galvanometer can easily be converted into an ammeter for measuring current, into a voltmeter for measuring potential difference. For accurate measurement of potential difference or emf, a potentiometer is more preferred. Resistances are accurately measured by using post office box or meter bridge which are based on the principle of "Wheatstone bridge". All these are discussed here one by one in brief.

Galvanometer

Many common devices including car instrument panels, battery chargers measure potential difference, current or resistance using d'Arsonval Galvanometer. It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

The maximum deflection, typically 90° to 120° is called full scale deflection. The essential electrical characteristics of the galvanometer are the current i_g required for full scale deflection (of the order of $10\,\mu$A to 10 mA) and the resistance G of the coil (of the order of 10 to 1000 Ω).

The galvanometer deflection is proportional to the current in the coil. If the coil obeys Ohm's law, the current is proportional to potential difference. The corresponding potential difference for full scale deflection is

$$\boxed{V = i_g G}$$

Ammeter

A current measuring instrument is called an ammeter. A galvanometer can be converted into an ammeter by connecting a small resistance S (called shunt) in parallel with it.

Suppose we want to convert a galvanometer with full scale current i_g and coil resistance G into an ammeter with full scale reading i. To determine the shunt resistance S needed, note that, at full scale deflection the total current through the parallel combination is i, the current through the galvanometer is i_g and the current through the shunt is $i - i_g$. The potential difference $V_{ab}\ (= V_a - V_b)$ is the same for both paths, so

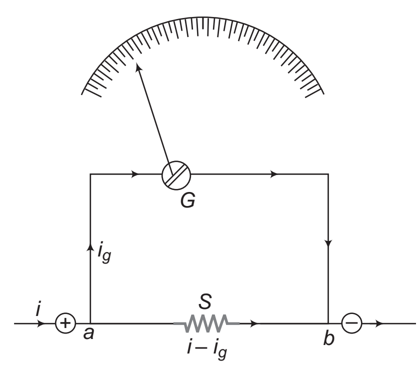


Fig. 23.68

$$i_g G = (i - i_g)\, S$$

$$\therefore \qquad \boxed{S = \left(\frac{i_g}{i - i_g}\right) G}$$

Voltmeter

A voltage measuring device is called a voltmeter. It measures the potential difference between two points.

A galvanometer can be converted into a voltmeter by connecting a high resistance (R) in series with it. The whole assembly called the voltmeter is connected in parallel between the points where potential difference has to be measured.

For a voltmeter with full scale reading V, we need a series resistor R such that

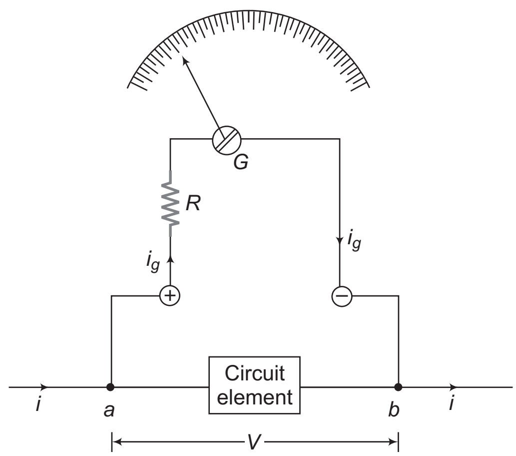


Fig. 23.69

$$V = i_g (G + R)$$

or

$$\boxed{R = \frac{V}{i_g} - G}$$

Extra Points to Remember

- **Conversion of galvanometer into an ammeter.**

  (i) A galvanometer is converted into an ammeter by connecting a low resistance (called shunt) in parallel with galvanometer. This assembly (called ammeter) is connected in series in the wire in which current is to be found. Resistance of an ideal ammeter should be zero.

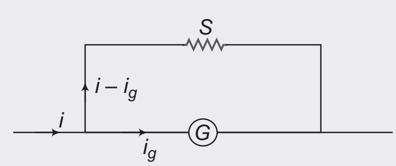
  

  Fig. 23.70

  In parallel, current distributes in the inverse ratio of resistance. Therefore,

  $$\frac{S}{G} = \frac{i_g}{i - i_g}$$

  $$\therefore \qquad S = \text{shunt} = \left(\frac{i_g}{i - i_g}\right) G$$

  (ii) Resistance of an ammeter is given by

  $$\frac{1}{A} = \frac{1}{G} + \frac{1}{S} \quad \Rightarrow \quad A = \frac{GS}{G + S}$$

(iii) The reading of an ammeter is always lesser than actual current in the circuit.

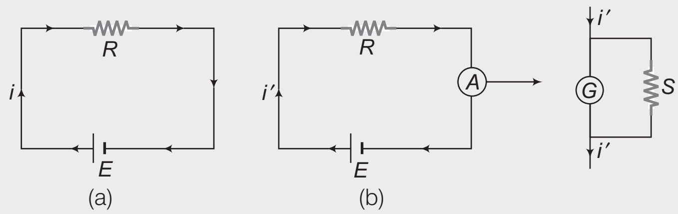


Fig. 23.71

For example, in Fig. (a), actual current through R is

$$i = \frac{E}{R} \quad \ldots(\text{i})$$

while the current after connecting an ammeter of resistance $A \left(= \frac{GS}{G+S}\right)$ in series with R is

$$i' = \frac{E}{R + A} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we see that $i' < i$ and $i' = i$ when $A = 0$.
i.e. resistance of an ideal ammeter should be zero.

- **Conversion of a galvanometer into a voltmeter**

(i) A galvanometer is converted into a voltmeter by connecting a high resistance in series with galvanometer. The whole assembly called voltmeter is connected in parallel across the two points between which potential difference is to be found. Resistance of an ideal voltmeter should be infinite.

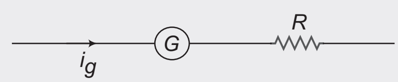


Fig. 23.72

$$V = I_g (G + R)$$

$\therefore$ $R =$ high resistance required in series

$$= \frac{V}{i_g} - G$$

(ii) Resistance of a voltmeter is $R_V = R + G$

(iii) The reading of a voltmeter is always lesser than the true value.

For example, if a current i is passing through a resistance r, the actual value is

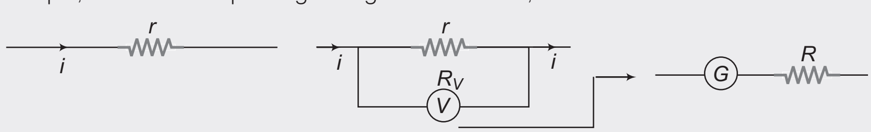


Fig. 23.73

$$V = ir \quad \ldots(\text{i})$$

Now, if a voltmeter of resistance R_V $(= G + R)$ is connected across the resistance r, the new value will be

$$V' = \frac{i \times (rR_V)}{r + R_V} \quad \text{or} \quad V' = \frac{ir}{1 + \frac{r}{R_V}} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we can see that, $V' < V$ and $V' = V$ if $R_V = \infty$

Thus, resistance of an ideal voltmeter should be infinite.

Example 23.25 *What shunt resistance is required to make the* $1.00\ mA, 20\ \Omega$ *galvanometer into an ammeter with a range of* 0 *to* $50.0\ mA$?

Solution Here, $i_g = 1.00\ \text{mA} = 10^{-3}\ \text{A}, \quad G = 20\ \Omega, \quad i = 50.0 \times 10^{-3}\ \text{A}$

Substituting in $S = \left(\dfrac{i_g}{i - i_g}\right) G = \dfrac{(10^{-3})(20)}{(50.0 \times 10^{-3}) - (10^{-3})}$

$$= 0.408\ \Omega$$ **Ans.**

Note *The resistance of ammeter is given by*

$$\frac{1}{A} = \frac{1}{G} + \frac{1}{S} = \frac{1}{20} + \frac{1}{0.408}$$

or $A = 0.4\ \Omega$

The shunt resistance is so small in comparison to the galvanometer resistance that the ammeter resistance is very nearly equal to the shunt resistance. This shunt resistance gives us a low resistance ammeter with the desired range of 0 *to* 50.0 *mA. At full scale deflection* $i = 50.0$ *mA, the current through the galvanometer is* 1.0 *mA while the current through the shunt is* 49.0 *mA. If the current i is less than* 50.0 *mA, the coil current and the deflection are proportionally less, but the ammeter resistance is still* $0.4\ \Omega$.

Example 23.26 *How can we make a galvanometer with* $G = 20\ \Omega$ *and* $i_g = 1.0\ mA$ *into a voltmeter with a maximum range of* $10\ V$?

Solution Using $R = \dfrac{V}{i_g} - G,$

We have, $$R = \frac{10}{10^{-3}} - 20$$

$$= 9980\ \Omega$$ **Ans.**

Thus, a resistance of $9980\ \Omega$ is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

Note *At full scale deflection current through the galvanometer, the voltage drop across the galvanometer*

$$V_g = i_g G = 20 \times 10^{-3}\ \text{volt} = 0.02\ \text{volt}$$

and the voltage drop across the series resistance R is

$$V = i_g R = 9980 \times 10^{-3}\ \text{volt} = 9.98\ \text{volt}$$

or we can say that most of the voltage appears across the series resistor.

Example 23.27 *Resistance of a milliammeter is* R_1 *of an ammeter is* R_2 *of a voltmeter is* R_3 *and of a kilovoltmeter is* R_4*. Find the correct order of* R_1, R_2, R_3 *and* R_4.

Solution To increase the range of an ammeter a low resistance has to be connected in parallel with galvanometer. Therefore, net resistance decreases. To increase the range of voltmeter, a high resistance has to be connected in series. So, net resistance further increases. Therefore, the correct order is

$$R_4 > R_3 > R_1 > R_2$$

Example 23.28 *A microammeter has a resistance of 100 Ω and full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination (s)* **(JEE 1991)**

(a) 50 V range with 10 kΩ resistance in series

(b) 10 V range with 200 kΩ resistance in series

(c) 5 mA range with 1 Ω resistance in parallel

(d) 10 mA range with 1 Ω resistance in parallel

Solution To increase the range of ammeter a parallel resistance (called shunt) is required which is given by

$$S = \left(\frac{i_g}{i - i_g}\right) G$$

For option (c), $$S = \left(\frac{50 \times 10^{-6}}{5 \times 10^{-3} - 50 \times 10^{-6}}\right)(100) \approx 1\,\Omega$$

To change it in voltmeter, a high resistance R is put in series, where R is given by $R = \frac{V}{i_g} - G$

For option (b), $$R = \frac{10}{50 \times 10^{-6}} - 100 \approx 200\ \text{k}\Omega$$

Therefore, options (b) and (c) are correct.

Example 23.29 *A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0-30 V. If connected to a $\frac{2n}{249}$ Ω resistance, it becomes an ammeter of range 0-1.5 A. The value of n is* **(JEE 2014)**

Solution

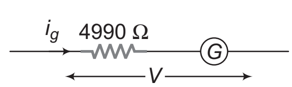


Fig. 23.74

$$i_g(G + 4990) = V$$

$$\Rightarrow \quad \frac{6}{1000}(G + 4990) = 30$$

$$\Rightarrow \quad G + 4990 = \frac{30000}{6} = 5000$$

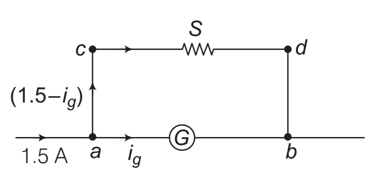


Fig. 23.75

$$\Rightarrow \qquad G = 10\,\Omega$$

$$V_{ab} = V_{cd}$$

$$\Rightarrow \qquad i_g G = (1.5 - i_g)\,S$$

$$\Rightarrow \qquad \frac{6}{1000} \times 10 = \left(1.5 - \frac{6}{1000}\right) S$$

$$\Rightarrow \qquad S = \frac{60}{1494} = \frac{2n}{249}$$

$$\Rightarrow \qquad n = \frac{249 \times 30}{1494}$$

$$= \frac{2490}{498} = 5$$

Ans.

INTRODUCTORY EXERCISE 23.9

1. The full scale deflection current of a galvanometer of resistance 1 Ω is 5 mA. How will you convert it into a voltmeter of range 5 V?
2. A micrometer has a resistance of 100 Ω and full scale deflection current of 50 μA. How can it be made to work as an ammeter of range 5 mA?
3. A voltmeter has a resistance G and range V. Calculate the resistance to be used in series with it to extend its range to nV.

Potentiometer

The potentiometer is an instrument that can be used to measure the emf or the internal resistance of an unknown source. It also has a number of other useful applications.

Principle of Potentiometer

The principle of potentiometer is schematically shown in figure.

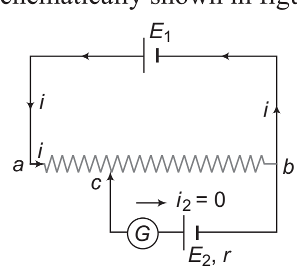


Fig. 23.76

A resistance wire ab of total resistance R_{ab} is permanently connected to the terminals of a source of known emf E_1. A sliding contact c is connected through the galvanometer G to a second source whose emf E_2 is to be measured. As contact c is moved along the potentiometer wire, the resistance R_{cb} between points c and b varies. If the resistance wire is uniform R_{cb} is proportional to the length of the wire between c and b. To determine the value of E_2, contact c is moved until a position is found at which the galvanometer shows no deflection. This corresponds to zero current passing through E_2. With $i_2 = 0$, Kirchhoff's second law gives

$$\boxed{E_2 = iR_{cb}}$$

With $i_2 = 0$, the current i produced by the emf E_1 has the same value no matter what the value of emf E_2. A potentiometer has the following applications.

To find emf of an unknown battery

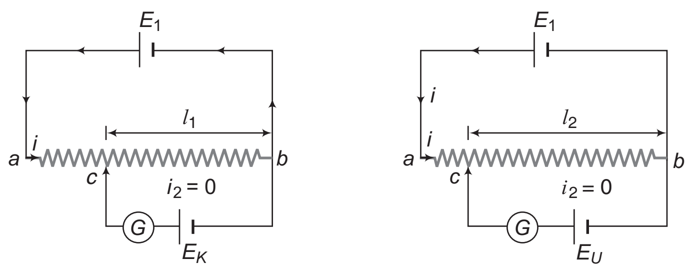


Fig. 23.77

We calibrate the device by replacing E_2 by a source of known emf E_K and then by unknown emf E_U. Let the null points are obtained at lengths l_1 and l_2. Then,

$$E_K = i\,(\rho l_1) \qquad \text{and} \qquad E_U = i\,(\rho l_2)$$

Here, ρ = resistance of wire ab per unit length.

$$\therefore \qquad \frac{E_K}{E_U} = \frac{l_1}{l_2} \qquad \text{or} \qquad \boxed{E_U = \left(\frac{l_2}{l_1}\right) E_K}$$

So, by measuring the lengths l_1 and l_2, we can find the emf of an unknown battery.

To find the internal resistance of an unknown battery

To find the internal resistance of an unknown battery let us derive a formula.

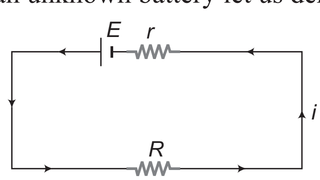


Fig. 23.78

In the circuit shown in figure,

$$i = \frac{E}{R + r} \qquad \text{...(i)}$$

and V = potential difference across the terminals of the battery

or $$V = E - ir = iR \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we can prove that

$$\boxed{r = R\left(\frac{E}{V} - 1\right)}$$

Thus, if a battery of emf E and internal resistance r is connected across a resistance R and the potential difference across its terminals comes out to be V then the internal resistance of the battery is given by the above formula. Now, let us apply it in a potentiometer for finding the internal resistance of the unknown battery. The circuit shown in Fig. 23.79 is similar to the previous one.

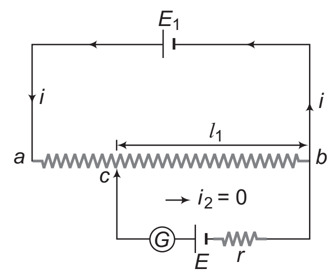


Fig. 23.79

Hence, $$E = i\rho l_1 \quad \text{...(i)}$$

Now, a known resistance R is connected across the terminals of the unknown battery as shown in Fig. 23.80.

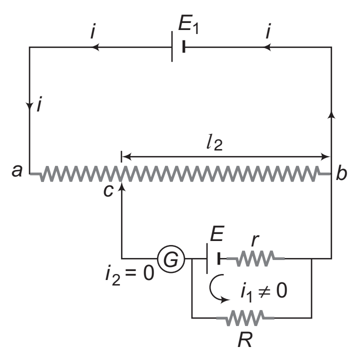


Fig. 23.80

This time $$V_{cb} \neq E, \quad \text{but} \quad V_{cb} = V$$

where, V = potential difference across the terminals of the unknown battery.

Hence, $$V = i\rho l_2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{E}{V} = \frac{l_1}{l_2}$$

Substituting in $r = R\left(\frac{E}{V} - 1\right)$, we get

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

So, by putting R, l_1 and l_2 we can determine the internal resistance r of unknown battery.

Extra Points to Remember

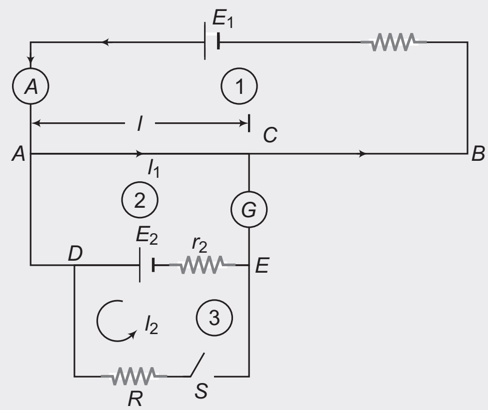
- 

Fig. 23.81

Under balanced condition (when $I_G = 0$) loop-1 and loop-3 are independent with each other. All problems in this condition can be solved by a single equation,

$$V_{AC} = V_{DE}$$

or
$$i_1 R_{AC} = E_2 - i_2 r_2$$

or
$$\boxed{i_1 \lambda l = E_2 - i_2 r_2} \quad \text{...(i)}$$

Here, λ is the resistance per unit length of potentiometer wire AB. Length l is called balance point length. Currents i_1 and i_2 are independent with each other. Current $i_2 = 0$, if switch is open.

- Under balanced condition, a part of potential difference of E_1 is balanced by the lower circuit. So, normally $E_2 < E_1$ for taking balance point length. Similarly, $V_{AC} = V_{DE} \Rightarrow V_A = V_D$ and $V_C = V_E$. Therefore, positive terminals of both batteries should be on same side and negative terminals on the other side.
- From Eq. (i), we can see that null point length is

$$l = \frac{E_2 - i_2 r_2}{i_1 \lambda}$$

Now, suppose E_1 is increased then i_1 will also increase and null point length l will decrease. Similarly, we can make some other cases also.

- If we do not get any balanced condition ($I_G \neq 0$), then the given circuit is simply a three loops problem, which can be solved with the help of Kirchhoff's laws.

Example 23.30 *A potentiometer wire of length* 100 cm *has a resistance of* 10 Ω. *It is connected in series with a resistance R and a cell of emf* 2 V *and of negligible internal resistance. A source of emf* 10 mV *is balanced against a length of* 40 cm *of the potentiometer wire. What is the value of R?*

Solution From the theory of potentiometer, $V_{cb} = E$, if no current is drawn from the battery

or
$$\left(\frac{E_1}{R + R_{ab}}\right) R_{cb} = E$$

Here, $E_1 = 2\,\text{V}$, $R_{ab} = 10\,\Omega$, $R_{cb} = \left(\frac{40}{100}\right) \times 10 = 4\,\Omega$

and $$E = 10 \times 10^{-3}\ \text{V}$$

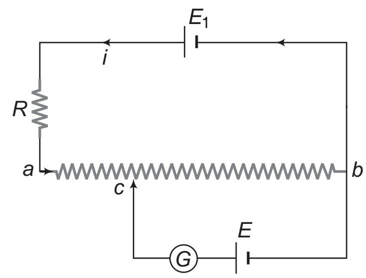


Fig. 23.82

Substituting in above equation, we get

$$R = 790\,\Omega$$ **Ans.**

Example 23.31 *When the switch is open in lowermost loop of a potentiometer, the balance point length is* 60 *cm. When the switch is closed with a known resistance of* $R = 4\,\Omega$*, the balance point length decreases to* 40 *cm. Find the internal resistance of the unknown battery.*

Solution Using the result,

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

$$= 4\left(\frac{60}{40} - 1\right)$$

$$= 2\,\Omega$$ **Ans.**

Example 23.32

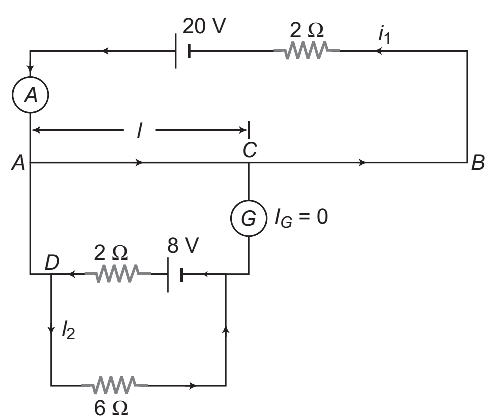


Fig. 23.83

In the figure shown, wire AB has a length of 100 *cm and resistance* $8\,\Omega$*. Find the balance point length l.*

Solution Using the equation, $$i_1\lambda l = E_2 - i_2 r_2$$

We have,

$$\left(\frac{20}{2+8}\right)\left(\frac{8}{100}\right)l = 8 - \left(\frac{8}{2+6}\right)(2)$$

Solving this equation, we get

$$l = 37.5 \text{ cm}$$ **Ans.**

INTRODUCTORY EXERCISE 23.10

1. In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of the cell are connected across 0.52 m of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω a balance is obtained when the cell is connected across 0.4 m of the wire. Find the internal resistance of the cell.
2. The potentiometer wire *AB* is 600 cm long.

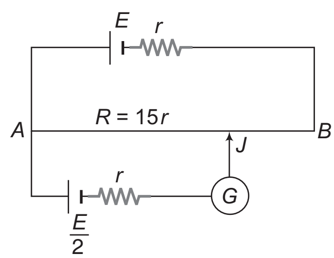


Fig. 23.84

(a) At what distance from *A* should the jockey *J* touch the wire to get zero deflection in the galvanometer.

(b) If the jockey touches the wire at a distance 560 cm from *A*, what will be the current through the galvanometer.

Principle of Wheatstone's Bridge

The scientist Wheatstone designed a circuit to find unknown resistance. Such a circuit is popularly known as Wheatstone's bridge. This is an arrangement of four resistances which can be used to measure one of them in terms of the rest. The figure shows the circuit designed by him. The bridge is said to be **balanced** when deflection in galvanometer is zero, i.e. $i_g = 0$, and hence,

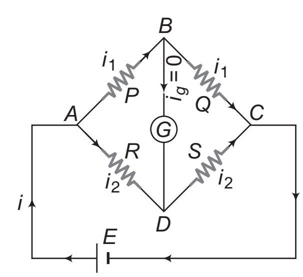


Fig. 23.85

$$V_B = V_D$$

Under this condition, $$V_A - V_B = V_A - V_D$$

or $$i_1 P = i_2 R$$

or $$\frac{i_1}{i_2} = \frac{R}{P} \quad \text{...(i)}$$

Similarly, $$V_B - V_C = V_D - V_C$$

or $$i_1Q = i_2S \quad \text{or} \quad \frac{i_1}{i_2} = \frac{S}{Q} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $$\frac{R}{P} = \frac{S}{Q} \quad \text{or} \quad \boxed{\frac{P}{Q} = \frac{R}{S}}$$

So, this is a condition for which a Wheatstone's bridge is balanced.

To measure the resistance of an unknown resistor, it is connected as one of the four resistors in the bridge. One of the other three should be a variable resistor. Let us suppose P is the unknown resistance and Q is the variable resistance. The value of Q is so adjusted that deflection through the galvanometer is zero. In this case, the bridge is balanced and

$$P = \left(\frac{R}{S}\right) \cdot Q$$

Knowing R, S and Q, the value of P is calculated. Following two points are important regarding a Wheatstone's bridge.

(i) In Wheatstone's bridge, cell and galvanometer arms are interchangeable.

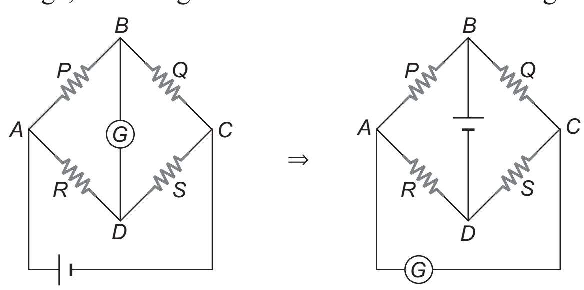


Fig. 23.86

In both the cases, condition of balanced bridge is

$$\frac{P}{Q} = \frac{R}{S}$$

(ii) If bridge is not balanced current will flow from D to B in Fig. 23.85 if,

$$PS > RQ$$

Exercise Try and prove the statements of both the points yourself.

Meter Bridge Experiment

Meter bridge works on Wheatstone's bridge principle and is used to find the unknown resistance (X) and its specific resistance (or resistivity).

Theory

As the meter bridge wire AC has uniform material density and area of cross-section, its resistance is proportional to its length. Hence, AB and BC are the ratio arms and their resistances correspond to P and Q respectively.

Thus, $$\frac{\text{Resistance of } AB}{\text{Resistance of } BC} = \frac{P}{Q} = \frac{\lambda l}{\lambda\,(100 - l)} = \frac{l}{100 - l}$$

Here, λ is the resistance per unit length of the bridge wire.

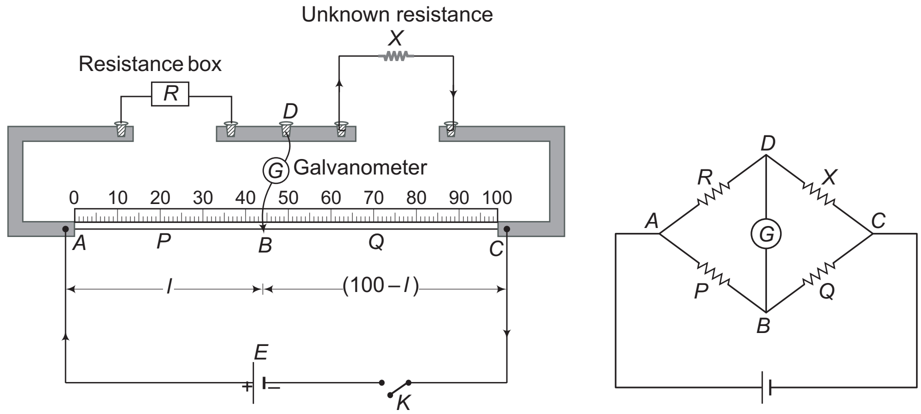


Fig. 23.87

Hence, according to Wheatstone's bridge principle,

When current through galvanometer is zero or bridge is balanced, then

$$\frac{P}{Q} = \frac{R}{X} \quad \text{or} \quad X = \frac{Q}{P}R$$

$$\therefore \qquad X = \left(\frac{100 - l}{l}\right)R \qquad \ldots\text{(i)}$$

So, by knowing R and l unknown resistance X can be determined.

Specific Resistance From resistance formula,

$$X = \rho \frac{L}{A}$$

or

$$\rho = \frac{XA}{L}$$

For a wire of radius r or diameter $D = 2r$,

$$A = \pi r^2 = \frac{\pi D^2}{4}$$

or

$$\rho = \frac{X\pi D^2}{4L} \qquad \ldots\text{(ii)}$$

By knowing X, D and L we can find specific resistance of the given wire by Eq. (ii).

Precautions

1. The connections should be clean and tight.
2. Null point should be brought between 40 cm and 60 cm.
3. At one place, diameter of wire (D) should be measured in two mutually perpendicular directions.
4. The jockey should be moved gently over the bridge wire so that it does not rub the wire.

End Corrections

In meter bridge, some extra length (under the metallic strips) comes at points A and C. Therefore, some additional length (α and β) should be included at the ends. Here, α and β are called the end corrections. Hence, in place of l we use $l+\alpha$ and in place of $100-l$ we use $100-l+\beta$.

To find α and β, use known resistors R_1 and R_2 in place of R and X and suppose we get null point length equal to l_1. Then,

$$\frac{R_1}{R_2}=\frac{l_1+\alpha}{100-l_1+\beta} \qquad \text{...(i)}$$

Now, we interchange the positions of R_1 and R_2 and suppose the new null point length is l_2. Then,

$$\frac{R_2}{R_1}=\frac{l_2+\alpha}{100-l_2+\beta} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\boxed{\begin{aligned}\alpha &= \frac{R_2l_1-R_1l_2}{R_1-R_2}\\ \text{and} \quad \beta &= \frac{R_1l_1-R_2l_2}{R_1-R_2}-100\end{aligned}}$$

Example 23.33 *If resistance R_1 in resistance box is 300 Ω, then the balanced length is found to be 75.0 cm from end A. The diameter of unknown wire is 1 mm and length of the unknown wire is 31.4 cm. Find the specific resistance of the unknown wire.*

Solution $\dfrac{R}{X}=\dfrac{l}{100-l}$

$$\Rightarrow \qquad X=\left(\frac{100-l}{l}\right)R$$

$$=\left(\frac{100-75}{75}\right)(300)=100\,\Omega$$

Now,
$$X=\frac{\rho l}{A}=\frac{\rho l}{(\pi d^2/4)}$$

$$\therefore \qquad \rho=\frac{\pi d^2 X}{4l}$$

$$=\frac{(22/7)(10^{-3})^2(100)}{(4)(0.314)}$$

$$=2.5\times10^{-4}\,\Omega\text{-m}$$ **Ans.**

Example 23.34 *In a meter bridge, null point is 20 cm, when the known resistance R is shunted by 10 Ω resistance, null point is found to be shifted by 10 cm. Find the unknown resistance X.*

Solution $$\frac{R}{X} = \frac{l}{100 - l}$$

$\therefore$ $$X = \left(\frac{100 - l}{l}\right) R$$

or $$X = \left(\frac{100 - 20}{20}\right) R = 4R \qquad \text{...(i)}$$

When known resistance R is shunted, its net resistance will decrease. Therefore, resistance parallel to this (i.e. P) should also decrease or its new null point length should also decrease.

$\therefore$ $$\frac{R'}{X} = \frac{l'}{100 - l'}$$

$$= \frac{20 - 10}{100 - (20 - 10)} = \frac{1}{9}$$

or $$X = 9R' \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$4R = 9R' = 9\left[\frac{10R}{10 + R}\right]$$

Solving this equation, we get

$$R = \frac{50}{4}\,\Omega$$

Now, from Eq. (i), the unknown resistance

$$X = 4R = 4\left(\frac{50}{4}\right)$$

or $$X = 50\,\Omega$$ **Ans.**

Note *R' is resultant of R and 10 Ω in parallel.*

$\therefore$ $$\frac{1}{R'} = \frac{1}{10} + \frac{1}{R}$$

or $$R' = \frac{10R}{10 + R}$$

Example 23.35 *If we use 100 Ω and 200 Ω in place of R and X we get null point deflection, $l = 33$ cm. If we interchange the resistors, the null point length is found to be 67 cm. Find end corrections α and β.*

Solution $$\alpha = \frac{R_2 l_1 - R_1 l_2}{R_1 - R_2}$$

$$= \frac{(200)(33) - (100)(67)}{100 - 200}$$

$$= 1\,\text{cm}$$ **Ans.**

$$\beta = \frac{R_1 l_1 - R_2 l_2}{R_1 - R_2} - 100$$

$$= \frac{(100)(33) - (200)(67)}{100 - 200} - 100$$

$$= 1 \text{cm}$$ **Ans.**

INTRODUCTORY EXERCISE 23.11

1. A resistance of $2\,\Omega$ is connected across one gap of a meter bridge (the length of the wire is 100 cm) and an unknown resistance, greater than $2\,\Omega$, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is **(JEE 2007)**

(a) $3\,\Omega$ (b) $4\,\Omega$
(c) $5\,\Omega$ (d) $6\,\Omega$

2. A meter bridge is setup as shown in figure, to determine an unknown resistance X using a standard $10\,\Omega$ resistor. The galvanometer shows null point when tapping key is at 52 cm mark. The end corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is

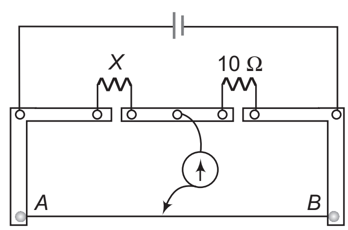


Fig. 23.88

(JEE 2011)

(a) $10.2\,\Omega$ (b) $10.6\,\Omega$
(c) $10.8\,\Omega$ (d) $11.1\,\Omega$

3. R_1, R_2, R_3 are different values of R. A, B and C are the null points obtained corresponding to R_1, R_2 and R_3 respectively. For which resistor, the value of X will be the most accurate and why? **(JEE 2005)**

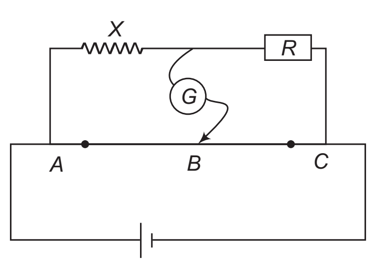


Fig. 23.89

Post Office Box

Post office box also works on the principle of Wheatstone's bridge.

In a Wheatstone's bridge circuit, if $\dfrac{P}{Q}=\dfrac{R}{X}$ then the bridge is balanced. So, unknown resistance $X=\dfrac{Q}{P}R$.

P and Q are set in arms AB and BC where we can have, $10\,\Omega, 100\,\Omega$ or $1000\,\Omega$ resistances to set any ratio $\dfrac{Q}{P}$.

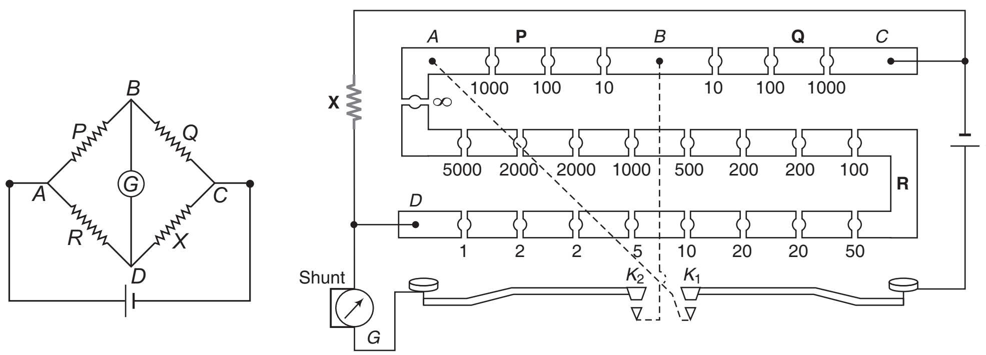


Fig. 23.90

These arms are called ratio arm, initially we take $Q=10\,\Omega$ and $P=10\,\Omega$ to set $\dfrac{Q}{P}=1$. The unknown resistance (X) is connected between C and D and battery is connected across A and C,

Now, adjust resistance in part A to D such that the bridge gets balanced. For this, keep on increasing the resistance with $1\,\Omega$ interval, check the deflection in galvanometer by first pressing key K_1 then galvanometer key K_2.

Suppose at $R=4\,\Omega$, we get deflection towards left and at $R=5\,\Omega$, we get deflection towards right. Then, we can say that for balanced condition, R should lie between $4\,\Omega$ to $5\,\Omega$.

Now, $X=\dfrac{Q}{P}R=\dfrac{10}{10}R=R=4\,\Omega$ to $5\,\Omega$

To get closer value of X, in the second observation, let us choose $\dfrac{Q}{P}=\dfrac{1}{10}$ i.e. $\left(\dfrac{P=100}{Q=10}\right)$

Suppose, now at $R=42$ we get deflection towards left and at $R=43$ deflection is towards right. So $R\in(42, 43)$.

Now, $X=\dfrac{Q}{P}R=\dfrac{10}{100}R=\dfrac{1}{10}R$, where $R\in(4.2, 4.3\,\Omega)$. Now, to get further closer value take $\dfrac{Q}{P}=\dfrac{1}{100}$ and so on.

The observation table is shown below.

Table 23.2

S.No	Resistance in the ratio arm AB (P) (ohm)	BC (Q) (ohm)	Resistance in arm (AD (R) (ohm)	Direction of deflection	Unknown resistance $X = \frac{Q}{P} \times R$ (ohm)
1	10	10	4	Left	4 to 5
			5	Right	
2	100	10	40	Left (large)	(4.2 to 4.3)
			50	Right (large)	
			42	Left	
			43	Right	
3	1000	10	420	Left	4.25
			424	Left	
			425	No deflection	
			426	Right	

So, the correct value of X is 4.25 Ω

Example 23.36 *To locate null point, deflection battery key* (K_1) *is pressed before the galvanometer key* (K_2)*. Explain why?*

Solution If galvanometer key K_2 is pressed first then just after closing the battery key K_1 current suddenly increases.

So, due to self-induction, a large back emf is generated in the galvanometer, which may damage the galvanometer.

Example 23.37 *What are the maximum and minimum values of unknown resistance X, which can be determined using the post office box shown in the Fig.* 23.90?

Solution

$$X = \frac{QR}{P}$$

$$\therefore \quad X_{max} = \frac{Q_{max} \ R_{max}}{P_{min}}$$

$$= \frac{1000}{10}(11110)$$

$$= 1111 \, k\Omega \qquad \textbf{Ans.}$$

$$X_{min} = \frac{Q_{min} \ R_{min}}{P_{max}}$$

$$= \frac{(10)(1)}{1000}$$

$$= 0.01 \, \Omega \qquad \textbf{Ans.}$$

INTRODUCTORY EXERCISE 23.12

1. In post office box experiment, $\frac{Q}{P} = \frac{1}{10}$. In R if 142 Ω is used then we get deflection towards right and if $R = 143\ \Omega$, then deflection is towards left. What is the range of unknown resistance?
2. What is the change in post office box experiment if battery is connected between B and C and galvanometer is connected across A and C?
3. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **(JEE 2004)**

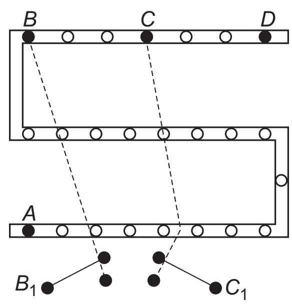


Fig. 23.91

(a) B and C (b) C and D (c) A and D (d) B_1 and C_1

Extra Topics For Other Examinations

23.13 Colour Codes for Resistors

Resistors are of the following two major types :

(i) wire bound resistors and (ii) carbon resistors

First type of resistors are made by winding the wires of an alloy like nichrome, manganin or constantan etc. Materials are so chosen that their resistivities are relatively less sensitive to temperature.

In carbon resistors, carbon with a suitable binding agent is molded into a cylinder. Wire leads are attached to this cylinder and the entire resistor is encased in a ceramic or plastic jacket. The two leads connect the resistor to a circuit. Carbon resistors are compact and inexpensive. Their values are given using a colour code.

Table 23.3

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	

Colour	Number	Multiplier	Tolerance (%)
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

The resistors have a set of four (or three) co-axial coloured rings, whose significance are listed in above table. The colours are noted from left to right.

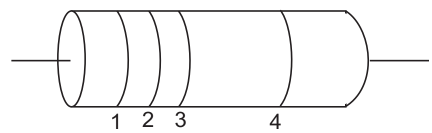


Fig. 23.92

Colour 1 $\rightarrow$ First significant figure

Colour 2 $\rightarrow$ Second significant figure

Colour 3 $\rightarrow$ Decimal multiplier

Colour 4 (or no colour) $\rightarrow$ Tolerance or possible variation in percentage.

Extra Points to Remember

- To remember the value of colour coding used for carbon resistor, the following sentences are found to be of great help (where bold letters stand for colours)
  B B ROY Great **B**ritain **V**ery **G**ood **W**ife wearing **Gold Silver** necklace.
  OR
  Black **B**rown **R**ods **O**f **Y**our **G**ate **B**ecome **V**ery **G**ood **W**hen **G**iven **S**ilver colour

Example 23.38 *The four colours on a resistor are : brown, yellow, green and gold as read from left to right. What is resistance corresponding to these colours.*

Solution From the table we can see that

$$\text{Brown colour} \rightarrow 1$$
$$\text{Yellow colour} \rightarrow 4$$
$$\text{Green colour} \rightarrow 10^5 \text{ and}$$
$$\text{Gold colour} \rightarrow 5\,\%$$
$$\therefore \quad R = (14 \times 10^5 \pm 5\%)\,\Omega \qquad \textbf{Ans.}$$

INTRODUCTORY EXERCISE 23.13

1. For the given carbon resistor, let the first strip be yellow, second strip be red, third strip be orange and fourth be gold. What is its resistance?
2. The resistance of the given carbon resistor is $(24 \times 10^6 \pm 5\%)\,\Omega$. What is the sequence of colours on the strips provided on resistor?

Final Touch Points

1. **Mobility** The physical significance of mobility means how mobile the charge carriers are for the current flow. If mobility of charge carriers is more than we can say that current flow will be more.

   In metals, the mobile charge carriers are electrons. In an ionised gas they are electrons and positive charged ions. In an electrolyte, these can be positive and negative ions. In semiconductors, charge carriers are electrons and holes. Later, we will see that mobility of electrons (in semiconductor) is more than the mobility of holes.

   Mobility (μ) is defined as the magnitude of drift velocity per unit electric field.

   Thus, $$\boxed{\mu = \frac{v_d}{E}}$$

   But, $$v_d = \frac{eE\tau}{m}$$

   $\therefore$ $$\boxed{\mu = \frac{e\tau}{m}}$$

   The SI units of mobility are $m^2/V\text{-}s$. Therefore, practical units of mobility is $cm^2/V\text{-}s$. Mobility of any charge carrier (whether it is electron, ion or hole) is always positive.

2. **Deduction of Ohm's law** We know that

   $$i = neAv_d$$

   where, $v_d = \dfrac{e\tau E}{m}$

   $\therefore$ $$i = \frac{ne^2\tau AE}{m}$$

   If V is the potential difference across the conductor and l is its length, then

   $$E = \frac{V}{l}$$

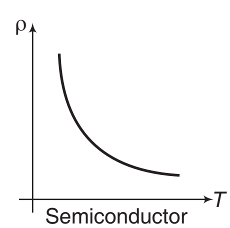
   $\therefore$ $$i = \frac{ne^2\tau AV}{ml} \quad \text{or} \quad V = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}i$$

   Here, $$\boxed{\frac{m}{ne^2\tau} = \frac{1}{\sigma} \text{ or } \rho}$$

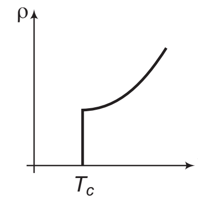
   $\therefore$ $$V = \left(\frac{\rho l}{A}\right)\cdot i \quad \text{or} \quad V = Ri$$ **Hence proved.**

   where, $R = \rho\dfrac{l}{A}$ is the resistance of the conductor.

3. **Thermistor** The temperature coefficient of resistivity is negative for semiconductors. This means that the resistivity decreases as we raise the temperature of such a material. The magnitude of the temperature coefficient of resistivity is often quite large for a semiconducting material. This fact is used to construct thermometers to detect small changes in temperatures. Such a device is called a thermistor. The variation of resistivity of a semiconductor with temperature is shown in figure. A typical thermistor can easily measure a change in temperature of the order of 10^{-3} °C



4. Superconductors Superconductivity was first discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes.



Superconductor

There are certain materials, including several metallic alloys and oxides for which as the temperature decreases, the resistivity first decreases smoothly, like that of any metal. But then at a certain critical temperature T_c a phase transition occurs, and the resistivity suddenly drops to zero as shown in figure.

Once, a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field. Possible applications of superconductors are ultrafast computer switches and transmission of electric power through superconducting power lines. However, the requirement of low temperature is posing difficulty. For instance the critical temperature for mercury is 4.2 K. Scientists are putting great effort to construct compounds and alloys which would be superconducting at room temperature (300 K). Superconductivity at around 125 K has already been achieved.

5. The ρ-T equation derived in article 23.6 can be derived from the relation,

$$\frac{d\rho}{dT} = \alpha\rho \quad \text{or} \quad \frac{d\rho}{\rho} = \alpha\, dT$$

$$\therefore \qquad \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \alpha \int_{T_0}^{T} dT \qquad (\text{if } \alpha = \text{constant}) \quad \ldots\text{(i)}$$

$$\therefore \qquad \ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$$

$$\therefore \qquad \rho = \rho_0 e^{\alpha(T - T_0)}$$

If α is small, $e^{\alpha(T - T_0)}$ can approximately be written as $1 + \alpha(T - T_0)$. Hence,

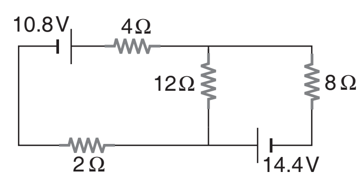
$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

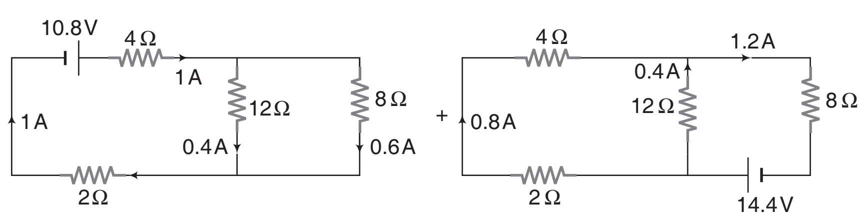
Which is the same result as we have discussed earlier.

In the above discussion, we have assumed α to be constant. If it is function of temperature it will come inside the integration in Eq. (i).

6. The principle of superposition Complex network problems can sometimes be solved easily by using the principle of superposition. This principle essentially states that when a number of emf's act in a network, the solution is the same as the superposition of the solutions for one emf acting at a time, the others being shorted.

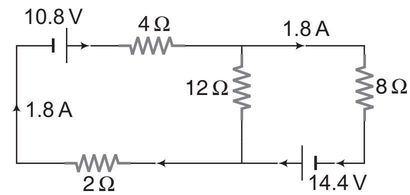
7. Figure shows a network with two loops. The currents in various branches can be calculated using Kirchhoff's laws. We can get the same solution by considering only one battery at a time and then superposing the two solutions. If a battery has an internal resistance, it must be left in place when the emf of the battery is removed. Figure shows how the superposition principle can be applied to the present problem.





(a) (b)

$\Downarrow$



(c)

The current values in Fig. (a) and (b) are easily verified. For example when the 10.8 V battery alone is acting, the total resistance in the circuit is

$$4 + \frac{12 \times 8}{12 + 8} + 2 = 10.8\ \Omega$$

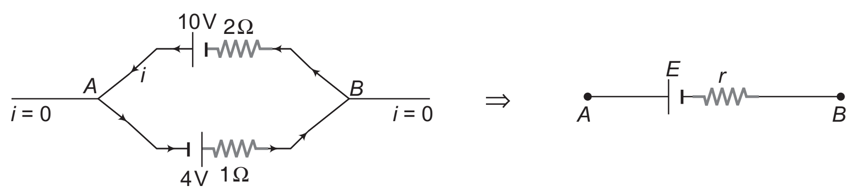
This makes the total current $\frac{10.8\ \text{V}}{10.8\ \Omega} = 1$ A. This current splits between 8 Ω and 12 Ω in the ratio 3 : 2. Similarly, the total resistance when only the 14.4 V battery is acting is

$$8 + \frac{12 \times 6}{12 + 6} = 12\ \Omega$$

Therefore, the total current is $\frac{14.4\ \text{V}}{12\ \Omega} = 1.2$ A.

The superposition principle shows that there is no current in the 12 Ω resistance. Only a current of 1.8 A flows through the outer loop. All these conclusions can be verified by analyzing the circuit using Kirchhoff's laws.

8. The equivalent emf of a cell can also be found by the following method.



Suppose we wish to find the equivalent emf of the above circuit. We apply the fact that

$$E = V$$

When no current is drawn from the cell. But current in the internal circuit may be non-zero. This current is,

$$i = \frac{10 + 4}{2 + 1} = \frac{14}{3}\ \text{A}$$

Now, $$V_A + 4 - 1 \times \frac{14}{3} = V_B$$

$\therefore$ $$V_A - V_B = \frac{14}{3} - 4 = \frac{2}{3}\ \text{V}$$

$\therefore$ $$E = V_A - V_B = V = \frac{2}{3}\ \text{V}$$

$\therefore$ $$E = \frac{2}{3}\ \text{V}$$

Further, $V_A - V_B$ is positive, i.e. $V_A > V_B$ or A is connected to the positive terminal of the battery and B to the negative.

Internal resistance of the equivalent battery is found by the normal procedure. For example, here 2 Ω and 1 Ω resistances are in parallel. Hence, their combined resistance is

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2} \quad \text{or} \quad r = \frac{2}{3}\ \Omega$$

Solved Examples

TYPED PROBLEMS

Type 1. *Based on potential difference across the terminals of a battery*

Concept

Potential difference V across the terminals of a battery is given by

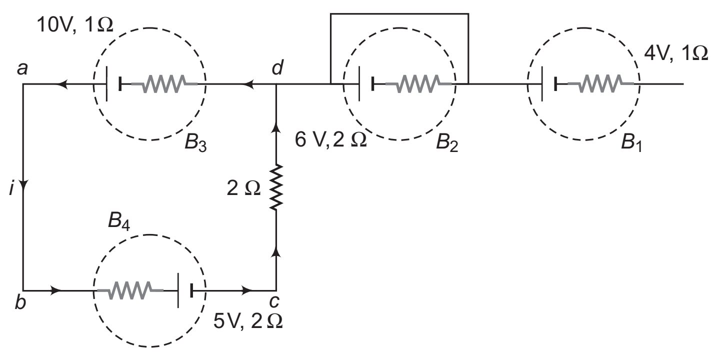
$V = E$ if $i = 0$

$V = 0$ if battery is short-circuited

$V = E - ir$ if normal current ($\xrightarrow{}\!|\,|\!\xleftarrow[i]{}$) flows through the battery and

$V = E + ir$ if current flows in the opposite direction ($\xrightarrow{}\!|\,|\!\xrightarrow[i]{}$)

Example 1 *Find the potential difference across each of the four batteries B_1, B_2, B_3 and B_4 as shown in the figure.*



Solution **Across B_1** This battery does not make any closed circuit.

$\therefore$
$$i = 0 \quad \text{or} \quad V = E = 4 \text{ volt} \qquad \textbf{Ans.}$$

Across B_2 This battery is short-circuited. Therefore,

$$V = 0 \qquad \textbf{Ans.}$$

Across B_3 and B_4 A current in anti-clockwise direction flows in the closed loop *abcda*. This current is

$$i = \frac{\text{net emf}}{\text{net resistance}} = \frac{10 - 5}{1 + 2 + 2}$$

$$= 1 \text{ A}$$

Now, current flows through B_3 in normal direction. Hence,

$$V = E - ir = 10 - 1 \times 1 = 9 \text{ volt} \qquad \textbf{Ans.}$$

From B_4, current flows in opposite direction. Hence,

$$V = E + ir = 5 + 1 \times 2 = 7 \text{ volt} \qquad \textbf{Ans.}$$

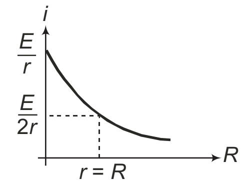
Example 2 *Draw (a) current versus load and (b) current versus potential difference (across its two terminals) graph for a cell.*

Solution (a) $i = \dfrac{E}{R + r}$

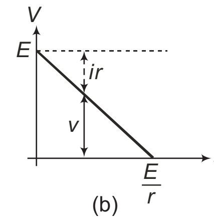
i *versus* R graph is shown in Fig. (a).

(b) $V = E - ir$

V *versus* i graph is shown in Fig. (b).



(a)



(b)

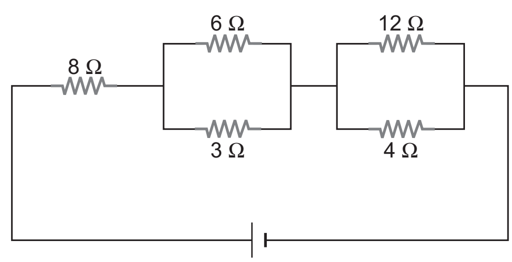
Type 2. *To find values of V, i and R across all resistors of a complex circuit if values across one resistance are known*

Concept

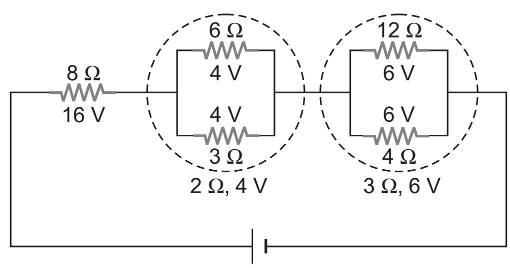
(i) In series, current remains same. But the potential difference distributes in the direct ratio of resistance.

(ii) In parallel, potential difference is same. But the current distributes in the inverse ratio of resistance.

Example 3 *In the circuit shown in figure potential difference across* $6\,\Omega$ *resistance is* 4 *volt. Find V and i values across each resistance. Also find emf E of the applied battery.*



Solution



$8\,\Omega, 2\,\Omega$ (Resultant of $6\,\Omega$ and $3\,\Omega$) and $3\,\Omega$ (resultant of $12\,\Omega$ and $4\,\Omega$) are in series. Therefore, potential drop across them should be in direct ratio of resistance. So, using this concept we can find the potential difference across other resistors. For example, potential across $2\,\Omega$ was 4 V. So, potential difference across $8\,\Omega$ (which is four times of $2\,\Omega$) should be 16 V. Similarly, potential difference across $3\,\Omega$ (which is 1.5 times of $2\,\Omega$) should be 1.5 times or 6 V.

Once V and R are known, we can find i across that resistance. For example,

$$i_{12\Omega} = \frac{6}{12} = \frac{1}{2}\text{ A} \qquad \left(i = \frac{V}{R}\right)$$

$$i_{8\Omega} = \frac{16}{8} = 2\text{ A etc}$$

Type 3. *To find equivalent value of temperature coefficient α if two or more than two resistors are connected in series or parallel*

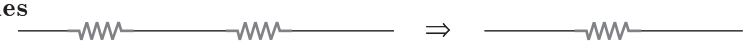
- This can be explained by the following example :

Example 4 *Two resistors with temperature coefficients of resistance α_1 and α_2 have resistances R_{01} and R_{02} at 0°C. Find the temperature coefficient of the compound resistor consisting of the two resistors connected*

(a) in series and

(b) in parallel.

Solution (a) **In Series**



At 0°C	R_{01}	R_{02}	$R_0 = R_{01} + R_{02}$
At t°C	$R_{01}(1+\alpha_1 t)$	$R_{02}(1+\alpha_2 t)$	$R_0(1+\alpha t)$

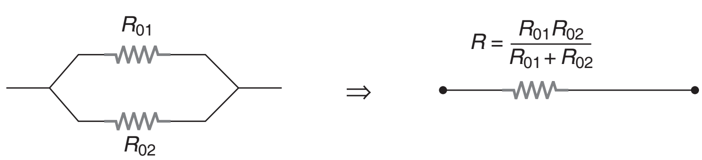
$$R_{01}(1+\alpha_1 t) + R_{02}(1+\alpha_2 t) = R_0(1+\alpha t)$$

or $$R_{01}(1+\alpha_1 t) + R_{02}(1+\alpha_2 t) = (R_{01}+R_{02})(1+\alpha t)$$

$\therefore$ $$R_{01} + R_{01}\alpha_1 t + R_{02} + R_{02}\alpha_2 t = R_{01} + R_{02} + (R_{01}+R_{02})\alpha t$$

or $$\alpha = \frac{R_{01}\alpha_1 + R_{02}\alpha_2}{R_{01}+R_{02}}$$ **Ans.**

(b) **In Parallel**



At t°C, $$\frac{1}{R_0(1+\alpha t)} = \frac{1}{R_{01}(1+\alpha_1 t)} + \frac{1}{R_{02}(1+\alpha_2 t)}$$

or $$\frac{R_{01}+R_{02}}{R_{01}R_{02}(1+\alpha t)} = \frac{1}{R_{01}(1+\alpha_1 t)} + \frac{1}{R_{02}(1+\alpha_2 t)}$$

Using the Binomial expansion, we have

$$\frac{1}{R_{02}}(1-\alpha t) + \frac{1}{R_{01}}(1-\alpha t) = \frac{1}{R_{01}}(1-\alpha_1 t) + \frac{1}{R_{02}}(1-\alpha_2 t)$$

i.e. $$\alpha t\left(\frac{1}{R_{01}} + \frac{1}{R_{02}}\right) = \frac{\alpha_1}{R_{01}}t + \frac{\alpha_2}{R_{02}}t$$

or $$\alpha = \frac{\alpha_1 R_{02} + \alpha_2 R_{01}}{R_{01}+R_{02}}$$ **Ans.**

Type 4. *Based on the verification of Ohm's law*

Concept

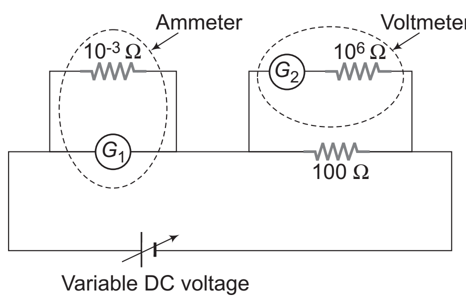
For verification of Ohm's law $\left(\frac{V}{i} = \text{constant} = R\right)$, we need an ohmic resistance, which follows this law. A voltmeter which will measure potential difference across this resistance, an ammeter which will measure current through this resistance and a variable battery which can provide a variable current in the circuit. Now, for different values of i, we have to measure different values of V and then prove that,

$$V \propto i \quad \text{or} \quad \frac{V}{i} = \text{constant}$$

and this constant is called resistance of that.

Example 5 *Draw the circuit for experimental verification of Ohm's law using a source of variable DC voltage, a main resistance of* $100\ \Omega$*, two galvanometers and two resistances of values* $10^6\ \Omega$ *and* $10^{-3}\ \Omega$ *respectively. Clearly show the positions of the voltmeter and the ammeter.* **(JEE 2004)**

Solution



Type 5. *Theory of bulbs or heater etc.*

Concept

(i) From the rated (written) values of power (P) and potential difference (V) we can determine resistance of filament of bulb.

$$P = \frac{V^2}{R}$$

$$\Rightarrow \qquad \boxed{R = \frac{V^2}{P}} \qquad \ldots\text{(i)}$$

For example, if rated values on a bulb are 220 V and 60 W, it means this bulb will consume 60 W of power (or 60 J in 1 s) if a potential difference of 220 V is applied across it. Resistance of this bulb will be

$$R = \frac{(220)^2}{60} = 806.67\ \Omega$$

(ii) Normally, rated value of V remains same in different bulbs. In India, it is 220 volt. Therefore, from Eq. (i)

$$R \propto \frac{1}{P} \quad \text{or} \quad R_{60\,\text{watt}} > R_{100\,\text{watt}}$$

(iii) Actual value of potential difference may be different from the rated value. Therefore, actual power consumption may also be different.

(iv) After finding resistance of the bulb using Eq. (i), we can apply normal Kirchhoff's laws for finding current passing through the bulb or actual power consumed by the bulb.

Example 6 *Prove that 60 W bulb glows more brightly than 100 W bulb if by mistake they are connected in series.*

Solution In series, we can use the formula

$$P = i^2R \quad \text{for the power consumption}$$

$$\Rightarrow \qquad P \propto R \qquad \text{(as } i \text{ is same in series)}$$

we have seen above that,

$$R_{60\,\text{W}} > R_{100\,\text{W}}$$

$$\therefore \qquad P_{60\,\text{W}} > P_{100\,\text{W}}$$

Hence Proved.

Note *In parallel 100 W bulb glows more brightly than 60 W bulb. Think why?*

Example 7 *The rated values of two bulbs are (P_1, V) and (P_2, V). Find actual power consumed by both of them if they are connected in*

(a) series

(b) parallel

and V potential difference is applied across both of them.

Solution (a) $\because \; R_1 = \dfrac{V^2}{P_1}$ and $R_2 = \dfrac{V^2}{P_2}$

In series,

$$P = \frac{V^2}{R_{\text{net}}} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{\dfrac{V^2}{P_1} + \dfrac{V^2}{P_2}} = \frac{P_1P_2}{P_1 + P_2}$$

or

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} \qquad \textbf{Ans.}$$

(b) In parallel,

$$P = \frac{V^2}{R_{\text{net}}} = V^2\left(\frac{1}{R_{\text{net}}}\right) = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or

$$P = V^2\left(\frac{P_1}{V^2} + \frac{P_2}{V^2}\right) \quad \text{or} \quad P = P_1 + P_2 \qquad \textbf{Ans.}$$

Example 8 *Heater-1, takes 3 minutes to boil a given amount of water. Heater-2 takes 6-minutes. Find the time taken if,*

(a) they are connected in series

(b) they are connected in parallel.

Potential difference V in all cases is same.

Solution Here, the heat required (say H) to boil water is same. Let P_1 and P_2 are the powers of the heaters. Then,

$$H = P_1 t_1 = P_2 t_2$$

$$\therefore \qquad P_1 = \frac{H}{t_1} = \frac{H}{3}$$

$$P_2 = \frac{H}{t_2} = \frac{H}{6}$$

(a) **In series,**

$$P = \frac{P_1 P_2}{P_1 + P_2} = \frac{(H/3)(H/6)}{(H/3) + (H/6)} \qquad \text{(Refer Example 7)}$$

$$= \frac{H}{9}$$

Now,

$$t = \frac{H}{P} = \frac{H}{(H/9)} = 9 \text{ min}$$ **Ans.**

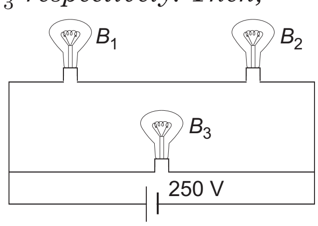
(b) **In parallel,**

$$P = P_1 + P_2$$

$$= \frac{H}{3} + \frac{H}{6} = \frac{H}{2}$$

$$\therefore \qquad t = \frac{H}{P} = \frac{H}{H/2} = 2 \text{ min}$$ **Ans.**

Example 9 *A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3 respectively. Then,* **(JEE 2002)**



(a) $W_1 > W_2 = W_3$ *(b)* $W_1 > W_2 > W_3$ *(c)* $W_1 < W_2 = W_3$ *(d)* $W_1 < W_2 < W_3$

Solution $P = \frac{V^2}{R}$ so, $R = \frac{V^2}{P}$

$$\therefore \qquad R_1 = \frac{V^2}{100} \quad \text{and} \quad R_2 = R_3 = \frac{V^2}{60}$$

Now,

$$W_1 = \frac{(250)^2}{(R_1 + R_2)^2} \cdot R_1$$

$$W_2 = \frac{(250)^2}{(R_1 + R_2)^2} \cdot R_2 \quad \text{and} \quad W_3 = \frac{(250)^2}{R_3}$$

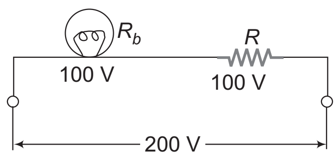
$$W_1 : W_2 : W_3 = 15 : 25 : 64 \quad \text{or} \quad W_1 < W_2 < W_3$$

$\therefore$ The correct option is (d).

Note *We have used $W = i^2 R$ for W_1 and W_2 and $W = \frac{V^2}{R}$ for W_3.*

Example 10 *An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb delivers 500 W is ......Ω.* **(JEE 1987)**

Solution Resistance of the given bulb

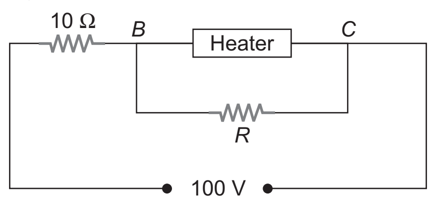


$$R_b = \frac{V^2}{P} = \frac{(100)^2}{500} = 20\ \Omega$$

To get 100 V out of 200 V across the bulb,

$$R = R_b = 20\ \Omega$$ **Ans.**

Example 11 *A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10 Ω and a resistance R, to a 100 V mains as shown in the figure. What will be the value of R so that the heater operates with a power of 62.5 W?* **(JEE 1978)**



Solution From $P = \dfrac{V^2}{R}$,

Resistance of heater, $$R = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\ \Omega$$

From $$P = i^2R$$

Current required across heater for power of 62.5 W,

$$i = \sqrt{\frac{P}{R}} = \sqrt{\frac{62.5}{10}} = 2.5\ \text{A}$$

Main current in the circuit, $$I = \frac{100}{10 + \dfrac{10R}{10+R}} = \frac{100\,(10+R)}{100+20R} = \frac{10\,(10+R)}{10+2R}$$

This current will distribute in inverse ratio of resistance between heater and R.

∴ $$i = \left(\frac{R}{10+R}\right) I$$

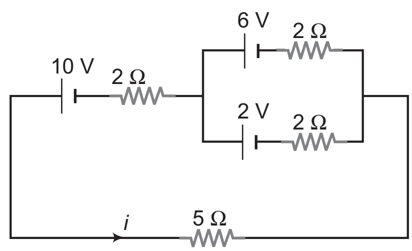
or $$2.5 = \left(\frac{R}{10+R}\right)\left[\frac{10\,(10+R)}{10+2R}\right] = \frac{10R}{10+2R}$$

Solving this equation, we get

$$R = 5\ \Omega$$ **Ans.**

Type 6. *To find current through a single external resistance in a complex circuit of batteries using the concepts of equivalent value of emf of battery*

Concept



In the above circuit, if we have to find only i then two parallel batteries may be converted into a single battery. Then, this battery is in series with the third battery of emf 10 volt. Now, we can find current i from the equation,

$$i = \frac{\text{Net emf}}{\text{Total resistance}}$$

Example 12 *Find the value of i in the circuit shown above.*

Solution Equivalent emf of the parallel combination is

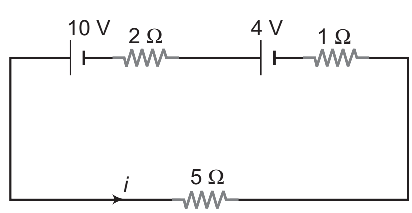
$$E = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2}$$

$$= \frac{6/2 + 2/2}{1/2 + 1/2} = 4 \text{ volt}$$

Equivalent internal resistance of the parallel combination is

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{(2)\,(2)}{2 + 2} = 1\ \Omega$$

Now, the equivalent simple circuit is as shown below



$$i = \frac{\text{Net emf}}{\text{Total resistance}}$$

$$= \frac{10 + 4}{2 + 1 + 5}$$

$$= 1.75 \text{ A}$$ **Ans.**

Note *By this concept, we can find only i. To find other currents (across 6V battery or 2V battery) we will have to apply Kirchhoff's laws.*

Miscellaneous Examples

Example 13 *Two sources of current of equal emf are connected in series and have different internal resistances r_1 and r_2 $(r_2 > r_1)$. Find the external resistance R at which the potential difference across the terminals of one of the sources becomes equal to zero.*

Solution $V = E - ir$

E and i for both the sources are equal. Therefore, potential difference (V) will be zero for a source having greater internal resistance, i.e. r_2.

$\therefore$ $$0 = E - ir_2$$

or $$E = ir_2 = \left(\frac{2E}{R + r_1 + r_2}\right) \cdot r_2$$

$\therefore$ $$2r_2 = R + r_1 + r_2$$

or $$R = r_2 - r_1$$ **Ans.**

Example 14

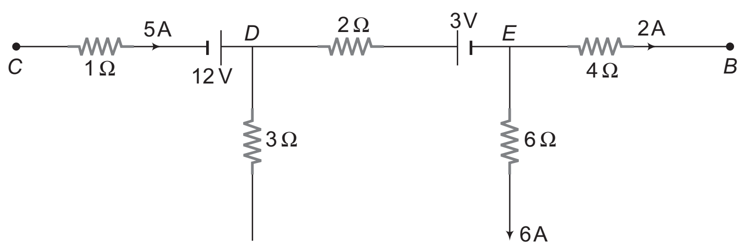
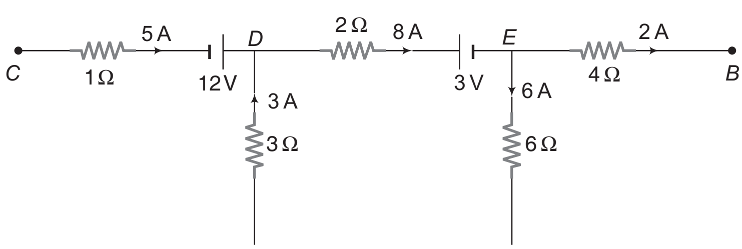


Figure shows the part of a circuit. Calculate the power dissipated in 3 Ω resistance. What is the potential difference $V_C - V_B$?

Solution Applying Kirchhoff's junction law at E current in wire DE is 8 A from D to E. Now further applying junction law at D, the current in 3 Ω resistance will be 3 A towards D.

Power dissipated in 3 Ω resistance $= i^2R = (3)^2 (3) = 27$ W **Ans.**



$\boldsymbol{V_C - V_B}$ $$V_C - 5 \times 1 + 12 - 8 \times 2 - 3 - 4 \times 2 = V_B$$

$\therefore$ $$V_C - V_B = 5 - 12 + 16 + 3 + 8$$

or $$V_C - V_B = 20 \text{ V}$$ **Ans.**

Example 15 *The emf of a storage battery is 90 V before charging and 100 V after charging. When charging began the current was 10 A. What is the current at the end of charging if the internal resistance of the storage battery during the whole process of charging may be taken as constant and equal to 2 Ω?*

Solution The voltage supplied by the charging plant is here constant which is equal to,

$$V = E_i + i_i \cdot r = (90) + (10)(2)$$
$$= 110 \text{ V}$$

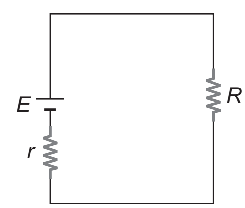
Let i_f be the current at the end of charging.

Then, $$V = E_f + i_f r$$

or $$i_f = \frac{V - E_f}{r}$$
$$= \frac{110 - 100}{2}$$
$$= 5 \text{ A}$$ **Ans.**

Example 16 *A battery has an open circuit potential difference of 6 V between its terminals. When a load resistance of 60 Ω is connected across the battery, the total power supplied by the battery is 0.4 W. What should be the load resistance R, so that maximum power will be dissipated in R. Calculate this power. What is the total power supplied by the battery when such a load is connected?*

Solution When the circuit is open, $V = E$



$\therefore$ $$E = 6 \text{ V}$$

Let r be the internal resistance of the battery.

Power supplied by the battery in this case is

$$P = \frac{E^2}{R + r}$$

Substituting the values, we have $$0.4 = \frac{(6)^2}{60 + r}$$

Solving this, we get $$r = 30\ \Omega$$

Maximum power is dissipated in the circuit when net external resistance is equal to net internal resistance or

$$R = r$$

$\therefore$ $$R = 30\ \Omega$$ **Ans.**

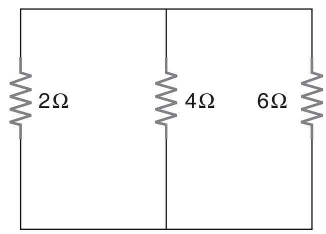
Further, total power supplied by the battery under this condition is

$$P_{\text{Total}} = \frac{E^2}{R + r} = \frac{(6)^2}{30 + 30}$$
$$= 0.6 \text{ W}$$ **Ans.**

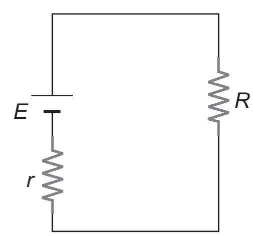
Of this 0.6 W half of the power is dissipated in R and half in r. Therefore, maximum power dissipated in R would be

$$\frac{0.6}{2} = 0.3 \text{ W}$$ **Ans.**

Example 17 *In which branch of the circuit shown in figure a 11 V battery be inserted so that it dissipates minimum power. What will be the current through the 2 Ω resistance for this position of the battery?*

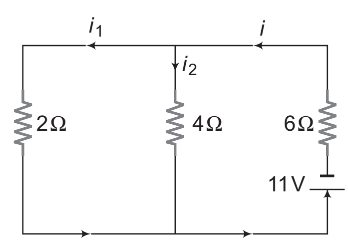


Solution Suppose, we insert the battery with 2 Ω resistance. Then, we can take 2 Ω as the internal resistance (r) of the battery and combined resistance of the other two as the external resistance (R). The circuit in that case is shown in figure,



Now power, $P = \dfrac{E^2}{R + r}$

This power will be minimum where $R + r$ is maximum and we can see that $(R + r)$ will be maximum when the battery is inserted with 6 Ω resistance as shown in figure.



Net resistance in this case is

$$6 + \frac{2 \times 4}{2 + 4} = \frac{22}{3}\ \Omega$$

$$\therefore \qquad i = \frac{11}{22/3} = 1.5\ \text{A}$$

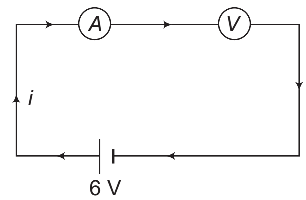
This current will be distributed in 2 Ω and 4 Ω in the inverse ratio of their resistances.

$$\therefore \qquad \frac{i_1}{i_2} = \frac{4}{2} = 2$$

$$\therefore \qquad i_1 = \left(\frac{2}{2 + 1}\right)(1.5) = 1.0\ \text{A}$$ **Ans.**

Example 18 *An ammeter and a voltmeter are connected in series to a battery of emf $E = 6.0\,V$. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decreases two times, whereas the reading of the ammeter increases the same number of times. Find the voltmeter reading after the connection of the resistance.*

Solution Let $R =$ resistance of ammeter



Potential difference across voltmeter = 6 – potential difference across ammeter

In first case, $$V = 6 - iR \qquad \text{...(i)}$$

In second case, $$\frac{V}{2} = 6 - (2i)\,R \qquad \text{...(ii)}$$

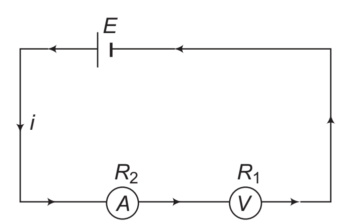
Solving these two equations, we get

$$V = 4 \text{ volt}$$

$\therefore$ $$V/2 = 2 \text{ volt}$$ **Ans.**

Example 19 *A voltmeter of resistance R_1 and an ammeter of resistance R_2 are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third. Find R_1 and R_2 in terms of R.*

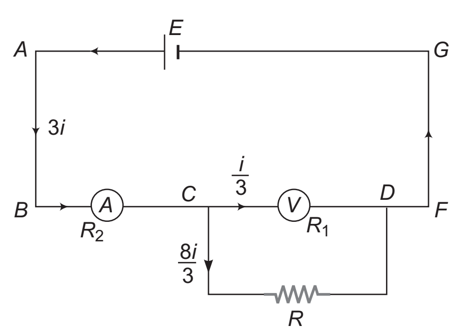
Solution Let E be the emf of the battery.



In the second case main current increases three times while current through voltmeter will reduce to $i/3$. Hence, the remaining $3i - i/3 = 8i/3$ passes through R as shown in figure.

$$V_C - V_D = \left(\frac{i}{3}\right) R_1 = \left(\frac{8i}{3}\right) R$$

or $$R_1 = 8R$$ **Ans.**



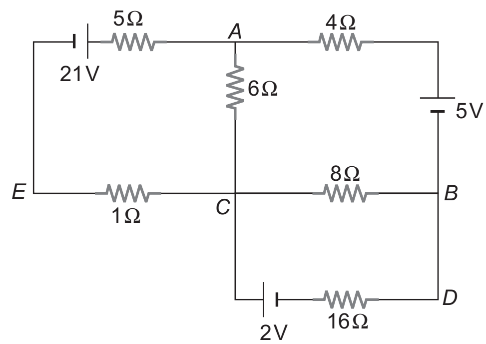
In the second case, main current becomes three times. Therefore, total resistance becomes $\frac{1}{3}$ times or

$$R_2 + \frac{RR_1}{R + R_1} = \frac{1}{3}(R_1 + R_2)$$

Substituting $R_1 = 8R$, we get

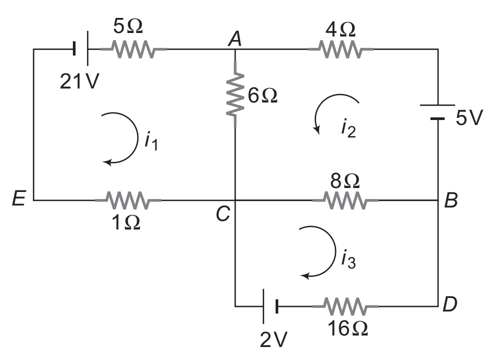
$$R_2 = \frac{8R}{3}$$ **Ans.**

Example 20 *Find the current in each branches of the circuit.*



Solution It is possible to use Kirchhoff's laws in a slightly different form, which may simplify the solution of certain problems. This method of applying Kirchhoff's laws is called the **loop current method.**

In this method, we assign a current to every closed loop in a network.



Suppose currents i_1, i_2 and i_3 are flowing in the three loops. The clockwise or anti-clockwise sense given to these currents is arbitrary. Applying Kirchhoff's second law to the three loops, we get

$$21 - 5i_1 - 6(i_1 + i_2) - i_1 = 0 \qquad \text{...(i)}$$

$$5 - 4i_2 - 6(i_1 + i_2) - 8(i_2 + i_3) = 0 \qquad \text{...(ii)}$$

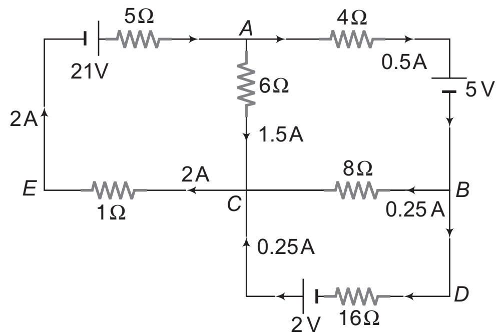
and

$$2 - 8(i_2 + i_3) - 16\, i_3 = 0 \qquad \text{...(iii)}$$

Solving these three equations, we get

$$i_1 = 2\text{ A}, \qquad i_2 = -\frac{1}{2}\text{ A} \qquad \text{and} \qquad i_3 = \frac{1}{4}\text{ A}$$

Therefore, current in different branches are as shown in figure given below.



Note *In wire AC, current is $i_1 + i_2$ and in CB it is $i_2 + i_3$.*

Example 21 *What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil*

(a) decreases down to zero uniformly during a time interval t_0?

(b) decreases down to zero halving its value every t_0 seconds?

HOW TO PROCEED *Heat generated in a resistance is given by*

$$H = i^2Rt$$

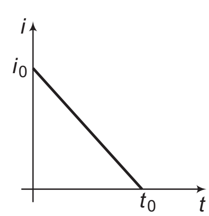
We can directly use this formula provided i is constant. Here, i is varying. So, first we will calculate i at any time t, then find a small heat dH in a short interval of time dt. Then by integrating it with proper limits we can obtain the total heat produced.

Solution (a) The corresponding i-t graph will be a straight line with i decreasing from a peak value (say i_0) to zero in time t_0.

i-t equation will be as

$$i = i_0 - \left(\frac{i_0}{t_0}\right) t \qquad (y = -mx + c) \qquad \ldots\text{(i)}$$

Here, i_0 is unknown, which can be obtained by using the fact that area under i-t graph gives the flow of charge. Hence,



$$q = \frac{1}{2}(t_0)(i_0)$$

$$\therefore \qquad i_0 = \frac{2q}{t_0}$$

Substituting in Eq. (i), we get

$$i = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right)$$

or $$i = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Now, at time t, heat produced in a short interval dt is

$$dH = i^2R\,dt$$

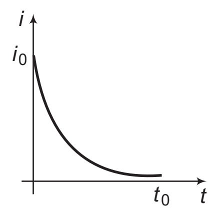
$$= \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 Rdt$$

$\therefore$ $$\text{Total heat produced} = \int_0^{t_0} dH$$

or $$H = \int_0^{t_0} \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\,dt$$

$$= \frac{4}{3}\frac{q^2R}{t_0}$$ **Ans.**

(b) Here, current decreases from some peak value (say i_0) to zero exponentially with half life t_0.



i-t equation in this case will be

$$i = i_0\,e^{-\lambda t}$$

Here, $$\lambda = \frac{\ln(2)}{t_0}$$

Now, $$q = \int_0^\infty i\,dt = \int_0^\infty i_0\,e^{-\lambda t}\,dt = \left(\frac{i_0}{\lambda}\right)$$

$\therefore$ $$i_0 = \lambda\,q$$

$\therefore$ $$i = (\lambda q)\,e^{-\lambda t}$$

$\therefore$ $$dH = i^2R\,dt = \lambda^2q^2e^{-2\lambda t}R\,dt$$

or $$H = \int_0^\infty dH = \lambda^2q^2R\int_0^\infty e^{-2\lambda t}\,dt = \frac{q^2\lambda R}{2}$$

Substituting $\lambda = \dfrac{\ln(2)}{t_0}$, we have $$H = \frac{q^2R\ln(2)}{2t_0}.$$ **Ans.**

Note *In radioactivity, half-life is given by*

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

$\therefore$ $$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both* ***Assertion*** *and* ***Reason*** *are true and the* ***Reason*** *is correct explanation of the* ***Assertion.***

(*b*) *If both* ***Assertion*** *and* ***Reason*** *are true but* ***Reason*** *is not the correct explanation of* ***Assertion.***

(*c*) *If* ***Assertion*** *is true, but the* ***Reason*** *is false.*

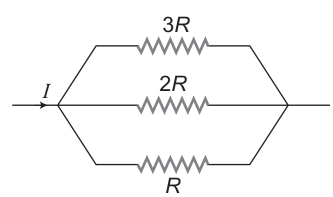
(*d*) *If* ***Assertion*** *is false but the* ***Reason*** *is true.*

1. Assertion : If potential difference across two points is zero, current between these two points should be zero.

Reason : Current passing from a resistor

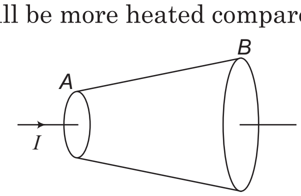
$$I = \frac{V}{R}$$

2. Assertion : In the part of the circuit shown in figure, maximum power is produced across R.



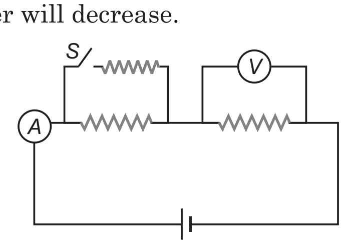
Reason : Power $P = \frac{V^2}{R}$

3. Assertion : Current I is flowing through a cylindrical wire of non-uniform cross-section as shown. Section of wire near A will be more heated compared to the section near B.



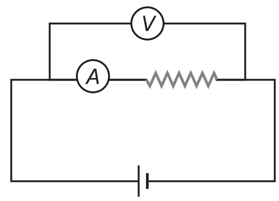
Reason : Current density near A is more.

4. Assertion : In the circuit shown in figure after closing the switch S reading of ammeter will increase while that of voltmeter will decrease.



Reason : Net resistance decreases as parallel combination of resistors is increase`d.

5. Assertion : In the circuit shown in figure ammeter and voltmeter are non-ideal. When positions of ammeter and voltmeter are changed, reading of ammeter will increase while that of voltmeter will decrease.

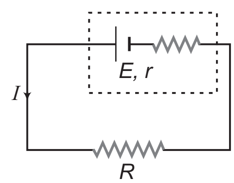


Reason : Resistance of an ideal ammeter is zero while that of an ideal voltmeter is infinite.

6. Assertion : In the part of a circuit shown in figure, given that $V_b > V_a$. The current should flow from b to a.

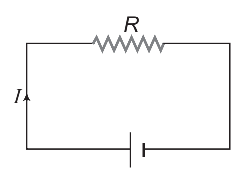
Reason : Direction of current inside a battery is always from negative terminal to positive terminal.

7. Assertion : In the circuit shown in figure R is variable. Value of current I is maximum when $R = r$.



Reason : At $R = r$, maximum power is produced across R.

8. Assertion : If variation in resistance due to temperature is taken into consideration, then current in the circuit I and power produced across the resistance P both will decrease with time.



Reason : $V = IR$ is Ohm's law.

9. Assertion : When a potential difference is applied across a conductor, free electrons start travelling with a constant speed called drift speed.

Reason : Due to potential difference an electric field is produced inside the conductor, in which electrons experience a force.

10. Assertion : When temperature of a conductor is increased, its resistance increases.

Reason : Free electrons collide more frequently.

11. Assertion : Two non-ideal batteries are connected in parallel with same polarities on same side. The equivalent emf is smaller than either of the two emfs.

Reason : Two non-ideal batteries are connected in parallel, the equivalent internal resistance is smaller than either of the two internal resistances.

Objective Questions

1. An ammeter should have very low resistance
(a) to show large deflection
(b) to generate less heat
(c) to prevent the galvanometer
(d) so that it may not change the value of the actual current in the circuit

2. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities which remain constant along the length of the conductor is/are
(a) current, electric field and drift speed
(b) drift speed only
(c) current and drift speed
(d) current only

3. If M = mass, L = length, T = time and I = electric current, then the dimensional formula of resistance R will be given by
(a) $[R] = [ML^2T^{-3}I^{-2}]$
(b) $[R] = [ML^2T^{-3}I^2]$
(c) $[R] = [ML^2T^3I^{-2}]$
(d) $[R] = [ML^2T^3I^2]$

4. The unit of electrical conductivity is
(a) ohm-m^{-2}
(b) ohm $\times$ m
(c) ohm^{-1}-m^{-1}
(d) None of these

5. Through an electrolyte an electrical current is due to drift of
(a) free electrons
(b) positive and negative ions
(c) free electrons and holes
(d) protons

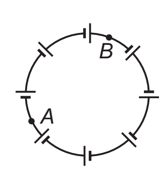
6. The current in a circuit with an external resistance of 3.75 Ω is 0.5 A. When a resistance of 1 Ω is introduced into the circuit, the current becomes 0.4 A. The emf of the power source is
(a) 1 V
(b) 2 V
(c) 3 V
(d) 4 V

7. The deflection in a galvanometer falls from 50 divisions to 20 divisions, when a 12 Ω shunt is applied. The galvanometer resistance is
(a) 18 Ω
(b) 24 Ω
(c) 30 Ω
(d) 36 Ω

8. If 2% of the main current is to be passed through the galvanometer of resistance G, the resistance of shunt required is
(a) $\frac{G}{49}$
(b) $\frac{G}{50}$
(c) $49\,G$
(d) $50\,G$

9. If the length of the filament of a heater is reduced by 10%, the power of the heater will
(a) increase by about 9%
(b) increase by about 11%
(c) increase by about 19%
(d) decrease by about 10%

10. N identical current sources each of emf E and internal resistance r are connected to form a closed loop as shown in figure. The potential difference between points A and B which divides the circuit into n and $(N - n)$ units is

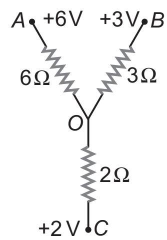


(a) NE
(b) $(N - n)E$
(c) nE
(d) zero

11. A 2.0 V potentiometer is used to determine the internal resistance of a 1.5 V cell. The balance point of the cell in the open circuit is 75 cm. When a resistor of 10 Ω is connected across the cell, the balance point shifts to 60 cm. The internal resistance of the cell is

(a) 1.5 Ω (b) 2.5 Ω
(c) 3.5 Ω (d) 4.5 Ω

12. Three resistances are joined together to form a letter Y, as shown in figure. If the potentials of the terminals A, B and C are 6 V, 3 V and 2 V respectively, then the potential of the point O will be



(a) 4 V (b) 3 V
(c) 2.5 V (d) 0 V

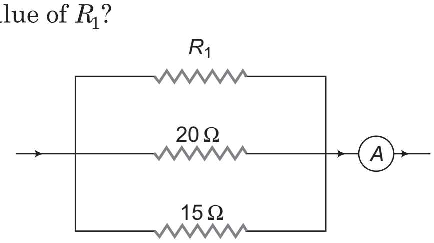
13. The drift velocity of free electrons in a conductor is v, when a current i is flowing in it. If both the radius and current are doubled, then the drift velocity will be

(a) v (b) $v/2$
(c) $v/4$ (d) $v/8$

14. A galvanometer is to be converted into an ammeter or voltmeter. In which of the following cases the resistance of the device is largest?

(a) an ammeter of range 10 A
(b) a voltmeter of range 5 V
(c) an ammeter of range 5 A
(d) a voltmeter of range 10 V

15. In the given circuit the current flowing through the resistance 20 Ω is 0.3 A, while the ammeter reads 0.8 A. What is the value of R_1?



(a) 30 Ω (b) 40 Ω
(c) 50 Ω (d) 60 Ω

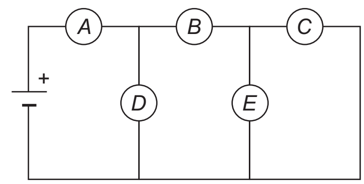
16. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V respectively. If a resistance is now joined in parallel with the voltmeter, then

(a) both A and V will increase
(b) both A and V will decrease
(c) A will decrease, V will increase
(d) A will increase, V will decrease

17. A resistor R has power of dissipation P with cell voltage E. The resistor is cut in n equal parts and all parts are connected in parallel with same cell. The new power dissipation is

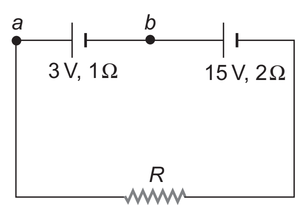
(a) nP (b) nP^2
(c) n^2P (d) n/P

18. In the circuit diagram shown in figure, a fuse bulb can cause all other bulbs to go out. Identify the bulb



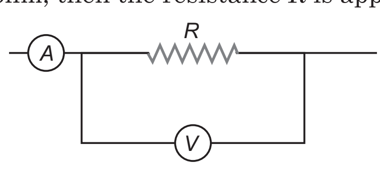
(a) B (b) C
(c) A (d) D or E

19. Two batteries one of the emf 3 V, internal resistance 1 Ω and the other of emf 15V, internal resistance 2 Ω are connected in series with a resistance R as shown. If the potential difference between points a and b is zero, the resistance R in Ω is



(a) 5 (b) 7
(c) 3 (d) 1

20. A part of a circuit is shown in figure. Here reading of ammeter is 5 A and voltmeter is 100 V. If voltmeter resistance is 2500 ohm, then the resistance R is approximately



(a) 20 Ω (b) 10 Ω
(c) 100 Ω (d) 200 Ω

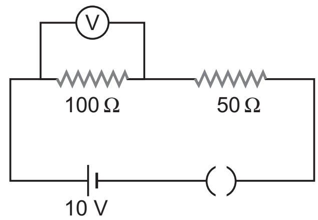
21. A copper wire of resistance R is cut into ten parts of equal length. Two pieces each are joined in series and then five such combinations are joined in parallel. The new combination will have a resistance

(a) R (b) $\frac{R}{4}$ (c) $\frac{R}{5}$ (d) $\frac{R}{25}$

22. Two resistances are connected in two gaps of a meter bridge. The balance point is 20 cm from the zero end. A resistance of 15 Ω is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in Ω is

(a) 3 (b) 6
(c) 9 (d) 12

23. In the given circuit, the voltmeter records 5 volt. The resistance of the voltmeter in Ω is

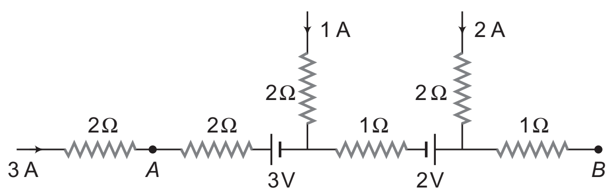


(a) 200
(b) 100
(c) 10
(d) 50

24. The wire of potentiometer has resistance 4 Ω and length 1 m. It is connected to a cell of emf 2 volt and internal resistance 1 Ω. If a cell of emf 1.2 volt is balanced by it, the balancing length will be

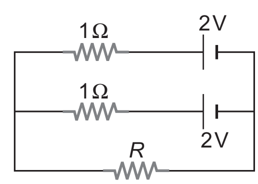
(a) 90 cm
(b) 60 cm
(c) 50 cm
(d) 75 cm

25. The potential difference between points *A* and *B*, in a section of a circuit shown, is



(a) 5 volt
(b) 1 volt
(c) 10 volt
(d) 17 volt

26. Two identical batteries, each of emf 2 V and internal resistance $r = 1\,\Omega$ are connected as shown. The maximum power that can be developed across *R* using these batteries is

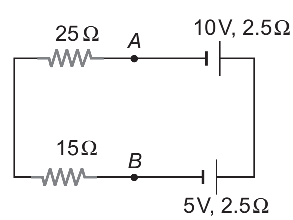


(a) 3.2 W
(b) 8.2 W
(c) 2 W
(d) 4 W

27. For a cell, the terminal potential difference is 2.2 V, when circuit is open and reduces to 1.8 V. When cell is connected to a resistance $R = 5\,\Omega$, the internal resistance of cell (r) is

(a) $\frac{10}{9}\,\Omega$
(b) $\frac{9}{10}\,\Omega$
(c) $\frac{11}{9}\,\Omega$
(d) $\frac{5}{9}\,\Omega$

28. The potential difference between points *A* and *B* in the circuit shown in figure, will be

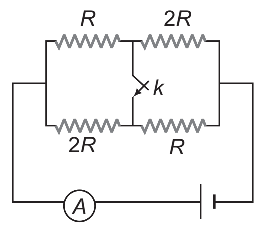


(a) 1 V
(b) 2 V
(c) – 3 V
(d) None of the above

29. Potentiometer wire of length 1 m is connected in series with 490 Ω resistance and 2 Vbattery. If 0.2 mV/cm is the potential gradient, then resistance of the potentiometer wire is approximately

(a) 4.9 Ω (b) 7.9 Ω (c) 5.9 Ω (d) 6.9 Ω

30. Find the ratio of currents as measured by ammeter in two cases when the key is open and when the key is closed



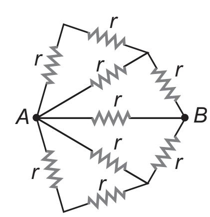
(a) 9/8 (b) 10/11 (c) 8/9 (d) None of these

31. A galvanometer has a resistance of 3663 Ω. A shunt S is connected across it such that (1/34) of the total current passes through the galvanometer. Then, the value of the shunt is

(a) 222 Ω (b) 111 Ω (c) 11 Ω (d) 22 Ω

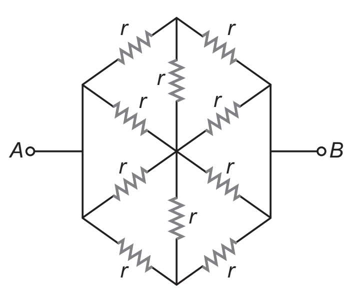
Note *Attempt the following questions after reading the chapter of capacitors.*

32. The network shown in figure is an arrangement of nine identical resistors. The resistance of the network between points A and B is 1.5 Ω. The resistance r is



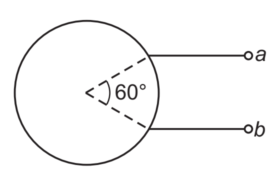
(a) 1.1 Ω
(b) 3.3 Ω
(c) 1.8 Ω
(d) 1.6 Ω

33. The equivalent resistance of the hexagonal network as shown in figure between points A and B is



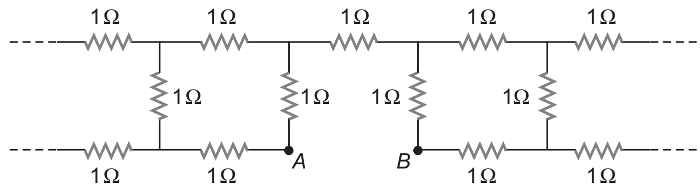
(a) r (b) $0.5\,r$ (c) $2\,r$ (d) $3\,r$

34. A uniform wire of resistance 18 Ω is bent in the form of a circle. The effective resistance across the points a and b is



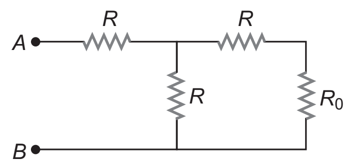
(a) 3 Ω
(b) 2 Ω
(c) 2.5 Ω
(d) 6 Ω

35. Each resistor shown in figure is an infinite network of resistance 1 Ω. The effective resistance between points A and B is



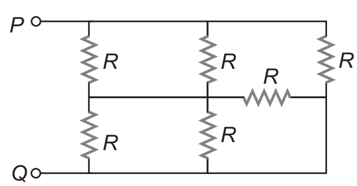
(a) less than 1 Ω (b) 1 Ω
(c) more than 1 Ω but less than 3 Ω (d) 3 Ω

36. In the circuit shown in figure, the total resistance between points A and B is R_0. The value of resistance R is



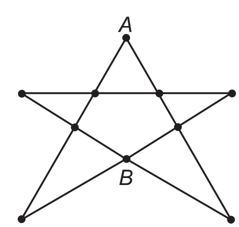
(a) R_0 (b) $\sqrt{3}\, R_0$ (c) $\frac{R_0}{2}$ (d) $\frac{R_0}{\sqrt{3}}$

37. In the circuit shown in the figure, $R = 55\ \Omega$, the equivalent resistance between the points P and Q is



(a) 30 Ω (b) 35 Ω
(c) 55 Ω (d) 25 Ω

38. The resistance of all the wires between any two adjacent dots is R. Then, equivalent resistance between A and B as shown in the figure is

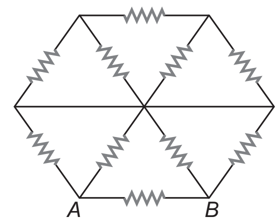


(a) (7/3) R
(b) (7/6) R
(c) (14/8) R
(d) None of the above

39. A uniform wire of resistance 4 Ω is bent into a circle of radius r. A specimen of the same wire is connected along the diameter of the circle. What is the equivalent resistance across the ends of this wire?

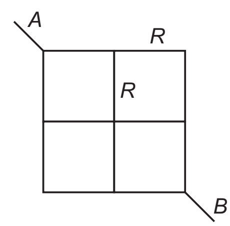
(a) $\frac{4}{(4+\pi)}\ \Omega$ (b) $\frac{3}{(3+\pi)}\ \Omega$
(c) $\frac{2}{(2+\pi)}\ \Omega$ (d) $\frac{1}{(1+\pi)}\ \Omega$

40. In the network shown in figure, each resistance is R. The equivalent resistance between points A and B is



(a) $\frac{20}{11}R$
(b) $\frac{19}{20}R$
(c) $\frac{8}{15}R$
(d) $\frac{R}{2}$

41. The equivalent resistance between the points A and B is (R is the resistance of each side of smaller square)



(a) R
(b) $\frac{3R}{2}$
(c) $2R$
(d) $\frac{R}{2}$

Subjective Questions

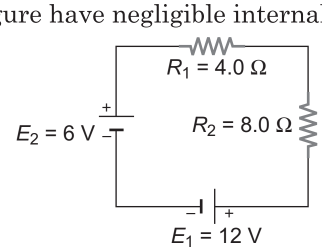
1. When a steady current passes through a cylindrical conductor, is there an electric field inside the conductor?

2. Electrons in a conductor have no motion in the absence of a potential difference across it. Is this statement true or false?

3. In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius 5×10^{-11} m, at a speed 2.2×10^{6} m/s. What is the current associated with electron motion?

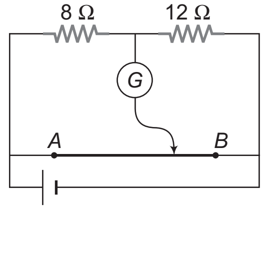
4. A 120 V house circuit has the following light bulbs switched on : 40 W, 60 W and 75 W. Find the equivalent resistance of these bulbs.

5. Assume that the batteries in figure have negligible internal resistance. Find

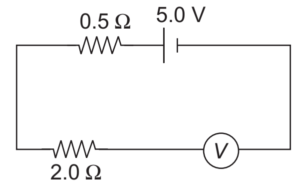


(a) the current in the circuit,
(b) the power dissipated in each resistor and
(c) the power of each battery, stating whether energy is supplied by or absorbed by it.

6. The potentiometer wire AB shown in figure is 40 cm long. Where the free end of the galvanometer should be connected on AB so that the galvanometer may show zero deflection?

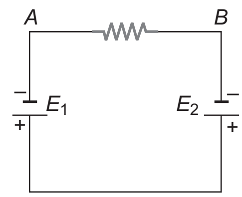


7. An ideal voltmeter V is connected to a $2.0\ \Omega$ resistor and a battery with emf 5.0 V and internal resistance $0.5\ \Omega$ as shown in figure :



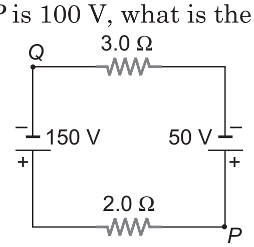
(a) What is the current in the $2.0\ \Omega$ resistor?
(b) What is the terminal voltage of the battery?
(c) What is the reading of the voltmeter?

8. In figure, $E_1 = 12$ V and $E_2 = 8$ V.



(a) What is the direction of the current in the resistor?
(b) Which battery is doing positive work?
(c) Which point, A or B, is at the higher potential?

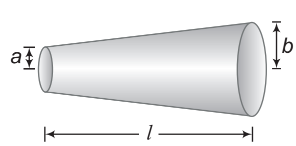
9. In figure, if the potential at point P is 100 V, what is the potential at point Q?



10. Copper has one conduction electron per atom. Its density is $8.89\ \text{g/cm}^3$ and its atomic mass is 63.54 g/mol. If a copper wire of diameter 1.0 mm carries a current of 2.0 A, what is the drift speed of the electrons in the wire?

11. An aluminium wire carrying a current has diameter 0.84 mm. The electric field in the wire is 0.49 V/m. What is
(a) the current carried by the wire?
(b) the potential difference between two points in the wire 12.0 m apart?
(c) the resistance of a 12.0 m length of this wire?
Specific resistance of aluminium is $2.75 \times 10^{-8}\ \Omega\text{-m}$.

12. A conductor of length l has a non-uniform cross-section. The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Find the resistance of the conductor across its ends.



13. If a battery of emf E and internal resistance r is connected across a load of resistance R. Show that the rate at which energy is dissipated in R is maximum when $R = r$ and this maximum power is $P = E^2/4r$.

14. Two identical batteries each of emf $E = 2$ volt and internal resistance $r = 1$ ohm are available to produce heat in an external resistance by passing a current through it. What is the maximum power that can be developed across an external resistance R using these batteries?

15. Two coils connected in series have resistance of $600\,\Omega$ and $300\,\Omega$ at $20°$ C and temperature coefficient of 0.001 and $0.004\,(°\text{C})^{-1}$ respectively. Find resistance of the combination at a temperature of $50°$ C. What is the effective temperature coefficient of combination?

16. An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is $0.017\,\mu\Omega$-m and that of the aluminium is $0.028\,\mu\,\Omega$-m.

17. The potential difference between two points in a wire 75.0 cm apart is 0.938 V, when the current density is 4.40×10^7 A/m^2. What is

(a) the magnitude of **E** in the wire?

(b) the resistivity of the material of which the wire is made?

18. A rectangular block of metal of resistivity ρ has dimensions $d \times 2d \times 3d$. A potential difference V is applied between two opposite faces of the block.

(a) To which two faces of the block should the potential difference V be applied to give the maximum current density? What is the maximum current density?

(b) To which two faces of the block should the potential difference V be applied to give the maximum current? What is this maximum current?

19. An electrical conductor designed to carry large currents has a circular cross-section 2.50 mm in diameter and is 14.0 m long. The resistance between its ends is $0.104\,\Omega$.

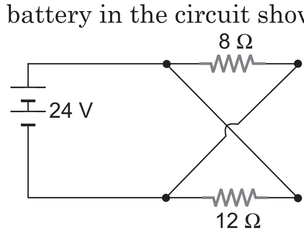
(a) What is the resistivity of the material?

(b) If the electric field magnitude in the conductor is 1.28 V/m, what is the total current?

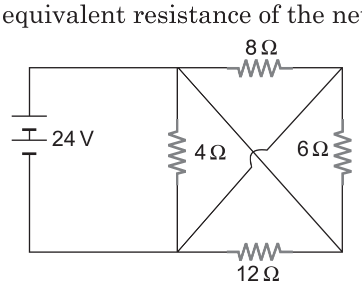
(c) If the material has 8.5×10^{28} free electrons per cubic metre, find the average drift speed under the conditions of part (b).

20. It is desired to make a $20.0\,\Omega$ coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20.00\,\Omega$ for all temperatures near $20°$ C. How large are R_1 and R_2? Given, $\alpha_C = -0.5 \times 10^{-3}\ \text{K}^{-1}$ and $\alpha_{Fe} = 5.0 \times 10^{-3}\ \text{K}^{-1}$.

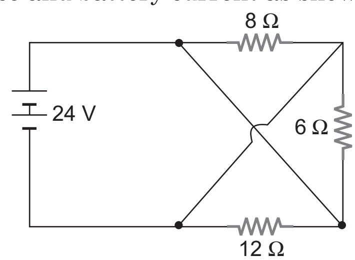
21. Find the current supplied by the battery in the circuit shown in figure.



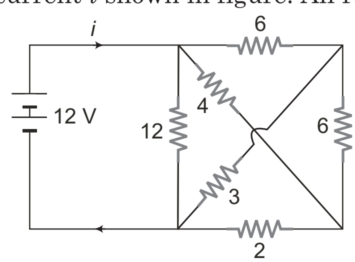
22. Calculate battery current and equivalent resistance of the network shown in figure.



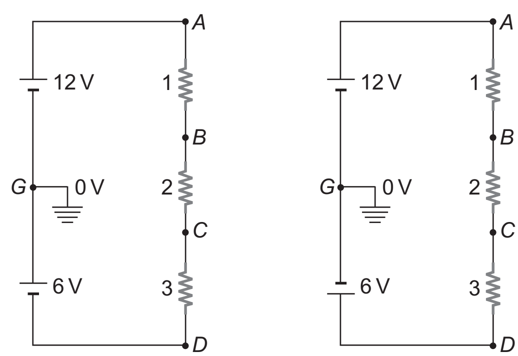
23. Compute total circuit resistance and battery current as shown in figure.



24. Compute the value of battery current i shown in figure. All resistances are in ohm.

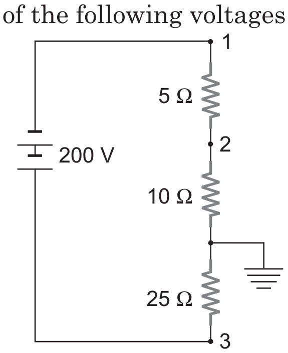


25. Calculate the potentials of points A, B, C and D as shown in Fig. (a). What would be the new potential values if connections of 6 V battery are reversed as shown in Fig. (b)? All resistances are in ohm.



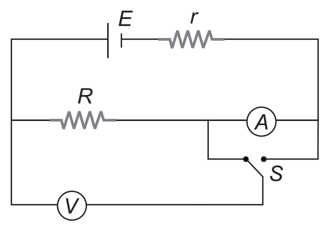
(a) (b)

26. Give the magnitude and polarity of the following voltages in the circuit of figure :



(i) V_1 (ii) V_2 (iii) V_3 (iv) V_{3-2}
(v) V_{1-2} (vi) V_{1-3}

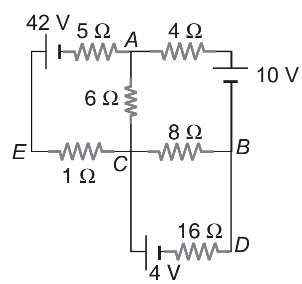
27. The emf E and the internal resistance r of the battery shown in figure are 4.3 V and 1.0 Ω respectively. The external resistance R is 50 Ω. The resistances of the ammeter and voltmeter are 2.0 Ω and 200 Ω, respectively.



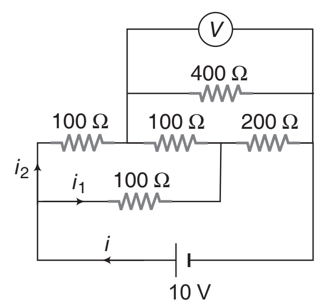
(a) Find the readings of the two meters.

(b) The switch is thrown to the other side. What will be the readings of the two meters now?

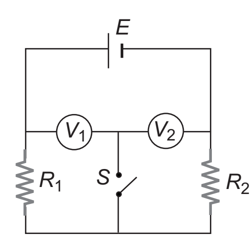
28. Find the current in each branch of the circuit shown in figure.



29. An electrical circuit is shown in figure. Calculate the potential difference across the resistor of 400 Ω as will be measured by the voltmeter V of resistance 400 Ω either by applying Kirchhoff's rules or otherwise. **(JEE 1996)**



30. In the circuit shown in figure V_1 and V_2 are two voltmeters of resistances 3000 Ω and 2000 Ω, respectively. In addition $R_1 = 2000\ \Omega$, $R_2 = 3000\ \Omega$ and $E = 200$ V, then



(a) Find the reading of voltmeters V_1 and V_2 when

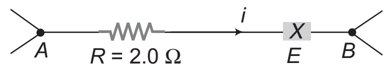
(i) switch S is open

(ii) switch S is closed

(b) Current through S, when it is closed

(Disregard the resistance of battery)

31. In figure, circuit section AB absorbs energy at the rate of 5.0 W when a current $i = 1.0$ A passes through it in the indicated direction.



(a) What is the potential difference between points A and B?

(b) Emf device X does not have internal resistance. What is its emf?

(c) What is its polarity (the orientation of its positive and negative terminals)?

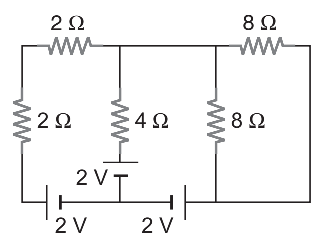
32. The potential difference across the terminals of a battery is 8.4 V when there is a current of 1.50 A in the battery from the negative to the positive terminal. When the current is 3.50 A in the reverse direction, the potential difference becomes 9.4 V.

(a) What is the internal resistance of the battery?

(b) What is the emf of the battery?

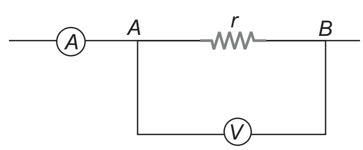
33. A battery of emf 2.0 V and internal resistance 0.10 Ω is being charged with a current of 5.0 A. Find the potential difference between the terminals of the battery?

34. Find the currents in different resistors shown in figure.

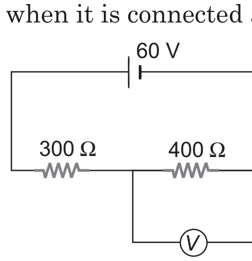


35. A resistance box, a battery and a galvanometer of resistance G ohm are connected in series. If the galvanometer is shunted by resistance of S ohm, find the change in resistance in the box required to maintain the current from the battery unchanged.

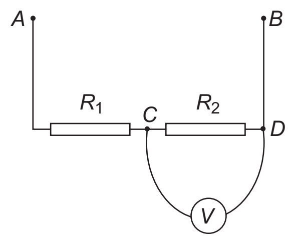
36. Determine the resistance r if an ammeter shows a current of $I = 5$ A and a voltmeter 100 V. The internal resistance of the voltmeter is $R = 2{,}500$ Ω.



37. In the circuit, a voltmeter reads 30 V when it is connected across 400 Ω resistance. Calculate what the same voltmeter will read when it is connected across the 300 Ω resistance?



38. Resistances R_1 and R_2, each $60\ \Omega$, are connected in series. The potential difference between points A and B is 120 V. Find the reading of voltmeter connected between points C and D if its resistance $r = 120\ \Omega$.



39. A moving coil galvanometer of resistance $20\ \Omega$ gives a full scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale. But the shunt of $0.005\ \Omega$ only is available. What resistance should be connected in series with the galvanometer coil?

40. A cell of emf 3.4 V and internal resistance $3\ \Omega$ is connected to an ammeter having resistance $2\ \Omega$ and to an external resistance of $100\ \Omega$. When a voltmeter is connected across the $100\ \Omega$ resistance, the ammeter reading is 0.04 A. Find the voltage reading by the voltmeter and its resistance. Had the voltmeter been an ideal one what would have been its reading?

41. (a) A voltmeter with resistance R_V is connected across the terminals of a battery of emf E and internal resistance r. Find the potential difference measured by the voltmeter.

(b) If $E = 7.50$ V and $r = 0.45\ \Omega$, find the minimum value of the voltmeter resistance R_V so that the voltmeter reading is within 1.0% of the emf of the battery.

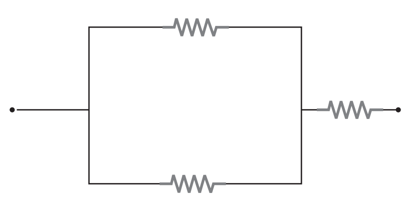
(c) Explain why your answer in part (b) represents a minimum value.

42. (a) An ammeter with resistance R_A is connected in series with a resistor R, a battery of emf ε and internal resistance r. The current measured by the ammeter is I_A. Find the current through the circuit if the ammeter is removed so that the battery and the resistor form a complete circuit. Express your answer in terms of I_A, r, R_A and R. Show that more "ideal" the ammeter, the smaller the difference between this current and the current I_A.

(b) If $R = 3.80\ \Omega$, $\varepsilon = 7.50$ V and $r = 0.45\ \Omega$, find the maximum value of the ammeter resistance R_A so that I_A is within 99% of the current in the circuit when the ammeter is absent.

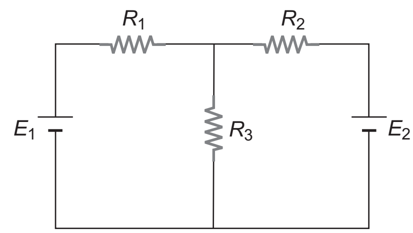
(c) Explain why your answer in part (b) represents a maximum value.

43. Each of three resistors in figure has a resistance of $2.4\ \Omega$ and can dissipate a maximum of 36 W without becoming excessively heated. What is the maximum power the circuit can dissipate?

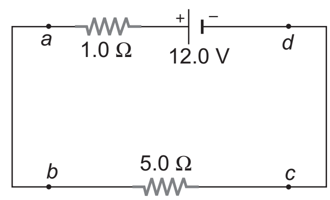


44. A storage battery with emf 2.6 V loaded with external resistance produces a current 1 A. In this case, the potential difference between the terminals of the storage battery equals 2 V. Find the thermal power generated in the battery and the net power supplied by the battery for external circuit.

45. In the circuit shown in figure $E_1 = 7$ V, $E_2 = 1$ V, $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$ respectively. Find the power supplied by the two batteries.



46. In the circuit shown in figure, find



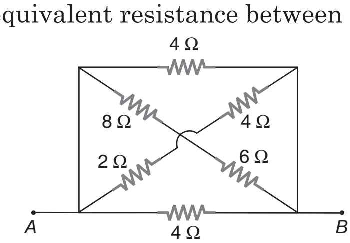
(a) the rate of conversion of internal (chemical) energy to electrical energy within the battery
(b) the rate of dissipation of electrical energy in the battery
(c) the rate of dissipation of electrical energy in the external resistor.

47. Three resistors having resistances of 1.60 Ω, 2.40 Ω and 4.80 Ω are connected in parallel to a 28.0 V battery that has negligible internal resistance. Find
(a) the equivalent resistance of the combination.
(b) the current in each resistor.
(c) the total current through the battery.
(d) the voltage across each resistor.
(e) the power dissipated in each resistor.
(f) which resistor dissipates the maximum power the one with the greatest resistance or the least resistance? Explain why this should be.

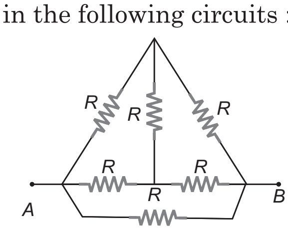
48. (a) The power of resistor is the maximum power the resistor can safely dissipate without too rise in temperature. The power rating of a 15 kΩ resistor is 5.0 W. What is the maximum allowable potential difference across the terminals of the resistor?
(b) A 9.0 kΩ resistor is to be connected across a 120 V potential difference. What power rating is required?

Note *Attempt the following questions after reading the chapter of capacitors.*

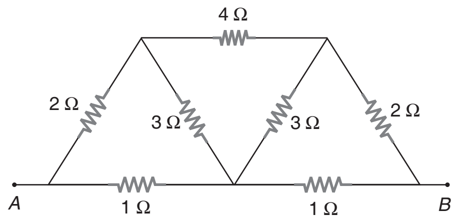
49. Find the equivalent resistance between points A and B in the following circuits :



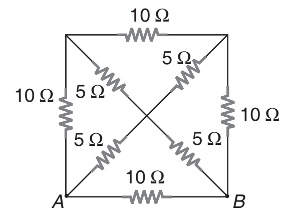
(a)



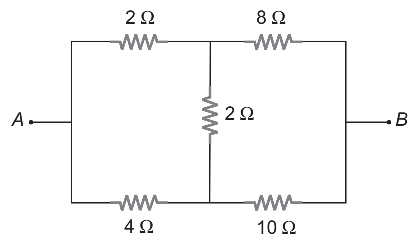
(b)



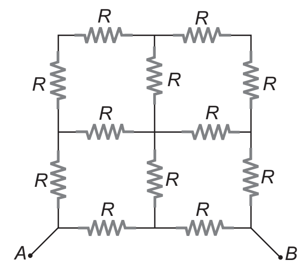
(c)



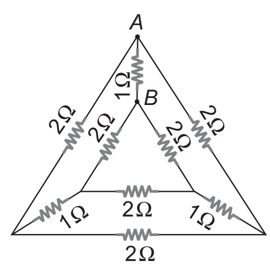
(d)



(e)

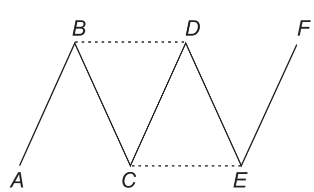


(f)

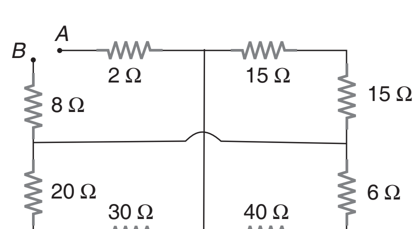


(g)

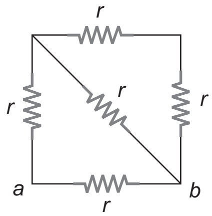
50. What will be the change in the resistance of a circuit between A and F consisting of five identical conductors, if two similar conductors are added as shown by the dashed line in figure?



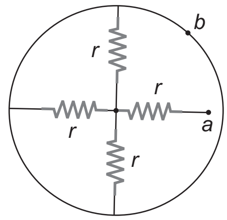
51. Find R_{AB} in the circuit, shown in figure.



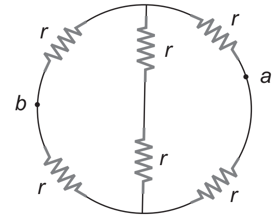
52. Find the equivalent resistance of the networks shown in figure between the points a and b.



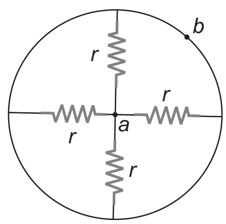
(a)



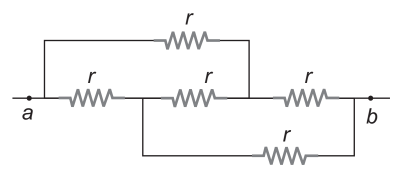
(b)



(c)

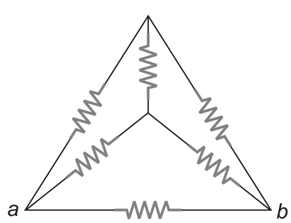


(d)

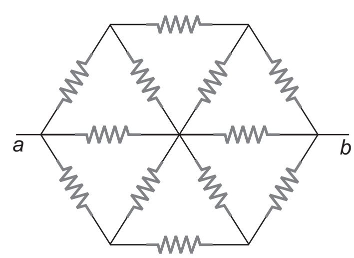


(e)

53. Find the equivalent resistance of the circuits shown in figure between the points a and b. Each resistor has a resistance r.



(a)

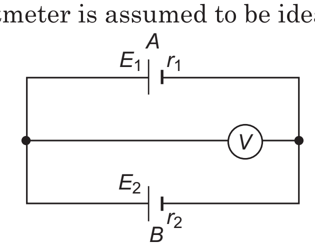


(b)

LEVEL 2

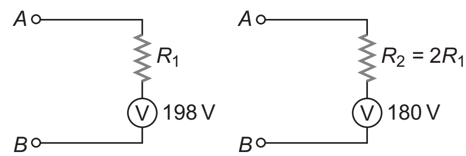
Single Correct Option

1. Two cells A and B of emf 1.3 V and 1.5 V respectively are arranged as shown in figure. The voltmeter reads 1.45 V. The voltmeter is assumed to be ideal. Then



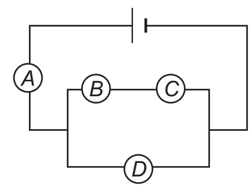
(a) $r_1 = 2r_2$ (b) $r_1 = 3r_2$ (c) $r_2 = 2r_1$ (d) $r_2 = 3r_1$

2. A voltmeter connected in series with a resistance R_1 to a circuit indicates a voltage $V_1 = 198$ V. When a series resistor $R_2 = 2R_1$ is used, the voltmeter indicates a voltage $V_2 = 180$ V. If the resistance of the voltmeter is $R_V = 900\ \Omega$, then the applied voltage across A and B is



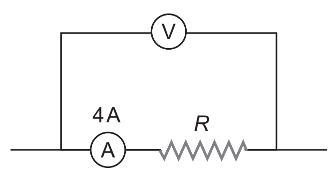
(a) 210 V
(b) 200 V
(c) 220 V
(d) 240 V

3. All bulbs in the circuit shown in figure are identical. Which bulb glows most brightly?



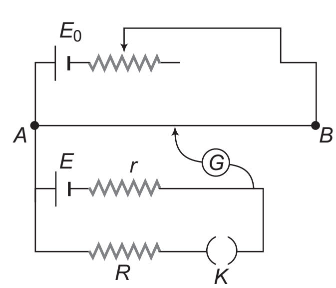
(a) B
(b) A
(c) D
(d) C

4. A student connects an ammeter A and a voltmeter V to measure a resistance R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4 A, then R is



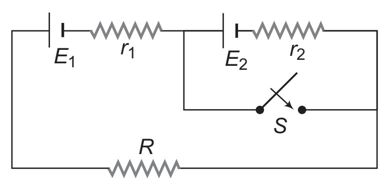
(a) equal to 5 Ω
(b) greater than 5 Ω
(c) less than 5 Ω
(d) greater or less than 5 Ω depending upon the direction of current

5. The given figure represents an arrangement of potentiometer for the calculation of internal resistance (r) of the unknown battery (E). The balance length is 70.0 cm with the key opened and 60.0 cm with the key closed. R is 132.40 Ω. The internal resistance (r) of the unknown cell will be



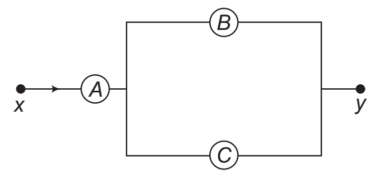
(a) 22.1 Ω
(b) 113.5 Ω
(c) 154.5 Ω
(d) 10 Ω

6. Switch S is closed at time $t = 0$. Which one of the following statements is correct?



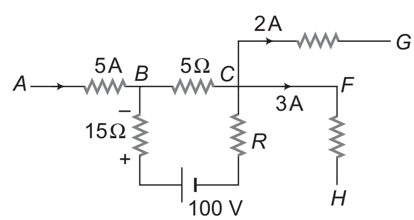
(a) Current in the resistance R increases if $E_1 r_2 < E_2(R + r_1)$
(b) Current in the resistance R increases if $E_1 r_2 > E_2(R + r_1)$
(c) Current in the resistance R decreases if $E_1 r_2 > E_2(R + r_1)$
(d) Current in the resistance R decreases if $E_1 r_2 = E_2(R + r_1)$

7. A, B and C are voltmeters of resistances R, $1.5R$ and $3R$ respectively. When some potential difference is applied between x and y, the voltmeter readings are V_A, V_B and V_C, then



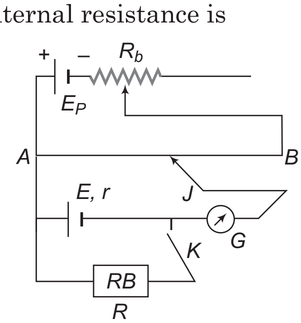
(a) $V_A = V_B = V_C$ (b) $V_A \neq V_B = V_C$ (c) $V_A = V_B \neq V_C$ (d) $V_A + V_B = V_C$

8. In the circuit shown, the voltage drop across the $15\,\Omega$ resistor is 30 V having the polarity as indicated. The ratio of potential difference across $5\,\Omega$ resistor and resistance R is



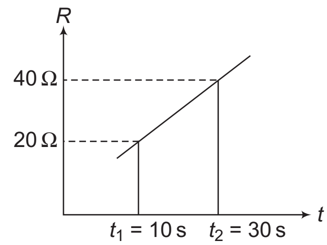
(a) 2/7 (b) 0.4 (c) 5/7 (d) 1

9. In an experiment on the measurement of internal resistance of a cell by using a potentiometer, when the key K is kept open then balancing length is obtained at y metre. When the key K is closed and some resistance R is inserted in the resistance box, then the balancing length is found to be x metre. Then, the internal resistance is



(a) $\dfrac{(x-y)}{y}R$ (b) $\dfrac{(y-x)}{x}R$ (c) $\dfrac{(y-x)}{y}R$ (d) $\dfrac{(x-y)}{x}R$

10. A source of emf $E = 10$ V and having negligible internal resistance is connected to a variable resistance. The resistance varies as shown in figure. The total charge that has passed through the resistor R during the time interval from t_1 to t_2 is

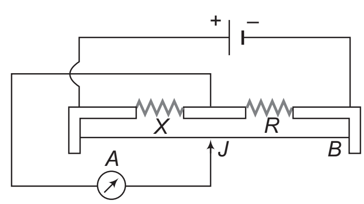


(a) $40 \log_e 4$
(b) $30 \log_e 3$
(c) $20 \log_e 2$
(d) $10 \log_e 2$

11. In order to increase the resistance of a given wire of uniform cross-section to four times its value, a fraction of its length is stretched uniformly till the full length of the wire becomes $\frac{3}{2}$ times the original length. What is the value of this fraction?

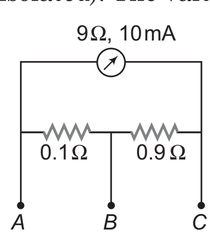
(a) $\frac{1}{4}$
(b) $\frac{1}{8}$
(c) $\frac{1}{16}$
(d) $\frac{1}{6}$

12. The figure shows a meter bridge circuit with $AB = 100$ cm, $X = 12\ \Omega$ and $R = 18\ \Omega$ and the jockey J in the position of balance. If R is now made $8\ \Omega$, through what distance will J have to be moved to obtain balance?



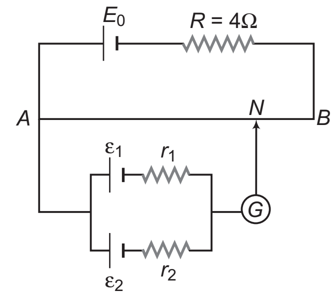
(a) 10 cm
(b) 20 cm
(c) 30 cm
(d) 40 cm

13. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The meter gives full scale deflection for current I when A and B are used as its terminals, i.e. current enters at A and leaves at B (C is left isolated). The value of I is



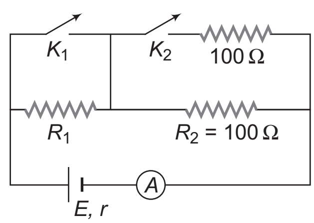
(a) 100 mA
(b) 900 mA
(c) 1 A
(d) 1.1 A

14. A battery of emf $E_0 = 12$ V is connected across a 4 m long uniform wire having resistance 4 Ω/m. The cell of small emfs $\varepsilon_1 = 2$ V and $\varepsilon_2 = 4$ V having internal resistance 2 Ω and 6 Ω respectively are connected as shown in the figure. If galvanometer shows no deflection at the point N, the distance of point N from the point A is equal to



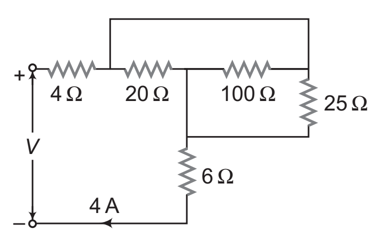
(a) $\frac{5}{3}$ m (b) $\frac{4}{3}$ m
(c) $\frac{3}{2}$ m (d) None of these

15. In the circuit shown, when keys K_1 and K_2 both are closed, the ammeter reads I_0. But when K_1 is open and K_2 is closed, the ammeter reads $I_0/2$. Assuming that ammeter resistance is much less than R_2, the values of r and R_1 in Ω are



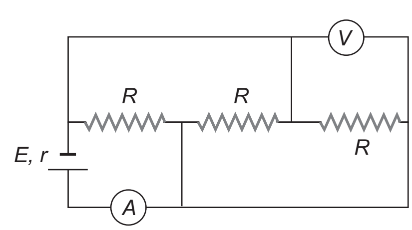
(a) 25, 50 (b) 25, 100
(c) 0, 100 (d) 0, 50

16. In the circuit shown in figure, V must be



(a) 50 V (b) 80 V
(c) 100 V (d) 1290 V

17. In the circuit shown in figure ammeter and voltmeter are ideal. If $E = 4$ V, $R = 9$ Ω and $r = 1$ Ω, then readings of ammeter and voltmeter are

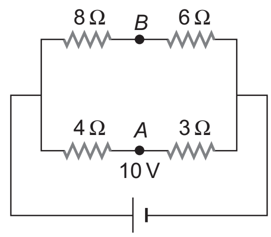


(a) 1 A, 3 V (b) 2 A, 3 V
(c) 3 A, 4 V (d) 4 A, 4 V

18. A moving coil galvanometer is converted into an ammeter reading up to 0.03 A by connecting a shunt of resistance $\frac{r}{4}$. What is the maximum current which can be sent through this galvanometer, if no shunt is used. (Here, r = resistance of galvanometer)

(a) 0.004 A (b) 0.005 A
(c) 0.006 A (d) 0.008 A

19. The potential difference between points A and B is



(a) $\frac{20}{7}$ V (b) $\frac{40}{7}$ V
(c) $\frac{10}{7}$ V (d) zero

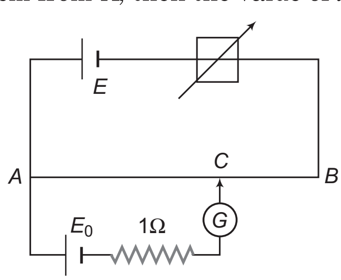
20. Two wires A and B made of same material and having their lengths in the ratio 6 : 1 are connected in series. The potential difference across the wires are 3 V and 2 V respectively. If r_A and r_B are the radii of A and B respectively, then $\frac{r_B}{r_A}$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

21. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with resistance of 2950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the above series resistance should be

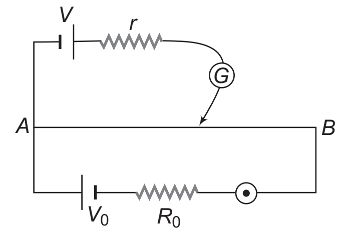
(a) 4450 Ω (b) 5050 Ω
(c) 5550 Ω (d) 6050 Ω

22. Figure shows a potentiometer arrangement with $R_{AB} = 10\,\Omega$ and rheostat of variable resistance x. For $x = 0$ null deflection point is found at 20 cm from A. For unknown value of x null deflection point was at 30 cm from A, then the value of x is



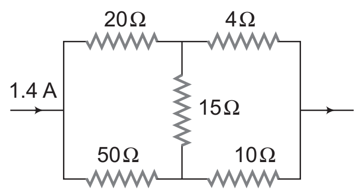
(a) 10 Ω (b) 5 Ω
(c) 2 Ω (d) 1 Ω

23. In the given potentiometer arrangement, the null point



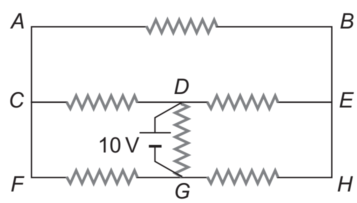
(a) can be obtained for any value of V
(b) can be obtained only if $V < V_0$
(c) can be obtained only if $V > V_0$
(d) can never be obtained

24. In the given figure the current through 4 Ω resistor is



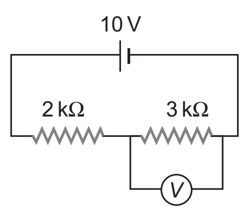
(a) 1.4 A
(b) 0.4 A
(c) 1.0 A
(d) 0.7 A

25. All resistances shown in circuit are 2 Ω each. The current in the resistance between D and E is



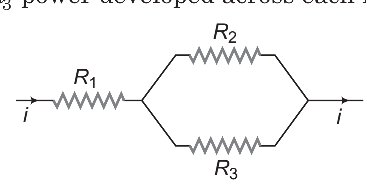
(a) 5 A
(b) 2.5 A
(c) 1 A
(d) 7.5 A

26. In the circuit shown in figure, the resistance of voltmeter is 6 kΩ. The voltmeter reading will be



(a) 6 V
(b) 5 V
(c) 4 V
(d) 3 V

27. For what ratio of R_1, R_2 and R_3 power developed across each resistor is equal?



(a) 1 : 1 : 1
(b) 4 : 4 : 1
(c) 4 : 1 : 1
(d) 1 : 4 : 4

More than One Correct Options

1. Two heaters designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in time t_1 and t_2 respectively. When used together across the same source, they produce H amount of heat in time t

(a) If they are in series, $t = t_1 + t_2$

(b) If they are in series, $t = 2(t_1 + t_2)$

(c) If they are in parallel, $t = \dfrac{t_1 t_2}{(t_1 + t_2)}$

(d) If they are in parallel, $t = \dfrac{t_1 t_2}{2(t_1 + t_2)}$

2. Two cells of emf $E_1 = 6$ V and $E_2 = 5$ V are joined in parallel with same polarity on same side, without any external load. If their internal resistances are $r_1 = 2\,\Omega$ and $r_2 = 3\,\Omega$ respectively, then

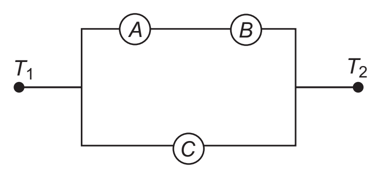
(a) terminal potential difference across any cell is less than 5 V

(b) terminal potential difference across any cell is 5.6 V

(c) current through the cells is 0.2 A

(d) current through the cells is zero if $E_1 = E_2$

3. Three ammeters A, B and C of resistances R_A, R_B and R_C respectively are joined as shown. When some potential difference is applied across the terminals T_1 and T_2, their readings are I_A, I_B and I_C respectively. Then,



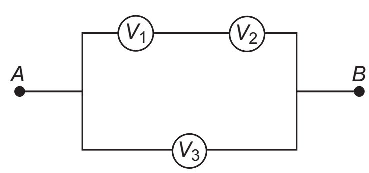
(a) $I_A = I_B$

(b) $I_A R_A + I_B R_B = I_C R_C$

(c) $\dfrac{I_A}{I_C} = \dfrac{R_C}{R_A}$

(d) $\dfrac{I_B}{I_C} = \dfrac{R_C}{R_A + R_B}$

4. Three voltmeters all having different resistances, are joined as shown. When some potential difference is applied across A and B, their readings are V_1, V_2 and V_3. Then,



(a) $V_1 = V_2$

(b) $V_1 \neq V_2$

(c) $V_1 + V_2 = V_3$

(d) $V_1 + V_2 > V_3$

5. Two conductors made of the same material have lengths L and $2L$ but have equal resistances. The two are connected in series in a circuit in which current is flowing. Which of the following is/are correct?

(a) The potential difference across the two conductors is the same

(b) The drift speed is larger in the conductor of length L

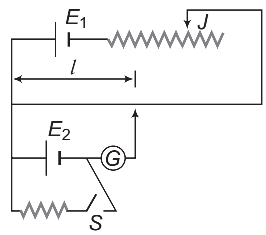
(c) The electric field in the first conductor is twice that in the second

(d) The electric field in the second conductor is twice that in the first

6. In the figure shown,

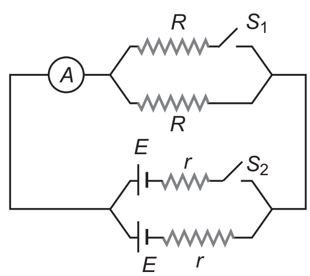
(a) current will flow from A to B
(b) current may flow A to B
(c) current may flow from B to A
(d) the direction of current will depend on E

7. In the potentiometer experiment shown in figure, the null point length is l. Choose the correct options given below.



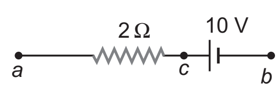
(a) If jockey J is shifted towards right, l will increase
(b) If value of E_1 is increased, l is decreased
(c) If value of E_2 is increased, l is increased
(d) If switch S is closed, l will decrease

8. In the circuit shown in figure, reading of ammeter will



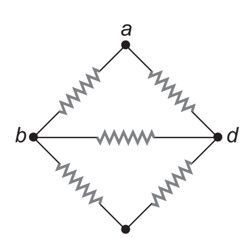
(a) increase if S_1 is closed
(b) decrease if S_1 is closed
(c) increase if S_2 is closed
(d) decrease if S_2 is closed\

9. In the circuit shown in figure it is given that $V_b - V_a = 2$ volt. Choose the correct options.



(a) Current in the wire is 6 A
(b) Direction of current is from a to b
(c) $V_a - V_c = 12$ volt
(d) $V_c - V_a = 12$ volt

10. Each resistance of the network shown in figure is r. Net resistance between



(a) a and b is $\frac{7}{3}r$
(b) a and c is r
(c) b and d is r
(d) b and d is $\frac{r}{2}$

Comprehension Based Questions

Passage (Q. No. 1 and 2)

The length of a potentiometer wire is 600 cm and it carries a current of 40 mA. For a cell of emf 2 V and internal resistance 10 Ω, the null point is found to be at 500 cm. On connecting a voltmeter across the cell, the balancing length is decreased by 10 cm.

1. The voltmeter reading will be

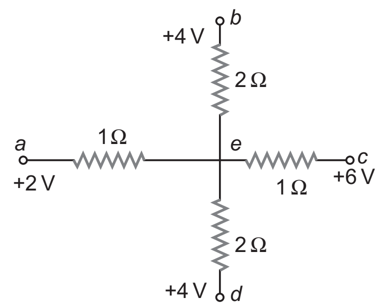
(a) 1.96 V (b) 1.8 V (c) 1.64 V (d) 0.96 V

2. The resistance of the voltmeter is

(a) 500 Ω (b) 290 Ω (c) 490 Ω (d) 20 Ω

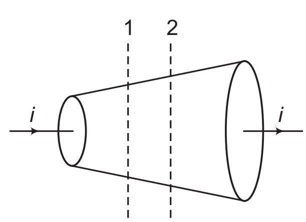
Match the Columns

1. For the circuit shown in figure, match the two columns.



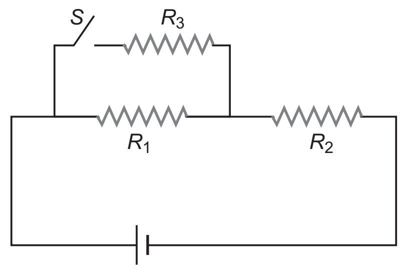
Column I	Column II
(a) current in wire *ae*	(p) 1 A
(b) current is wire *be*	(q) 2 A
(c) current in wire *ce*	(r) 0.5 A
(d) current in wire *de*	(s) None of these

2. Current *i* is flowing through a wire of non-uniform cross-section as shown. Match the following two columns.



Column I	Column II
(a) Current density	(p) is more at 1
(b) Electric field	(q) is more at 2
(c) Resistance per unit length	(r) is same at both sections 1 and 2
(d) Potential difference per unit length	(s) data is insufficient

3. In the circuit shown in figure, after closing the switch S, match the following two columns.

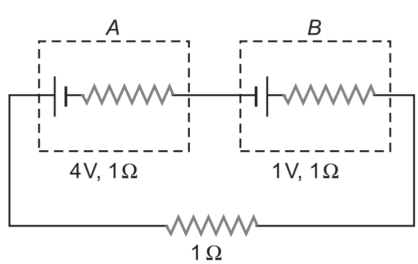


Column I	Column II
(a) current through R_1	(p) will increase
(b) current through R_2	(q) will decrease
(c) potential difference across R_1	(r) will remain same
(d) potential difference across R_2	(s) data insufficient

4. Match the following two columns.

Column I	Column II
(a) Electrical resistance	(p) $[MLT^{-2}A^{2}]$
(b) Electric potential	(q) $[ML^2T^{-3}A^{-2}]$
(c) Specific resistance	(r) $[ML^2T^{-3}A^{-1}]$
(d) Specific conductance	(s) None of these

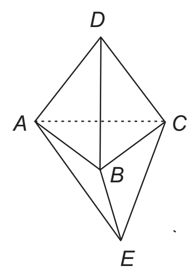
5. In the circuit shown in figure, match the following two columns :



Column I	Column II (In SI units)
(a) potential difference across battery A	(p) zero
(b) potential difference across battery B	(q) 1
(c) net power supplied/ consumed by A	(r) 2
(d) net power supplied/ consumed by B	(s) 3

Subjective Questions

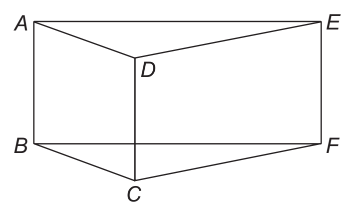
1. Find the equivalent resistance of the triangular bipyramid between the points.



(a) A and C (b) D and E

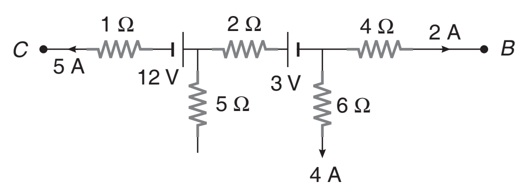
Assume the resistance of each branch to be R.

2. Nine wires each of resistance r are connected to make a prism as shown in figure. Find the equivalent resistance of the arrangement across



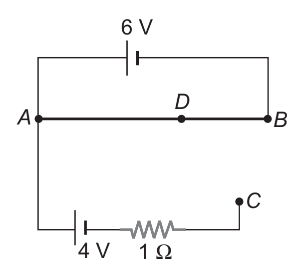
(a) AD (b) AB

3. The figure shows part of certain circuit, find :



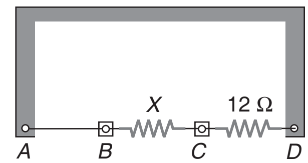
(a) Power dissipated in 5 Ω resistance.
(b) Potential difference $V_C - V_B$.
(c) Which battery is being charged?

4. A 6 V battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 4 V and internal resistance 1 Ω is joined to the point A as shown in figure. Take the potential at B to be zero.



(a) What are the potentials at the points A and C?
(b) At which point D of the wire AB, the potential is equal to the potential at C?
(c) If the points C and D are connected by a wire, what will be the current through it?
(d) If the 4V battery is replaced by 7.5 V battery, what would be the answers of parts (a) and (b)?

5. A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of $12\,\Omega$ are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X. Using the principle of Wheatstone bridge answer the following questions :

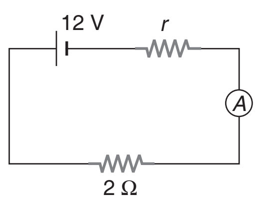


(a) Are there positive and negative terminals on the galvanometer?

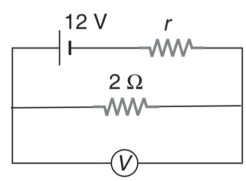
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

6. A galvanometer (coil resistance $99\,\Omega$) is converted into an ammeter using a shunt of $1\,\Omega$ and connected as shown in figure (a). The ammeter reads 3 A. The same galvanometer is converted into a voltmeter by connecting a resistance of $101\,\Omega$ in series. This voltmeter is connected as shown in figure (b). Its reading is found to be 4/5 of the full scale reading. Find :



(a)



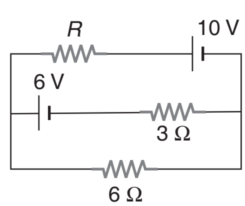
(b)

(a) internal resistance r of the cell

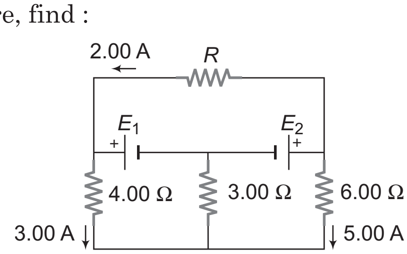
(b) range of the ammeter and voltmeter

(c) full scale deflection current of the galvanometer.

7. In a circuit shown in figure if the internal resistances of the sources are negligible then at what value of resistance R will the thermal power generated in it will be the maximum. What is the value of maximum power?

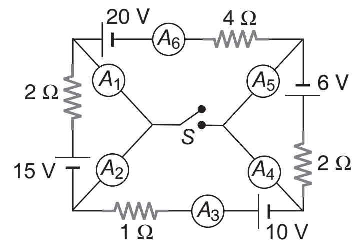


8. In the circuit shown in figure, find :



(a) the current in the 3.00 Ω resistor, (b) the unknown emfs E_1 and E_2 and (c) the resistance R.

9. In the circuit shown, all the ammeters are ideal.



(a) If the switch S is open, find the reading of all ammeters and the potential difference across the switch.

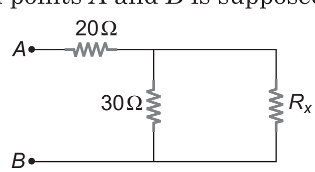
(b) If the switch S is closed, find the current through all ammeters and the switch also.

10. An accumulator of emf 2 V and negligible internal resistance is connected across a uniform wire of length 10 m and resistance 30 Ω. The appropriate terminals of a cell of emf 1.5 V and internal resistance 1 Ω is connected to one end of the wire and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What is the length of the wire that will be required to produce zero deflection of the galvanometer? How will the balancing length change?

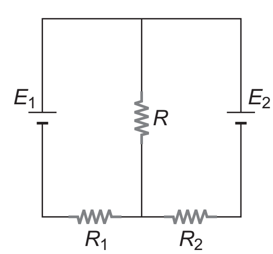
(a) When a coil of resistance 5 Ω is placed in series with the accumulator.

(b) The cell of 1.5 V is shunted with 5 Ω resistor?

11. A circuit shown in the figure has resistances 20 Ω and 30 Ω. At what value of resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance? The voltage between points A and B is supposed to be constant in this case.



12. In the circuit shown in figure, the emfs of batteries are E_1 and E_2 which have internal resistances R_1 and R_2. At what value of the resistance R will the thermal power generated in it be the highest? What it is?



13. A conductor has a temperature independent resistance R and a total heat capacity C. At the moment $t = 0$ it is connected to a DC voltage V. Find the time dependence of the conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$, where k is a constant, T_0 is the surrounding temperature (equal to conductor's temperature at the initial moment).

Answers

Introductory Exercise 23.1

1. 4.375×10^{18} **2.** 38880 C **3.** (a) 337.5 C (b) 2.1×10^{21} **4.** 6.6×10^{15} rps, 1.06 mA

5. 300 C **6.** Yes, from left to right

Introductory Exercise 23.2

1. False

Introductory Exercise 23.3

1. 6.0×10^{-4} m/s **2.** 0.735 μ m/s, 431.4 yr.

Introductory Exercise 23.4

1. 0.18 Ω **2.** True **3.** 15 g **4.** (c)

Introductory Exercise 23.5

1. (d) **2.** 85° C

Introductory Exercise 23.6

1. 5 A, 2.5 A **2.** 0, 2 V, 5 V, 15 V, 3 A from C to B, 7.5 A from D to A.

3. 5 V **4.** $\frac{1}{2}$ A **5.** Zero, 1 A

Introductory Exercise 23.7

1. 3 A, 2 Ω, – 5 V **2.** 36 W, 12 W

Introductory Exercise 23.8

1. $V = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$, $r = \frac{r_1 r_2}{r_1 + r_2}$ **2.** 2 V **3.** 7.5 V, 0.5 Ω

Introductory Exercise 23.9

1. By connecting a resistance of 999 Ω in series with galvanometer

2. By connecting 1 Ω resistance in parallel with it **3.** $(n - 1)G$

Introductory Exercise 23.10

1. 1.5 Ω **2.** (a) 320 cm (b) $\frac{3E}{22r}$

Introductory Exercise 23.11

1. (a) **2.** (b) **3.** B is most accurate

Introductory Exercise 23.12

1. 14.2 Ω to 14.3 Ω **2.** See the hints **3.** (c)

Introductory Exercise 23.13

1. $(42 \times 10^3 \pm 5\%)\,\Omega$ **2.** Red, Yellow, Blue, Gold

Exercises

LEVEL 1

Assertion and Reason

1. (d) **2.** (a,b) **3.** (b) **4.** (d) **5.** (b) **6.** (c) **7.** (d) **8.** (c) **9.** (d) **10.** (a)
11. (d)

Objective Questions

1.(d) **2.**(d) **3.**(a) **4.**(c) **5.**(b) **6.**(b) **7.**(a) **8.**(a) **9.**(b) **10.**(d)
11.(b) **12.**(b) **13.**(b) **14.**(d) **15.**(d) **16.**(d) **17.**(c) **18.**(c) **19.**(c) **20.**(a)
21.(d) **22.**(c) **23.**(b) **24.**(d) **25.**(d) **26.**(c) **27.**(a) **28.**(d) **29.**(a) **30.**(c)
31.(b) **32.**(b) **33.**(b) **34.**(c) **35.**(c) **36.**(d) **37.**(d) **38.**(b) **39.**(a) **40.**(d)
41.(b)

Subjective Questions

1. Yes **2.** False **3.** 1.12 mA **4.** 82 Ω

5. (a) $\frac{1}{2}$ A (b) 1 W, 2 W (c) 6 W (supplied), 3 W (absorbed) **6.** 16 cm from A

7. (a) zero (b) 5.0 V (c) 5.0 V **8.** (a) Anti-clockwise (b) E_1 (c) Point B

9. –10 V **10.** 1.9×10^{-4} m/s **11.** (a) 9.9 A (b) 5.88 V (c) 0.60 Ω

12. $\frac{\rho l}{\pi ab}$ **14.** 2 W **15.** 954 Ω, 0.002 /°C **16.** 0.569 mm

17. (a) 1.25 V/m (b) $2.84\times 10^{-8}\ \Omega\cdot m$ **18.** (a) $2d \times 3d, \frac{V}{\rho d}$ (b) $2d \times 3d, \frac{6vd}{\rho}$

19. (a) $3.65\times 10^{-8}\ \Omega\cdot m$ (b) 172.3 A (c) 2.58×10^{-3} m/s

20. $R_1 = 18.18\ \Omega, R_2 = 1.82\ \Omega$ **21.** 5 A **22.** 15 A, $\frac{8}{5}\ \Omega$ **23.** $\frac{8}{3}\ \Omega$, 9 A **24.** $\frac{13}{3}$ A

25. $V_A = 12\text{ V}, V_B = 9\text{ V}, V_C = 3\text{ V}, V_D = -6\text{ V}, V_A' = 12\text{ V}, V_B' = 11\text{ V}, V_C' = 9\text{ V}, V_D' = 6\text{ V}$

26. – 75 V, –50 V, 125 V, 175 V, –25 V, –200 V **27.** (a) 0.1 A, 4.0 V (b) 0.08 A, 4.2 V

28.

Resistance	5 Ω	8 Ω	6 Ω	16 Ω	4 Ω	1 Ω
Current	4 A	0.5 A	3.0 A	0.5 A	1.0 A	4 A
Towards	A	C	C	C	B	E

29. $\frac{20}{3}$ V **30.** (a) (i) 120 V, 80 V (ii) 100 V, 100 V (b) $\frac{1}{60}$ A

31. (a) 5 V (b) 3 V (c) positive terminal on left side **32.** (a) 0.20 Ω (b) 8.7 V

33. 2.5 V **34.** current in all resistors is zero **35.** $\frac{G^2}{G+S}$ **36.** 20.16 Ω **37.** 22.5 V

38. 48 V **39.** 80 Ω **40.** 400 Ω, 3.2 V, 3.238 V **41.** (a) $\frac{ER_v}{R_v + r}$ (b) $4.5 \times 10^{-3}\ \Omega$

42. (a) $I_A\left[1 + \frac{R_A}{R + r}\right]$ (b) 0.0045 Ω **43.** 54 W **44.** 0.6 W, 2 W

45. +14 W, –1 W **46.** (a) 24 W (b) 4 W (c) 20 W

47. (a) 0.80 Ω (b) 1.60 Ω resistor 17.5 A, 2.40 Ω resistor 11.7 A, 4.80 Ω resistor 5.8 A
(c) 35.0 A (d) 28.0 V for each
(e) 1.60 Ω resistor 490 W, 2.40 Ω resistor 327 W, 4.80 Ω resistor 163 W (f) least resistance

48. (a) 273.8 V (b) 1.6 W

49. (a) $\frac{42}{31}\,\Omega$ (b) $\frac{R}{2}$ (c) $\frac{32}{21}\,\Omega$ (d) $\frac{25}{6}\,\Omega$ (e) 6.194 Ω (f) $\frac{5R}{4}$ (g) $\frac{5}{7}\,\Omega$

50. The new equivalent resistance will become 0.6 times **51.** 23.32 Ω

52. (a) $\frac{5}{8}r$ (b) $\frac{4}{3}r$ (c) r (d) $\frac{r}{4}$ (e) r **53.** (a) $r/2$ (b) $4r/5$

LEVEL 2

Single Correct Option

1.(b) **2.**(c) **3.**(b) **4.**(c) **5.**(a) **6.**(b) **7.**(a) **8.**(d) **9.**(b) **10.**(d)
11.(b) **12.**(b) **13.**(c) **14.**(d) **15.**(d) **16.**(b) **17.**(a) **18.**(c) **19.**(d) **20.**(b)
21.(a) **22.**(b) **23.**(d) **24.**(c) **25.**(b) **26.**(b) **27.**(d)

More than One Correct Options

1. (a,c) **2.** (b,c,d) **3.** (a,b,d) **4.** (b,c) **5.** (a,b,c) **6.** (b,c,d) **7.** (a,b,c,d) **8.** (a,c) **9.** (a,d) **10.** (b,d)

Comprehension Based Questions

1. (a) **2.** (c)

Match the Columns

1. (a) → q (b) → s (c) → q (d) → s
2. (a) → p (b) → p (c) → p (d) → p
3. (a) → q (b) → p (c) → q (d) → p
4. (a) → q (b) → r (c) → s (d) → s
5. (a) → s (b) → r (c) → s (d) → r

Subjective Questions

1. (a) $\frac{2}{5}R$ (b) $\frac{2}{3}R$ **2.** (a) $\frac{8}{15}r$ (b) $\frac{3}{5}r$ **3.** (a) 605 W (b) 6 V (c) both

4. (a) 6 V, 2V (b) AD = 66.7 cm (c) zero (d) 6 V, –1.5 V, no such point D exists.

5. (a) No (c) 8 Ω **6.** (a) 1.01 Ω (b) 5 A, 9.95 V (c) 0.05 A

7. 2 Ω, 4.5 W **8.** (a) 8 A (b) 36 V, 54 V (c) 9 Ω

9. (a) 9.5 A, 9.5 A, 2 A, 5 A, 5 A, 2 A, 12 V (b) 12.5 A, 2.5 A, 10 A, 7 A, 8 A, 5 A, 15 A

10. 7.5 m (a) 8.75 m (b) 6.25 m **11.** 12 Ω **12.** $R = \frac{R_1R_2}{R_1 + R_2}$, $P_{max} = \frac{(E_1R_2 + E_2R_1)^2}{4R_1R_2\,(R_1 + R_2)}$

13. $T = T_0 + (1 - e^{-kt/C})\frac{V^2}{kR}$

24

Electrostatics

Chapter Contents

24.1 Introduction

When we comb our hair on a dry day and bring the comb close to tiny pieces of paper, we note that they are swiftly attracted by the comb. Similar phenomena occur if we rub a glass rod or an amber rod with a cloth or with a piece of fur. Why does this happens? What really happens in an electric circuit? How do electric motors and generators work?

The answers to all these questions come from a branch of physics known as **electromagnetism**, the study of electric and magnetic interactions. These interactions involve particles that have a property called electric charge, an inherent property of matter that is as fundamental as mass.

We begin our study of electromagnetism in this chapter by the electric charge. We will see that it is quantized and obeys a conservation principle. Then we will study the interactions of electric charges that are at rest, called **electrostatic** interactions. These interactions are governed by a simple relationship known as **Coulomb's law.** This law is more conveniently described by using the concept of **electric field.**

24.2 Electric Charge

The electrical nature of matter is inherent in atomic structure. An atom consists of a small, relatively massive nucleus that contains particles called protons and neutrons. A proton has a mass 1.673×10^{-27} kg, while a neutron has a slightly greater mass 1.675×10^{-27} kg. Surrounding the nucleus is a diffuse cloud of orbiting particles called electrons. An electron has a mass of 9.11×10^{-31} kg.

Like mass, **electric charge** is an intrinsic property of protons and electrons, and only two types of charge have been discovered positive and negative. A proton has a positive charge, and an electron has a negative charge. A neutron has no net electric charge.

The magnitude of the charge on the proton exactly equals the magnitude of the charge on the electron. The proton carries a charge $+e$ and the electron carries a charge $-e$. The SI unit of charge is coulomb (C) and e has the value

$$\boxed{e = 1.6 \times 10^{-19}\,\text{C}}$$

Regarding charge the following points are worth noting:

1. Like charges repel each other and unlike charges attract each other.
2. Charge is a scalar and can be of two types positive or negative.
3. Charge is quantized. The quantum of charge is e. The charge on any body will be some integral multiple of e, i.e.

$$\boxed{q = \pm ne} \qquad \text{where,} \quad n = 1, 2, 3 \ldots$$

Charge on any body can never be $\left(\frac{1}{3}e\right)$, $1.5e$, etc.

Note *(i) Apart from charge, energy, angular momentum and mass are also quantized. The quantum of energy is $h\nu$ and that of angular momentum is $\frac{h}{2\pi}$. Quantum of mass is not known till date.*

(ii) The protons and neutrons are combination of other entities called ***quarks****, which have charges $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$. However, isolated quarks have not been observed. So, quantum of charge is still e.*

4. During any process, the net electric charge of an isolated system remains constant or we can say that charge is conserved. Pair production and pair annihilation are two examples of conservation of charge.
5. A charged particle at rest produces electric field. A charged particle in an unaccelerated motion produces both electric and magnetic fields but does not radiate energy. But an accelerated charged particle not only produces an electric and magnetic fields but also radiates energy in the form of electromagnetic waves.

Example 24.1 *How many electrons are there in one coulomb of negative charge?*

Solution The negative charge is due to the presence of excess electrons, since they carry negative charge. Because an electron has a charge whose magnitude is $e = 1.6 \times 10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron. Therefore, the number n of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$ **Ans.**

24.3 Conductors and Insulators

For the purpose of electrostatic theory, all substances can be divided into two main groups, conductors and insulators. In conductors, electric charges are free to move from one place to another, whereas in insulators they are tightly bound to their respective atoms. In an uncharged body, there are equal number of positive and negative charges.

The examples of conductors of electricity are the metals, human body and the earth and that of insulators are glass, hard rubber and plastics. In metals, the free charges are free electrons known as conduction electrons.

Semiconductors are a third class of materials and their electrical properties are somewhere between those of insulators and conductors. Silicon and germanium are well known examples of semiconductors.

24.4 Charging of a Body

Mainly there are the following three methods of charging a body :

Charging by Rubbing

The simplest way to experience electric charges is to rub certain bodies against each other. When a glass rod is rubbed with a silk cloth, the glass rod acquires some positive charge and the silk cloth acquires negative charge by the same amount. The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body pass onto the other body. The body that donates the electrons becomes positively charged while that which receives the electrons becomes negatively charged. For example, when glass rod is rubbed with silk cloth, glass rod becomes positively charged because it donates the electrons while the silk cloth becomes negatively charged because it receives electrons. Electricity so obtained by rubbing two objects is also known as **frictional electricity.** The other places where the frictional electricity can be observed are when amber is rubbed with wool or a comb is passed through a dry hair. Clouds also become charged by friction.

Charging by Contact

When a negatively charged ebonite rod is rubbed on a metal object, such as a sphere, some of the excess electrons from the rod are transferred to the sphere. Once the electrons are on the metal sphere, where they can move readily, they repel one another and spread out over the sphere's surface. The insulated stand prevents them from flowing to the earth. When the rod is removed, the sphere is left with a negative charge distributed over its surface. In a similar manner, the sphere will be left with a positive charge after being rubbed with a positively charged rod. In this case, electrons from the sphere would be transferred to the rod. The process of giving one object a net electric charge by placing it in contact with another object that is already charged is known as **charging by contact.**

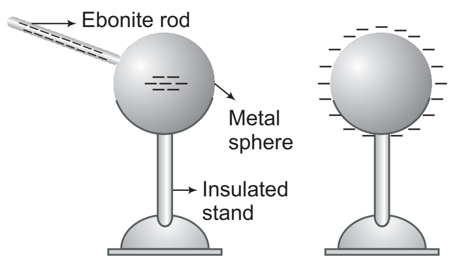


Fig. 24.1

Charging by Induction

It is also possible to charge a conductor in a way that does not involve contact.

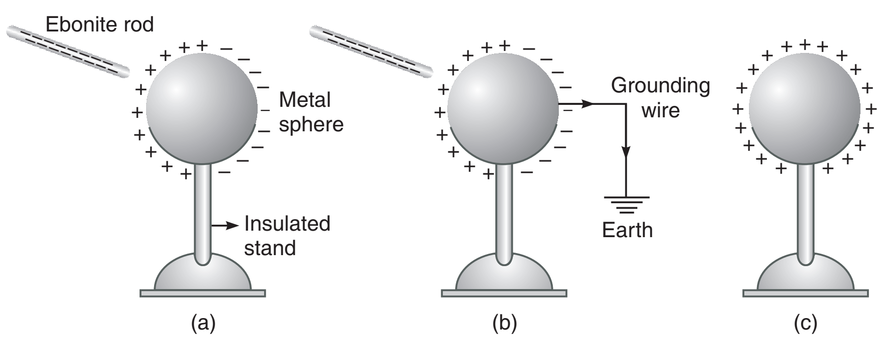


Fig. 24.2

In Fig. (a), a negatively charged rod brought close to (but does not touch) a metal sphere. In the sphere, the free electrons close to the rod move to the other side (by repulsion). As a result, the part of the sphere nearer to the rod becomes positively charged and the part farthest from the rod negatively charged. This phenomenon is called induction. Now, if the rod is removed, the free electrons return to their original places and the charged regions disappear. Under most conditions the earth is a good electric conductor. So, when a metal wire is attached between the sphere and the ground as in figure (b) some of the free electrons leave the sphere and distribute themselves on the much larger earth. If the grounding wire is then removed, followed by the ebonite rod, the sphere is left with a net positive charge.

The process of giving one object a net electric charge without touching the object to a second charged object is called **charging by induction.** The process could also be used to give the sphere a net negative charge, if a positively charged rod were used. Then, electrons would be drawn up from the ground through the grounding wire and onto the sphere.

If the sphere were made from an insulating material like plastic, instead of metal, the method of producing a net charge by induction would not work, because very little charge would flow through the insulating material and down the grounding wire. However, the electric force of the charged rod would have some effect as shown in figure. The electric force would cause the positive and negative charges in the molecules of the insulating material to separate slightly, with the negative charges being pushed away from the negative rod. The surface of the plastic sphere does acquire a slight induced positive charge, although no net charge is created.

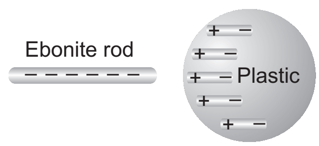


Fig. 24.3

Example 24.2 *If we comb our hair on a dry day and bring the comb near small pieces of paper, the comb attracts the pieces, why?*

Solution This is an example of frictional electricity and induction. When we comb our hair, it gets positively charged by rubbing. When the comb is brought near the pieces of paper some of the electrons accumulate at the edge of the paper piece which is closer to the comb. At the farther end of the piece there is deficiency of electrons and hence, positive charge appears there. Such a redistribution of charge in a material, due to presence of a nearby charged body is called inducion. The comb exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charge. This is because the negative charges are closer to the comb. Hence, there is a net attraction between the comb and the paper piece.

Example 24.3 *Does the attraction between the comb and the piece of papers last for longer period of time?*

Solution No, because the comb loses its net charge after some time. The excess charge of the comb transfers to earth through our body after some time.

Example 24.4 *Can two similarly charged bodies attract each other?*

Solution Yes, when the charge on one body (q_1) is much greater than that on the other (q_2) and they are close enough to each other so that force of attraction between q_1 and induced charge on the other exceeds the force of repulsion between q_1 and q_2. However, two similar point charges can never attract each other because no induction will take place here.

Example 24.5 *Does in charging the mass of a body change?*

Solution Yes, as charging a body means addition or removal of electrons and electron has a mass.

Example 24.6 *Why a third hole in a socket provided for grounding?*

Solution All electric appliances may end with some charge due to faulty connections. In such a situation charge will be accumulated on the appliance. When the user touches the appliance, he may get a shock. By providing the third hole for grounding all accumulated charge is discharged to the ground and the appliance is safe.

INTRODUCTORY EXERCISE 24.1

1. Is attraction a true test of electrification?
2. Is repulsion a true test of electrification?
3. Why does a phonograph record attract dust particles just after it is cleaned?
4. What is the total charge, in coulombs, of all the electrons in three gram mole of hydrogen atom?

24.5 Coulomb's Law

The law that describes how charges interact with one another was discovered by Charles Augustin de Coulomb in 1785. With a sensitive torsion balance, Coulomb measured the electric force between charged spheres. In Coulomb's experiment, the charged spheres were much smaller than the distance between them so that the charges could be treated as point charges. The results of the experiments of Coulomb and others are summarized in **Coulomb's law.**

The electric force $\mathbf{F}_e$ *exerted by one point charge on another acts along the line between the charges. It varies inversely as the square of the distance separating the charges and is proportional to the product of charges. The force is repulsive if the charges have the same sign and attractive if the charges have opposite signs.*

The magnitude of the electric force exerted by a charge q_1 on another charge q_2 a distance r away is thus, given by

$$F_e = \frac{k\,|q_1 q_2|}{r^2} \qquad \text{...(i)}$$

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In SI units the constant k is

$$k = 8.987551787 \times 10^9 \ \frac{\text{N-m}^2}{\text{C}^2}$$

$$\approx 8.988 \times 10^9 \ \frac{\text{N-m}^2}{\text{C}^2}$$

The value of k is known to such a large number of significant digits because this value is closely related to the speed of light in vacuum. This speed is defined to be exactly $c = 2.99792458 \times 10^8$ m/s. The numerical value of k is defined in terms of c to be precisely.

$$k = \left(10^{-7} \ \frac{\text{N-s}^2}{\text{C}^2}\right) c^2$$

This constant k is often written as $\dfrac{1}{4\pi\varepsilon_0}$, where ε_0 ("epsilon-nought") is another constant. This appears to complicate matters, but it actually simplifies many formulae that we will encounter in later chapters. Thus, Eq. (i) can be written as

$$\boxed{F_e = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}} \qquad \ldots\text{(ii)}$$

Here, $$\frac{1}{4\pi\varepsilon_0} = \left(10^{-7}\,\frac{\text{N-s}^2}{\text{C}^2}\right)c^2$$

Substituting value of $c = 2.99792458 \times 10^8$ m/s, we get

$$\boxed{\frac{1}{4\pi\,\varepsilon_0} = 8.99 \times 10^9\,\text{N-m/C}^2}$$

In examples and problems, we will often use the approximate value

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ \text{N-m}^2/\text{C}^2$$

Here, the quantity ε_0 is called the **permittivity of free space.** It has the value,

$$\boxed{\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2/\text{N-m}^2}$$

Regarding Coulomb's law, the following points are worth noting:

1. Coulomb's law stated above describes the interaction of two point charges. When two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges. Thus,

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n$$

2. The electric force is an action reaction pair, i.e. the two charges exert equal and opposite forces on each other.
3. The electric force is conservative in nature.
4. Coulomb's law as we have stated above can be used for point charges in **vacuum.** If some dielectric is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening medium. We will describe this effect later. Here at this moment it is enough to say that the force decreases K times if the medium extends till infinity. Here, K is a dimensionless constant which depends on the medium and called dielectric constant of the medium. Thus,

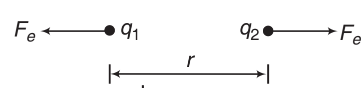


Fig. 24.4

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \qquad \text{(in vacuum)}$$

$$F_e' = \frac{F_e}{K} = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon}\cdot\frac{q_1 q_2}{r^2} \qquad \text{(in medium)}$$

Here, $\varepsilon = \varepsilon_0 K$ is called permittivity of the medium.

Extra Points to Remember

- In few problems of electrostatics **Lami's theorem** is very useful.
  According to this theorem, "if three concurrent forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ as shown in Fig. 24.5 are in equilibrium or if $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, then

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

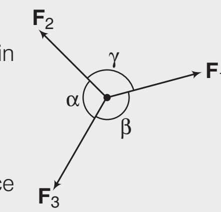


Fig. 24.5

- Suppose the position vectors of two charges q_1 and q_2 are $\mathbf{r}_1$ and $\mathbf{r}_2$, then electric force on charge q_1 due to charge q_2 is,

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} (\mathbf{r}_1 - \mathbf{r}_2)$$

  Similarly, electric force on q_2 due to charge q_1 is

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3} (\mathbf{r}_2 - \mathbf{r}_1)$$

  Here, q_1 and q_2 are to be substituted with sign. $\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$ and $\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$ where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the coordinates of charges q_1 and q_2.

Example 24.7 *What is the smallest electric force between two charges placed at a distance of 1.0 m?*

Solution $$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad \text{...(i)}$$

For F_e to be minimum $q_1 q_2$ should be minimum. We know that

$$(q_1)_{\min} = (q_2)_{\min} = e = 1.6 \times 10^{-19} \text{ C}$$

Substituting in Eq. (i), we have

$$(F_e)_{\min} = \frac{(9.0 \times 10^9)(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(1.0)^2}$$

$$= 2.304 \times 10^{-28} \text{ N}$$ **Ans.**

Example 24.8 *Three charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed on the vertices of an equilateral triangle of side 1.0 m. Find the net electric force acting on charge q_1.*

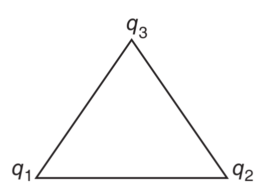


Fig. 24.6

HOW TO PROCEED *Charge q_2 will attract charge q_1 (along the line joining them) and charge q_3 will repel charge q_1. Therefore, two forces will act on q_1, one due to q_2 and another due to q_3. Since, the force is a vector quantity both of these forces (say $\mathbf{F}_1$ and $\mathbf{F}_2$) will be added by vector method. The following are two methods of their addition.*

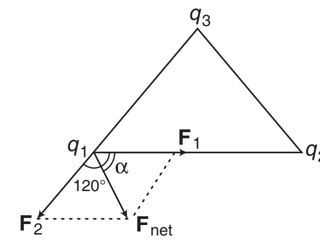


Fig. 24.7

Solution **Method 1.** In the figure,

$$|\mathbf{F}_1| = F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

$$= \text{magnitude of force between } q_1 \text{ and } q_2$$

$$= \frac{(9.0\times 10^9)(1.0\times 10^{-6})(2.0\times 10^{-6})}{(1.0)^2}$$

$$= 1.8\times 10^{-2}\ \text{N}$$

Similarly,

$$|\mathbf{F}_2| = F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_3}{r^2}$$

$$= \text{magnitude of force between } q_1 \text{ and } q_3$$

$$= \frac{(9.0\times 10^9)(1.0\times 10^{-6})(3.0\times 10^{-6})}{(1.0)^2}$$

$$= 2.7\times 10^{-2}\ \text{N}$$

Now,

$$|\mathbf{F}_{net}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

$$= \left(\sqrt{(1.8)^2 + (2.7)^2 + 2\,(1.8)\,(2.7)\left(-\frac{1}{2}\right)}\right)\times 10^{-2}\ \text{N}$$

$$= 2.38\times 10^{-2}\ \text{N}$$

and

$$\tan\alpha = \frac{F_2\sin 120°}{F_1 + F_2\cos 120°}$$

$$= \frac{(2.7\times 10^{-2})(0.87)}{(1.8\times 10^{-2}) + (2.7\times 10^{-2})\left(-\frac{1}{2}\right)}$$

or

$$\alpha = 79.2°$$

Thus, the net force on charge q_1 is 2.38×10^{-2} N at an angle $\alpha = 79.2°$ with a line joining q_1 and q_2 as shown in the figure. **Ans.**

Method 2. In this method let us assume a coordinate axes with q_1 at origin as shown in figure. The coordinates of q_1, q_2 and q_3 in this coordinate system are (0, 0, 0), (1 m, 0, 0) and (0.5 m, 0.87 m, 0) respectively. Now,

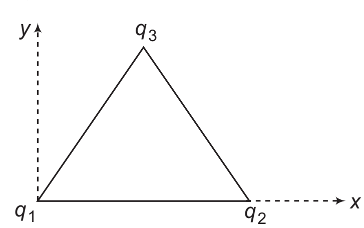


Fig. 24.8

$$\mathbf{F}_1 = \text{force on } q_1 \text{ due to charge } q_2$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2)$$
$$= \frac{(9.0 \times 10^9)(1.0 \times 10^{-6})(-2.0 \times 10^{-6})}{(1.0)^3}[(0-1)\hat{\mathbf{i}} + (0-0)\hat{\mathbf{j}} + (0-0)\hat{\mathbf{k}}]$$
$$= (1.8 \times 10^{-2}\,\hat{\mathbf{i}})\text{ N}$$

and
$$\mathbf{F}_2 = \text{force on } q_1 \text{ due to charge } q_3$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}(\mathbf{r}_1 - \mathbf{r}_3)$$
$$= \frac{(9.0 \times 10^9)(1.0 \times 10^{-6})(3.0 \times 10^{-6})}{(1.0)^3}[(0-0.5)\hat{\mathbf{i}} + (0-0.87)\hat{\mathbf{j}} + (0-0)\hat{\mathbf{k}}]$$
$$= (-1.35\,\hat{\mathbf{i}} - 2.349\,\hat{\mathbf{j}}) \times 10^{-2}\text{ N}$$

Therefore, net force on q_1 is $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$$= (0.45\,\hat{\mathbf{i}} - 2.349\,\hat{\mathbf{j}}) \times 10^{-2}\text{ N}$$ **Ans.**

Note *Once you write a vector in terms of* $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ *and* $\hat{\mathbf{k}}$, *there is no need of writing the magnitude and direction of vector separately.*

Example 24.9 *Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.*

Solution Each ball is in equilibrium under the following three forces :

(i) tension, (ii) electric force and (iii) weight

So, Lami's theorem can be applied.

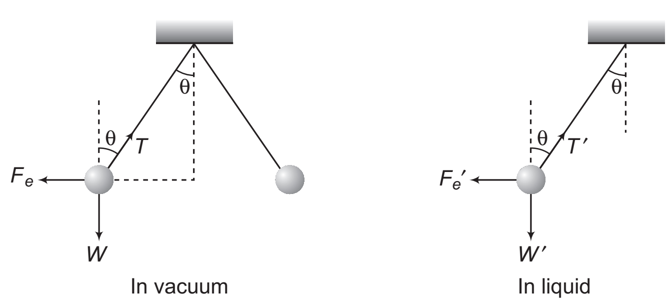


Fig. 24.9

In the liquid, $$F_e' = \frac{F_e}{K}$$

where, K = dielectric constant of liquid and $W' = W - \text{upthrust}$

Applying Lami's theorem in vacuum

$$\frac{W}{\sin(90^\circ + \theta)} = \frac{F_e}{\sin(180^\circ - \theta)}$$

or
$$\frac{W}{\cos\theta} = \frac{F_e}{\sin\theta} \qquad \text{...(i)}$$

Similarly in liquid,
$$\frac{W'}{\cos\theta} = \frac{F_e'}{\sin\theta} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{W}{W'} = \frac{F_e}{F_e'}$$

or
$$K = \frac{W}{W - \text{upthrust}} \qquad \left(\text{as } \frac{F_e}{F_e'} = K\right)$$

$$= \frac{V\rho g}{V\rho g - V\sigma g} \qquad (V = \text{volume of ball})$$

or
$$K = \frac{\rho}{\rho - \sigma}$$ **Ans.**

Note *In the liquid F_e and W have changed. Therefore, T will also change.*

INTRODUCTORY EXERCISE 24.2

1. The mass of an electron is 9.11×10^{-31} kg, that of a proton is 1.67×10^{-27} kg. Find the ratio F_e/F_g of the electric force and the gravitational force exerted by the proton on the electron.
2. Find the dimensions and units of ε_0.
3. Three point charges q are placed at three vertices of an equilateral triangle of side a. Find magnitude of electric force on any charge due to the other two.
4. Three point charges each of value $+q$ are placed on three vertices of a square of side a metre. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the centre of the square?
5. Coulomb's law states that the electric force becomes weaker with increasing distance. Suppose that instead, the electric force between two charged particles were independent of distance. In this case, would a neutral insulator still be attracted towards the comb.
6. A metal sphere is suspended from a nylon thread. Initially, the metal sphere is uncharged. When a positively charged glass rod is brought close to the metal sphere, the sphere is drawn towards the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain, why the sphere is first attracted then repelled?
7. Is there any lower limit to the electric force between two particles placed at a certain distance?
8. Does the force on a charge due to another charge depend on the charges present nearby?
9. The electric force on a charge q_1 due to q_2 is $(4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ N. What is the force on q_2 due to q_1?

24.6 Electric Field

A charged particle cannot directly interact with another particle kept at a distance. A charge produces something called an electric field in the space around it and this electric field exerts a force on any other charge (except the source charge itself) placed in it.

Thus, the region surrounding a charge or distribution of charge in which its electrical effects can be observed is called the **electric field** of the charge or distribution of charge. Electric field at a point can be defined in terms of either a vector function **E** called **'electric field strength'** or a scalar function V called **'electric potential'.** The electric field can also be visualised graphically in terms of **'lines of force'.** Note that all these are functions of position **r** (x, y, z). The field propagates through space with the speed of light, c. Thus, if a charge is suddenly moved, the force it exerts on another charge a distance r away does not change until a time r/c later. In our forgoing discussion, we will see that electric field strength **E** and electric potential V are interrelated. It is similar to a case where the acceleration, velocity and displacement of a particle are related to each other.

Electric Field Strength (E)

Like its gravitational counterpart, the electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force $\mathbf{F}_e$ per unit positive charge. Thus, if the electrostatic force experienced by a small test charge q_0 is $\mathbf{F}_e$, then field strength at that point is defined as

$$\boxed{\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}_e}{q_0}}$$

The electric field is a vector quantity and its direction is the same as the direction of the force $\mathbf{F}_e$ on a positive test charge. The SI unit of electric field is N/C. Here, it should be noted that the test charge q_0 should be infinitesimally small so that it does not disturb other charges which produces **E**. With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in this field.

An Electric Field Leads to a Force

Suppose there is an electric field strength **E** at some point in an electric field, then the electrostatic force acting on a charge $+q$ is qE in the direction of **E**, while on the charge $-q$ it is qE in the opposite direction of **E**.

Example 24.10 *An electric field of 10^5 N/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of $+2\,\mu C$ and $-5\,\mu C$ at this spot?*

Solution Force on $+2\,\mu\text{C} = qE = (2\times 10^{-6})(10^5)$

$= 0.2\,\text{N}$ (due west) **Ans.**

Force on $-5\,\mu\text{C} = (5\times 10^{-6})(10^5)$

$= 0.5\,\text{N}$ (due east) **Ans.**

Electric Field Due to a Point Charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. First note that the magnitude of the force exerted by the charge q on a test charge q_0 is

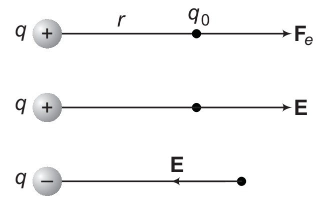


Fig. 24.10

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2}$$

then divide this value by q_0 to obtain the magnitude of the field.

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

If q is positive, $\mathbf{E}$ is directed away from q. On the other hand, if q is negative, then $\mathbf{E}$ is directed towards q.

The electric field at a point is a vector quantity. Suppose $\mathbf{E}_1$ is the field at a point due to a charge q_1 and $\mathbf{E}_2$ in the field at the same point due to a charge q_2. The resultant field when both the charges are present is

$$\boxed{\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2}$$

If the given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point.

Example 24.11 *Two positive point charges $q_1 = 16\ \mu C$ and $q_2 = 4\ \mu C$, are separated in vacuum by a distance of 3.0 m. Find the point on the line between the charges where the net electric field is zero.*

Solution Between the charges the two field contributions have opposite directions, and the net electric field is zero at a point (say P) where the magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal. However, since $q_2 < q_1$, point P must be closer to q_2, in order that the field of the smaller charge can balance the field of the larger charge.

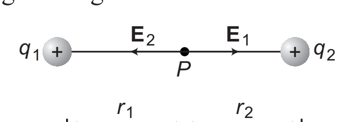


Fig. 24.11

At P, $E_1 = E_2$

or
$$\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2^2}$$

$$\therefore \qquad \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}} = \sqrt{\frac{16}{4}} = 2 \qquad \text{...(i)}$$

Also, $$r_1 + r_2 = 3.0\text{ m} \qquad \text{...(ii)}$$

Solving these equations, we get

$$r_1 = 2\text{ m} \quad \text{and} \quad r_2 = 1\text{ m}$$

Thus, the point P is at a distance of 2 m from q_1 and 1 m from q_2. **Ans.**

Electric Field of a Ring of Charge

A conducting ring of radius R has a total charge q uniformly distributed over its circumference. We are interested in finding the electric field at point P that lies on the axis of the ring at a distance x from its centre.

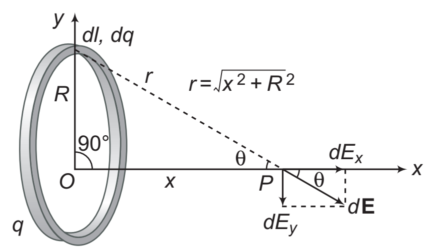


Fig. 24.12

We divide the ring into infinitesimal segments of length dl. Each segment has a charge dq and acts as a point charge source of electric field.

Let $d\mathbf{E}$ be the electric field from one such segment; the net electric field at P is then the sum of all contributions $d\mathbf{E}$ from all the segments that make up the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\mathbf{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\mathbf{E}$ will have only a component along the ring's symmetry axis (the x-axis) with no component perpendicular to that axis (i.e. no y or z-component). So, the field at P is described completely by its x-component E_x.

Calculation of E_x

$$dq = \left(\frac{q}{2\pi R}\right) \cdot dl$$

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2}$$

$$\therefore \qquad dE_x = dE\cos\theta = \left(\frac{1}{4\pi\,\varepsilon_0}\right)\left(\frac{dq}{x^2+R^2}\right)\left(\frac{x}{\sqrt{x^2+R^2}}\right)$$

$$= \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{(dq)\,x}{(x^2+R^2)^{3/2}}$$

$$\therefore \qquad E_x = \int dE_x = \frac{x}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}} \int dq$$

or

$$\boxed{E_x = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{qx}{(x^2 + R^2)^{3/2}}}$$

From the above expression, we can see that

(i) $E_x = 0$ at $x = 0$, i.e. field is zero at the centre of the ring. We should expect this, charges on opposite sides of the ring would push in opposite directions on a test charge at the centre, and the forces would add to zero.

(ii) $E_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$ for $x \gg R$, i.e. when the point P is much farther from the ring, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

(iii) E_x will be maximum where $\frac{dE_x}{dx} = 0$. Differentiating E_x w.r.t. x and putting it equal to zero we get $x = \frac{R}{\sqrt{2}}$ and E_{max} comes out to be, $\frac{2}{3\sqrt{3}}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}\right)$.

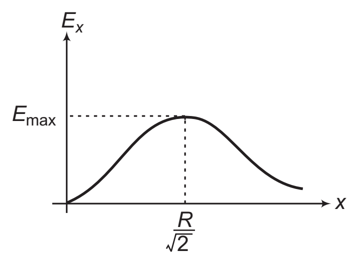


Fig. 24.13

Electric Field of a Line Charge

Positive charge q is distributed uniformly along a line with length $2a$, lying along the y-axis between $y = -a$ and $y = +a$. We are here interested in finding the electric field at point P on the x-axis.

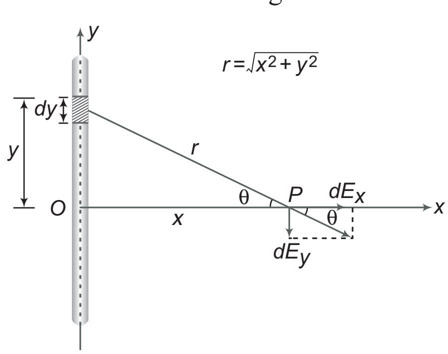


Fig. 24.14

$$\lambda = \text{charge per unit length} = \frac{q}{2a}$$

$$dq = \lambda\, dy = \frac{q}{2a}\, dy$$

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} = \frac{q}{4\pi\varepsilon_0} \frac{dy}{2a\,(x^2 + y^2)}$$

$$dE_x = dE \cos\theta = \frac{q}{4\pi\varepsilon_0} \cdot \frac{x\, dy}{2a\,(x^2 + y^2)^{3/2}}$$

$$dE_y = -\, dE \sin\theta = -\frac{q}{4\pi\varepsilon_0} \cdot \frac{y\, dy}{2a\,(x^2 + y^2)^{3/2}}$$

$\therefore$

$$E_x = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{qx}{2a} \int_{-a}^{a} \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{x\sqrt{x^2 + a^2}}$$

and

$$E_y = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2a} \int_{-a}^{a} \frac{y\, dy}{(x^2 + y^2)^{3/2}} = 0$$

Thus, electric field is along x-axis only and which has a magnitude,

$$\boxed{E_x = \frac{q}{4\pi\varepsilon_0 x\sqrt{x^2 + a^2}}} \qquad \ldots\text{(i)}$$

From the above expression, we can see that

(i) if $x >> a$, $E_x = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{x^2}$, i.e. if point P is very far from the line charge, the field at P is the same as that of a point charge.

(ii) if we make the line of charge longer and longer, adding charge in proportion to the total length so that λ, the charge per unit length remains constant. In this case, Eq. (i) can be written as

$$E_x = \frac{1}{2\pi\varepsilon_0} \cdot \left(\frac{q}{2a}\right) \cdot \frac{1}{x\sqrt{x^2/a^2 + 1}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0\, x\sqrt{x^2/a^2 + 1}}$$

Now, $x^2/a^2 \to 0$ as $a >> x$, $E_x = \dfrac{\lambda}{2\pi\varepsilon_0 x}$

Thus, the magnitude of electric field depends only on the distance of point P from the line of charge, so we can say that at any point P at a perpendicular distance r from the line in any direction, the field has magnitude

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} \qquad \text{(due to infinite line of charge)}$$

or $$E \propto \frac{1}{r}$$

Thus, E-r graph is as shown in Fig. 24.15.

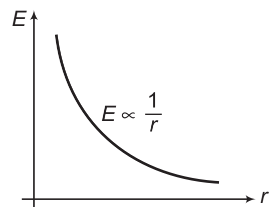


Fig. 24.15

The direction of **E** is radially outward from the line.

Note *Suppose a charge q is placed at a point whose position vector is* $\mathbf{r}_q$ *and we want to find the electric field at a point P whose position vector is* $\mathbf{r}_P$. *Then, in vector form the electric field is given by*

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}_P - \mathbf{r}_q|^3} (\mathbf{r}_P - \mathbf{r}_q)$$

Here, $$\mathbf{r}_P = x_P \hat{\mathbf{i}} + y_P \hat{\mathbf{j}} + z_P \hat{\mathbf{k}}$$

and $$\mathbf{r}_q = x_q \hat{\mathbf{i}} + y_q \hat{\mathbf{j}} + z_q \hat{\mathbf{k}}$$

In this equation, q is to be substituted with sign.

Example 24.12 *A charge* $q = 1\,\mu C$ *is placed at point* (1 *m*, 2 *m*, 4 *m*). *Find the electric field at point P* (0, – 4 *m*, 3 *m*).

Solution Here, $$\mathbf{r}_q = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

and $$\mathbf{r}_P = -4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$\therefore$ $$\mathbf{r}_P - \mathbf{r}_q = -\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

or $$|\mathbf{r}_P - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38}\text{ m}$$

Now, $$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}_P - \mathbf{r}_q|^3} (\mathbf{r}_P - \mathbf{r}_q)$$

Substituting the values, we have

$$\mathbf{E} = \frac{(9.0 \times 10^9)(1.0 \times 10^{-6})}{(38)^{3/2}} (-\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= (-38.42\,\hat{\mathbf{i}} - 230.52\,\hat{\mathbf{j}} - 38.42\,\hat{\mathbf{k}})\,\text{N/C}$$ **Ans.**

Electric Field Lines

As we have seen, electric charges create an electric field in the space surrounding them. It is useful to have a kind of "map" that gives the direction and indicates the strength of the field at various places. **Field lines,** a concept introduced by Michael Faraday, provide us with an easy way to visualize the electric field.

"An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point. The relative closeness of the lines at some place give an idea about the intensity of electric field at that point."

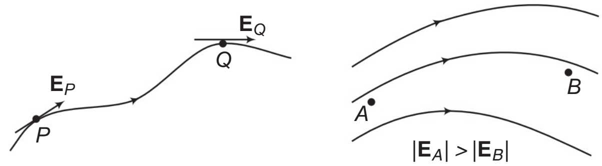


Fig. 24.16

The electric field lines have the following properties :

1. The tangent to a line at any point gives the direction of **E** at that point. This is also the path on which a positive test charge will tend to move if free to do so.
2. Electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in mid-space.
3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. This means, for example that if 100 lines are drawn leaving a $+4\,\mu C$ charge then 75 lines would have to end on a $-3\,\mu C$ charge.
4. Two lines can never intersect. If it happens then two tangents can be drawn at their point of intersection, i.e. intensity at that point will have two directions which is absurd.
5. In a uniform field, the field lines are straight parallel and uniformly spaced.

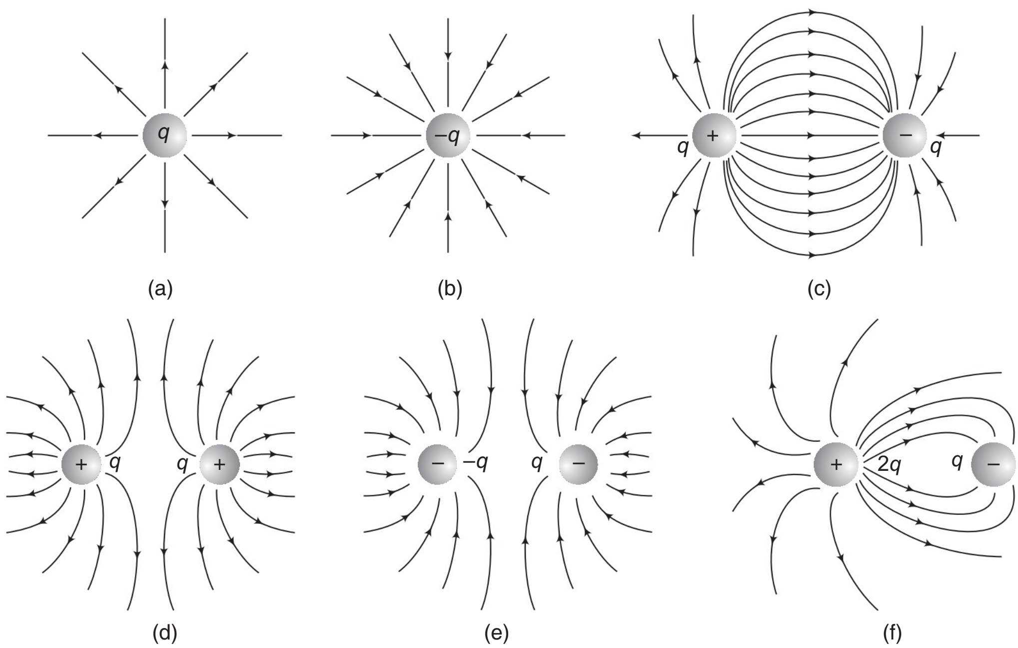


Fig. 24.17

6. The electric field lines can never form closed loops as a line can never start and end on the same charge.

7. Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)
8. Electric field lines always flow from higher potential to lower potential.
9. In a region where there is no electric field, lines are absent. This is why inside a conductor (where electric field is zero) there, cannot be any electric field line.
10. Electric lines of force ends or starts normally from the surface of a conductor.

INTRODUCTORY EXERCISE 24.3

1. The electric field of a point charge is uniform. Is it true or false?
2. Electric field lines are shown in Fig. 24.18. State whether the electric potential is greater at A or B.

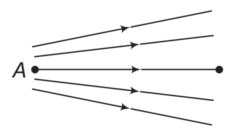


Fig. 24.18

3. A charged particle always move in the direction of electric field. Is this statement true or false?
4. The trajectory of a charged particle is the same as a field line. Is this statement true or false?
5. Figure shows some of the electric field lines due to three point charges q_1, q_2 and q_3 of equal magnitude. What are the signs of each of the three charges?

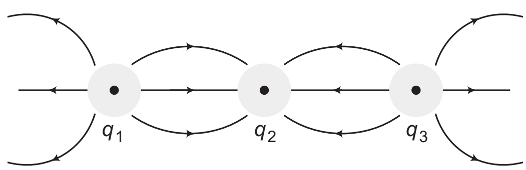


Fig. 24.19

6. Four particles each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.
7. A charge $q = -2.0\ \mu C$ is placed at origin. Find the electric field at (3 m, 4 m, 0).

24.7 Electric Potential Energy

The electric force between two charges is directed along the line of the charges and depends on the inverse square of their separation, the same as the gravitational force between two masses. Like the gravitational force, the electric force is conservative, so there is a potential energy function U associated with it.

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. This work can always be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field, when a force **F** acts on a particle that moves from point a to point b, the work $W_{a \to b}$ done by the force is given by

$$W_{a \to b} = \int_a^b \mathbf{F} \cdot d\mathbf{s} = \int_a^b F \cos\theta \, ds$$

where, $d\mathbf{s}$ is an infinitesimal displacement along the particle's path and θ is the angle between **F** and **ds** at each point along the path.

Second, if the force **F** is conservative, the work done by **F** can always be expressed in terms of a **potential energy** $\boldsymbol{U}$. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is, $\Delta U = U_b - U_a$. This is related by the work $W_{a \to b}$ as

$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \qquad \text{...(i)}$$

Here, $W_{a\to b}$ is the work done in displacing the particle from a to b by the **conservative force** (here electrostatic) not by us. Moreover we can see from Eq. (i) that if $W_{a \to b}$ is positive, ΔU is negative and the potential energy decreases. So, **whenever the work done by a conservative force is positive, the potential energy of the system decreases and *vice-versa*.** That's what happens when a particle is thrown upwards, the work done by gravity is negative, and the potential energy increases.

Example 24.13 *A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in this field from $y_i = a$ to $y_f = 2a$ along the y-axis.*

Solution Electrostatic force on the test charge,

$$F_e = q_0 E_0 \qquad \text{(along positive } y\text{-direction)}$$

$\therefore$ $$W_{i-f} = -\Delta U$$

or $$\Delta U = -W_{i-f} = -[q_0 E_0 (2a - a)]$$

$$= -q_0 E_0 a \qquad \textbf{Ans.}$$

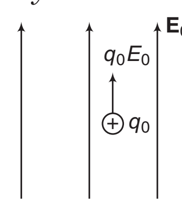


Fig. 24.20

Note *Here, work done by electrostatic force is positive. Hence, the potential energy is decreasing.*

Electric Potential Energy of Two Charges

The idea of electric potential energy is not restricted to the special case of a uniform electric field as in example 24.13. Let us now calculate the work done on a test charge q_0 moving in a non-uniform electric field caused by a single, stationary point charge q.

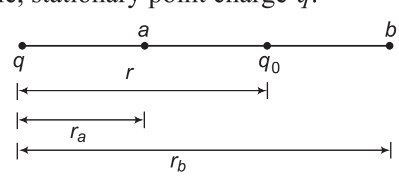


Fig. 24.21

The Coulomb's force on q_0 at a distance r from a fixed charge q is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2}$$

If the two charges have same signs, the force is repulsive and if the two charges have opposite signs, the force is attractive. The force is not constant during the displacement, so we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b.

$$\therefore \qquad W_{a\to b} = \int_{r_a}^{r_b} F\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\, dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

Being a conservative force this work is path independent. From the definition of potential energy,

$$U_b - U_a = -W_{a-b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

We choose the potential energy of the two charge system to be zero when they have infinite separation. This means $U_\infty = 0$. The potential energy when the separation is r is U_r

$$\therefore \qquad U_r - U_\infty = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{\infty}\right)$$

or

$$\boxed{U_r = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r}}$$

This is the expression for electric potential energy of two point charges kept at a separation r. In this expression both the charges q and q_0 are to be substituted with sign. The potential energy is positive if the charges q and q_0 have the same sign and negative if they have opposite signs. Note that the above equation is derived by assuming that one of the charges is fixed and the other is displaced. However, the potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles. We emphasize that the potential energy U given by the above equation is a shared property of two charges q and q_0, it is a consequence of the interaction between these two charges. If the distance between the two charges is changed from r_a to r_b, the change in the potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason we will never use the phrase 'the electric potential energy of a point charge'.

Electric Potential Energy of a System of Charges

The electric potential energy of a system of charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

This sum extends over all pairs of charges. We don't let $i = j$, because that would be an interaction of a charge with itself, and we include only terms with $i < j$ to make sure that we count each pair only once.

Thus, to account for the interaction between q_5 and q_4, we include a term with $i = 4$ and $j = 5$ but not a term with $i = 5$ and $j = 4$.

For example, electric potential energy of four point charges q_1, q_2, q_3 and q_4 would be given by

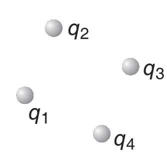


Fig. 24.22

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_4q_3}{r_{43}}+\frac{q_4q_2}{r_{42}}+\frac{q_4q_1}{r_{41}}+\frac{q_3q_2}{r_{32}}+\frac{q_3q_1}{r_{31}}+\frac{q_2q_1}{r_{21}}\right] \qquad \ldots\text{(ii)}$$

Here, all the charges are to be substituted with sign.

Note *Total number of pairs formed by n point charges are* $\frac{n(n-1)}{2}$.

Example 24.14 *Four charges $q_1 = 1\,\mu C$, $q_2 = 2\,\mu C$, $q_3 = -3\,\mu C$ and $q_4 = 4\,\mu C$ are kept on the vertices of a square of side 1m. Find the electric potential energy of this system of charges.*

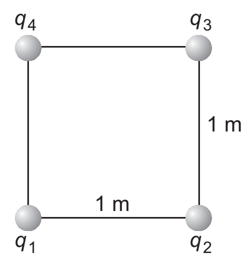


Fig. 24.23

Solution In this problem,

$$r_{41} = r_{43} = r_{32} = r_{21} = 1\,\text{m}$$

and
$$r_{42} = r_{31} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}\,\text{m}$$

Substituting the proper values with sign in Eq. (ii), we get

$$U = (9.0 \times 10^9)(10^{-6})(10^{-6})\left[\frac{(4)(-3)}{1} + \frac{(4)(2)}{\sqrt{2}} + \frac{(4)(1)}{1} + \frac{(-3)(2)}{1} + \frac{(-3)(1)}{\sqrt{2}} + \frac{(2)(1)}{1}\right]$$

$$= (9.0 \times 10^{-3})\left[-12 + \frac{5}{\sqrt{2}}\right]$$

$$= -7.62 \times 10^{-2}\,\text{J}$$ **Ans.**

Note *Here, negative sign of U implies that positive work has been done by electrostatic forces in assembling these charges at respective distances from infinity.*

Example 24.15 *Two point charges are located on the x-axis, $q_1 = -1\,\mu C$ at $x = 0$ and $q_2 = +1\,\mu C$ at $x = 1$ m.*

(a) Find the work that must be done by an external force to bring a third point charge $q_3 = +1\,\mu C$ from infinity to $x = 2$ m.

(b) Find the total potential energy of the system of three charges.

Solution (a) The work that must be done on q_3 by an external force is equal to the difference of potential energy U when the charge is at $x = 2\,\text{m}$ and the potential energy when it is at infinity.

$$\therefore \quad W = U_f - U_i$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} + \frac{q_2 q_1}{(r_{21})_f}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_2}{(r_{32})_i} + \frac{q_3 q_1}{(r_{31})_i} + \frac{q_2 q_1}{(r_{21})_i}\right]$$

Here,
$$(r_{21})_i = (r_{21})_f$$

and
$$(r_{32})_i = (r_{31})_i = \infty$$

$$\therefore \quad W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f}\right]$$

Substituting the values, we have

$$W = (9.0\times 10^{9})(10^{-12})\left[\frac{(1)(1)}{(1.0)}+\frac{(1)(-1)}{(2.0)}\right]$$

$$= 4.5\times 10^{-3}\ \text{J}$$ **Ans.**

(b) The total potential energy of the three charges is given by,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3 q_2}{r_{32}}+\frac{q_3 q_1}{r_{31}}+\frac{q_2 q_1}{r_{21}}\right)$$

$$= (9.0\times 10^{9})\left[\frac{(1)(1)}{(1.0)}+\frac{(1)(-1)}{(2.0)}+\frac{(1)(-1)}{(1.0)}\right](10^{-12})$$

$$= -4.5\times 10^{-3}\ \text{J}$$ **Ans.**

Example 24.16 *Two point charges $q_1 = q_2 = 2\,\mu C$ are fixed at $x_1 = +3$ m and $x_2 = -3$ m as shown in figure. A third particle of mass 1 g and charge $q_3 = -4\,\mu C$ are released from rest at $y = 4.0$ m. Find the speed of the particle as it reaches the origin.*

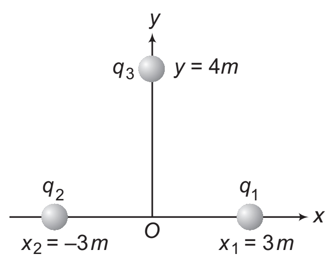


Fig. 24.24

HOW TO PROCEED *Here, the charge q_3 is attracted towards q_1 and q_2 both. So, the net force on q_3 is towards origin.*

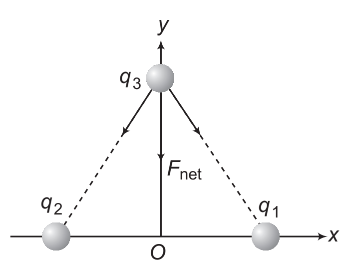


Fig. 24.25

By this force, charge is accelerated towards origin, but this acceleration is not constant. So, to obtain the speed of particle at origin by kinematics we will have to first find the acceleration at some intermediate position and then will have to integrate it with proper limits. On the other hand, it is easy to use energy conservation principle, as the only forces are conservative.

Solution Let v be the speed of particle at origin. From conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

or
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right] + 0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] + \frac{1}{2}mv^2$$

Here,
$$(r_{21})_i = (r_{21})_f$$

Substituting the proper values, we have

$$(9.0\times 10^9)\left[\frac{(-4)(2)}{(5.0)} + \frac{(-4)(2)}{(5.0)}\right]\times 10^{-12} = (9.0\times 10^9)\left[\frac{(-4)(2)}{(3.0)} + \frac{(-4)(2)}{(3.0)}\right]\times 10^{-12} + \frac{1}{2}\times 10^{-3}\times v^2$$

∴
$$(9\times 10^{-3})\left(-\frac{16}{5}\right) = (9\times 10^{-3})\left(-\frac{16}{3}\right) + \frac{1}{2}\times 10^{-3}\times v^2$$

$$(9\times 10^{-3})(16)\left(\frac{2}{15}\right) = \frac{1}{2}\times 10^{-3}\times v^2$$

∴
$$v = 6.2 \text{ m/s}$$
Ans.

INTRODUCTORY EXERCISE 24.4

1. A point charge $q_1 = 1.0\,\mu\text{C}$ is held fixed at origin. A second point charge $q_2 = -2.0\,\mu\text{C}$ and a mass 10^{-4} kg is placed on the x-axis, 1.0 m from the origin. The second point charge is released from rest. What is its speed when it is 0.5 m from the origin?
2. A point charge $q_1 = -1.0\,\mu\text{C}$ is held stationary at the origin. A second point charge $q_2 = +2.0\,\mu\text{C}$ moves from the point (1.0 m, 0, 0) to (2.0 m, 0, 0). How much work is done by the electric force on q_2?
3. A point charge q_1 is held stationary at the origin. A second charge q_2 is placed at a point a, and the electric potential energy of the pair of charges is -6.4×10^{-8} J. When the second charge is moved to point b, the electric force on the charge does 4.2×10^{-8} J of work. What is the electric potential energy of the pair of charges when the second charge is at point b?
4. Is it possible to have an arrangement of two point charges separated by finite distances such that the electric potential energy of the arrangement is the same as if the two charges were infinitely far apart? What if there are three charges?

24.8 Electric Potential

As we have discussed in Article 24.6 that an electric field at any point can be defined in two different ways:

(i) by the field strength **E**, and
(ii) by the electric potential V at the point under consideration.

Both **E** and V are functions of position and there is a fixed relationship between these two. Of these, the field strength **E** is a vector quantity while the electric potential V is a scalar quantity. In this article, we will discuss about the electric potential and in the next, the relationship between **E** and V. "**Potential** is the potential energy per unit charge." Electric potential at any point in an electric field is defined as the potential energy per unit charge, same as the field strength is defined as the force per unit charge. Thus,

$$\boxed{V = \frac{U}{q_0} \quad \text{or} \quad U = q_0 V}$$

The SI unit of potential is **volt** (V) which is equal to joule per coulomb. So,

$$1\text{ V} = 1\text{ J/C}$$

The work done by the electrostatic force in displacing a test charge q_0 from a to b in an electric field is defined as the negative of change in potential energy between them, or

$$\Delta U = -W_{a-b}$$

∴ $$U_b - U_a = -W_{a-b}$$

We divide this equation by q_0 $$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\frac{W_{a-b}}{q_0}$$

or $$V_a - V_b = \frac{W_{a-b}}{q_0}$$

as $$V = \frac{U}{q_0}$$

Thus, the work done per unit charge by the **electric force** when a charged body moves from a to b is equal to the potential at a minus the potential at b. We sometimes abbreviate this difference as $V_{ab} = V_a - V_b$.

Another way to interpret the potential difference V_{ab} is that the potential at a minus potential at b, equals the work that must be done to move a unit positive charge slowly from b to a against the electric force.

$$\boxed{V_a - V_b = \frac{(W_{b-a})_{\text{external force}}}{q_0}}$$

Absolute Potential at Some Point

Suppose we take the point b at infinity and as a reference point assign the value $V_b = 0$, the above equations can be written as

$$V_a - V_b = \frac{(W_{a-b})_{\text{electric force}}}{q_0} = \frac{(W_{b-a})_{\text{external force}}}{q_0}$$

or $$V_a = \frac{(W_{a-\infty})_{\text{electric force}}}{q_0} = \frac{(W_{\infty-a})_{\text{external force}}}{q_0}$$

Thus, the absolute electric potential at point a in an electric field can be defined as the work done in displacing a unit positive test charge from infinity to a by the external force or the work done per unit positive charge in displacing it from a to infinity.

Note *The following three formulae are very useful in the problems related to work done in electric field.*

$$(W_{a-b})_{\text{electric force}} = q_0 (V_a - V_b)$$
$$(W_{a-b})_{\text{external force}} = q_0 (V_b - V_a) = -(W_{a-b})_{\text{electric force}}$$
$$(W_{\infty-a})_{\text{external force}} = q_0 V_a$$

Here, q_0, V_a and V_b are to be substituted with sign.

Example 24.17 *The electric potential at point A is 20 V and at B is – 40 V. Find the work done by an external force and electrostatic force in moving an electron slowly from B to A.*

Solution Here, the test charge is an electron, i.e.

$$q_0 = -1.6 \times 10^{-19}\ \text{C}$$
$$V_A = 20\ \text{V}$$

and $$V_B = -40\ \text{V}$$

Work done by external force

$$(W_{B-A})_{\text{external force}} = q_0 (V_A - V_B)$$
$$= (-1.6 \times 10^{-19})[(20) - (-40)]$$
$$= -9.6 \times 10^{-18}\ \text{J}$$ **Ans.**

Work done by electric force

$$(W_{B-A})_{\text{electric force}} = -(W_{B-A})_{\text{external force}}$$
$$= -(-9.6 \times 10^{-18}\ \text{J})$$
$$= 9.6 \times 10^{-18}\ \text{J}$$ **Ans.**

Note *Here, we can see that the electron (a negative charge) moves from B (lower potential) to A (higher potential) and the work done by electric force is positive. Therefore, we may conclude that whenever a negative charge moves from a lower potential to higher potential work done by the electric force is positive or when a positive charge moves from lower potential to higher potential the work done by the electric force is negative.*

Example 24.18 *Find the work done by some external force in moving a charge $q = 2\ \mu C$ from infinity to a point where electric potential is 10^4 V.*

Solution Using the relation,

$$(W_{\infty-a})_{\text{external force}} = qV_a$$

We have, $$(W_{\infty-a})_{\text{external force}} = (2 \times 10^{-6})(10^4)$$
$$= 2 \times 10^{-2}\ \text{J}$$ **Ans.**

Electric Potential Due to a Point Charge *q*

From the definition of potential, $$V = \frac{U}{q_0} = \frac{\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q q_0}{r}}{q_0}$$

or $$\boxed{V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}}$$

Here, r is the distance from the point charge q to the point at which the potential is evaluated.

If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$.

Electric Potential Due to a System of Charges

Just as the electric field due to a collection of point charges is the vector sum of the fields produced by each charge, the electric potential due to a collection of point charges is the scalar sum of the potentials due to each charge.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}}$$

In this expression, r_i is the distance from the i^{th} charge, q_i, to the point at which V is evaluated. For a continuous distribution of charge along a line, over a surface or through a volume, we divide the charge into elements dq and the sum in the above equation becomes an integral,

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}}$$

Note *In the equation* $V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$ *or* $V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$, *if the whole charge is at equal distance* r_0 *from the point where V is to be evaluated, then we can write,*

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_{net}}{r_0}$$

where, q_{net} *is the algebraic sum of all the charges of which the system is made.*

Here there are few examples :

Example (i) *Four charges are placed on the vertices of a square as shown in figure. The electric potential at centre of the square is zero as all the charges are at same distance from the centre and*

$$q_{net} = 4\,\mu C - 2\,\mu C + 2\,\mu C - 4\,\mu C = 0$$

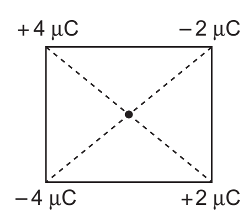


Fig. 24.26

Example (ii) *A charge q is uniformly distributed over the circumference of a ring in Fig. (a) and is non-uniformly distributed in Fig. (b).*

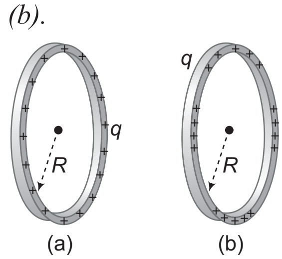


Fig. 24.27

The electric potential at the centre of the ring in both the cases is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \quad \text{(where, } R = \text{radius of ring)}$$

and at a distance r from the centre of ring on its axis would be

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{R^2 + r^2}}$$

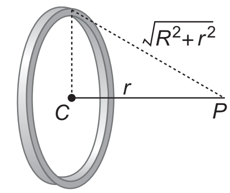


Fig. 24.28

Example 24.19 *Three point charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed at (1 m, 0, 0), (0, 2 m, 0) and (0, 0, 3 m) respectively. Find the electric potential at origin.*

Solution The net electric potential at origin is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

Substituting the values, we have

$$V = (9.0 \times 10^9)\left(\frac{1}{1.0} - \frac{2}{2.0} + \frac{3}{3.0}\right) \times 10^{-6}$$

$$= 9.0 \times 10^3 \text{ V}$$ **Ans.**

Example 24.20 *A charge $q = 10\,\mu C$ is distributed uniformly over the circumference of a ring of radius 3 m placed on x-y plane with its centre at origin. Find the electric potential at a point P (0, 0, 4 m).*

Solution The electric potential at point P would be

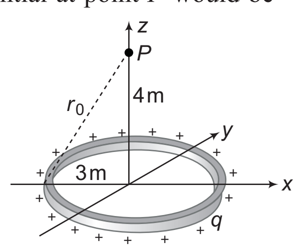


Fig. 24.29

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_0}$$

Here, r_0 = distance of point P from the circumference of ring

$$= \sqrt{(3)^2 + (4)^2} = 5 \text{ m}$$

and $q = 10\,\mu\text{C} = 10^{-5}$ C

Substituting the values, we have

$$V = \frac{(9.0 \times 10^9)(10^{-5})}{(5.0)} = 1.8 \times 10^4 \text{ V}$$ **Ans.**

Variation of Electric Potential on the Axis of a Charged Ring

We have discussed earlier that the electric potential at the centre of a charged ring (whether charged uniformly or non-uniformly) is $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$ and at a distance r from the centre on the axis of the ring is $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{R^2+r^2}}$. From these expressions, we can see that electric potential is maximum at the centre and decreases as we move away from the centre on the axis. Thus, potential varies with distance r as shown in figure.

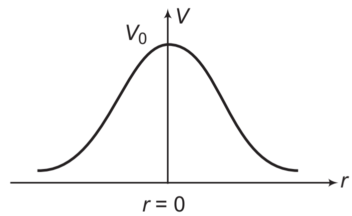


Fig. 24.30

In the figure,
$$V_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

Electric Potential on the Axis of a Uniformly Charged Disc

Let us find the electric potential at any point P, a distance x on the axis of a uniformly charged circular disc, having surface charge density σ. Let us divide the disc into a large number of thin circular strips and consider a strip of radius r and width dr. Each point of this strip can be assumed to be at equal distance $\sqrt{r^2+x^2}$ from point P. Potential at P due to this circular strip is

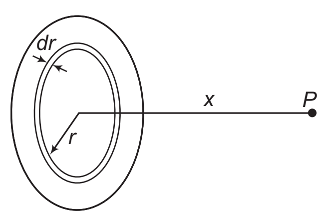


Fig. 24.31

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{\sqrt{r^2+x^2}}$$

Here, $$dq = \sigma\,(\text{area of strip}) \quad \text{or} \quad dq = \sigma\,(2\pi r dr)$$

$\therefore$ $$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma\,(2\pi r dr)}{\sqrt{r^2+x^2}}$$

Thus, the potential due to the whole disc is

$$V = \int_0^R dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r dr}{\sqrt{r^2+x^2}} \quad \text{or} \quad \boxed{V = \frac{\sigma}{2\varepsilon_0}[\sqrt{R^2+x^2}-x]}$$

(i) At the centre of the disc, $x = 0$

$$\therefore \qquad \boxed{V\,(\text{centre}) = \frac{\sigma R}{2\varepsilon_0}} \qquad \ldots\text{(i)}$$

(ii) For $x >> R$, using the Binomial expansion for

$$\sqrt{R^2 + x^2} = x\left(1 + \frac{R^2}{x^2}\right)^{1/2} \approx x + \frac{R^2}{2x}$$

$$\therefore \qquad V = \frac{\sigma}{2\varepsilon_0}\left(x + \frac{R^2}{2x} - x\right) = \frac{\sigma R^2}{4\varepsilon_0 x} = \frac{\pi R^2 \sigma}{4\pi\varepsilon_0 x}$$

$$\text{or} \qquad \boxed{V = \frac{q}{4\pi\varepsilon_0 x}}$$

as $\pi R^2 \sigma = q$, the total charge on the disc.

This is the relation as obtained due to a point charge. Thus, at far away points, the distribution of charge becomes insignificant. It is difficult to calculate the potential at the points other than on the axis. However, potential on the edge of the disc can be calculated as under.

Potential on the Edge of the Disc

To calculate the potential at point P, let us divide the disc in large number of rings with P as centre. The potential due to one segment between r and $r + dr$ is given as

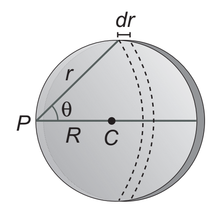


Fig. 24.32

$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r}$$

Here, $\qquad dq = \sigma\,(\text{Area of ring})$

$$= \sigma\,(2r\theta)\,dr$$

$$\therefore \qquad dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma\,(2r\theta)\,dr}{r}$$

$$= \frac{\sigma}{2\pi\varepsilon_0} \cdot \theta\,dr$$

Further, $\qquad r = 2R\cos\theta$

$$\therefore \qquad dr = -2R\sin\theta\,d\theta$$

Hence, $\qquad dV = -\dfrac{\sigma}{2\pi\varepsilon_0} 2R\theta\sin\theta\,d\theta$

$$\therefore \qquad V = \int_{\pi/2}^{0} dV = \frac{\sigma R}{\pi\varepsilon_0}\int_0^{\pi/2} \theta\sin\theta\,d\theta$$

Solving, we get $\qquad \boxed{V = \dfrac{\sigma R}{\pi\varepsilon_0}} \qquad \ldots\text{(ii)}$

Comparing Eqs. (i) and (ii), we see that potential at the centre of the disc is greater than the potential at the edge.

Example 24.21 *Find out the points on the line joining two charges + q and – 3q (kept at a distance of 1.0 m) where electric potential is zero.*

Solution Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,

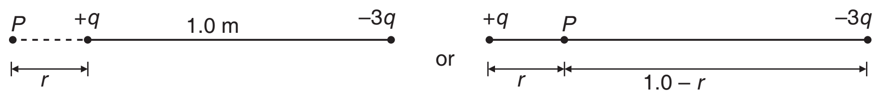


Fig. 24.33

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 (1+r)} = 0$$

Solving this, we get $r = 0.5$ m

Further,
$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 \,(1-r)} = 0$$

which gives $r = 0.25$ m

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$. **Ans.**

INTRODUCTORY EXERCISE 24.5

1. Find V_{ba} if 12 J of work has to be done against an electric field to take a charge of 10^{-2} C from a to b.
2. A rod of length L lies along the x-axis with its left end at the origin. It has a non-uniform charge density $\lambda = \alpha x$, where α is a positive constant.
   (a) What are the units of α?
   (b) Calculate the electric potential at point A where $x = -d$.
3. A charge q is uniformly distributed along an insulating straight wire of length $2l$ as shown in Fig. 24.34. Find an expression for the electric potential at a point located a distance d from the distribution along its perpendicular bisector.

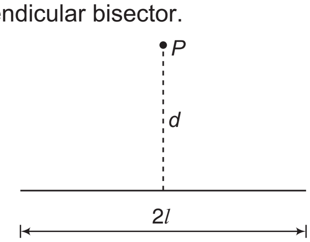


Fig. 24.34

4. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the work done in bringing a small test charge q from infinity to the apex of the cone. The cone has a slope length L.

24.9 Relation Between Electric Field and Potential

As we have discussed above, an invisible space is produced across a charge or system of charges in which any other test charge experiences an electrical force. The vector quantity related to this force is known as electric field. Further, a work is done by this electrostatic force when this test charge is moved from one point to another point. The scalar quantity related to this work done is called potential. Electric field (**E**) and potential (V) are different at different positions. So, they are functions of position.

In a cartesian coordinate system, position of a particle can be represented by three variable coordinates x, y and z. Therefore, **E** and V are functions of three variables x, y and z. In physics, we normally keep least number of variables. So, sometimes E and V are the functions of a single variable x or r. Here, x is the x-coordinate along x-axis and r normally a distance from a point charge or from the centre of a charged sphere or charged spherical shell. From the x-coordinate, we can cover only x-axis. But, from the variable r, we can cover the whole space.

Now, **E** and V functions are related to each other either by differentiation or integration. As far as differentiation is concerned, if there are more than one variables then partial differentiation is done and in case of single variable direct differentiation is required. In case of integration, some limit is required. Limit means value of the function which we get after integration should be known to us at some position. For example, after integrating **E**, we get V. So, value of V should be known at some given position. Without knowing some limit, an unknown in the form of constant of integration remains in the equation. One known limit of V is : potential is zero at infinity.

Conversion of *V* function into E function

This requires differentiation.

Case 1 **When variables are more than one**

In this case,

$$\mathbf{E} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} + E_z\,\hat{\mathbf{k}}$$

Here,

$$E_x = -\frac{\partial V}{\partial x} = -\text{ (partial derivative of } V \text{ w.r.t. } x)$$

$$E_y = -\frac{\partial V}{\partial y} = -\text{ (partial derivative of } V \text{ w.r.t. } y)$$

$$E_z = -\frac{\partial V}{\partial z} = -\text{ (partial derivative of } V \text{ w.r.t. } z)$$

$\therefore$

$$\boxed{\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]}$$

This is also sometimes written as

$$\boxed{\mathbf{E} = -\text{ gradient } V = -\text{ grad } V = -\nabla V}$$

Example 24.22 *The electric potential in a region is represented as*

$$V = 2x + 3y - z$$

obtain expression for electric field strength.

Solution
$$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

Here,
$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$$
$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$
$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$$

$\therefore$
$$\mathbf{E} = -2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$$
Ans.

Case 2 **When variable is only one** In this case, electric potential is function of only one variable (say r) and we can write the expression like :

$$\boxed{E = -\frac{dV}{dr}}$$

or
$$E = -\text{ slope of } V\text{-}r \text{ graph}$$

Example Electric potential due to a point charge q at distance r is given as

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \Rightarrow \frac{dV}{dr} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$\therefore$
$$E = -\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

and we know that this is the expression of electric field due to a point charge.

Note *E is a vector quantity. In the above method, if single variable is x and E comes out to be positive, then direction of E is towards positive x-axis. Negative value of E means direction is towards negative x-axis. If variable is r, then positive value of E means away from the point charge or away from the centre of charged spherical body and negative value of E means towards the charge or towards the centre of charged spherical body.*

Let us take an another example : We wish to find E-r graph corresponding to V-r graph shown in Fig. 24.35.

Electric field $E = -5$ V/m for $0 \le r \le 2$ m as slope of V-r graph is 5 V/m. $E = 0$ for 2 m $\le r \le$ 4 m as slope of V-r graph in this region is zero. Similarly, $E = 5$ V/m for 4 m $\le r \le$ 6 m as slope in this region is – 5 V/m.

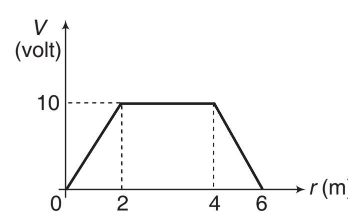


Fig. 24.35

So, the corresponding E-r graph is as shown in Fig. 24.36.

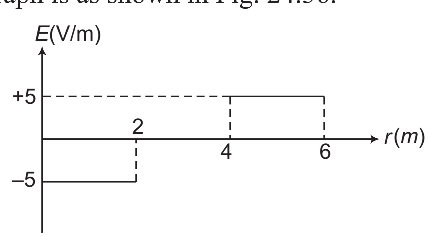


Fig. 24.36

Example 24.23 *The electric potential V at any point x, y, z (all in metre) in space is given by* $V = 4x^2$ *volt. The electric field at the point (1m, 0, 2 m) is ....V/m.* **(JEE 1992)**

Solution $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right] \Rightarrow V = 4x^2$

Therefore, $$\frac{\partial V}{\partial x} = 8x \text{ and } \frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}$$

$$\mathbf{E} = -8x\hat{\mathbf{i}}$$

or **E** at (1 m, 0, 2 m) is $-8\hat{\mathbf{i}}$ V/m.

Conversion of E into *V*

We have learnt, how to find electric field **E** from the electrostatic potential V. Let us now discuss how to calculate potential difference or absolute potential if electric field **E** is known. For this, use the relation

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

or $$\int_A^B dV = -\int_A^B \mathbf{E} \cdot d\mathbf{r}$$

or $$\boxed{V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r}}$$

Here, $$d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$$

When E is Uniform

Let us take this case with the help of an example.

Example 24.24 *Find* V_{ab} *in an electric field* $\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\frac{N}{C}$,

where $\mathbf{r}_a = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\ m$ *and* $\mathbf{r}_b = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})\ m$

Solution Here, the given field is uniform (constant). So using,

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

or
$$V_{ab} = V_a - V_b = -\int_b^a \mathbf{E} \cdot d\mathbf{r}$$
$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$
$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\,dx + 3\,dy + 4\,dz)$$
$$= -[2x + 3y + 4z]_{(2,1,-2)}^{(1,-2,1)}$$
$$= -1\text{ V}$$ **Ans.**

Note *In uniform electric field, we can also apply* $\boxed{V = Ed}$
Here, V is the potential difference between any two points, E is the magnitude of uniform electric field and d is the projection of the distance between two points along the electric field.

For example, in the figure for finding the potential difference between points A and B we will have to keep two points in mind,

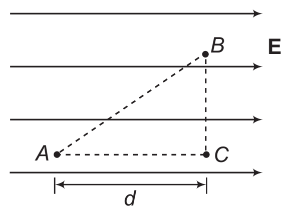


Fig. 24.37

(i) $V_A > V_B$ as electric lines always flow from higher potential to lower potential.

(ii) $d \neq AB$ but $d = AC$

Hence, in the above figure, $\quad V_A - V_B = Ed$

Example 24.25 *In uniform electric field* $\mathbf{E} = 10\ N/C$, *find*

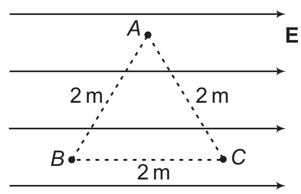


Fig. 24.38

(a) $V_A - V_B$ $\qquad$ *(b)* $V_B - V_C$

Solution (a) $V_B > V_A$, So, $V_A - V_B$ will be negative.

Further $d_{AB} = 2\cos 60° = 1\,\text{m}$

$\therefore \qquad V_A - V_B = -Ed_{AB} = (-10)(1) = -10$ volt **Ans.**

(b) $V_B > V_C$, so $V_B - V_C$ will be positive.

Further, $\qquad d_{BC} = 2.0\,\text{m}$

$\therefore \qquad V_B - V_C = (10)(2) = 20$ volt **Ans.**

Example 24.26 *A uniform electric field of* 100 V/m *is directed at* 30° *with the positive x-axis as shown in figure. Find the potential difference* V_{BA} *if* $OA = 2$ *m and* $OB = 4$ *m.*

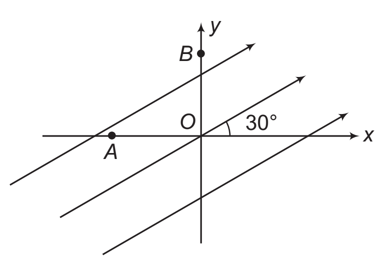


Fig. 24.39

Solution This problem can be solved by both the methods discussed above.

Method 1. Electric field in vector form can be written as

$$\mathbf{E} = (100\cos 30^\circ\,\hat{\mathbf{i}} + 100\sin 30^\circ\,\hat{\mathbf{j}})\ \text{V/m}$$

$$= (50\sqrt{3}\,\hat{\mathbf{i}} + 50\hat{\mathbf{j}})\ \text{V/m}$$

$$A \equiv (-2m, 0, 0)$$

and $$B \equiv (0, 4m, 0)$$

$\therefore$ $$V_{BA} = V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{r}$$

$$= -\int_{(-2m,\,0,\,0)}^{(0,\,4m,\,0)} (50\sqrt{3}\,\hat{\mathbf{i}} + 50\,\hat{\mathbf{j}})\cdot(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$

$$= -[50\sqrt{3}\,x + 50\,y]_{(-2m,\,0,\,0)}^{(0,\,4m,\,0)}$$

$$= -100\,(2+\sqrt{3})\ \text{V}$$ **Ans.**

Method 2. We can also use, $$V = Ed$$

With the view that $V_A > V_B$ or $V_B - V_A$ will be negative.

Here, $$d_{AB} = OA\cos 30^\circ + OB\sin 30^\circ$$

$$= 2\times\frac{\sqrt{3}}{2} + 4\times\frac{1}{2} = (\sqrt{3}+2)$$

$\therefore$ $$V_B - V_A = -Ed_{AB} = -100\,(2+\sqrt{3})$$ **Ans.**

Example 24.27 *A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then, the potentials at the points A, B and C satisfy* **(JEE 2001)**

(a) $V_A < V_B$ *(b)* $V_A > V_B$

(c) $V_A < V_C$ *(d)* $V_A > V_C$

Solution Potential decreases in the direction of electric field. Dotted lines are equipotential lines.

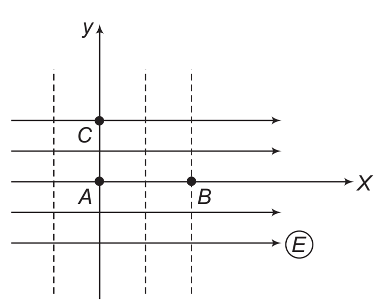


Fig. 24.40

$\therefore$ $$V_A = V_C \quad \text{and} \quad V_A > V_B$$

Hence, the correct option is (b).

Example 24.28 *A non-conducting ring of radius* 0.5 *m carries a total charge of* 1.11×10^{-10} *C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral* $\int_{l=\infty}^{l=0} -\mathbf{E}\cdot d\mathbf{l}$ *($l=0$ being centre of the ring) in volt is* **(JEE 1997)**

(a) + 2 (b) − 1 (c) − 2 (d) zero

Solution $-\int_{l=\infty}^{l=0}\mathbf{E}\cdot d\mathbf{l}=\int_{l=\infty}^{l=0} dV = V\text{ (centre)} - V\text{ (infinity)}$

but $V\text{ (infinity)} = 0$

$\therefore$ $-\int_{l=\infty}^{l=0}\mathbf{E}\cdot d\mathbf{l}$ corresponds to potential at centre of ring.

and $$V\text{ (centre)}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}=\frac{(9\times10^{9})(1.11\times10^{-10})}{0.5}\approx 2\,\text{V}$$

Therefore, the correct answer is (a).

INTRODUCTORY EXERCISE 24.6

1. Determine the electric field strength vector if the potential of this field depends on x, y coordinates as

   (a) $V = a(x^2 - y^2)$ (b) $V = axy$

   where, a is a constant.

2. The electrical potential function for an electrical field directed parallel to the x-axis is shown in the given graph.

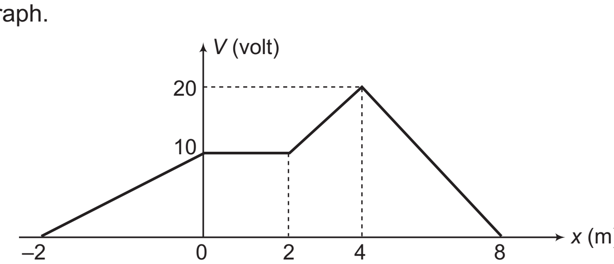


Fig. 24.41

   Draw the graph of electric field strength.

3. The electric potential decreases uniformly from 100 V to 50 V as one moves along the x-axis from $x = 0$ to $x = 5$ m. The electric field at $x = 2$ m must be equal to 10 V/m. Is this statement true or false.

4. In the uniform electric field shown in figure, find :

   (a) $V_A - V_D$
   (b) $V_A - V_C$
   (c) $V_B - V_D$
   (d) $V_C - V_D$

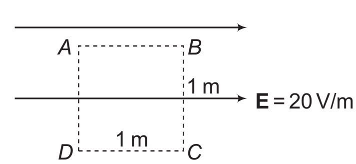


Fig. 24.42

24.10 Equipotential Surfaces

The equipotential surfaces in an electric field have the same basic idea as topographic maps used by civil engineers or mountain climbers. On a topographic map, contour lines are drawn passing through the points having the same elevation. The potential energy of a mass m does not change along a contour line as the elevation is same everywhere.

By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the electric potential V is the same at every point on it. An equipotential surface has the following characteristics.

1. Potential difference between any two points in an equipotential surface is zero.
2. If a test charge q_0 is moved from one point to the other on such a surface, the electric potential energy q_0V remains constant.
3. No work is done by the electric force when the test charge is moved along this surface.
4. Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not possible.
5. As the work done by electric force is zero when a test charge is moved along the equipotential surface, it follows that **E** must be perpendicular to the surface at every point so that the electric force $q_0\,\mathbf{E}$ will always be perpendicular to the displacement of a charge moving on the surface. Thus, **field lines and equipotential surfaces are always mutually perpendicular.** Some equipotential surfaces are shown in Fig. 24.43.

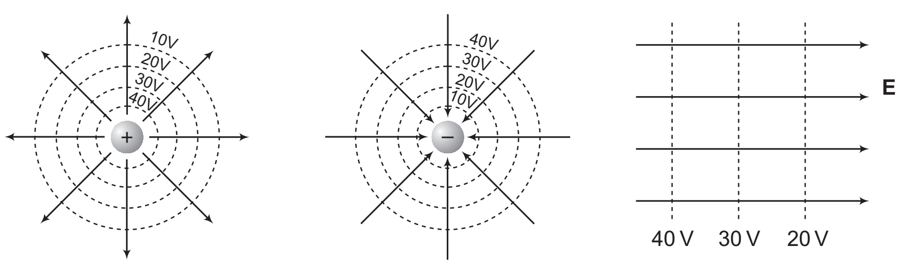


Fig. 24.43

The equipotential surfaces are a family of concentric spheres for a point charge or a sphere of charge and are a family of concentric cylinders for a line of charge or cylinder of charge. For a special case of a uniform field, where the field lines are straight, parallel and equally spaced the equipotential surfaces are parallel planes perpendicular to the field lines.

Note *While drawing the equipotential surfaces we should keep in mind the two main points.*
(i) These are perpendicular to field lines at all places.
(ii) Field lines always flow from higher potential to lower potential.

Example 24.29 *Equipotential spheres are drawn round a point charge. As we move away from the charge, will the spacing between two spheres having a constant potential difference decrease, increase or remain constant.*

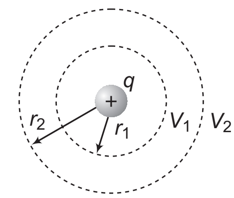


Fig. 24.44

Solution $V_1 > V_2$

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1} \quad \text{and} \quad V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_2}$$

Now, $$V_1 - V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$\therefore$ $$(r_2 - r_1) = \frac{(4\pi\varepsilon_0)(V_1 - V_2)}{q}(r_1 r_2)$$

For a constant potential difference $(V_1 - V_2)$,

$$r_2 - r_1 \propto r_1 r_2$$

i.e. the spacing between two spheres $(r_2 - r_1)$ increases as we move away from the charge, because the product $r_1 r_2$ will increase.

24.11 Electric Dipole

A pair of equal and opposite point charges $\pm q$, that are separated by a fixed distance is known as **electric dipole.** Electric dipole occurs in nature in a variety of situations. The hydrogen fluoride molecule (HF) is typical. When a hydrogen atom combines with a fluorine atom, the single electron of the former is strongly attracted to the later and spends most of its time near the fluorine atom. As a result, the molecule consists of a strongly negative fluorine ion some (small) distance away from a strongly positive ion, though the molecule is electrically neutral overall.

Every electric dipole is characterized by its **electric dipole moment** which is a vector **p** directed from the negative to the positive charge.

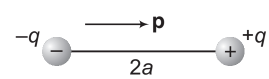


Fig. 24.45

The magnitude of dipole moment is

$$\boxed{p = (2a)\,q}$$

Here, $2a$ is the distance between the two charges.

Electric Potential and Field Due to an Electric Dipole

Consider an electric dipole lying along positive y-direction with its centre at origin.

$$\mathbf{p} = 2aq\,\hat{\mathbf{j}}$$

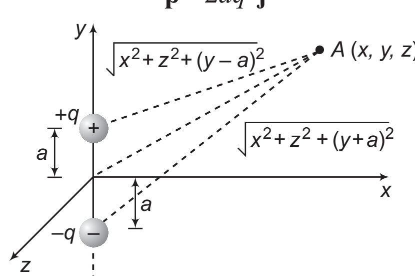


Fig. 24.46

The electric potential due to this dipole at point $A\,(x, y, z)$ as shown is simply the sum of the potentials due to the two charges. Thus,

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+(y-a)^2+z^2}} - \frac{q}{\sqrt{x^2+(y+a)^2+z^2}}\right]}$$

By differentiating this function, we obtain the electric field of the dipole.

$$E_x = -\frac{\partial V}{\partial x} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{x}{[x^2+(y-a)^2+z^2]^{3/2}} - \frac{x}{[x^2+(y+a)^2+z^2]^{3/2}}\right\}$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{y-a}{[x^2+(y-a)^2+z^2]^{3/2}} - \frac{y+a}{[x^2+(y+a)^2+z^2]^{3/2}}\right\}$$

$$E_z = -\frac{\partial V}{\partial z} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{z}{[x^2+(y-a)^2+z^2]^{3/2}} - \frac{z}{[x^2+(y+a)^2+z^2]^{3/2}}\right\}$$

Special Cases

1. **On the axis of the dipole** (say, along y-axis)

$$x = 0, \quad z = 0$$

$\therefore$
$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{y-a} - \frac{1}{y+a}\right] = \frac{2aq}{4\pi\varepsilon_0\,(y^2-a^2)}$$

or
$$V = \frac{p}{4\pi\varepsilon_0\,(y^2-a^2)} \qquad (\text{as } 2aq = p)$$

i.e. at a distance r from the centre of the dipole ($y = r$)

$$V = \frac{p}{4\pi\varepsilon_0\,(r^2-a^2)} \quad \text{or} \quad \boxed{V_{\text{axis}} \approx \frac{p}{4\pi\varepsilon_0 r^2}} \qquad (\text{for } r >> a)$$

V is positive when the point under consideration is towards positive charge and negative if it is towards negative charge.

Moreover the components of electric field are as under

$$E_x = 0, \quad E_z = 0 \qquad (\text{as } x = 0,\ z = 0)$$

and
$$E_y = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}\right]$$

$$= \frac{4ayq}{4\pi\varepsilon_0\,(y^2-a^2)^2} \quad \text{or} \quad E_y = \frac{1}{4\pi\varepsilon_0}\,\frac{2py}{(y^2-a^2)^2}$$

Note that E_y is along positive y-direction or parallel to **p**.

Further, at a distance r from the centre of the dipole ($y = r$).

$$E_y = \frac{1}{4\pi\varepsilon_0}\,\frac{2pr}{(r^2-a^2)^2} \quad \text{or} \quad \boxed{E_{\text{axis}} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}} \qquad (\text{for } r >> a)$$

2. On the perpendicular bisector of dipole

Say along x-axis (it may be along z-axis also).

$$y=0, \quad z=0$$

$\therefore$
$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+a^2}}-\frac{q}{\sqrt{x^2+a^2}}\right]=0$$

or
$$V_{\perp \text{ bisector}}=0$$

Moreover the components of electric field are as under,

$$E_x=0, \quad E_z=0$$

and
$$E_y=\frac{q}{4\pi\varepsilon_0}\left\{\frac{-a}{(x^2+a^2)^{3/2}}-\frac{a}{(x^2+a^2)^{3/2}}\right\}$$

$$=\frac{-2aq}{4\pi\varepsilon_0\,(x^2+a^2)^{3/2}}$$

or
$$E_y=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(x^2+a^2)^{3/2}}$$

Here, negative sign implies that the electric field is along negative y-direction or antiparallel to **p**.

Further, at a distance r from the centre of dipole ($x=r$), the magnitude of electric field is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}} \quad \text{or} \quad \boxed{E_{\perp \text{ bisector}}\approx\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}} \qquad (\text{for } r >> a)$$

Electric Dipole in Uniform Electric Field

As we have said earlier also, uniform electric field means, at every point the direction and magnitude of electric field is constant. A uniform electric field is shown by parallel equidistant lines. The field due to a point charge or due to an electric dipole is non-uniform in nature. Uniform electric field is found between the plates of a parallel plate capacitor. Now, let us discuss the behaviour of a dipole in uniform electric field.

Force on Dipole

Suppose an electric dipole of dipole moment $|\mathbf{p}|=2aq$ is placed in a uniform electric field **E** at an angle θ. Here, θ is the angle between **p** and **E**. A force $\mathbf{F}_1=q\mathbf{E}$ will act on positive charge and $\mathbf{F}_2=-q\mathbf{E}$ on negative charge. Since, $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in magnitude but opposite in direction.

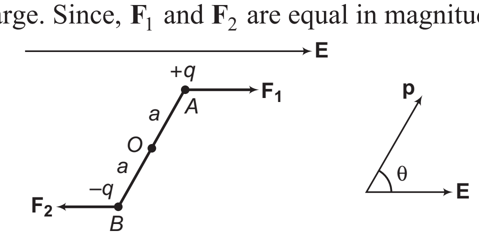


Fig. 24.47

Hence, $\mathbf{F}_1 + \mathbf{F}_2 = 0$ or $\mathbf{F}_{net} = 0$

Thus, net force on a dipole in uniform electric field is zero. While in a non-uniform electric field it may or may not be zero.

Torque on Dipole

The torque of $\mathbf{F}_1$ about O, $\tau_1 = \mathbf{OA} \times \mathbf{F}_1 = q\,(\mathbf{OA} \times \mathbf{E})$

and torque of $\mathbf{F}_2$ about O is, $\tau_2 = \mathbf{OB} \times \mathbf{F}_2 = -\,q(\mathbf{OB} \times \mathbf{E})$

$$= q(\mathbf{BO} \times \mathbf{E})$$

The net torque acting on the dipole is

$$\tau = \tau_1 + \tau_2 = q\,(\mathbf{OA} \times \mathbf{E}) + q\,(\mathbf{BO} \times \mathbf{E})$$

$$= q\,(\mathbf{OA} + \mathbf{BO}) \times \mathbf{E}$$

$$= q\,(\mathbf{BA} \times \mathbf{E})$$

or

$$\boxed{\tau = \mathbf{p} \times \mathbf{E}}$$

Thus, the magnitude of torque is $\tau = pE \sin\theta$. The direction of torque is perpendicular to the plane of paper inwards. Further this torque is zero at $\theta = 0°$ or $\theta = 180°$, i.e. when the dipole is parallel or antiparallel to $\mathbf{E}$ and maximum at $\theta = 90°$.

Potential Energy of Dipole

When an electric dipole is placed in an electric field $\mathbf{E}$, a torque $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. If we rotate the dipole through a small angle $d\theta$, the work done by the torque is

$$dW = \tau\, d\theta$$

$$dW = -\,pE \sin\theta\, d\theta$$

The work is negative as the rotation $d\theta$ is opposite to the torque. The change in electric potential energy of the dipole is therefore

$$dU = -\,dW = pE \sin\theta\, d\theta$$

Now, at angle $\theta = 90°$, the electric potential energy of the dipole may be assumed to be zero as net work done by the electric forces in bringing the dipole from infinity to this position will be zero.

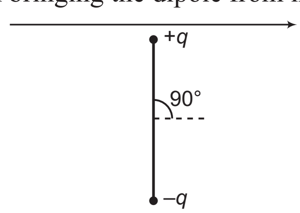


Fig. 24.48

Integrating, $dU = pE \sin\theta\, d\theta$

From 90° to θ, we have $\int_{90°}^{\theta} dU = \int_{90°}^{\theta} pE \sin\theta\, d\theta$

or $U(\theta) - U(90°) = pE\,[-\cos\theta]_{90°}^{\theta}$

∴ $\boxed{U(\theta) = -\,pE\cos\theta = -\,\mathbf{p} \cdot \mathbf{E}}$

If the dipole is rotated from an angle θ_1 to θ_2, then

$$\text{Work done by external forces} = U(\theta_2) - U(\theta_1)$$

or
$$W_{\text{ext. forces}} = -pE\cos\theta_2 - (-pE\cos\theta_1)$$

or
$$\boxed{W_{\text{ext. forces}} = pE(\cos\theta_1 - \cos\theta_2)}$$

and work done by electric forces,

$$\boxed{W_{\text{electric force}} = -W_{\text{ext. force}} = pE(\cos\theta_2 - \cos\theta_1)}$$

Equilibrium of Dipole

When an electric dipole is placed in a uniform electric field net force on it is zero for any position of the dipole in the electric field. But torque acting on it is zero only at $\theta = 0°$ and $180°$. Thus, we can say that at these two positions of the dipole, net force and torque on it is zero or the dipole is in equilibrium

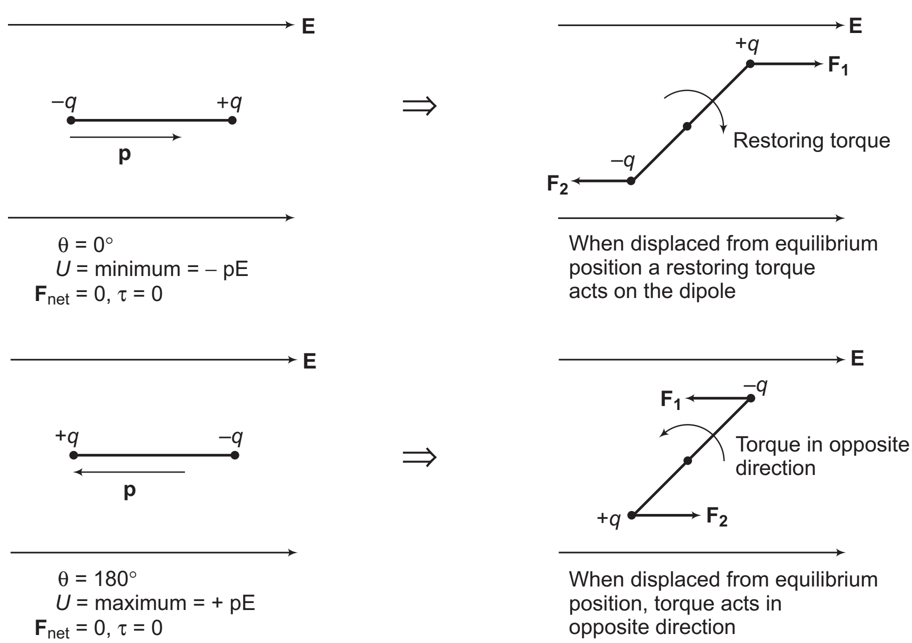


Fig. 24.49

Of this, $\theta = 0°$ is the **stable equilibrium position** of the dipole because potential energy in this position is minimum ($U = -pE\cos 0° = -pE$) and when displaced from this position a torque starts acting on it which is restoring in nature and which has a tendency to bring the dipole back in its equilibrium position. On the other hand, at $\theta = 180°$, the potential energy of the dipole is maximum ($U = -pE\cos 180° = +pE$) and when it is displaced from this position, the torque has a tendency to rotate it in other direction. This torque is not restoring in nature. So, this equilibrium is known as **unstable equilibrium position.**

Important Formulae

1. As there are too many formulae in electric dipole, we have summarised them as under :

$$|\mathbf{p}| = (2a)q$$

Direction of $\mathbf{p}$ is from $-q$ to $+q$.

2. If a dipole is placed along y-axis with its centre at origin, then

$$V(x, y, z) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + (y-a)^2 + z^2}} - \frac{q}{\sqrt{x^2 + (y+a)^2 + z^2}}\right]$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}$$

and $$E_z = -\frac{\partial V}{\partial z}$$

3. On the axis of dipole $x = 0, \; z = 0$

(i) $$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{(y^2 - a^2)}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2 - a^2} \qquad \text{if } y = r$$

or $$V_{\text{axis}} \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \qquad \text{if } r >> a$$

(ii) $$E_x = 0 = E_z \text{ and}$$

$$E = E_y = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2py}{(y^2 - a^2)^2} \qquad (\text{along } \mathbf{p})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - a^2)^2} \qquad \text{if } y = r$$

or $$E_{\text{axis}} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \qquad \text{for } r >> a$$

4. On the perpendicular bisector of dipole Along x-axis, $y = 0, \; z = 0$

(i) $$V_{\perp \text{ bisector}} = 0$$

(ii) $$E_x = 0, \quad E_z = 0 \text{ and}$$

$$E_y = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(x^2 + a^2)^{3/2}}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + a^2)^{3/2}} \qquad (\text{opposite to } \mathbf{p})$$

$$\approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3} \qquad \text{for } r >> a$$

5. Dipole in uniform electric field

(i) $\mathbf{F}_{\text{net}} = 0$

(ii) $\tau = \mathbf{p} \times \mathbf{E}$ and $|\tau| = pE\sin\theta$

(iii) $U(\theta) = -\mathbf{p}\cdot\mathbf{E} = -pE\cos\theta$ with $U(90°) = 0$

(iv) $(W_{\theta_1 \to \theta_2})_{\text{ext. force}} = pE(\cos\theta_1 - \cos\theta_2)$

(v) $(W_{\theta_1 \to \theta_2})_{\text{electric force}} = pE(\cos\theta_2 - \cos\theta_1) = -(W_{\theta_1 \to \theta_2})_{\text{ext. force}}$

(vi) At $\theta = 0°$, $\mathbf{F}_{\text{net}} = 0$, $\tau_{\text{net}} = 0$, U = minimum (stable equilibrium position)

(vii) At $\theta = 180°$, $\mathbf{F}_{\text{net}} = 0$, $\tau_{\text{net}} = 0$, U = maximum (unstable equilibrium position)

Example 24.30 *Draw electric lines of forces due to an electric dipole.*

Solution Electric lines of forces due to an electric dipole are as shown in figure.

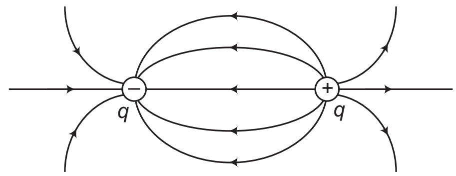


Fig. 24.50

Example 24.31 *Along the axis of a dipole, direction of electric field is always in the direction of electric dipole moment* **p**. *Is this statement true or false?*

Solution False. In the above figure, we can see that direction of electric field is in the opposite direction of **p** between the two charges.

Example 24.32 *At a far away distance r along the axis from an electric dipole electric field is* **E**. *Find the electric field at distance* $2r$ *along the perpendicular bisector.*

Solution Along the axis of dipole,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \qquad \text{...(i)}$$

This electric field is in the direction of **p**. Along the perpendicular bisector at a distance $2r$,

$$E' = \frac{1}{4\pi\varepsilon_0}\frac{p}{(2r)^3} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we can see that

$$E' = \frac{E}{16}$$

Moreover, E' is in the opposite direction of **p**. Hence,

$$\mathbf{E}' = -\frac{\mathbf{E}}{16}$$ **Ans.**

24.12 Gauss's Law

Gauss's law is a tool of simplifying electric field calculations where there is symmetrical distribution of charge. Many physical systems have symmetry, for example a cylindrical body doesn't look any different if we rotate it around its axis.

Before studying the detailed discussion of Gauss's law let us understand electric flux.

Electric Flux (ϕ)

(i) Electric flux is a measure of the field lines crossing a surface.

(ii) It is a scalar quantity with SI units $\frac{\text{N}}{\text{C}}\text{-}\,\text{m}^2$ or V- m.

(iii) Electric flux passing through a small surface dS is given by

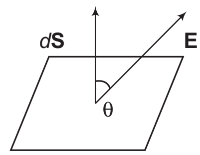


Fig. 24.51

$$\boxed{d\phi = \mathbf{E} \cdot d\mathbf{S} = E\, dS \cos\theta} \qquad \ldots\text{(i)}$$

Here, $d\mathbf{S}$ is an area vector, whose magnitude is equal to dS and whose direction is perpendicular to the surface.

Note *If the surface is open, then d**S** can be taken in either of the two directions perpendicular to the surface, but it should not change even if we rotate the surface.*

*If the surface is closed then by convention, d**S** is normally taken in outward direction.*

(iv) From Eq. (i), we can see that maximum value of $d\phi$ is $E\,dS$, if $\theta = 90°$ or electric lines are perpendicular to the surface. Electric flux is zero, if $\theta = 90°$ or electric lines are tangential to the surface.

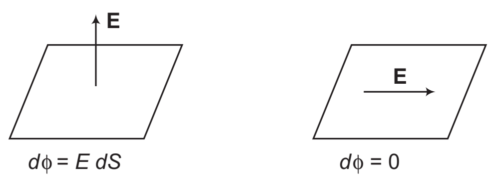


Fig. 24.52

(v) Electric flux passing through a large surface is given by

$$\boxed{\phi = \int d\phi = \int \mathbf{E} \cdot d\mathbf{S} = \int E\, dS \cos\theta} \qquad \ldots\text{(ii)}$$

This is basically surface integral of electric flux over the given surface. But normally we do not study surface integral in detail in physics.

Here, are two special cases for calculating the electric flux passing through a surface S of finite size (whether closed or open)

Case 1 $\boxed{\phi = ES}$

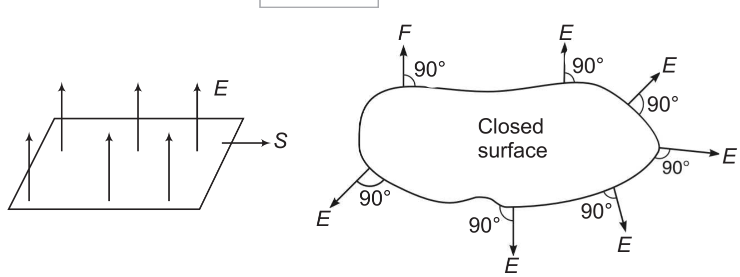


Fig. 24.53

If at every point on the surface, the magnitude of electric field is constant and perpendicular (to the surface).

Case 2 $\boxed{\phi = 0}$

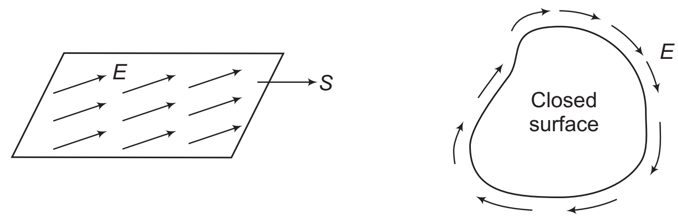


Fig. 24.54

If at all points on the surface the electric field is tangential to the surface.

Gauss's Law

This law gives a relation between the net electric flux through a closed surface and the charge enclosed by the surface. According to this law,

"the net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0." In symbols, it can be written as

$$\boxed{\phi_e = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0}} \qquad \text{...(i)}$$

where, q_{in} represents the net charge inside the closed surface and $\mathbf{E}$ represents the electric field at any point on the surface.

In principle, Gauss's law is valid for the electric field of any system of charges or continuous distribution of charge. In practice however, the technique is useful for calculating the electric field only in situations where the degree of symmetry is high. Gauss's law can be used to evaluate the electric field for charge distributions that have spherical, cylindrical or plane symmetry.

Simplified Form of Gauss's Theorem

Gauss's law in simplified form can be written as under

$$\boxed{ES = \frac{q_{\text{in}}}{\varepsilon_0} \quad \text{or} \quad E = \frac{q_{\text{in}}}{S\varepsilon_0}} \qquad \text{...(ii)}$$

but this form of Gauss's law is applicable only under the following two conditions :

(i) The electric field at every point on the surface is either perpendicular or tangential.

(ii) Magnitude of electric field at every point where it is perpendicular to the surface has a constant value (say E).

Here, S is the area where electric field is perpendicular to the surface.

Applications of Gauss's Law

As Gauss's law does not provide expression for electric field but provides only for its flux through a closed surface. To calculate E we choose an imaginary closed surface (called Gaussian surface) in which Eq. (ii) can be applied easily. Let us discuss few simple cases.

Electric field due to a point charge

The electric field due to a point charge is everywhere radial. We wish to find the electric field at a distance r from the charge q. We select Gaussian surface, a sphere at distance r from the charge. At every point of this sphere the electric field has the same magnitude E and it is perpendicular to the surface itself. Hence, we can apply the simplified form of Gauss's law,

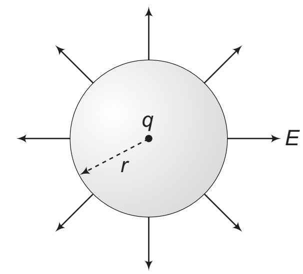


Fig. 24.55

$$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, S = area of sphere = $4\pi r^2$ and

q_{in} = net charge enclosing the Gaussian surface = q

$\therefore$ $$E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$\therefore$ $$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

It is nothing but Coulomb's law.

Electric field due to a linear charge distribution

Consider a long line charge with a linear charge density (charge per unit length) λ. We have to calculate the electric field at a point, a distance r from the line charge. We construct a Gaussian surface, a cylinder of any arbitrary length l of radius r and its axis coinciding with the axis of the line charge. This cylinder have three surfaces. One is curved surface and the two plane parallel surfaces. Field lines at plane parallel surfaces are tangential (so flux passing through these surfaces is zero). The magnitude of electric field is having the same magnitude (say E) at curved surface and simultaneously the electric field is perpendicular at every point of this surface.

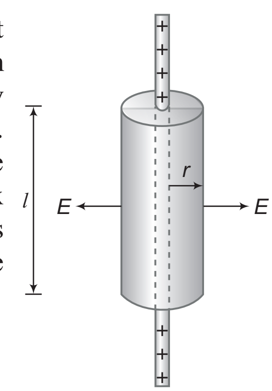


Fig. 24.56

Hence, we can apply the Gauss's law as

$$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, S = area of curved surface = $(2\pi rl)$

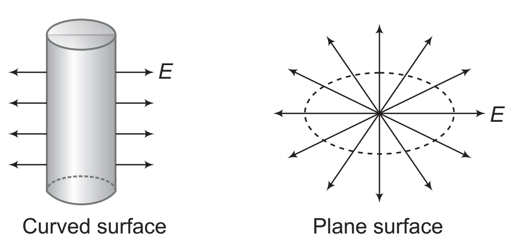


Fig. 24.57

and $\quad q_{in}$ = net charge enclosing this cylinder = λl

$\therefore \quad E\,(2\pi r l) = \dfrac{\lambda l}{\varepsilon_0}$

$\therefore \quad \boxed{E = \dfrac{\lambda}{2\pi\varepsilon_0 r}}$

i.e. $\quad E \propto \dfrac{1}{r}$

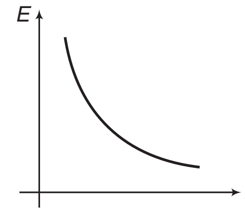


Fig. 24.58

or $\quad$ *E-r* graph is a rectangular hyperbola as shown in Fig. 24.58.

Electric field due to a plane sheet of charge

Figure shows a portion of a flat thin sheet, infinite in size with constant surface charge density σ (charge per unit area). By symmetry, since the sheet is infinite, the field must have the same magnitude and the opposite directions at two points equidistant from the sheet on opposite sides. Let us draw a Gaussian surface (a cylinder) with one end on one side and other end on the other side and of cross-sectional area S_0. Field lines will be tangential to the curved surface, so flux passing through this surface is zero. At plane surfaces electric field has same magnitude and perpendicular to surface.

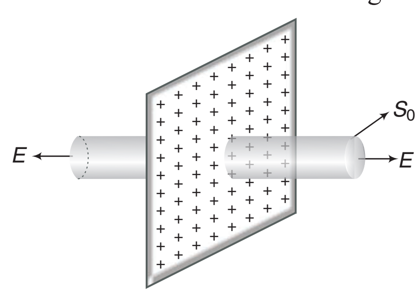


Fig. 24.59

Hence, using $\quad ES = \dfrac{q_{in}}{\varepsilon_0}$

$\therefore \quad E\,(2S_0) = \dfrac{(\sigma)\,(S_0)}{\varepsilon_0}$

$\therefore \quad \boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Thus, we see that the magnitude of the field is independent of the distance from the sheet. Practically, an infinite sheet of charge does not exist. This result is correct for real charge sheets if points under consideration are not near the edges and the distances from the sheet are small compared to the dimensions of sheet.

Electric field near a charged conducting surface

When a charge is given to a conducting plate, it distributes itself over the entire outer surface of the plate. The surface density σ is uniform and is the same on both surfaces if plate is of uniform thickness and of infinite size.

This is similar to the previous one the only difference is that this time charges are on both sides.

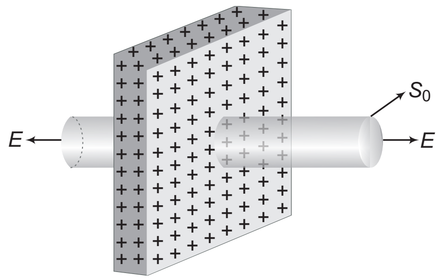


Fig. 24.60

Hence, applying $$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, $$S = 2S_0 \quad \text{and} \quad q_{in} = (\sigma)(2S_0)$$

$\therefore$ $$E(2S_0) = \frac{(\sigma)(2S_0)}{\varepsilon_0}$$

$\therefore$ $$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Thus, field due to a charged conducting plate is twice the field due to plane sheet of charge. It also has same limitations.

Later, we will see that the electric field near a charged conducting surface of any shape is σ/ε_0 and it is normal to the surface.

Note *In case of closed symmetrical body with charge q at its centre, the electric flux linked with each half will be* $\frac{\phi}{2} = \frac{q}{2\varepsilon_0}$*. If the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be* $\frac{\phi}{n} = \frac{q}{n\,\varepsilon_0}$.

Extra Points to Remember

- Net electric flux passing through a closed surface in uniform electric field is zero.

Example 24.33 *An electric dipole is placed at the centre of a sphere. Find the electric flux passing through the sphere.*

Solution Net charge inside the sphere $q_{in} = 0$. Therefore, according to Gauss's law net flux passing through the sphere is zero. **Ans.**

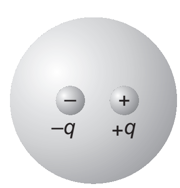


Fig. 24.61

Example 24.34 *A point charge q is placed at the centre of a cube. What is the flux linked*

(a) with all the faces of the cube?

(b) with each face of the cube?

(c) if charge is not at the centre, then what will be the answers of parts (a) and (b)?

Solution (a) According to Gauss's law,

$$\phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{q}{\varepsilon_0} \qquad \textbf{Ans.}$$

(b) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be

$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\varepsilon_0} \qquad \textbf{Ans.}$$

(c) If charge is not at the centre, the answer of part (a) will remain same while that of part (b) will change.

INTRODUCTORY EXERCISE 24.7

1. In figure (a), a charge q is placed just outside the centre of a closed hemisphere. In figure (b), the same charge q is placed just inside the centre of the closed hemisphere and in figure (c), the charge is placed at the centre of hemisphere open from the base. Find the electric flux passing through the hemisphere in all the three cases.

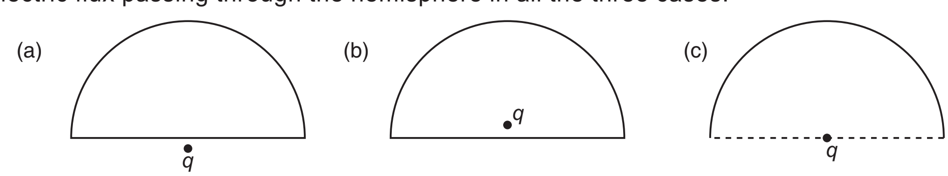


Fig. 24.62

2. Net charge within an imaginary cube drawn in a uniform electric field is always zero. Is this statement true or false?
3. A hemispherical body of radius R is placed in a uniform electric field E. What is the flux linked with the curved surface if, the field is (a) parallel to the base, (b) perpendicular to the base.
4. A cube has sides of length $L = 0.2$ m. It is placed with one corner at the origin as shown in figure. The electric field is uniform and given by $\mathbf{E} = (2.5 \text{ N/C})\,\hat{\mathbf{i}} - (4.2 \text{ N/C})\,\hat{\mathbf{j}}$. Find the electric flux through the entire cube.

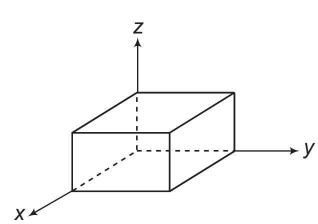


Fig. 24.63

24.13 Properties of a Conductor

Conductors (such as metals) possess free electrons. If a resultant electric field exists in the conductor these free charges will experience a force which will set a current flow. When no current flows, the resultant force and the electric field must be zero. Thus, under **electrostatic conditions** the value of **E** at all points within a conductor is zero. This idea, together with the Gauss's law can be used to prove several interesting facts regarding a conductor.

Excess Charge on a Conductor Resides on its Outer Surface

Consider a charged conductor carrying a charge q and no currents are flowing in it. Now, consider a Gaussian surface inside the conductor everywhere on which $\mathbf{E} = 0$.

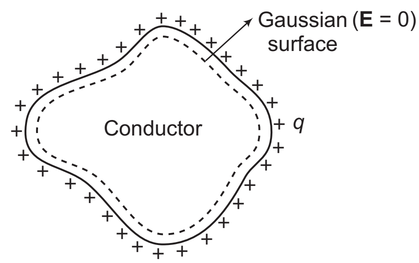


Fig. 24.64

Thus, from Gauss's law,

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0}$$

We get, $$q_{\text{in}} = 0, \text{ as } \mathbf{E} = 0$$

Thus, the sum of all charges inside the Gaussian surface is zero. This surface can be taken just inside the surface of the conductor, hence, any charge on the conductor must be on the surface of the conductor. In other words,

"Under electrostatic conditions, the excess charge on a conductor resides on its outer surface."

Electric Field at Any Point Close to the Charged Conductor is $\dfrac{\sigma}{\varepsilon_0}$

Consider a charged conductor of irregular shape. In general, surface charge density will vary from point to point. At a small surface ΔS, let us assume it to be a constant σ. Let us construct a Gaussian surface in the form of a cylinder of cross-section ΔS. One plane face of the cylinder is inside the conductor and other outside the conductor close to it. The surface inside the conductor does not contribute to the flux as **E** is zero everywhere inside the conductor. The curved surface outside the conductor also does not contribute to flux as **E** is always normal to the charged conductor and hence parallel to the curved surface. Thus, the only contribution to the flux is through the plane face outside the conductor. Thus, from Gauss's law,

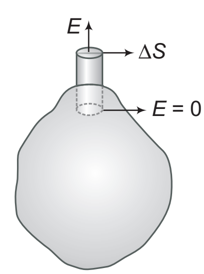


Fig. 24.65

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0}$$

or $$E\Delta S = \frac{(\sigma)(\Delta S)}{\varepsilon_0}$$

or $$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Note *(i) Electric field changes discontinuously at the surface of a conductor. Just inside the conductor it is zero and just outside the conductor it is* $\frac{\sigma}{\varepsilon_0}$*. In fact, the field gradually decreases from* $\frac{\sigma}{\varepsilon_0}$ *to zero in a small thickness of about 4 to 5 atomic layers at the surface.*

(ii) For a non-uniform conductor the surface charge density (σ) *varies inversely as the radius of curvature* (ρ) *of that part of the conductor, i.e.*

$$\boxed{\sigma \propto \frac{1}{\textit{Radius of curvature}\,(\rho)}}$$

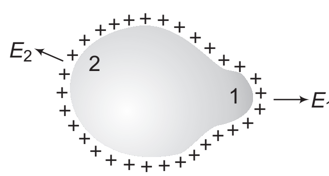


Fig. 24.66

For example in the figure, $\rho_1 < \rho_2 \quad \therefore \quad \sigma_1 > \sigma_2$

or $E_1 > E_2 \quad$ *as* $\quad E = \frac{\sigma}{\varepsilon_0}$

Electric Field and Field Lines are Normal to the Surface of a Conductor

Net field inside a conductor is zero. It implies that no field lines enter a conductor. On the surface of a conductor, electric field and hence field lines are normal to the surface of the conductor.

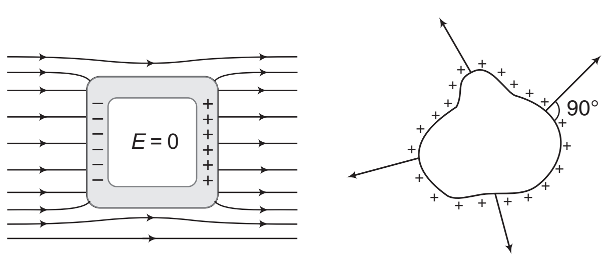


Fig. 24.67

If a conducting box is immersed in a uniform electric field, the field lines near the box are somewhat distorted. Similarly, if a conductor is positively charged, the field lines originate from the surface and are normal at every point and if it is negatively charged the field lines terminate on the surface normally at every point.

Cavity Inside a Conductor

Consider a charge $+q$ suspended in a cavity in a conductor. Consider a Gaussian surface just outside the cavity and inside the conductor. $\mathbf{E} = 0$ on this Gaussian surface as it is inside the conductor. Hence, from Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0} \quad \text{gives } q_{\text{in}} = 0$$

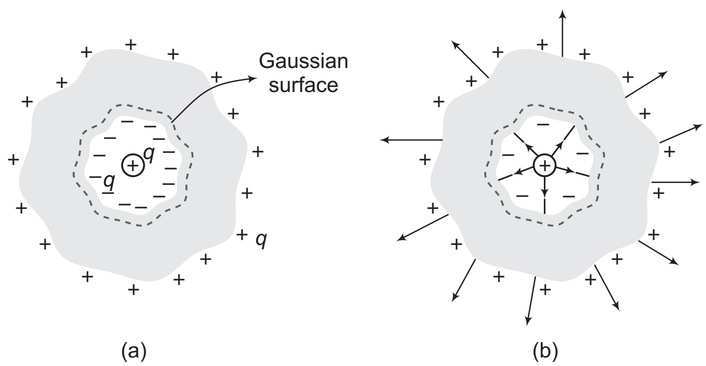


Fig. 24.68

This concludes that a charge of $-q$ must reside on the metal surface of the cavity so that the sum of this induced charge $-q$ and the original charge $+q$ within the Gaussian surface is zero. In other words, a charge q suspended inside a cavity in a conductor induces an equal and opposite charge $-q$ on the surface of the cavity. Further as the conductor is electrically neutral a charge $+q$ is induced on the outer surface of the conductor. As field inside the conductor is zero, the field lines coming from q cannot penetrate into the conductor. The field lines will be as shown in Fig. (b).

The same line of approach can be used to show that the field inside the cavity of a conductor is zero when no charge is suspended in it.

Electrostatic shielding

Suppose we have a very sensitive electronic instrument that we want to protect from external electric fields that might cause wrong measurements. We surround the instrument with a conducting box or we keep the instrument inside the cavity of a conductor. By doing this charge in the conductor is so distributed that the net electric field inside the cavity becomes zero and the instrument is protected from the external fields. This is called electronic shielding.

The Potential of a Charged Conductor Throughout its Volume is Same

In any region in which $\mathbf{E} = 0$ at all points, such as the region very far from all charges or the interior of a charged conductor, the line integral of $\mathbf{E}$ is zero along any path. It means that the potential difference between any two points in the conductor are at the same potential or the interior of a charged conductor is an equipotential region.

24.14 Electric Field and Potential Due to Charged Spherical Shell or Solid Conducting Sphere

Electric Field

At all points inside the charged spherical conductor or hollow spherical shell, electric field $\mathbf{E} = 0$, as there is no charge inside such a sphere. In an isolated charged spherical conductor any excess charge on it is distributed uniformly over its outer surface same as that of charged spherical shell or hollow sphere. The field at external points has the same symmetry as that of a point charge. We can construct a Gaussian surface (a sphere) of radius $r > R$. At all points of this sphere the magnitude of electric field is the same and its direction is perpendicular to the surface.

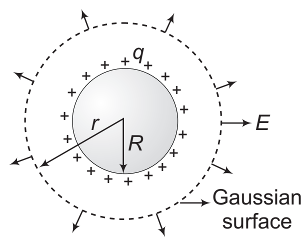


Fig. 24.69

Thus, we can apply

$$ES = \frac{q_{\text{in}}}{\varepsilon_0} \quad \text{or} \quad E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Hence, the electric field at any external point is the same as if the total charge is concentrated at centre.

At the surface of sphere $r = R$,

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

Thus, we can write

$$\boxed{\begin{aligned} E_{\text{inside}} &= 0 \\ E_{\text{surface}} &= \frac{1}{4\pi\varepsilon_0}\,\frac{q}{R^2} \\ E_{\text{outside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \end{aligned}}$$

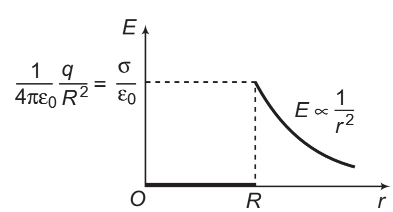


Fig. 24.70

The variation of electric field (E) with the distance from the centre (r) is as shown in Fig. 24.70.

Note *(i) At the surface graph is discontinuous*

(ii) $E_{surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = \frac{q/4\pi R^2}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Potential

As we have seen,
$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$\therefore$
$$\left(\frac{-dV_{\text{outside}}}{dr}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad \left(E = -\frac{dV}{dr}\right)$$

$\therefore$
$$\int_0^V dV_{\text{outside}} = \frac{-q}{4\pi\varepsilon_0} \int_\infty^r \frac{dr}{r^2} \qquad (V_\infty = 0)$$

$\therefore$
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \quad \text{or} \quad V \propto \frac{1}{r}$$

Thus, at external points, the potential at any point is the same when the whole charge is assumed to be concentrated at the centre. At the surface of the sphere, $r = R$

$\therefore$
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

At some internal point electric field is zero everywhere, therefore, the potential is same at all points which is equal to the potential at surface. Thus, we can write

$$\boxed{\begin{aligned} V_{\text{inside}} &= V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \\ V_{\text{outside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \end{aligned}}$$

and

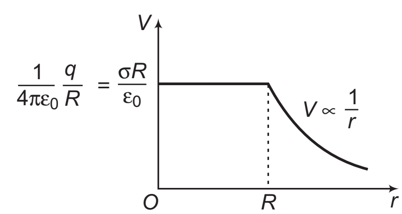


Fig. 24.71

The potential (V) varies with the distance from the centre (r) as shown in Fig. 24.71.

24.15 Electric Field and Potential Due to a Solid Sphere of Charge

Electric Field

Positive charge q is uniformly distributed throughout the volume of a solid sphere of radius R. For finding the electric field at a distance r ($< R$) from the centre let us choose as our Gaussian surface a sphere of radius r, concentric with the charge distribution. From symmetry, the magnitude E of electric field has the same value at every point on the Gaussian surface and the direction of **E** is radial at every point on the surface. So, applying Gauss's law

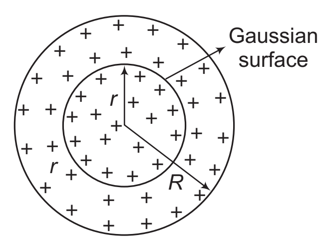


Fig. 24.72

$$ES = \frac{q_{\text{in}}}{\varepsilon_0} \qquad \ldots\text{(i)}$$

Here,
$$S = 4\pi r^2 \quad \text{and} \quad q_{\text{in}} = (\rho)\left(\frac{4}{3}\pi r^3\right)$$

Here,
$$\rho = \text{charge per unit volume} = \frac{q}{\frac{4}{3}\pi R^3}$$

Substituting these values in Eq. (i)

We have, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r \quad \text{or} \quad E \propto r$$

At the centre $\quad r = 0, \quad \text{so} \quad E = 0$

At surface $\quad r = R, \quad \text{so} \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius $r\ (> R)$. This surface encloses the entire charged sphere, so $q_{in} = q$, and Gauss's law gives

$$E\ (4\pi r^2) = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

or $$E \propto \frac{1}{r^2}$$

Notice that if we set $r = R$ in either of the two expressions for E (outside and inside the sphere), we get the same result, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

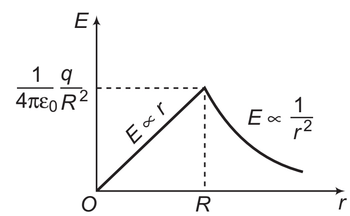


Fig. 24.73

this is because E is **continuous function** of r in this case. By contrast, for the charged conducting sphere the magnitude of electric field is discontinuous at $r = R$ (it jumps from $E = 0$ to $E = \sigma/\varepsilon_0$).

Thus, for a uniformly charged solid sphere we have the following formulae for magnitude of electric field :

$$\boxed{\begin{aligned} E_{\text{inside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r \\ E_{\text{surface}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} \\ E_{\text{outside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \end{aligned}}$$

The variation of electric field (E) with the distance from the centre of the sphere (r) is shown in Fig. 24.73.

Potential

The field intensity outside the sphere is

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$\frac{dV_{\text{outside}}}{dr} = -E_{\text{outside}}$$

$$\therefore \quad dV_{\text{outside}} = -E_{\text{outside}}\ dr$$

or $$\int_{\infty}^{V} dV_{\text{outside}} = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\, dr$$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \quad \text{as} \quad V_{\infty} = 0 \quad \text{or} \quad V \propto \frac{1}{r}$$

At $r = R$, $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

i.e. at the surface of the sphere potential is $V_S = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$

The electric intensity inside the sphere,

$$E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r$$

$$\frac{dV_{\text{inside}}}{dr} = -E_{\text{inside}}$$

$\therefore$ $$dV_{\text{inside}} = -E_{\text{inside}}\, dr$$

$\therefore$ $$\int_{V_S}^{V} dV_{\text{inside}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \int_{R}^{r} r\, dr$$

$\therefore$ $$V - V_S = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \left[\frac{r^2}{2}\right]_R^r$$

Substituting $V_S = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$, we get

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^3} (1.5\, R^2 - 0.5\, r^2)$$

At the centre $r = 0$ and $V_c = \dfrac{3}{2}\left(\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}\right) = \dfrac{3}{2} V_s$, i.e. potential at the centre is 1.5 times the potential at surface.

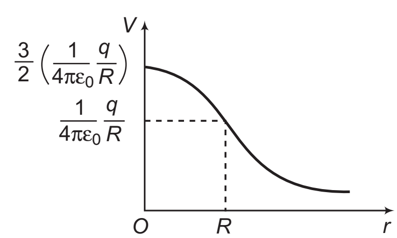


Fig. 24.74

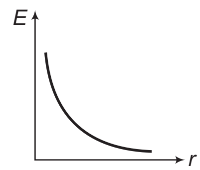
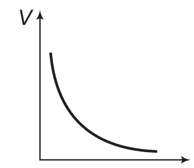
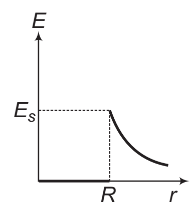
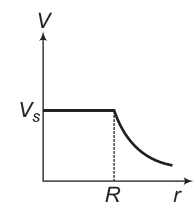
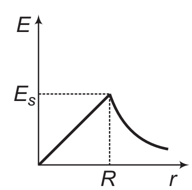
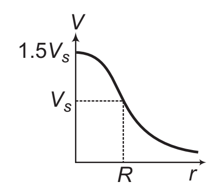
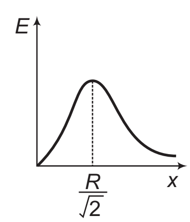
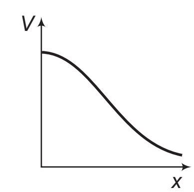
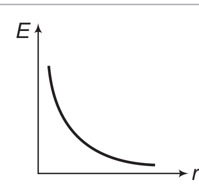
Thus, for a uniformly charged solid sphere we have the following formulae for potential :

$$\boxed{\begin{aligned} V_{\text{outside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \\ V_{\text{surface}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \\ \text{and} \quad V_{\text{inside}} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right] \end{aligned}}$$

The variation of potential (V) with distance from the centre (r) is as shown in Fig. 24.74. For inside points variation is parabolic.

List of formulae for field strength E and potential V $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

Table 24.1

S.No.	Charge Distribution	E Formula	E Graph	V Formula	V Graph
1.	Point charge	$E = \frac{kq}{r^2}$		$V = \frac{kq}{r}$	
2.	Uniformly charged spherical shell	$E_i = 0$ $E_s = k \cdot \frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$ $E_o = \frac{kq}{r^2}$		$V_i = V_s = \frac{kq}{R}$ $= \frac{\sigma R}{\varepsilon_0}$ $V_o = \frac{kq}{r}$	
3.	Solid sphere of charge	$E_i = \frac{kqr}{R^3}$ $E_s = \frac{kq}{R^2}$ $E_o = \frac{Kq}{r^2}$		$V_i = \frac{kq}{R^3}(1.5 R^2 - 0.5 r^2)$ $V_s = \frac{kq}{R}$ $V_o = \frac{kq}{r}$	
4.	On the axis of uniformly charged ring	$E = \frac{kqx}{(R^2 + x^2)^{3/2}}$ At centre $x = 0$ $\therefore \quad E = 0$		$V = \frac{kq}{\sqrt{R^2 + x^2}}$ At centre $x = 0$ $\therefore \quad V = \frac{kq}{R}$	
5.	Infinitely long line charge	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$		$\text{PD} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right)$	Not required

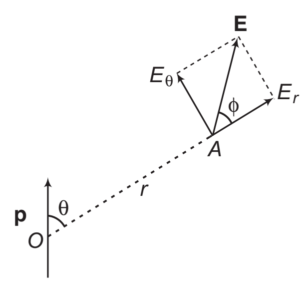
Final Touch Points

1. **Permittivity** Permittivity or absolute permittivity is a measure of resistance that is encountered when forming an electric field in a medium. Thus, permittivity relates to a material's ability to resist an electric field (while unfortunately, the word "permit" suggests the inverse quantity).

   The permittivity of a medium describes how much electric field (more correctly, flux) is generated per unit charge in that medium. More electric flux (per unit charge) exists in a medium with a low permittivity. Vacuum has the lowest permittivity (therefore maximum electric flux per unit charge). Any other dielectric medium has K -times (K = dielectric constant) the permittivity of vacuum. This is because, due to polarization effects electric flux per unit charge deceases K - times ($K > 1$).

2. **Dielectric constant (K)** Also known as relative permittivity of a given material is the ratio of permittivity of the material to the permittivity of vacuum. This is the factor by which the electric force between the two charges is decreased relative to vacuum. Similarly, in the chapter of capacitors we will see that it is the ratio of capacitance of a capacitor using that material as a dielectric compared to a similar capacitor that has vacuum as its dielectric.

3. **Electric field and potential due to a dipole at polar coordinates (r, θ)**



or $$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$

The electric field **E** can be resolved into two components E_r and E_θ, where

or $$\boxed{E_r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p\cos\theta}{r^3}}$$

and $$\boxed{E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}}$$

The magnitude of resultant electric field $E = \sqrt{E_r^2 + E_\theta^2}$

or $$\boxed{E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}}$$

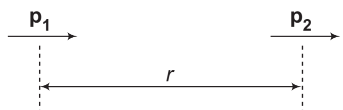
Its inclination ϕ to OA is given by

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{p\sin\theta/4\pi\varepsilon_0 r^3}{2p\cos\theta/4\pi\varepsilon_0 r^3}$$

or $$\boxed{\tan\phi = \frac{\tan\theta}{2}}$$

4. Force between two dipoles The force between two dipoles varies inversely with the fourth power of the distance between their centres or

$$F \propto \frac{1}{r^4}$$



In the, figure, a dipole on left with dipole moment $\mathbf{p}_1$ interacts with the dipole on the right with dipole moment $\mathbf{p}_2$. We assume that the distance between them is quite large. The electric field of the dipole on the left hand side exerts a net force on the dipole on the right hand side. Let us now calculate the net force on the dipole on right hand side.

The electric field at the centre of this dipole

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1}{r^3}$$

$$\therefore \qquad dE = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1}{r^4}\, dr$$

Now, the electric field at the point where $-q$ charge of the dipole lies is given by

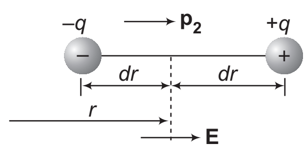
$$E_1 = E + |dE|$$

and force on $-q$ is qE_1 (towards left)

Similarly, electric field at the point where $+q$ charge of the dipole lies is

$$E_2 = E - |dE|$$

and force on $+q$ is qE_2 (towards right)



$\therefore$ Net force on the dipole is

$$F = q\,E_1 - q\,E_2 \qquad \text{(towards left)}$$

$$= 2q\,|dE|$$

$$= \frac{6\,(2qdr)\,p_1}{4\pi\varepsilon_0\, r^4}$$

or $$F = \frac{6p_1 p_2}{4\pi\varepsilon_0\, r^4} \qquad [\text{as } 2q\,(dr) = p_2]$$

Thus, if $\mathbf{p}_1 \,||\, \mathbf{p}_2$, the two dipoles attract each other with a force given by the above relation.

5. Earthing a conductor Potential of earth is often taken to be zero. If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e. zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

Solved Examples

TYPED PROBLEMS

Type 1. *To find electric potential due to charged spherical shells*

Concept

To find the electric potential due to a conducting sphere (or shell) we should keep in mind the following two points

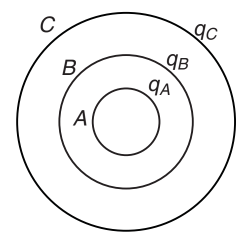
(i) Electric potential on the surface and at any point inside the sphere is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \qquad (R = \text{radius of sphere})$$

(ii) Electric potential at any point outside the sphere is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad (r = \text{distance of the point from the centre})$$

For example, in the figure shown, potential at A is

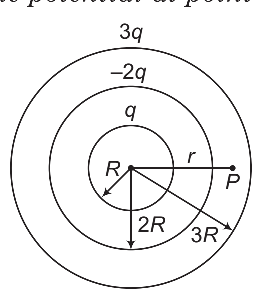


$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A} + \frac{q_B}{r_B} + \frac{q_C}{r_C}\right]$$

Similarly, potential at B is $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B} + \frac{q_B}{r_B} + \frac{q_C}{r_C}\right]$

and potential at C is, $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_C} + \frac{q_B}{r_C} + \frac{q_C}{r_C}\right]$

Example 1 *Three conducting spherical shells have charges q, $-2q$ and $3q$ as shown in figure. Find electric potential at point P as shown in figure.*



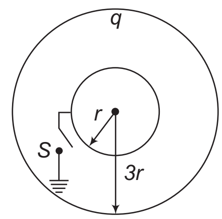
Solution Potential at P,

$$V_P = V_q + V_{-2q} + V_{3q}$$
$$= \frac{kq}{r} - \frac{k(2q)}{r} + \frac{k(3q)}{3R}$$

$$= kq\left(\frac{1}{R} - \frac{1}{r}\right)$$ **Ans.**

Here, $$k = \frac{1}{4\pi\varepsilon_0}$$

Example 2 *Figure shows two conducting thin concentric shells of radii r and 3r. The outer shell carries a charge q. Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.*



Solution Let q' be the charge on inner shell when it is earthed.

Potential of inner shell is zero.

$\therefore$ $$\frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{r} + \frac{q}{3r}\right] = 0$$

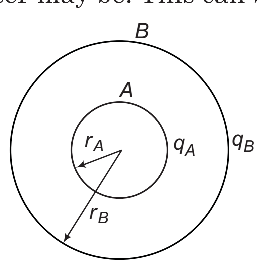
$\therefore$ $$q' = -\frac{q}{3}$$

i.e. $+\dfrac{q}{3}$ charge will flow from inner shell to earth. **Ans.**

Type 2. *Based on the principle of generator*

Concept

A generator is an instrument for producing high voltages in the million volt region. Its design is based on the principle that if a charged conductor (say A) is brought into contact with a hollow conductor (say B), all of its charge transfers to the hollow conductor no matter how high the potential of the later may be. This can be shown as under:

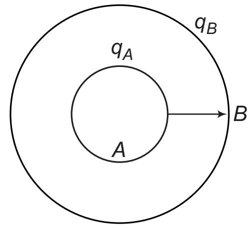


In the figure, $$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A} - \frac{q_B}{r_B}\right]$$

and $$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B} - \frac{q_B}{r_B}\right]$$

$$\therefore \qquad V_A - V_B = \frac{q_A}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

From this expression the following conclusions can be drawn :

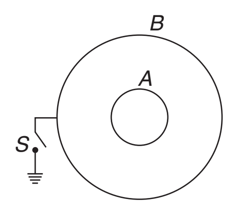


(i) The potential difference (PD) depends on q_A only. It does not depend on q_B.

(ii) If q_A is positive, then $V_A - V_B$ is positive (as $r_A < r_B$), i.e. $V_A > V_B$. So if the two spheres are connected by a conducting wire charge flows from inner sphere to outer sphere (positive charge flows from higher potential to lower potential) till $V_A = V_B$ or $V_A - V_B = 0$. But potential difference will become zero only when $q_A = 0$, i.e. all charge q_A flows from inner sphere to outer sphere.

(iii) If q_A is negative, $V_A - V_B$ is negative, i.e. $V_A < V_B$. Hence, when the two spheres are connected by a thin wire all charge q_A will flow from inner sphere to the outer sphere. Because negative charge flows from lower potential to higher potential. Thus, we see that the whole charge q_A flows from inner sphere to the outer sphere, no matter how high q_B is. Charge always flows from A to B, whether $q_A > q_B$ or $q_B > q_A$, $V_A > V_B$ or $V_B > V_A$.

Example 3 *Initially the spheres A and B are at potentials V_A and V_B. Find the potential of A when sphere B is earthed.*



Solution As we have studied above that the potential difference between these two spheres depends on the charge on the inner sphere only. Hence, the PD will remain unchanged because by earthing the sphere B charge on A remains constant. Let V'_A be the new potential at A. Then,

$$V_A - V_B = V'_A - V'_B$$

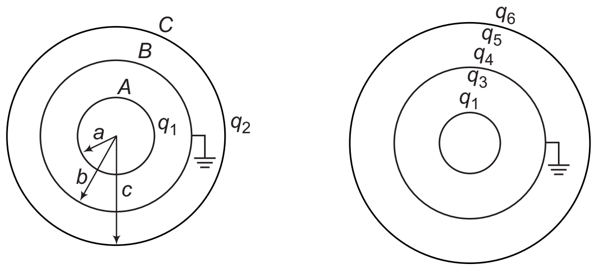
but $V'_B = 0$ as it is earthed. Hence,

$$V'_A = V_A - V_B$$ **Ans.**

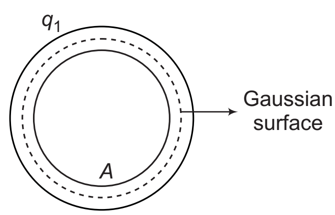
Type 3. *Based on the charges appearing on different surfaces of concentric spherical shells*

Concept

Figure shows three concentric thin spherical shells A, B and C of radii a, b and c. The shells A and C are given charges q_1 and q_2 and the shell B is earthed. We are interested in finding the charges on inner and outer surfaces of A, B and C. To solve such type of problems we should keep the following points in mind :



(i) The whole charge q_1 will come on the outer surface of A unless some charge is kept inside A. To understand it let us consider a Gaussian surface (a sphere) through the material of A. As the electric field in a conducting material is zero. The flux through this Gaussian surface is zero. Using Gauss's law, the total charge enclosed must be zero.



(ii) Similarly, if we draw a Gaussian surface through the material of B we can see that

$$q_3 + q_1 = 0 \quad \text{or} \quad q_3 = -q_1$$

and if we draw a Gaussian surface through the material of C, then

$$q_5 + q_4 + q_3 + q_1 = 0 \quad \text{or} \quad q_5 = -q_4$$

(iii) $q_5 + q_6 = q_2$. As q_2 charge was given to shell C.

(iv) Potential of B should be zero, as it is earthed. Thus,

$$V_B = 0$$

or
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{b} + \frac{q_3 + q_4}{b} + \frac{q_5 + q_6}{c}\right] = 0$$

So, using the above conditions we can find charges on different surfaces.

We can summarise the above points as under

1. Net charge inside a closed Gaussian surface drawn in any shell is zero. (provided the shell is conducting).

2. Potential of the conductor which is earthed is zero.

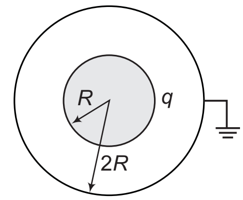
3. If two conductors are connected, they are at same potential.

4. Charge remains constant in all conductors except those which are earthed.

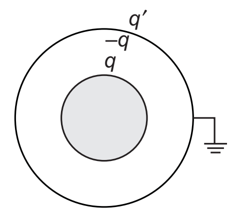
5. Charge on the inner surface of the innermost shell is zero provided no charge is kept inside it. In all other shells charge resides on both the surfaces.

6. Equal and opposite charges appear on opposite faces.

Example 4 *A charge q is distributed uniformly on the surface of a solid sphere of radius R. It is covered by a concentric hollow conducting sphere of radius 2R. Find the charges on inner and outer surfaces of hollow sphere if it is earthed.*



Solution The charge on the inner surface of the hollow sphere should be $-q$, because if we draw a closed Gaussian surface through the material of the hollow sphere the total charge enclosed by this Gaussian surface should be zero. Let q' be the charge on the outer surface of the hollow sphere.



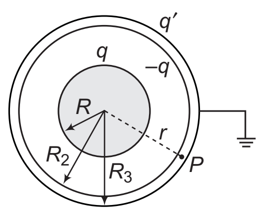
Since, the hollow sphere is earthed, its potential should be zero. The potential on it is due to the charges q, $-q$ and q', Hence,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} - \frac{q}{2R} + \frac{q'}{2R}\right] = 0$$

$\therefore$ $$q' = 0$$ **Ans.**

Therefore, there will be no charge on the outer surface of the hollow sphere.

Example 5 *Solve the above problem if thickness of the hollow sphere is considerable.*



Solution In this case, we can set $V = 0$ at any point on the hollow sphere. Let us select a point P a distance r from the centre, were $R_2 < r < R_3$. So,

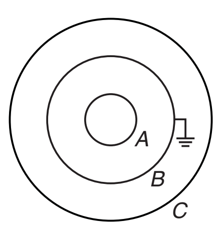
$$V_P = 0$$

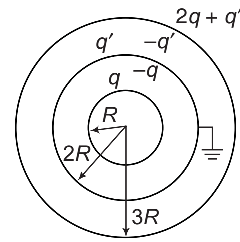
$\therefore$ $$\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} - \frac{q}{r} + \frac{q'}{R_3}\right] = 0$$

$\therefore$ $$q' = 0$$ **Ans.**

i.e. in this case also there will be no charge on the outer surface of the hollow sphere.

Example 6 *Figure shows three concentric thin spherical shells A, B and C of radii R, 2R and 3R. The shell B is earthed and A and C are given charges q and 2q, respectively. Find the charges appearing on all the surfaces of A, B and C.*





Solution Since, there is no charge inside A. The whole charge q given to the shell A will appear on its outer surface. Charge on its inner surface will be zero. Moreover if a Gaussian surface is drawn on the material of shell B, net charge enclosed by it should be zero. Therefore, charge on its inner surface will be $-q$. Now let q' be the charge on its outer surface, then charge on the inner surface of C will be $-q'$ and on its outer surface will be, $2q-(-q')=2q+q'$ as total charge on C is $2q$.

Shell B is earthed. Hence, its potential should be zero.

$$V_B = 0$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R}-\frac{q}{2R}+\frac{q'}{2R}-\frac{q'}{3R}+\frac{2q+q'}{3R}\right]=0$$

Solving this equation, we get

$$q' = -\frac{4}{3}q$$

$$\therefore \quad 2q+q' = 2q-\frac{4}{3}q = \frac{2}{3}q$$

Therefore, charges on different surfaces in tabular form are given below :

Table 24.2

	A	B	C
Inner surface	0	$-q$	$\frac{4}{3}q$
Outer surface	q	$-\frac{4}{3}q$	$\frac{2}{3}q$

Type 4. *Based on finding electric field due to spherical charge distribution*

Concept

According to Gauss's theorem, at a distance r from centre of sphere,

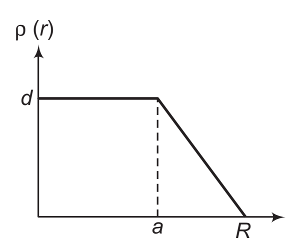
$$E = \frac{kq_{in}}{r^2} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

Here, q_{in} is the net charge inside the sphere of radius r. If volume charge density (say ρ) is constant, then

$$q_{in} = (\text{volume of sphere of radius } r)(\rho) = \frac{4}{3}\pi r^3 \rho$$

If ρ is variable, then q_{in} can be obtained by integration.

Passage (Ex. 7 to Ex. 9)



The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.

Example 7 *The electric field at $r = R$ is* **(JEE 2008)**

(a) independent of a

(b) directly proportional to a

(c) directly proportional to a^2

(d) inversely proportional to a

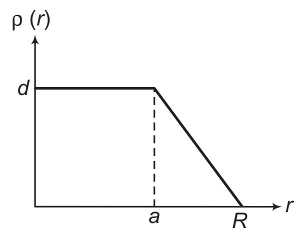
Solution At $r = R$, from Gauss's law

$$E\,(4\pi R^2) = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{Ze}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze}{R^2}$$

E is independent of a.

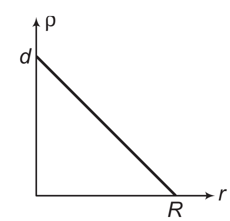
$\therefore$ The correct option is (a).

Example 8 *For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is* **(JEE 2008)**



(a) $\dfrac{3Ze}{4\pi R^3}$ (b) $\dfrac{3Ze}{\pi R^3}$ (c) $\dfrac{4Ze}{3\pi R^3}$ (d) $\dfrac{Ze}{3\pi R^3}$

Solution For $a = 0$,



$$\rho(r) = \left(-\frac{d}{R}\cdot r + d\right)$$

Now,
$$\int_0^R (4\pi r^2)\left(d - \frac{d}{R}r\right)dr = \text{net charge} = Ze$$

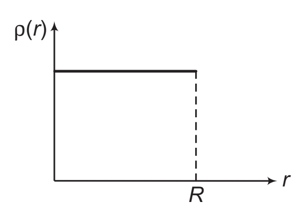
Solving this equation, we get
$$d = \frac{3Ze}{\pi R^3}$$

$\therefore$ The correct option is (b).

Example 9 *The electric field within the nucleus is generally observed to be linearly dependent on r. This implies* **(JEE 2008)**

(a) $a = 0$ (b) $a = \dfrac{R}{2}$ (c) $a = R$ (d) $a = \dfrac{2R}{3}$

Solution In case of solid sphere of charge of uniform volume density



$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^3}\cdot r \quad \text{or} \quad E \propto r$$

Thus, for E to be linearly dependent on r, volume charge density should be constant.

or
$$a = R$$

$\therefore$ The correct option is (c).

Type 5. *Based on calculation of electric flux*

Concept

(i) To find electric flux from any closed surface, direct result of Gauss's theorem can be used,

$$\phi = \frac{q_{\text{in}}}{\varepsilon_0}$$

(ii) To find electric flux from an open surface, result of Gauss's theorem and concept of symmetry can be used.

(iii) To find electric flux from a plane surface in uniform electric field,

$$\phi = \mathbf{E} \cdot \mathbf{S} \quad \text{or} \quad ES \cos\theta$$

can be used.

(iv) Net electric flux from a closed surface in uniform electric field is always zero.

Example 10 *The electric field in a region is given by $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. Here, a and b are constants. Find the net flux passing through a square area of side l parallel to y-z plane.*

Solution A square area of side l parallel to y-z plane in vector form can be written as,

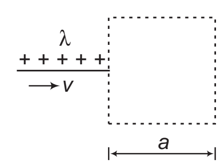
$$\mathbf{S} = l^2\hat{\mathbf{i}}$$

Given, $$\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$$

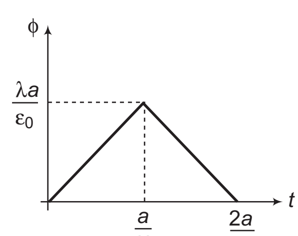
∴ Electric flux passing through the given area will be,

$$\phi = \mathbf{E} \cdot \mathbf{S}$$
$$= (a\hat{\mathbf{i}} + b\hat{\mathbf{j}}) \cdot (l^2\hat{\mathbf{i}})$$
$$= al^2$$ **Ans.**

Example 11 *Figure shows an imaginary cube of side a. A uniformly charged rod of length a moves towards right at a constant speed v. At $t = 0$, the right end of the rod just touches the left face of the cube. Plot a graph between electric flux passing through the cube versus time.*



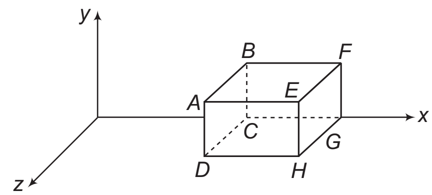
Solution The electric flux passing through a closed surface depends on the net charge inside the surface. Net charge in this case first increases, reaches a maximum value and finally decreases to zero. The same is the case with the electric flux. The electric flux ϕ *versus* time graph is as shown in figure below.



Example 12 *The electric field in a region is given by* $\mathbf{E} = \alpha x \hat{\mathbf{i}}$. *Here,* α *is a constant of proper dimensions. Find*

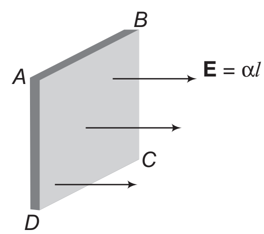
(a) the total flux passing through a cube bounded by the surfaces, $x = l$, $x = 2l$, $y = 0$, $y = l$, $z = 0$, $z = l$.

(b) the charge contained inside the above cube.



Solution (a) Electric field is along positive x-direction. Therefore, field lines are perpendicular to faces $ABCD$ and $EFGH$. At all other four faces field lines are tangential. So, net flux passing through these four faces will be zero.

Flux entering at face *ABCD* At this face $x = l$

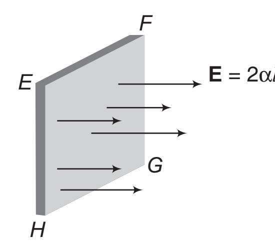


$\therefore$ $$\mathbf{E} = \alpha l \hat{\mathbf{i}}$$

$\therefore$ Flux entering the cube from this face,

$$\phi_1 = \mathbf{ES} = (\alpha l)(l^2) = \alpha l^3$$

Flux leaving the face *EFGH* At this face $x = 2l$



$\therefore$ $$\mathbf{E} = 2\alpha l \hat{\mathbf{i}}$$

$\therefore$ Flux coming out of this face

$$\phi_2 = \mathbf{ES} = (2\alpha l)(l^2) = 2\alpha l^3$$

$\therefore$ Net flux passing through the cube,

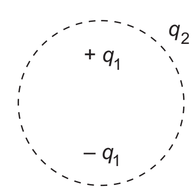
$$\phi_{net} = \phi_2 - \phi_1 = 2\alpha l^3 - \alpha l^3 = \alpha l^3$$ **Ans.**

(b) From Gauss's law,

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0}$$

$$q_{in} = (\phi_{net})(\varepsilon_0) = \alpha \varepsilon_0 l^3$$ **Ans.**

Example 13 *Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to* **(JEE 2004)**



(a) q_2

(b) only the positive charges

(c) all the charges

(d) $+q_1$ *and* $-q_1$

Solution At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2

$\therefore$ The correct option is (c)..

Note *Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.*

Example 14 *A point charge q is placed on the top of a cone of semi vertex angle* θ. *Show that the electric flux through the base of the cone is* $\dfrac{q(1-\cos\theta)}{2\varepsilon_0}$.

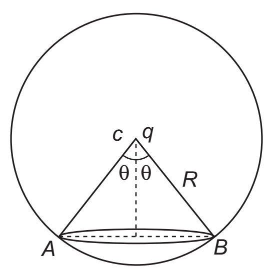
HOW TO PROCEED *This problem can be solved by the method of symmetry. Consider a Gaussian surface, a sphere with its centre at the top and radius the slant length of the cone. The flux through the whole sphere is* q/ε_0. *Therefore, the flux through the base of the cone can be calculated by using the following formula,*

$$\phi_e = \left(\frac{S}{S_0}\right)\cdot\frac{q}{\varepsilon_0}$$

Here, S_0 = *area of whole sphere*

and S = *area of sphere below the base of the cone.*

Solution Let R = slant length of cone = radius of Gaussian sphere



$\therefore$ S_0 = area of whole sphere = $(4\pi R^2)$

S = area of sphere below the base of the cone

$= 2\pi R^2 (1-\cos\theta)$

$\therefore$ The desired flux is,

$$\phi = \left(\frac{S}{S_0}\right)\cdot\frac{q}{\varepsilon_0}$$

$$= \frac{(2\pi R^2)(1-\cos\theta)}{(4\pi R^2)}\cdot\frac{q}{\varepsilon_0}$$

$$= \frac{q(1-\cos\theta)}{2\varepsilon_0}$$

Proved

Note $S = 2\pi R^2 (1-\cos\theta)$ *can be calculated by integration.*

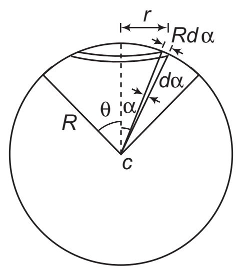
At $\theta = 0°$, $S = 2\pi R^2 (1-\cos 0°) = 0$

$\theta = 90°$, $S = 2\pi R^2 (1-\cos 90°) = 2\pi R^2$

and $\theta = 180°$, $S = 2\pi R^2 (1-\cos 180°) = 4\pi R^2$

Proof

$$dS = (2\pi r)\,R d\alpha$$
$$= (2\pi R \sin\alpha)\,R d\alpha \qquad \text{as } r = R\sin\alpha$$
$$= (2\pi R^2)\sin\alpha\; d\alpha$$
$$\therefore \qquad S = \int_0^{\theta} (2\pi R^2)\sin\alpha\; d\alpha$$
$$\therefore \qquad S = 2\pi R^2\,(1-\cos\theta)$$



Students are advised to remember this result.

Type 6. *Based on E-r and V-r graphs due to two point charges*

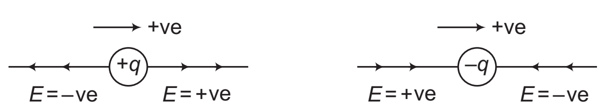
Concept

(i) $$E = \frac{kq}{r^2} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

and $$V = \pm\frac{kq}{r} \qquad \text{(due to a point charge)}$$

(ii) As $r \to 0$, $E \to \infty$ and $V \to \pm\infty$
As $r \to \infty$, $E \to 0$ and $V \to 0$

(iii) E is a vector quantity. Due to a point charge, its direction is away from the charge and due to negative charge it is towards the charge. Along one dimension if one direction is taken as positive direction then the other direction is taken as the negative direction.

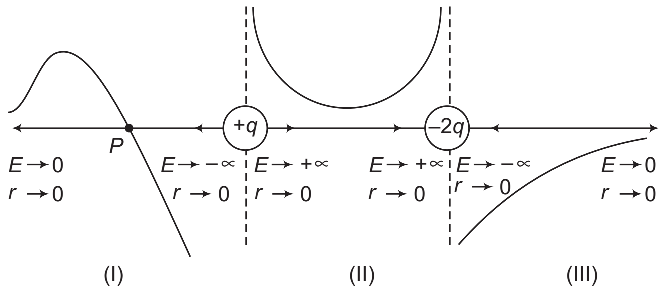


(iv) V is a scalar quantity. On both sides of a positive charge it is positive and it is negative due to negative charge.

(v) Between zero and zero value, normally we get either a maximum or minimum value.

Example 15 *Draw E - r and V - r graphs due to two point charges $+q$ and $-2q$ kept at some distance along the line joining these two charges.*

Solution ***E* - *r* graph**

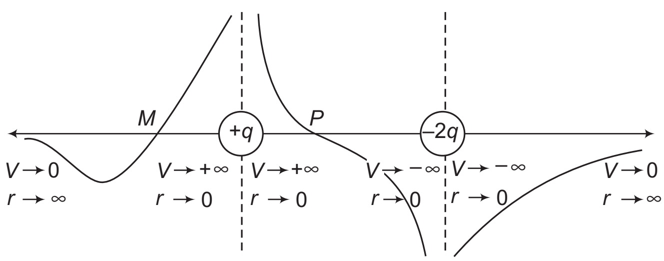


In region I E due to $+q$ is towards left (so negative) and E due to $-2q$ is towards right (so positive). Near $+q$, electric field of $+q$ will dominate. So, net value will be negative. At some point say P both positive and negative values are equal. So, $E_p = 0$. Beyond this point, electric field due to $-2q$ will dominate due to its higher magnitude. So, net value will be positive. $E_p = 0$ and E_∞ (towards left) is also zero. Between zero and zero we will get a maximum positive value.

In region II E due to $+q$ and due to $-2q$ is towards right (so positive). Between the value $+\infty$ and $+\infty$ the graph is as shown in figure.

In region III E due to $+q$ is towards right (so positive) and E due to $-2q$ is towards left (so negative). But electric field of $-2q$ will dominate due to its higher magnitude and lesser distance. Hence, net electric field is always negative.

***V* - *r* graph**



The logics developed in E - r graph can also be applied here with V - r graph. At point P, positive potential due to $+q$ is equal to negative potential due to $-2q$. Hence, $V_p = 0$, so this point is near $2q$. Same is the case at M.

Type 7. *E - r and V - r graphs due to charged spherical shells of negligible thickness*

Concept

According to Gauss's theorem,

$$E = \frac{kq_{\text{in}}}{r^2} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

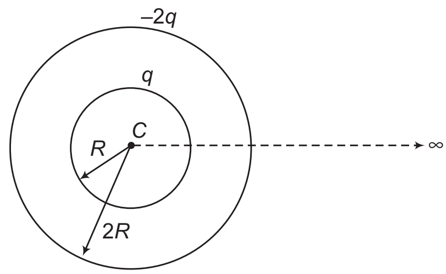
So, only inside charges contribute in the electric field.

$$V = \frac{kq}{R} = \text{constant} \qquad \text{(inside the shell)}$$

$$V = \frac{kq}{r} \neq \text{constant} \qquad \text{(outside the shell)}$$

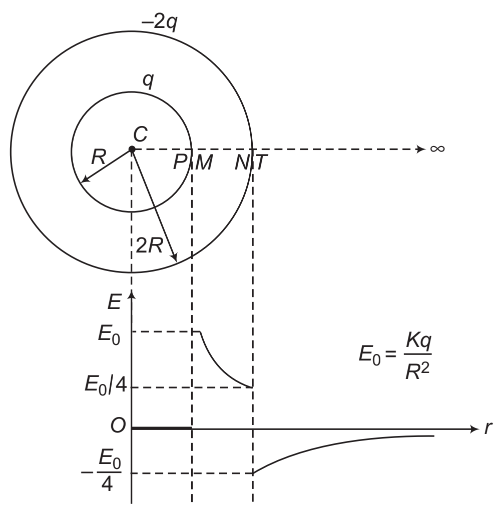
Here, q is the charge on shell.

Example 16 *Draw E - r and V - r graphs due to two charged spherical shells as shown in figure (along the line between C and ∞).*



Solution

***E*-*r* graph**



C to P $\quad q_{\text{in}} = 0 \quad \Rightarrow \quad E = 0$

At M $\quad E = \dfrac{kq}{R^2}$ (radially outwards, say positive) $= E_0$ (say)

At N $\quad E = \dfrac{kq}{(2R)^2} = \dfrac{kq}{4R^2} = \dfrac{E_0}{4}$ (radially outwards)

From M to N Value will decrease from E_0 to $\dfrac{E_0}{4}$

At T

$$E = \frac{k(-2q+q)}{(2R)^2} \quad \text{(radially inwards)}$$

$$= -\frac{E_0}{4}$$

From T to ∞ Value changes from $-\dfrac{E_0}{4}$ to zero.

***V*-*r* graph**

From C to P Points are lying inside both the shells. Hence, potential due to both shells is constant.

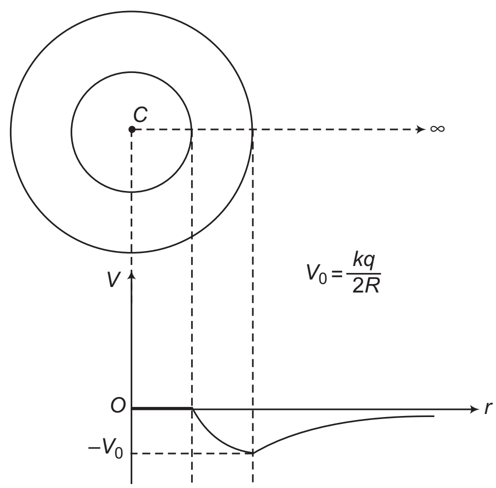
$$\therefore \qquad V = \frac{kq}{R} - \frac{k(2q)}{2R} = 0$$

From M to N Potential of $-2q$ will remain constant but potential of q will decrease. So, net value comes out to be negative. At N or T

$$V = \frac{kq}{2R} - \frac{k(2q)}{2R}$$

$$= -\frac{kq}{2R} = -V_0 \qquad \text{(say)}$$

From T to ∞ Value will change from $-V_0$ to zero. The correct graph is as shown below.



Type 8. *Based on motion of a charged particle in uniform electric field*

Concept

(i) In uniform electric field, force on the charged particle is

$$\mathbf{F} = q\mathbf{E}$$

or qE force acts in the direction of electric field if q is positive and in the opposite direction of electric field if q is negative.

(ii) Acceleration of the particle is therefore,

$$a = \frac{F}{m} = \frac{qE}{m}$$

This acceleration is constant. So, path is therefore either a straight line or parabola. If initial velocity is zero or parallel to acceleration or antiparallel to acceleration, then path is straight line. Otherwise in all other cases, path is a parabola.

Example 17 *An electron with a speed of 5.00×10^6 m/s enters an electric field of magnitude 10^3 N/C, travelling along the field lines in the direction that retards its motion.*

(a) How far will the electron travel in the field before stopping momentarily?

(b) *How much time will have elapsed?*

(c) *If the region with the electric field is only 8.00 mm long (too short from the electron to stop with in it), what fraction of the electron's initial kinetic energy will be lost in that region?*

Solution (a) $$s = \frac{u^2}{2a} = \frac{u^2}{2\,(qE/m)} = \frac{mu^2}{2qE}$$

$$= \frac{(9.1 \times 10^{-31})\,(5 \times 10^6)^2}{2 \times 1.6 \times 10^{-19} \times 10^3}$$

$$= 7.1 \times 10^{-2}\ \text{m} = 7.1\ \text{cm}$$ **Ans.**

(b) $$t = \frac{u}{a} = \frac{u}{qE/m} = \frac{mu}{qE}$$

$$= \frac{(9.1 \times 10^{-31})\,(5 \times 10^6)}{(1.6 \times 10^{-19})\,(10^3)}$$

$$= 2.84 \times 10^{-8}\ \text{s}$$ **Ans.**

(c) Loss of energy (in fraction)

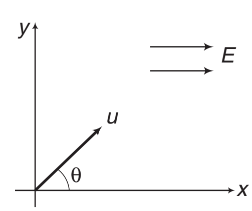
$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - \frac{u^2}{u^2}$$

$$= 1 - \frac{u^2 - 2as}{u^2} = \frac{2as}{u^2} = \frac{2qEs}{mu^2}$$

$$= \frac{2 \times 1.6 \times 10^{-19} \times 10^3 \times 8 \times 10^{-3}}{9.1 \times 10^{-31} \times (5 \times 10^6)^2}$$

$$= 0.11$$ **Ans.**

Example 18 *A charged particle of mass $m = 1$ kg and charge $q = 2\ \mu C$ is thrown from a horizontal ground at an angle $\theta = 45°$ with speed 20 m/s. In space a horizontal electric field $E = 2 \times 10^7$ V/m exist. Find the range on horizontal ground of the projectile thrown.*



Solution The path of the particle will be a parabola, but along x-axis also motion of the particle will be accelerated. Time of flight of the projectile is

$$T = \frac{2u_y}{a_y} = \frac{2u_y}{g} = \frac{2 \times 20 \cos 45°}{10} = 2\sqrt{2}\ \text{s}$$

Horizontal range of the particle will be

$$R = u_x T + \frac{1}{2} a_x T^2$$

Here, $$a_x = \frac{qE}{m} = \frac{(2 \times 10^{-6})\,(2 \times 10^7)}{1} = 40\ \text{m/s}^2$$

∴ $$R = (20 \cos 45°)\,(2\sqrt{2}) + \frac{1}{2}(40)\,(2\sqrt{2})^2$$

$$= 40 + 160$$

$$= 200\ \text{m}$$ **Ans.**

Type 9. *To find potential difference between two points when electric field is known*

Concept

In Article 24.9, we have already read the relation between **E** and V. There we have taken a simple case when electric field was uniform. Here, two more cases are possible depending on the nature of **E**.

When E has a Function Like $f_1(x)\hat{\mathbf{i}} + f_2(y)\hat{\mathbf{j}} + f_3(z)\hat{\mathbf{k}}$

In this case also, we will use the same approach. Let us take an example.

Example 19 *Find the potential difference V_{AB} between $A(2\text{m}, 1\text{m}, 0)$ and $B(0, 2\text{m}, 4\text{m})$ in an electric field,*

$$\mathbf{E} = (x\hat{\mathbf{i}} - 2y\hat{\mathbf{j}} + z\hat{\mathbf{k}})\ V/m$$

Solution

$$dV = -\mathbf{E}\cdot d\mathbf{r}$$

$$\int_B^A dV = -\int_{(0,\,2,\,4)}^{(2,\,1,\,0)} (x\hat{\mathbf{i}} - 2y\hat{\mathbf{j}} + z\hat{\mathbf{k}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$\therefore \quad V_A - V_B = -\int_{(0,\,2,\,4)}^{(2,\,1,\,0)} (x\,dx - 2y\,dy + z\,dz)$$

or

$$V_{AB} = -\left[\frac{x^2}{2} - y^2 + \frac{z^2}{2}\right]_{(0,\,2,\,4)}^{(2,\,1,\,0)}$$

$$= 3 \text{ volt}$$ **Ans.**

When E · dr becomes a Perfect Differential.

Same method is used when $\mathbf{E}\cdot d\mathbf{r}$ becomes a perfect differential. The following example will illustrate the theory.

Example 20 *Find potential difference V_{AB} between $A(0, 0, 0)$ and $B(1m, 1m, 1m)$ in an electric field (a) $\mathbf{E} = y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$ (b) $\mathbf{E} = 3x^2y\hat{\mathbf{i}} + x^3\hat{\mathbf{j}}$*

Solution (a)

$$dV = -\mathbf{E}\cdot d\mathbf{r}$$

$$\therefore \quad \int_B^A dV = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} (y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

or

$$V_A - V_B = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} (y\,dx + x\,dy)$$

or

$$V_{AB} = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} d(xy) \qquad [\text{as } y\,dx + x\,dy = d(xy)]$$

$$\therefore \quad V_{AB} = -[xy]_{(1,\,1,\,1)}^{(0,\,0,\,0)} = 1\text{ V}$$ **Ans.**

(b)

$$dV = -\mathbf{E}\cdot d\mathbf{r}$$

$$\therefore \quad \int_B^A dV = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} (3x^2y\hat{\mathbf{i}} + x^3\hat{\mathbf{j}})\bullet(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

or

$$V_A - V_B = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} (3x^2y\,dx + x^3\,dy)$$

$$= -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)} d(x^3y)$$

$$\therefore \quad V_{AB} = -[x^3y]_{(1,\,1,\,1)}^{(0,\,0,\,0)} = 1\text{ V}$$ **Ans.**

Type 10. *Based on oscillations of a dipole*

Concept

In uniform electric field, net force on a dipole is zero at all angles. But net torque is zero for $\theta = 0°$ or $180°$. Here, $\theta = 0°$ is the stable equilibrium position and $\theta = 180°$ is unstable equilibrium position. If the dipole is released from any angle other than 0° or 180°, it rotates towards 0°. In this process electrostatic potential energy of the dipole decreases. But rotational kinetic energy increases. At two angles θ_1 and θ_2, we can apply the equation

$$U_{\theta_1} + K_{\theta_1} = U_{\theta_2} + K_{\theta_2}$$

or

$$-pE\cos\theta_1 + \frac{1}{2}I\omega_1^2 = -pE\cos\theta_2 + \frac{1}{2}I\omega_2^2$$

Moreover, if the dipole is displaced from stable equilibrium position ($\theta = 0°$), then it starts rotational oscillations. For small value of θ, these oscillations are simple harmonic in nature.

Example 21 *An electric dipole of dipole moment p is placed in a uniform electric field E in stable equilibrium position. Its moment of inertia about the centroidal axis is I. If it is displaced slightly from its mean position, find the period of small oscillations.*

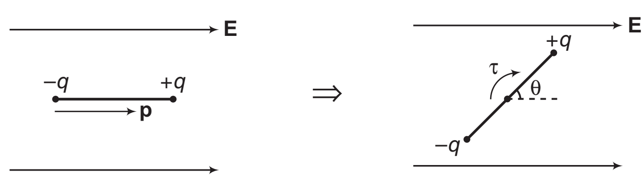
Solution When displaced at an angle θ from its mean position, the magnitude of restoring torque is

$$\tau = -pE\sin\theta$$

For small angular displacement $\sin\theta \approx \theta$

$$\tau = -pE\,\theta$$

The angular acceleration is $\alpha = \frac{\tau}{I} = -\left(\frac{pE}{I}\right)\theta = -\omega^2\theta$



where, $\omega^2 = \frac{pE}{I} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}}$ **Ans.**

Type 11. *Based on the work done (by external forces) in moving a charge from one point to another point*

Concept

If kinetic energy of the particle is not changed, then

$$W = \Delta U = U_f - U_i = q(V_f - V_i) \quad \text{or} \quad q(\Delta U)$$

Here, q is the charge to be displaced and V_i and V_f are the initial and final potentials.

Example 22 *Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is* **(JEE 1992)**

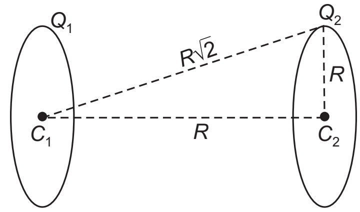
(a) zero

(b) $\dfrac{q(Q_1 - Q_2)(\sqrt{2}-1)}{\sqrt{2}\,(4\pi\varepsilon_0 R)}$

(c) $\dfrac{q\sqrt{2}(Q_1 + Q_2)}{(4\pi\varepsilon_0 R)}$

(d) $q(Q_1 / Q_2)(\sqrt{2}+1)\sqrt{2}\,(4\pi\varepsilon_0 R)$

Solution $V_{C_1} = V_{Q_1} + V_{Q_2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1}{R} + \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_2}{R\sqrt{2}} = \dfrac{1}{4\pi\varepsilon_0 R}\left(Q_1 + \dfrac{Q_2}{\sqrt{2}}\right)$



Similarly, $$V_{C_2} = \frac{1}{4\pi\varepsilon_0 R}\left(Q_2 + \frac{Q_1}{\sqrt{2}}\right)$$

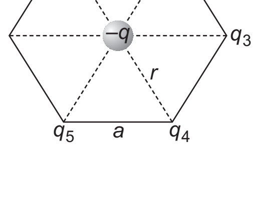
$\therefore$ $$\Delta V = V_{C_1} - V_{C_2}$$
$$= \frac{1}{4\pi\varepsilon_0 R}\left[(Q_1 - Q_2) - \frac{1}{\sqrt{2}}(Q_1 - Q_2)\right]$$
$$= \frac{Q_1 - Q_2}{\sqrt{2}(4\pi\varepsilon_0 R)}(\sqrt{2}-1)$$
$$W = q\Delta V = q\,(Q_1 - Q_2)\,(\sqrt{2}-1)/\sqrt{2}(4\pi\varepsilon_0 R)$$

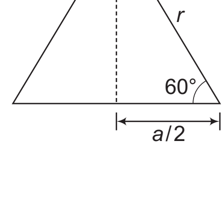
$\therefore$ The correct option is (b).

Miscellaneous Examples

Example 23 *Five point charges each of value + q are placed on five vertices of a regular hexagon of side 'a' metre. What is the magnitude of the force on a point charge of value – q coulomb placed at the centre of the hexagon?*

Solution





$$\frac{a/2}{r} = \cos 60° = \frac{1}{2}$$

$\therefore$ $$a = r$$

$$q_1 = q_2 = \ldots = q_5 = q$$

Net force on $-q$ is only due to q_3 because forces due to q_1 and due to q_4 are equal and opposite so cancel each other. Similarly, forces due to q_2 and q_5 also cancel each other. Hence, the net force on $-q$ is

$$F = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{(q)\,(q)}{r^2} \qquad \text{(towards } q_3\text{)}$$

or $$F = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q^2}{r^2}$$ **Ans.**

Example 24 *A point charge $q_1 = 9.1\,\mu C$ is held fixed at origin. A second point charge $q_2 = -0.42\,\mu C$ and a mass 3.2×10^{-4} kg is placed on the x-axis, 0.96 m from the origin. The second point charge is released at rest. What is its speed when it is 0.24 m from the origin?*

Solution From conservation of mechanical energy, we have

Decrease in electrostatic potential energy = Increase in kinetic energy

or $$\frac{1}{2} mv^2 = U_i - U_f = \frac{q_1 q_2}{4\pi\,\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{q_1 q_2}{4\pi\,\varepsilon_0}\left(\frac{r_f - r_i}{r_i r_f}\right)$$

$\therefore$ $$v = \sqrt{\frac{q_1 q_2}{2\pi\,\varepsilon_0 m}\left(\frac{r_f - r_i}{r_i r_f}\right)}$$

$$= \sqrt{\frac{(9.1 \times 10^{-6})\,(-0.42 \times 10^{-6}) \times 2 \times 9 \times 10^9}{3.2 \times 10^{-4}}\left(\frac{0.24 - 0.96}{(0.24)\,(0.96)}\right)}$$

$$= 26 \text{ m/s}$$ **Ans.**

Example 25 *A point charge $q_1 = -5.8\,\mu C$ is held stationary at the origin. A second point charge $q_2 = +4.3\,\mu C$ moves from the point (0.26 m, 0, 0) to (0.38 m, 0, 0). How much work is done by the electric force on q_2?*

Solution Work done by the electrostatic forces $= U_i - U_f$

$$= \frac{q_1 q_2}{4\pi\,\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{q_1 q_2}{4\pi\,\varepsilon_0}\left(\frac{r_f - r_i}{r_i\, r_f}\right)$$

$$= \frac{(-5.8 \times 10^{-6})\,(4.3 \times 10^{-6})\,(9 \times 10^9)\,(0.38 - 0.26)}{(0.38)\,(0.26)}$$

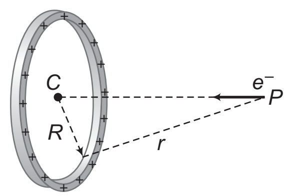
$$= -0.272 \text{ J}$$ **Ans.**

Example 26 *A uniformly charged thin ring has radius* 10.0 *cm and total charge* + 12.0 μ*C. An electron is placed on the ring's axis a distance* 25.0 *cm from the centre of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest.*

(a) describe the subsequent motion of the electron.

(b) find the speed of the electron when it reaches the centre of the ring.

Solution (a) The electron will be attracted towards the centre C of the ring. At C net force is zero, but on reaching C, electron has some kinetic energy and due to inertia it crosses C, but on the other side it is further attracted towards C. Hence, motion of electron is oscillatory about point C.



(b) As the electron approaches C, its speed (hence, kinetic energy) increases due to force of attraction towards the centre C. This increase in kinetic energy is at the cost of electrostatic potential energy. Thus,

$$\frac{1}{2}mv^2 = U_i - U_f$$

$$= U_P - U_C = (-e)[V_P - V_C] \qquad ...(i)$$

Here, V is the potential due to ring.

$$V_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad (q = \text{charge on ring})$$

$$= \frac{(9\times10^9)(12\times10^{-9})}{(\sqrt{(10)^2+(25)^2})\times10^{-2}} = 401 \text{ V}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

$$= \frac{(9\times10^9)(12\times10^{-9})}{10\times10^{-2}} = 1080 \text{ V}$$

Substituting the proper values in Eq. (i), we have

$$\frac{1}{2}\times9.1\times10^{-31}\times v^2 = (-1.6\times10^{-19})(401-1080)$$

$$\therefore \qquad v = 15.45\times10^6 \text{ m/s} \qquad \textbf{Ans.}$$

Example 27 *Two points A and B are* 2 *cm apart and a uniform electric field E acts along the straight line AB directed from A to B with* $E = 200$ *N/C. A particle of charge* $+10^{-6}$ *C is taken from A to B along AB. Calculate*

(a) the force on the charge

(b) the potential difference $V_A - V_B$ *and*

(c) the work done on the charge by **E**

Solution (a) Electrostatic force on the charge,

$$F = qE = (10^{-6})(200)$$
$$= 2 \times 10^{-4} \text{ N} \qquad \textbf{Ans.}$$

(b) In uniform electric field,

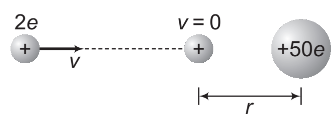
PD, $\quad V = E \cdot d$

or $\quad V_A - V_B = 200 \times 2 \times 10^{-2}$
$$= 4 \text{ V} \qquad \textbf{Ans.}$$

(c) $W = (2 \times 10^{-4})(2 \times 10^{-2}) \cos 0°$
$$= 4 \times 10^{-6} \text{ J} \qquad \textbf{Ans.}$$

Example 28 *An alpha particle with kinetic energy* 10 *MeV is heading towards a stationary tin nucleus of atomic number* 50. *Calculate the distance of closest approach. Initially they were far apart.*

Solution Due to repulsion by the tin nucleus, the kinetic energy of the α-particle gradually decreases at the expense of electrostatic potential energy.



∴ Decrease in kinetic energy = increase in potential energy

or
$$\frac{1}{2} mv^2 = U_f - U_i$$

or
$$\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r} - 0$$

∴
$$r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(50e)}{(\text{KE})}$$

Substituting the values,

$$r = \frac{(9 \times 10^9)(2 \times 1.6 \times 10^{-19})(1.6 \times 10^{-19} \times 50)}{10 \times 10^6 \times 1.6 \times 10^{-19}}$$
$$= 14.4 \times 10^{-15} \text{ m} \qquad \textbf{Ans.}$$

Example 29 *Three point charges of* 1 *C*, 2 *C and* 3 *C are placed at the corners of an equilateral triangle of side* 1 *m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side* 0.5 *m.*

Solution Work done $= U_f - U_i$

$$= \frac{1}{4\pi\,\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)[q_3 q_2 + q_3 q_1 + q_2 q_1]$$
$$= 9 \times 10^9 \left(\frac{1}{0.5} - \frac{1}{1}\right)[(3)(2) + (3)(1) + (2)(1)]$$
$$= 99 \times 10^9 \text{ J} \qquad \textbf{Ans.}$$

Note *Work done by electrostatic forces is $U_i - U_f$ but work done by external forces is $U_f - U_i$. Sometimes in a simple way it is asked, find the work done. It means $U_f - U_i$.*

Example 30 *Consider a spherical surface of radius 4 m centred at the origin. Point charges + q and – 2q are fixed at points A (2 m, 0, 0) and B (8 m, 0, 0) respectively. Show that every point on the spherical surface is at zero potential.*

Solution Let P (x, y, z) be any point on the sphere. From the property of the sphere,

$$x^2 + y^2 + z^2 = (4^2) = 16 \qquad \text{...(i)}$$

Further, $$PA = \sqrt{(x-2)^2 + y^2 + z^2} \qquad \text{...(ii)}$$

and $$PB = \sqrt{(x-8)^2 + y^2 + z^2} \qquad \text{...(iii)}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PA} - \frac{2q}{PB}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(x-2)^2 + y^2 + z^2}} - \frac{2q}{\sqrt{(x-8)^2 + y^2 + z^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + y^2 + z^2 + 4 - 4x}} - \frac{2q}{\sqrt{x^2 + y^2 + z^2 + 64 - 16x}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{16 + 4 - 4x}} - \frac{2q}{\sqrt{16 + 64 - 16x}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{20 - 4x}} - \frac{q}{\sqrt{20 - 4x}}\right]$$

$$= 0$$

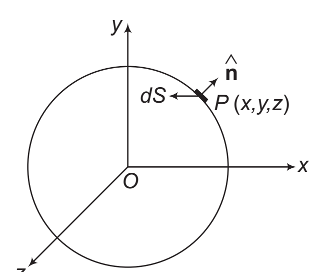
Proved

Example 31 *The intensity of an electric field depends only on the coordinates x and y as follows*

$$\mathbf{E} = \frac{a\,(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}})}{x^2 + y^2}$$

where, a is a constant and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are the unit vectors of the x and y-axes. Find the charge within a sphere of radius R with the centre at the origin.

Solution At any point P (x, y, z) on the sphere a unit vector perpendicular to the sphere radially outwards is



$$\hat{\mathbf{n}} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}\,\hat{\mathbf{i}} + \frac{y}{\sqrt{x^2 + y^2 + z^2}}\,\hat{\mathbf{j}} + \frac{z}{\sqrt{x^2 + y^2 + z^2}}\,\hat{\mathbf{k}}$$

$$= \frac{x}{R}\,\hat{\mathbf{i}} + \frac{y}{R}\,\hat{\mathbf{j}} + \frac{z}{R}\,\hat{\mathbf{k}} \quad \text{as} \quad x^2 + y^2 + z^2 = R^2$$

Let us find the electric flux passing through a small area dS at point P on the sphere,

$$d\phi = \mathbf{E}\cdot\hat{\mathbf{n}}\;dS = \left\{\frac{ax^2}{R\,(x^2 + y^2)} + \frac{ay^2}{R\,(x^2 + y^2)}\right\} dS$$

$$= \left(\frac{a}{R}\right) dS$$

Here, we note that $d\phi$ is independent of the coordinates x, y and z. Therefore, total flux passing through the sphere

$$\phi = \int d\phi = \frac{a}{R}\int dS = \left(\frac{a}{R}\right)(4\pi R^2)$$

$$= 4\pi aR$$

From Gauss's law,

$$\phi = \frac{q_{\text{in}}}{\varepsilon_0} \quad \text{or} \quad (4\pi aR) = \frac{q_{\text{in}}}{\varepsilon_0}$$

$\therefore$ $$q_{\text{in}} = 4\pi\varepsilon_0 aR$$ **Ans.**

Example 32 *Find the electric field caused by a disc of radius a with a uniform surface charge density σ (charge per unit area), at a point along the axis of the disc a distance x from its centre.*

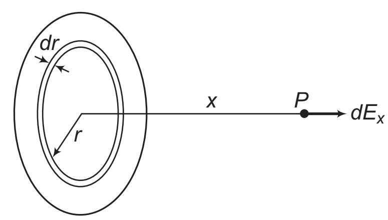
Solution We can assume this charge distribution as a collection of concentric rings of charge.

$$dA = (2\pi r)\, dr$$

$$dq = \sigma\, dA = (2\pi\sigma r)\, dr$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(dq)x}{(x^2+r^2)^{3/2}}$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(2\pi\sigma r\, dr)\, x}{(x^2+r^2)^{3/2}}$$



$\therefore$ $$E_x = \int_0^a dE_x$$

$$= \int_0^a \frac{(2\pi\sigma r\, dr)\, x}{4\pi\,\varepsilon_0\,(x^2+r^2)^{3/2}}$$

$$= \frac{\sigma x}{2\varepsilon_0}\int_0^a \frac{r\, dr}{(x^2+r^2)^{3/2}}$$

or $$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{a^2/x^2+1}}\right]$$

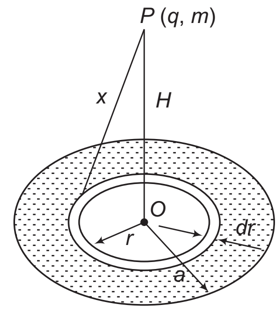
If the charge distribution gets very large, i.e. $a >> x$, the term $\dfrac{1}{\sqrt{a^2/x^2+1}}$ becomes negligibly small, and we get $E = \dfrac{\sigma}{2\varepsilon_0}$.

Thus, we can say that electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet. Thus, the field is uniform, its direction is everywhere perpendicular to the sheet.

Example 33 *A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0\, g/\sigma$.*

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.



Solution Potential at a height H **on the axis of the disc** $V(P)$. The charge dq contained in the ring shown in figure, $dq = (2\pi r dr)\sigma$

Potential of P due to this ring

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{x}, \quad \text{where } x = \sqrt{H^2 + r^2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2\pi r dr)\,\sigma}{\sqrt{H^2 + r^2}} = \frac{\sigma}{2\varepsilon_0}\,\frac{r dr}{\sqrt{H^2 + r^2}}$$

$\therefore$ Potential due to the complete disc,

$$V_P = \int_{r=0}^{r=a} dV = \frac{\sigma}{2\varepsilon_0}\int_{r=0}^{r=a}\frac{r dr}{\sqrt{H^2 + r^2}}$$

$$V_P = \frac{\sigma}{2\varepsilon_0}\,[\sqrt{a^2 + H^2} - H]$$

Potential at centre, O will be

$$V_O = \frac{\sigma a}{2\varepsilon_0} \qquad (H = 0)$$

(a) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy

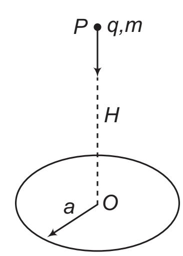
decrease in gravitational potential energy = increase in electrostatic potential energy

(Δ KE = 0 because $K_i = K_f = 0$)

$\therefore$ $$mgH = q\,[V_O - V_P]$$

or $$gH = \left(\frac{q}{m}\right)\left(\frac{\sigma}{2\varepsilon_0}\right)[a - \sqrt{a^2 + H^2} + H] \qquad \ldots\text{(i)}$$

$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma} \Rightarrow \frac{q\sigma}{2\varepsilon_0 m} = 2g$$



Substituting in Eq. (i), we get

$$gH = 2g\,[a + H - \sqrt{a^2 + H^2}]$$

or $$\frac{H}{2} = (a + H) - \sqrt{a^2 + H^2}$$

or $$\sqrt{a^2 + H^2} = a + \frac{H}{2}$$

or $$a^2 + H^2 = a^2 + \frac{H^2}{4} + aH$$

or $$\frac{3}{4}H^2 = aH \quad \text{or} \quad H = \frac{4}{3}a \quad \text{and} \quad H = 0$$

$\therefore$ $$H = (4/3)\,a$$ **Ans.**

(b) Potential energy of the particle at height H = Electrostatic potential energy + gravitational potential energy

$\therefore$ $$U = qV + mgH$$

Here, V = Potential at height H

$\therefore$ $$U = \frac{\sigma q}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H] + mgH \qquad \ldots\text{(ii)}$$

At equilibrium position, $F = \frac{-dU}{dH} = 0$

Differentiating Eq. (ii) w.r.t. H,

or $$mg + \frac{\sigma q}{2\varepsilon_0}\left[\left(\frac{1}{2}\right)(2H)\frac{1}{\sqrt{a^2+H^2}} - 1\right] = 0 \qquad \left[\frac{\sigma q}{2\varepsilon_0} = 2mg\right]$$

$\therefore$ $$mg + 2mg\left[\frac{H}{\sqrt{a^2+H^2}} - 1\right] = 0$$

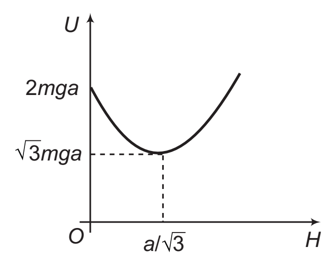
or $$1 + \frac{2H}{\sqrt{a^2+H^2}} - 2 = 0 \Rightarrow \frac{2H}{\sqrt{a^2+H^2}} = 1$$

or $$\frac{H^2}{a^2+H^2} = \frac{1}{4} \quad \text{or} \quad 3H^2 = a^2$$

or $$H = \frac{a}{\sqrt{3}}$$ **Ans.**

From Eq. (ii), we can see that

$$U = 2\ mga \text{ at } H = 0 \text{ and}$$
$$U = U_{\min} = \sqrt{3}\ mga \text{ at } H = \frac{a}{\sqrt{3}}$$



Therefore, U-H graph will be as shown.

Note that at $H = \frac{a}{\sqrt{3}}$, U is minimum.

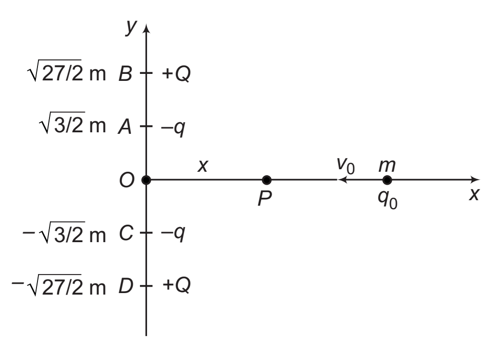
Therefore, $H = \frac{a}{\sqrt{3}}$ is stable equilibrium position.

Example 34 *Four point charges $+8\,\mu C$, $-1\,\mu C$, $-1\,\mu C$ and $+8\,\mu C$ are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the Y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\,\mu C$ moves along the –X direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free.*

Solution In the figure,

$$q = 1\,\mu\text{C} = 10^{-6}\ \text{C}$$
$$q_0 = +0.1\,\mu\text{C} = 10^{-7}\ \text{C}$$
$$m = 6 \times 10^{-4}\ \text{kg}$$

and $$Q = 8\,\mu\text{C} = 8 \times 10^{-6}\ \text{C}$$



Let P be any point at a distance x from origin O. Then,

$$AP = CP = \sqrt{\frac{3}{2} + x^2}$$

$$BP = DP = \sqrt{\frac{27}{2} + x^2}$$

Electric potential at point P will be

$$V = \frac{2kQ}{BP} - \frac{2kq}{AP}$$

where, $$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$\therefore$ $$V = 2 \times 9 \times 10^9 \left[\frac{8 \times 10^{-6}}{\sqrt{\frac{27}{2} + x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2} + x^2}}\right]$$

$$V = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2} + x^2}} - \frac{1}{\sqrt{\frac{3}{2} + x^2}}\right] \qquad \text{...(i)}$$

$\therefore$ Electric field at P is

$$E = -\frac{dV}{dx} = -1.8 \times 10^4 \left[(8)\left(-\frac{1}{2}\right)\left(\frac{27}{2} + x^2\right)^{-3/2} - (1)\left(-\frac{1}{2}\right)\left(\frac{3}{2} + x^2\right)^{-3/2}\right](2x)$$

$E = 0$ on x-axis where

$$\frac{8}{\left(\frac{27}{2} + x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2} + x^2\right)^{3/2}}$$

$\Rightarrow$ $$\frac{(4)^{3/2}}{\left(\frac{27}{2} + x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2} + x^2\right)^{3/2}}$$

$\Rightarrow$ $$\left(\frac{27}{2} + x^2\right) = 4\left(\frac{3}{2} + x^2\right)$$

This equation gives $$x = \pm\sqrt{\frac{5}{2}} \text{ m}$$

The least value of kinetic energy of the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m because

at $x = +\sqrt{\frac{5}{2}}$ m, $E = 0$ $\Rightarrow$ Electrostatic force on charge q_0 is zero or $F_e = 0$

for $x > \sqrt{\frac{5}{2}}$ m, E is repulsive (towards positive x-axis)

and for $x < \sqrt{\frac{5}{2}}$ m, E is attractive (towards negative x-axis)

Now, from Eq. (i), potential at $$x = \sqrt{\frac{5}{2}} \text{ m}$$

$$V = 1.8. \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2} + \frac{5}{2}}} - \frac{1}{\sqrt{\frac{3}{2} + \frac{5}{2}}}\right]$$

$$V = 2.7 \times 10^4 \text{ V}$$

Applying energy conservation at $x = \infty$ and $x = \sqrt{\frac{5}{2}}$ m

$$\frac{1}{2} m v_0^2 = q_0 V \qquad \text{...(ii)}$$

$\therefore$
$$v_0 = \sqrt{\frac{2 q_0 V}{m}}$$

Substituting the values,
$$v_0 = \sqrt{\frac{2 \times 10^{-7} \times 2.7 \times 10^{4}}{6 \times 10^{-4}}}$$

$$v_0 = 3 \text{ m/s}$$ **Ans.**

$\therefore$ Minimum value of v_0 is 3 m/s.

From Eq. (i), potential at origin $(x = 0)$ is

$$V_0 = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2}}} - \frac{1}{\sqrt{\frac{3}{2}}} \right] = 2.4 \times 10^4 \text{ V}$$

Let T be the kinetic energy of the particle at origin.

Applying energy conservation at $x = 0$ and at $x = \infty$

$$T + q_0 V_0 = \frac{1}{2} m v_0^2$$

But,
$$\frac{1}{2} m v_0^2 = q_0 V \qquad \text{[from Eq. (ii)]}$$

$\therefore$
$$T = q_0 (V - V_0)$$
$$T = (10^{-7})(2.7 \times 10^4 - 2.4 \times 10^4)$$
$$T = 3 \times 10^{-4} \text{ J}$$ **Ans.**

Note *$E = 0$ or F_e on q_0 is zero at $x = 0$ and $x = \pm\sqrt{\frac{5}{2}}$ m. Of these $x = 0$ is stable equilibrium position and $x = \pm\sqrt{\frac{5}{2}}$ m is unstable equilibrium position.*

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(c) *If **Assertion** is true, but the **Reason** is false.*

(d) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : An independent negative charge moves itself from point A to point B. Then, potential at A should be less than potential at B.

Reason : While moving from A to B kinetic energy of electron will increase.

2. Assertion : When two unlike charges are brought nearer, their electrostatic potential energy decreases.

Reason : All conservative forces act in the direction of decreasing potential energy.

3. Assertion : At a point electric potential is decreasing along x-axis at a rate of 10 V/m. Therefore, x-component of electric field at this point should be 10 V/m along x-axis.

Reason : Magnitude of $E_x = \frac{\partial V}{\partial x}$

4. Assertion : Electric potential on the surface of a charged sphere of radius R is V. Then electric field at a distance $r = \frac{R}{2}$ from centre is $\frac{V}{2R}$. Charge is distributed uniformly over the volume.

Reason : From centre to surface, electric field varies linearly with r. Here, r is distance from centre.

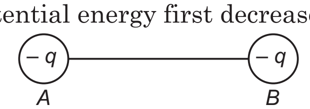
5. Assertion : Gauss's theorem can be applied only for a closed surface.

Reason : Electric flux can be obtained passing from an open surface also.

6. Assertion : In the electric field $\mathbf{E} = (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N/C, electric potential at $A(4\text{ m}, 0)$ is more than the electric potential at $B(0, 4\text{ m})$.

Reason : Electric lines of forces always travel from higher potential to lower potential.

7. Assertion : Two charges $-q$ each are fixed at points A and B. When a third charge $-q$ is moved from A to B, electrical potential energy first decreases than increases.



Reason : Along the line joining A and B, the third charge is in stable equilibrium position at centre.

8. **Assertion :** A small electric dipole is moved translationally from higher potential to lower potential in uniform electric field. Work done by electric field is positive.

   Reason : When a positive charge is moved from higher potential to lower potential, work done by electric field is positive.

9. **Assertion :** In case of charged spherical shells, *E-r* graph is discontinuous while *V-r* graph is continuous.

   Reason : According to Gauss's theorem only the charge inside a closed surface can produce electric field at some point.

10. **Assertion :** If we see along the axis of a charged ring, the magnitude of electric field is minimum at centre and magnitude of electric potential is maximum.

    Reason : Electric field is a vector quantity while electric potential is scalar.

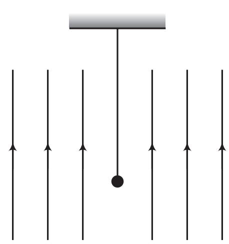
Objective Questions

1. Units of electric flux are

   (a) $\frac{\text{N-m}^2}{\text{C}^2}$ (b) $\frac{\text{N}}{\text{C}^2\text{-m}^2}$

   (c) volt-m (d) volt- m^3

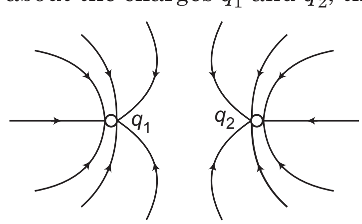
2. A neutral pendulum oscillates in a uniform electric field as shown in figure. If a positive charge is given to the pendulum, then its time period



   (a) will increase (b) will decrease

   (c) will remain constant (d) will first increase then decrease

3. Identify the correct statement about the charges q_1 and q_2, then



   (a) q_1 and q_2 both are positive (b) q_1 and q_2 both are negative

   (c) q_1 is positive q_2 is negative (d) q_2 is positive and q_1 is negative

4. Three identical charges are placed at corners of an equilateral triangle of side l. If force between any two charges is F, the work required to double the dimensions of triangle is

   (a) $-3\ Fl$ (b) $3\ Fl$

   (c) $(-3/2)\ Fl$ (d) $(3/2)\ Fl$

5. A proton, a deuteron and an alpha particle are accelerated through potentials of V, 2 V and 4 V respectively. Their velocity will bear a ratio

(a) $1:1:1$ (b) $1:\sqrt{2}:1$
(c) $\sqrt{2}:1:1$ (d) $1:1:\sqrt{2}$

6. Electric potential at a point P, r distance away due to a point charge q kept at point A is V. If twice of this charge is distributed uniformly on the surface of a hollow sphere of radius $4r$ with centre at point A, the potential at P now is

(a) V (b) $V/2$
(c) $V/4$ (d) $V/8$

7. Four charges $+q, -q, +q$ and $-q$ are placed in order on the four consecutive corners of a square of side a. The work done in interchanging the positions of any two neighbouring charges of the opposite sign is

(a) $\frac{q^2}{4\pi\varepsilon_0 a}(-4+\sqrt{2})$ (b) $\frac{q^2}{4\pi\varepsilon_0 a}(4+2\sqrt{2})$
(c) $\frac{q^2}{4\pi\varepsilon_0 a}(4-2\sqrt{2})$ (d) $\frac{q^2}{4\pi\varepsilon_0 a}(4+\sqrt{2})$

8. Two concentric spheres of radii R and $2R$ are charged. The inner sphere has a charge of $1\,\mu C$ and the outer sphere has a charge of $2\,\mu C$ of the same sign. The potential is 9000 V at a distance $3R$ from the common centre. The value of R is

(a) 1 m (b) 2 m
(c) 3 m (d) 4 m

9. A ring of radius R is having two charges q and $2q$ distributed on its two half parts. The electric potential at a point on its axis at a distance of $2\sqrt{2}\,R$ from its centre is $\left(k=\frac{1}{4\pi\varepsilon_0}\right)$

(a) $\frac{3kq}{R}$ (b) $\frac{kq}{3R}$
(c) $\frac{kq}{R}$ (d) $\frac{kq}{\sqrt{3}R}$

10. A particle A having a charge of 2.0×10^{-6} C and a mass of 100 g is fixed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle B having same charge and mass, be placed on the inclined plane so that B may remain in equilibrium?

(a) 8 cm from the bottom (b) 13 cm from the bottom
(c) 21 cm from the bottom (d) 27 cm from the bottom

11. Four positive charges $(2\sqrt{2}-1)Q$ are arranged at the four corners of a square. Another charge q is placed at the centre of the square. Resulting force acting on each corner charge is zero if q is

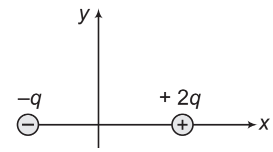
(a) $-\frac{7Q}{4}$ (b) $-\frac{4Q}{7}$
(c) $-Q$ (d) $-(\sqrt{2}+1)Q$

12. A proton is released from rest, 10 cm from a charged sheet carrying charged density of $-2.21\times10^{-9}\,C/m^2$. It will strike the sheet after the time (approximately)

(a) $4\,\mu s$ (b) $2\,\mu s$
(c) $2\sqrt{2}\,\mu s$ (d) $4\sqrt{2}\,\mu s$

13. Two point charges $+q$ and $-q$ are placed a distance x apart. A third charge is so placed that all the three charges are in equilibrium. Then,
(a) unknown charge is $-4q/9$
(b) unknown charge is $-9q/4$
(c) it should be at $(x/3)$ from smaller charge between them
(d) None of the above

14. Charges $2q$ and $-q$ are placed at $(a, 0)$ and $(-a, 0)$ as shown in the figure. The coordinates of the point at which electric field intensity is zero will be $(x, 0)$, where



(a) $-a < x < a$ (b) $x < -a$
(c) $x > -a$ (d) $0 < x < a$

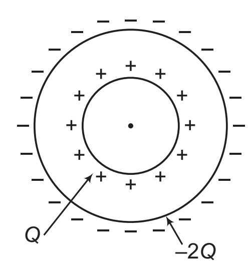
15. Five point charges ($+q$ each) are placed at the five vertices of a regular hexagon of side $2a$. What is the magnitude of the net electric field at the centre of the hexagon?
(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ (b) $\frac{q}{16\pi\varepsilon_0 a^2}$
(c) $\frac{\sqrt{2}q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{5q}{16\pi\varepsilon_0 a^2}$

16. Two identical small conducting spheres having unequal positive charges q_1 and q_2 are separated by a distance r. If they are now made to touch each other and then separated again to the same distance, the electrostatic force between them in this case will be
(a) less than before (b) same as before
(c) more than before (d) zero

17. Three concentric conducting spherical shells carry charges $+4Q$ on the inner shell $-2Q$ on the middle shell and $+6Q$ on the outer shell. The charge on the inner surface of the outer shell is
(a) 0 (b) $4Q$
(c) $-Q$ (d) $-2Q$

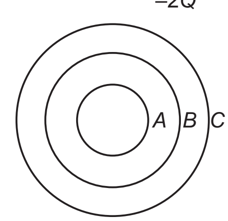
18. 1000 drops of same size are charged to a potential of 1 V each. If they coalesce to form a single drop, its potential would be
(a) V (b) 10 V
(c) 100 V (d) 1000 V

19. Two concentric conducting spheres of radii R and $2R$ are carrying charges Q and $-2Q$, respectively. If the charge on inner sphere is doubled, the potential difference between the two spheres will



(a) become two times
(b) become four times
(c) be halved
(d) remain same

20. Charges Q, $2Q$ and $-Q$ are given to three concentric conducting spherical shells A, B and C respectively as shown in figure. The ratio of charges on the inner and outer surfaces of shell C will be



(a) $+\frac{3}{4}$ (b) $\frac{-3}{4}$
(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

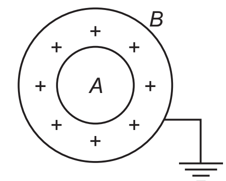
21. The electric field in a region of space is given by $\mathbf{E} = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ N/C. The flux of $\mathbf{E}$ due to this field through an area $1\,\text{m}^2$ lying in the y-z plane, in SI units, is

(a) 5 (b) 10
(c) 2 (d) $5\sqrt{29}$

22. A charge Q is placed at each of the two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant force on each charge q is zero, then

(a) $q = \sqrt{2}\,Q$ (b) $q = -\sqrt{2}\,Q$
(c) $q = 2\sqrt{2}\,Q$ (d) $q = -2\sqrt{2}\,Q$

23. A and B are two concentric spherical shells. If A is given a charge $+q$ while B is earthed as shown in figure, then



(a) charge on the outer surface of shell B is zero
(b) the charge on B is equal and opposite to that of A
(c) the field inside A and outside B is zero
(d) All of the above

24. A solid sphere of radius R has charge 'q' uniformly distributed over its volume. The distance from its surface at which the electrostatic potential is equal to half of the potential at the centre is

(a) R (b) $2R$
(c) $\frac{R}{3}$ (d) $\frac{R}{2}$

25. Four dipoles each of magnitudes of charges $\pm e$ are placed inside a sphere. The total flux of $\mathbf{E}$ coming out of the sphere is

(a) zero (b) $\frac{4e}{\varepsilon_0}$
(c) $\frac{8e}{\varepsilon_0}$ (d) None of these

26. A pendulum bob of mass m carrying a charge q is at rest with its string making an angle θ with the vertical in a uniform horizontal electric field E. The tension in the string is

(a) $\frac{mg}{\sin\theta}$ (b) mg
(c) $\frac{qE}{\sin\theta}$ (d) $\frac{qE}{\cos\theta}$

27. Two isolated charged conducting spheres of radii a and b produce the same electric field near their surfaces. The ratio of electric potentials on their surfaces is

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$
(c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

28. Two point charges $+q$ and $-q$ are held fixed at $(-a, 0)$ and $(a, 0)$ respectively of a x-y coordinate system, then

(a) the electric field $\mathbf{E}$ at all points on the x-axis has the same direction
(b) $\mathbf{E}$ at all points on the y-axis is along $\hat{\mathbf{i}}$
(c) positive work is done in bringing a test charge from infinity to the origin
(d) All of the above

29. A conducting shell S_1 having a charge Q is surrounded by an uncharged concentric conducting spherical shell S_2. Let the potential difference between S_1 and that S_2 be V. If the shell S_2 is now given a charge $-3Q$, the new potential difference between the same two shells is

(a) V
(b) $2V$
(c) $4V$
(d) $-2V$

30. At a certain distance from a point charge, the field intensity is 500 V/m and the potential is – 3000 V. The distance to the charge and the magnitude of the charge respectively are

(a) 6 m and 6 μC
(b) 4 m and 2 μC
(c) 6 m and 4 μC
(d) 6 m and 2 μC

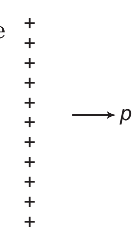
31. Two point charges q_1 and q_2 are placed at a distance of 50 m from each other in air, and interact with a certain force. The same charges are now put in oil whose relative permittivity is 5. If the interacting force between them is still the same, their separation now is

(a) 16.6 m
(b) 22.3 m
(c) 28.4 m
(d) 25.0 cm

32. An infinite line of charge λ per unit length is placed along the y-axis. The work done in moving a charge q from $A(a, 0)$ to $B(2a, 0)$ is

(a) $\frac{q\lambda}{2\pi\varepsilon_0}\ln 2$
(b) $\frac{q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{1}{2}\right)$
(c) $\frac{q\lambda}{4\pi\varepsilon_0}\ln\sqrt{2}$
(d) $\frac{q\lambda}{4\pi\varepsilon_0}\ln 2$

33. An electric dipole is placed perpendicular to an infinite line of charge at some distance as shown in figure. Identify the correct statement.

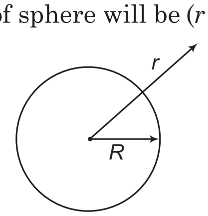


(a) The dipole is attracted towards the line charge
(b) The dipole is repelled away from the line charge
(c) The dipole does not experience a force
(d) The dipole experiences a force as well as a torque

34. An electrical charge 2×10^{-8} C is placed at the point (1, 2, 4) m. At the point (4, 2, 0) m, the electric

(a) potential will be 36 V
(b) field will be along y-axis
(c) field will increase if the space between the points is filled with a dielectric
(d) All of the above

35. If the potential at the centre of a uniformly charged hollow sphere of radius R is V, then electric field at a distance r from the centre of sphere will be $(r > R)$



(a) $\frac{VR}{r^2}$
(b) $\frac{Vr}{R^2}$
(c) $\frac{VR}{r}$
(d) $\frac{VR}{R^2 + r^2}$

36. There is an electric field E in x-direction. If the work done on moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with x-axis is 4 J, then what is the value of E?

(a) $\sqrt{3}$ N/C (b) 4 N/C
(c) 5 N/C (d) 20 N/C

37. Two thin wire rings each having radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+Q$ and $-Q$. The potential difference between the centres of the two rings is

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\frac{Q}{4\pi\varepsilon_0 d^2}$ (d) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

38. The electric field at a distance 2 cm from the centre of a hollow spherical conducting shell of radius 4 cm having a charge of 2×10^{-3} C on its surface is

(a) 1.1×10^{10} V/m (b) 4.5×10^{-10} V/m
(c) 4.5×10^{10} V/m (d) zero

39. Charge Q is given a displacement $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ in an electric field $\mathbf{E} = E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}}$. The work done is

(a) $Q(E_1a + E_2b)$ (b) $Q\sqrt{(E_1a)^2+(E_2b)^2}$

(c) $Q(E_1+E_2)\sqrt{a^2+b^2}$ (d) $Q\sqrt{E_1^2+E_2^2}\,\sqrt{a^2+b^2}$

Subjective Questions

Note *You can take approximations in the answers.*

1. A certain charge Q is divided into two parts q and $Q-q$, which are then separated by a certain distance. What must q be in terms of Q to maximize the electrostatic repulsion between the two charges?

2. An α-particle is the nucleus of a helium atom. It has a mass $m = 6.64\times10^{-27}$ kg and a charge $q = +2e = 3.2\times10^{-19}$ C. Compare the force of the electric repulsion between two α-particles with the force of gravitational attraction between them.

3. What is the charge per unit area in $\mathrm{C/m^2}$ of an infinite plane sheet of charge if the electric field produced by the sheet of charge has magnitude 3.0 N/C?

4. A circular wire loop of radius R carries a total charge q distributed uniformly over its length. A small length x ($\ll R$) of the wire is cut off. Find the electric field at the centre due to the remaining wire.

5. Two identical conducting spheres, fixed in space, attract each other with an electrostatic force of 0.108 N when separated by 50.0 cm, centre-to-centre. A thin conducting wire then connects the spheres. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. What were the initial charges on the spheres?

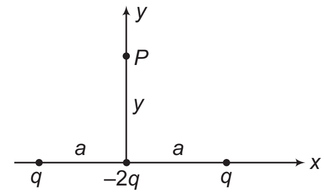
6. Show that the torque on an electric dipole placed in a uniform electric field is

$$\tau = \mathbf{p}\times\mathbf{E}$$

independent of the origin about which torque is calculated.

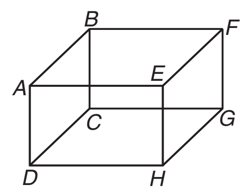
7. Three point charges $q, -2q$ and q are located along the x-axis as shown in figure. Show that the electric field at P ($y >> a$) along the y-axis is,

$$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{3qa^2}{y^4}\hat{\mathbf{j}}$$



Note *This charge distribution which is essentially that of two electric dipoles is called an electric quadrupole. Note that* **E** *varies as* r^{-4} *for a quadrupole compared with variations of* r^{-3} *for the dipole and* r^{-2} *for a monopole (a single charge).*

8. A charge q is placed at point D of the cube. Find the electric flux passing through the face $EFGH$ and face $AEHD$.



9. Point charges q_1 and q_2 lie on the x-axis at points $x = -a$ and $x = +a$ respectively.

(a) How must q_1 and q_2 be related for the net electrostatic force on point charge $+Q$, placed at $x = +a/2$, to be zero?

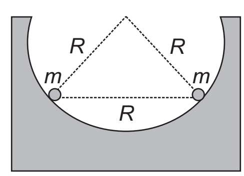
(b) With the same point charge $+Q$ now placed at $x = +3a/2$.

10. Two particles (free to move) with charges $+q$ and $+4q$ are a distance L apart. A third charge is placed so that the entire system is in equilibrium.

(a) Find the location, magnitude and sign of the third charge.

(b) Show that the equilibrium is unstable.

11. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, non-conducting walls, the beads move, and at equilibrium they are a distance R apart (figure). Determine the charge on each bead.



12. Three identical small balls, each of mass 0.1 g, are suspended at one point on silk thread having a length of $l = 20\,\text{cm}$. What charges should be imparted to the balls for each thread to form an angle of $\alpha = 30°$ with the vertical?

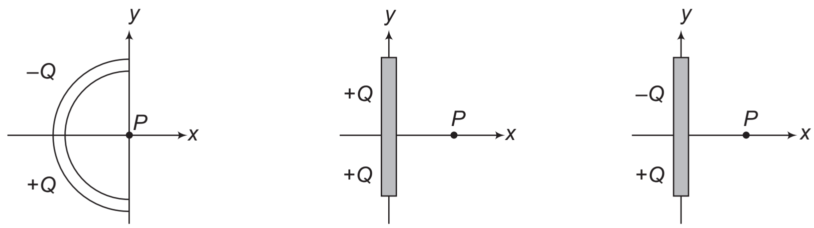
13. Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at fourth corner.

14. A point charge $q = -8.0$ nC is located at the origin. Find the electric field vector at the point $x = 1.2$ m, $y = -1.6$ m.

15. Find the electric field at the centre of a uniformly charged semicircular ring of radius R. Linear charge density is λ.

16. Find the electric field at a point P on the perpendicular bisector of a uniformly charged rod. The length of the rod is L, the charge on it is Q and the distance of P from the centre of the rod is a.

17. Find the direction of electric field at point P for the charge distribution as shown in figure.

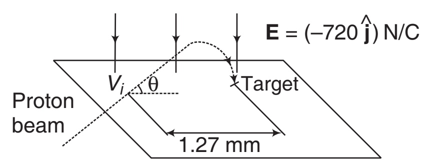


(a) (b) (c)

18. A clock face has charges $-q, -2q, -3q, \ldots -12q$ fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the direction of the electric field at the centre of the dial.

19. A charged particle of mass $m = 1$ kg and charge $q = 2\,\mu\text{C}$ is thrown from a horizontal ground at an angle $\theta = 45°$ with the speed 25 m/s. In space, a horizontal electric field $E = 2 \times 10^7$ V/m exists in the direction of motion. Find the range on horizontal ground of the projectile thrown. Take $g = 10\ \text{m/s}^2$.

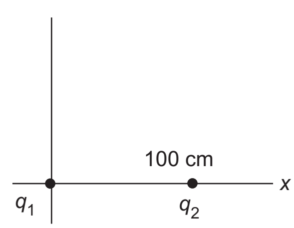
20. Protons are projected with an initial speed $v_i = 9.55 \times 10^3$ m/s into a region where a uniform electric field $\mathbf{E} = (-720\,\hat{\mathbf{j}})$ N/C is present, as shown in figure. The protons are to hit a target that lies at a horizontal distance of 1.27 mm from the point where the protons are launched. Find



(a) the two projection angles θ that result in a hit and
(b) the total time of flight for each trajectory.

21. At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 1.5 \times 10^5$ m/s and $v_y = 3.0 \times 10^6$ m/s. Suppose that the electric field between the plates is given by $\mathbf{E} = (120\ \text{N/C})\,\hat{\mathbf{j}}$.
(a) What is the acceleration of the electron?
(b) What will be the velocity of the electron after its x-coordinate has changed by 2.0 cm?

22. A point charge $q_1 = +2\,\mu\text{C}$ is placed at the origin of coordinates. A second charge, $q_2 = -3\,\mu\text{C}$, is placed on the x-axis at $x = 100$ cm. At what point (or points) on the x-axis will the absolute potential be zero?



23. A charge Q is spread uniformly in the form of a line charge density $\lambda = \dfrac{Q}{3a}$ on the sides of an equilateral triangle of perimeter $3a$. Calculate the potential at the centroid C of the triangle.

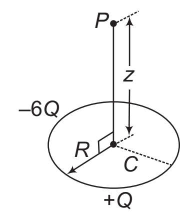
24. A uniform electric field of magnitude 250 V/m is directed in the positive x-direction. A $+12\mu C$ charge moves from the origin to the point $(x, y) = (20.0 \text{ cm}, 5.0 \text{ cm})$.

(a) What was the change in the potential energy of this charge?

(b) Through what potential difference did the charge move?

25. A small particle has charge $-5.00\,\mu C$ and mass 2.00×10^{-4} kg. It moves from point A, where the electric potential is $V_A = +200$ V, to point B, where the electric potential is $V_B = +800$ V. The electric force is the only force acting on the particle. The particle has speed 5.00 m/s at point A. What is its speed at point B? Is it moving faster or slower at B than at A? Explain.

26. A plastic rod has been formed into a circle of radius R. It has a positive charge $+Q$ uniformly distributed along one-quarter of its circumference and a negative charge of $-6Q$ uniformly distributed along the rest of the circumference (figure). With $V = 0$ at infinity, what is the electric potential



(a) at the centre C of the circle and

(b) at point P, which is on the central axis of the circle at distance z from the centre?

27. A point charge $q_1 = +2.40\,\mu C$ is held stationary at the origin. A second point charge $q_2 = -4.30\,\mu C$ moves from the point $x = 0.150$ m, $y = 0$ to the point $x = 0.250$ m, $y = 0.250$ m. How much work is done by the electric force on q_2?

28. A point charge $q_1 = 4.00$ nC is placed at the origin, and a second point charge $q_2 = -3.00$ nC is placed on the x-axis at $x = +20.0$ cm. A third point charge $q_3 = 2.00$ nC is placed on the x-axis between q_1 and q_2. (Take as zero the potential energy of the three charges when they are infinitely far apart).

(a) What is the potential energy of the system of the three charges if q_3 is placed at $x = +10.0$ cm?

(b) Where should q_3 be placed to make the potential energy of the system equal to zero?

29. Three point charges, which initially are infinitely far apart, are placed at the corners of an equilateral triangle with sides d. Two of the point charges are identical and have charge q. If zero net work is required to place the three charges at the corners of the triangles, what must the value of the third charge be?

30. The electric field in a certain region is given by $\mathbf{E} = (5\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ kV/m . Find the difference in potential $V_B - V_A$. If A is at the origin and point B is at (a) (0, 0, 5) m, (b) (4, 0, 3) m.

31. In a certain region of space, the electric field is along $+y$-direction and has a magnitude of 400 V/m . What is the potential difference from the coordinate origin to the following points?

(a) $x = 0$, $y = 20$ cm, $z = 0$

(b) $x = 0$, $y = -30$ cm, $z = 0$

(c) $x = 0$, $y = 0$, $z = 15$ cm

32. An electric field of 20 N/C exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by

(a) $A = (0, 0)$, $B = (4 \text{ m}, 2 \text{ m})$

(b) $A = (4 \text{ m}, 2 \text{ m})$, $B = (6 \text{ m}, 5 \text{ m})$

33. The electric potential existing in space is $V(x, y, z) = A(xy + yz + zx)$.

(a) Write the dimensional formula of A.

(b) Find the expression for the electric field.

(c) If A is 10 SI units, find the magnitude of the electric field at (1 m, 1 m, 1 m)

34. An electric field $\mathbf{E} = (20\,\hat{\mathbf{i}} + 30\,\hat{\mathbf{j}})$ N/C exists in the space. If the potential at the origin is taken to be zero, find the potential at $(2\,m, 2\,m)$.

35. In a certain region of space, the electric potential is $V(x, y, z) = Axy - Bx^2 + Cy$, where A, B and C are positive constants.

(a) Calculate the x, y and z-components of the electric field.

(b) At which points is the electric field equal to zero?

36. A sphere centered at the origin has radius 0.200 m. A $-500\,\mu\text{C}$ point charge is on the x-axis at $x = 0.300$ m. The net flux through the sphere is 360 N-m^2/ C. What is the total charge inside the sphere?

37. (a) A closed surface encloses a net charge of $-3.60\,\mu\text{C}$. What is the net electric flux through the surface?

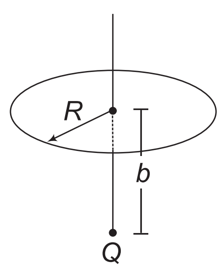
(b) The electric flux through a closed surface is found to be 780 N-m^2/C. What quantity of charge is enclosed by the surface?

(c) The closed surface in part (b) is a cube with sides of length 2.50 cm. From the information given in part (b), is it possible to tell where within the cube the charge is located? Explain.

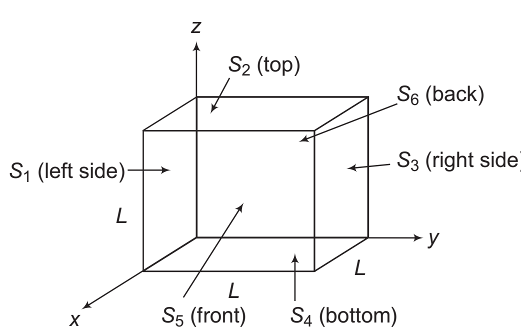
38. The electric field in a region is given by $\mathbf{E} = \frac{3}{5}E_0\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the y-z plane.

39. The electric field in a region is given by $\mathbf{E} = \frac{E_0 x}{l}\hat{\mathbf{i}}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N/C, $l = 2$ cm and $a = 1$ cm.

40. A point charge Q is located on the axis of a disc of radius R at a distance b from the plane of the disc (figure). Show that if one-fourth of the electric flux from the charge passes through the disc, then $R = \sqrt{3}\,b$.



41. A cube has sides of length L. It is placed with one corner at the origin as shown in figure. The electric field is uniform and given by $\mathbf{E} = -B\hat{\mathbf{i}} + C\hat{\mathbf{j}} - D\hat{\mathbf{k}}$, where B, C and D are positive constants.



(a) Find the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5 and S_6.

(b) Find the electric flux through the entire cube.

42. Two point charges q and $-q$ are separated by a distance $2l$. Find the flux of electric field strength vector across the circle of radius R placed with its centre coinciding with the mid-point of line joining the two charges in the perpendicular plane.

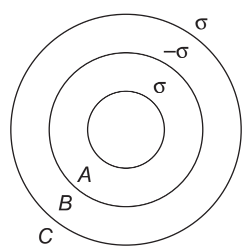
43. A point charge q is placed at the origin. Calculate the electric flux through the open hemispherical surface : $(x-a)^2 + y^2 + z^2 = a^2, x \geq a$

44. A charge Q is distributed over two concentric hollow spheres of radii r and $R\,(> r)$ such that the surface charge densities are equal. Find the potential at the common centre.

45. A charge q_0 is distributed uniformly on a ring of radius R. A sphere of equal radius R is constructed with its centre on the circumference of the ring. Find the electric flux through the surface of the sphere.

46. Two concentric conducting shells A and B are of radii R and $2R$. A charge $+q$ is placed at the centre of the shells. Shell B is earthed and a charge q is given to shell A. Find the charge on outer surface of A and B.

47. Three concentric metallic shells A, B and C of radii a, b and $c\,(a < b < c)$ have surface charge densities, $\sigma, -\sigma$ and σ respectively.



(a) Find the potentials of three shells A, B and C.

(b) It is found that no work is required to bring a charge q from shell A to shell C, then obtain the relation between the radii a, b and c.

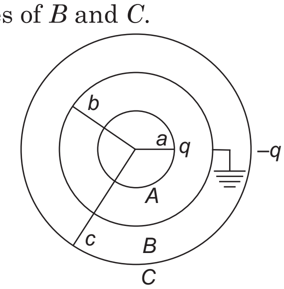
48. A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a,

(a) Find the surface charge density on the inner surface and on the outer surface.

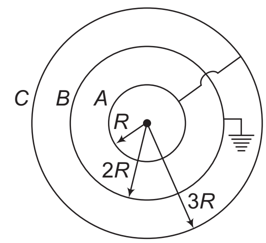
(b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces?

(c) Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

49. Figure shows three concentric thin spherical shells A, B and C of radii a, b and c respectively. The shells A and C are given charges q and $-q$ respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.

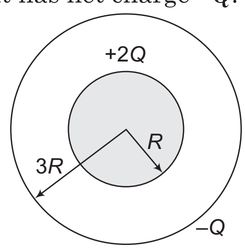


50. Three spherical shells have radii R, $2R$ and $3R$ respectively. Total charge on A and C is $3q$. Find the charges on different surfaces of A, B and C. The connecting wire does not touch the shell B.



51. In the above problem, the charges on different surfaces if a charge q is placed at the centre of the shell with all other conditions remaining the same.

52. A solid sphere of radius R has a charge $+2Q$. A hollow spherical shell of radius $3R$ placed concentric with the first sphere that has net charge $-Q$.



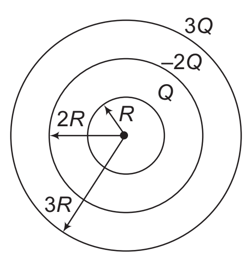
(a) Find the electric field between the spheres at a distance r from the centre of the inner sphere. $[R < r < 3R]$

(b) Calculate the potential difference between the spheres.

(c) What would be the final distribution of charges, if a conducting wire joins the spheres?

(d) Instead of (c), if the inner sphere is earthed, what is the charge on it?

53. Three concentric conducting spherical shells of radii R, $2R$ and $3R$ carry charges Q, $-2Q$ and $3Q$, respectively.



(a) Find the electric potential at $r = R$ and $r = 3R$, where r is the radial distance from the centre.

(b) Compute the electric field at $r = \frac{5}{2}R$

(c) Compute the total electrostatic energy stored in the system.

The inner shell is now connected to the external one by a conducting wire, passing through a very small hole in the middle shell.

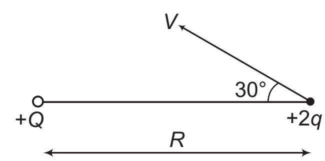
(d) Compute the charges on the spheres of radii R and $3R$.

(e) Compute the electric field at $r = \frac{5}{2}R$.

LEVEL 2

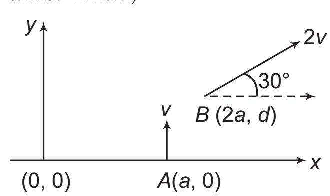
Single Correct Option

1. In the diagram shown, the charge $+Q$ is fixed. Another charge $+2q$ and mass M is projected from a distance R from the fixed charge. Minimum separation between the two charges if the velocity becomes $\frac{1}{\sqrt{3}}$ times of the projected velocity, at this moment is (Assume gravity to be absent)



(a) $\frac{\sqrt{3}}{2}R$ (b) $\frac{1}{\sqrt{3}}R$ (c) $\frac{1}{2}R$ (d) None of these

2. A uniform electric field of strength $\mathbf{E}$ exists in a region. An electron enters a point A with velocity v as shown. It moves through the electric field and reaches at point B. Velocity of particle at B is $2\,v$ at 30° with x-axis. Then,



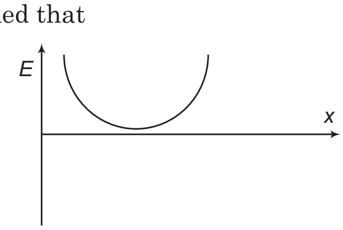
(a) electric field $\mathbf{E} = -\frac{3mv^2}{2ea}\,\hat{\mathbf{i}}$

(b) rate of doing work done by electric field at B is $\frac{3mv^3}{2ea}$

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

3. Two point charges a and b whose magnitudes are same, positioned at a certain distance along the positive x-axis from each other. a is at origin. Graph is drawn between electrical field strength and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph it can be decided that

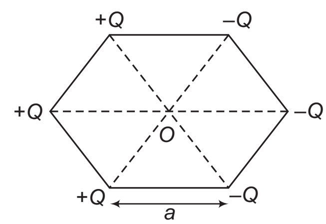


(a) a is positive, b is negative (b) a and b both are positive

(c) a and b both are negative (d) a is negative, b is positive

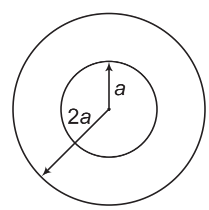
Note *Graph is drawn only between a and b.*

4. Six charges are placed at the vertices of a rectangular hexagon as shown in the figure. The electric field on the line passing through point O and perpendicular to the plane of the figure as a function of distance x from point O is $(x >> a)$



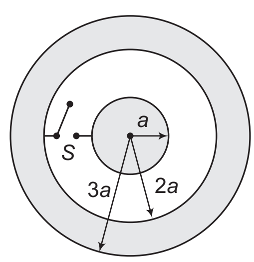
(a) 0 (b) $\dfrac{Qa}{\pi \varepsilon_0 x^3}$ (c) $\dfrac{2Qa}{\pi \varepsilon_0 x^3}$ (d) $\dfrac{\sqrt{3}Qa}{\pi \varepsilon_0 x^3}$

5. If the electric potential of the inner shell is 10 V and that of the outer shell is 5 V, then the potential at the centre will be



(a) 10 V (b) 5 V (c) 15 V (d) zero

6. A solid conducting sphere of radius a having a charge q is surrounded by a concentric conducting spherical shell of inner radius $2a$ and outer radius $3a$ as shown in figure. Find the amount of heat produced when switch is closed $\left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$



(a) $\dfrac{kq^2}{2a}$ (b) $\dfrac{kq^2}{3a}$ (c) $\dfrac{kq^2}{4a}$ (d) $\dfrac{kq^2}{6a}$

7. There are four concentric shells A, B, C and D of radii $a, 2a, 3a$ and $4a$ respectively. Shells B and D are given charges $+q$ and $-q$ respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is $\left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

(a) $\dfrac{kq}{2a}$ (b) $\dfrac{kq}{3a}$ (c) $\dfrac{kq}{4a}$ (d) $\dfrac{kq}{6a}$

8. Potential difference between centre and surface of the sphere of radius R and uniform volume charge density ρ within it will be

(a) $\dfrac{\rho R^2}{6\,\varepsilon_0}$ (b) $\dfrac{\rho R^2}{4\,\varepsilon_0}$

(c) $\dfrac{\rho R^2}{3\,\varepsilon_0}$ (d) $\dfrac{\rho R^2}{2\,\varepsilon_0}$

9. A positively charged disc is placed on a horizontal plane. A charged particle is released from a certain height on its axis. The particle just reaches the centre of the disc. Select the correct alternative.

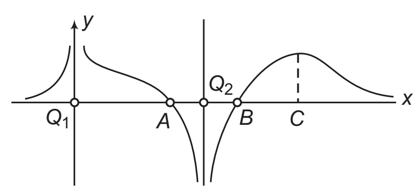
(a) Particle has negative charge on it

(b) Total potential energy (gravitational + electrostatic) of the particle first increases, then decreases

(c) Total potential energy of the particle first decreases, then increases

(d) Total potential energy of the particle continuously decreases

10. The curve represents the distribution of potential along the straight line joining the two charges Q_1 and Q_2 (separated by a distance r) then which of the following statements are correct?



1. $|Q_1| > |Q_2|$
2. Q_1 is positive in nature
3. A and B are equilibrium points
4. C is a point of unstable equilibrium

(a) 1 and 2 (b) 1, 2 and 3

(c) 1, 2 and 4 (d) 1, 2, 3 and 4

11. A point charge $q_1 = q$ is placed at point P. Another point charge $q_2 = -q$ is placed at point Q. At some point $R(R \neq P, R \neq Q)$, electric potential due to q_1 is V_1 and electric potential due to q_2 is V_2. Which of the following is correct?

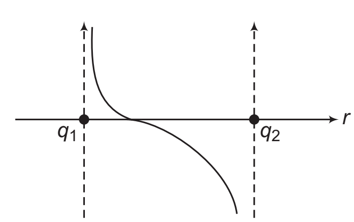
(a) Only for some points $V_1 > V_2$

(b) Only for some points $V_2 > V_1$

(c) For all points $V_1 > V_2$

(d) For all points $V_2 > V_1$

12. The variation of electric field between two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightward direction of electric field as positive) as shown in the figure. Then, the correct statement is



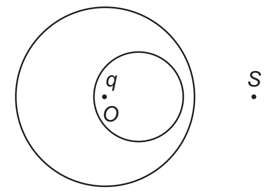
(a) q_1 and q_2 are positive and $q_1 < q_2$

(b) q_1 and q_2 are positive and $q_1 > q_2$

(c) q_1 is positive and q_2 is negative $q_1 < |q_2|$

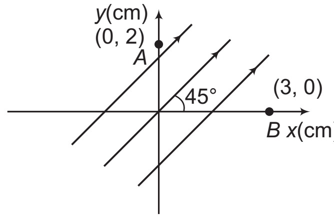
(d) q_1 and q_2 are negative and $|q_1| < |q_2|$

13. A charge q is placed at O in the cavity in a spherical uncharged conductor. Point S is outside the conductor. If q is displaced from O towards S (still remaining within the cavity)



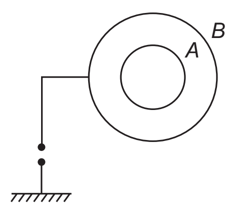
(a) electric field at S will increase
(b) electric field at S will decrease
(c) electric field at S will first increase and then decrease
(d) electric field at S will not change

14. A uniform electric field of 400 V/m is directed at 45° above the x-axis as shown in the figure. The potential difference $V_A - V_B$ is given by



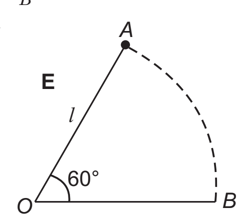
(a) 0
(b) 4 V
(c) 6.4 V
(d) 2.8 V

15. Initially the spheres A and B are at potentials V_A and V_B respectively. Now, sphere B is earthed by closing the switch. The potential of A will now become



(a) 0
(b) V_A
(c) $V_A - V_B$
(d) V_B

16. A particle of mass m and charge q is fastened to one end of a string of length l. The other end of the string is fixed to the point O. The whole system lies on a frictionless horizontal plane. Initially, the mass is at rest at A. A uniform electric field in the direction shown is then switched on. Then,



(a) the speed of the particle when it reaches B is $\sqrt{\dfrac{2qEl}{m}}$
(b) the speed of the particle when it reaches B is $\sqrt{\dfrac{qEl}{m}}$
(c) the tension in the string when the particle reaches at B is qE
(d) the tension in the string when the particle reaches at B is zero

17. A charged particle of mass m and charge q is released from rest from the position $(x_0, 0)$ in a uniform electric field $E_0\hat{\mathbf{j}}$. The angular momentum of the particle about origin

(a) is zero
(b) is constant
(c) increases with time
(d) decreases with time

18. A charge $+Q$ is uniformly distributed in a spherical volume of radius R. A particle of charge $+q$ and mass m projected with velocity v_0 from the surface of the spherical volume to its centre inside a smooth tunnel dug across the sphere. The minimum value of v_0 such that it just reaches the centre (assume that there is no resistance on the particle except electrostatic force) of the spherical volume is

(a) $\sqrt{\dfrac{Qq}{2\pi\varepsilon_0 mR}}$ (b) $\sqrt{\dfrac{Qq}{\pi\varepsilon_0 mR}}$

(c) $\sqrt{\dfrac{2Qq}{\pi\varepsilon_0 mR}}$ (d) $\sqrt{\dfrac{Qq}{4\pi\varepsilon_0 mR}}$

19. Two identical coaxial rings each of radius R are separated by a distance of $\sqrt{3}R$. They are uniformly charged with charges $+Q$ and $-Q$ respectively. The minimum kinetic energy with which a charged particle (charge $+q$) should be projected from the centre of the negatively charged ring along the axis of the rings such that it reaches the centre of the positively charged ring is

(a) $\dfrac{Qq}{4\pi\varepsilon_0 R}$ (b) $\dfrac{Qq}{2\pi\varepsilon_0 R}$ (c) $\dfrac{Qq}{8\pi\varepsilon_0 R}$ (d) $\dfrac{3Qq}{4\pi\varepsilon_0 R}$

20. A uniform electric field exists in x-y plane. The potential of points $A(2\text{ m}, 2\text{ m})$, $B(-2\text{ m}, 2\text{ m})$ and $C(2\text{ m}, 4\text{ m})$ are 4 V, 16 V and 12 V respectively. The electric field is

(a) $(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ V/m (b) $(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ V/m

(c) $-(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ V/m (d) $(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$ V/m

21. Two fixed charges $-2Q$ and $+Q$ are located at points $(-3a, 0)$ and $(+3a, 0)$ respectively. Then, which of the following statement is correct?

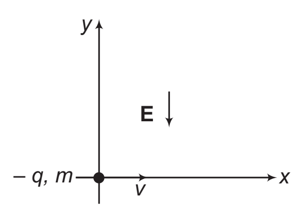
(a) Points where the electric potential due to the two charges is zero in x-y plane, lie on a circle of radius $4a$ and centre $(5a, 0)$

(b) Potential is zero at $x = a$ and $x = 9a$

(c) Both (a) and (b) are wrong

(d) Both (a) and (b) are correct

22. A particle of mass m and charge $-q$ is projected from the origin with a horizontal speed v into an electric field of intensity E directed downward. Choose the **wrong** statement. Neglect gravity



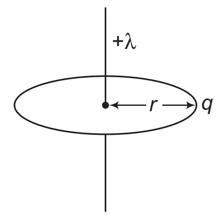
(a) The kinetic energy after a displacement y is qEy

(b) The horizontal and vertical components of acceleration are $a_x = 0, a_y = \dfrac{qE}{m}$

(c) The equation of trajectory is $y = \dfrac{1}{2}\left(\dfrac{qEx^2}{mv^2}\right)$

(d) The horizontal and vertical displacements x and y after a time t are $x = vt$ and $y = \dfrac{1}{2}a_y t^2$

23. A particle of charge $-q$ and mass m moves in a circle of radius r around an infinitely long line charge of linear charge density $+\lambda$. Then, time period will be



(a) $T = 2\pi r\sqrt{\dfrac{m}{2k\lambda q}}$

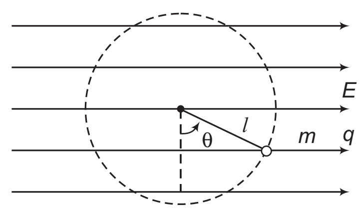
(b) $T^2 = \dfrac{4\pi^2 m}{2k\lambda q} r^3$

(c) $T = \dfrac{1}{2\pi r}\sqrt{\dfrac{2k\lambda q}{m}}$

(d) $T = \dfrac{1}{2\pi r}\sqrt{\dfrac{m}{2k\lambda q}}$

where, $\quad k = \dfrac{1}{4\pi\varepsilon_0}$

24. A small ball of mass m and charge $+q$ tied with a string of length l, rotating in a vertical circle under gravity and a uniform horizontal electric field E as shown. The tension in the string will be minimum for



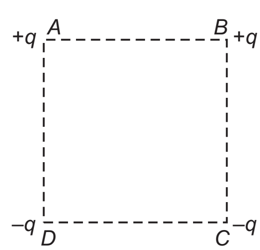
(a) $\theta = \tan^{-1}\left(\dfrac{qE}{mg}\right)$

(b) $\theta = \pi$

(c) $\theta = 0°$

(d) $\theta = \pi + \tan^{-1}\left(\dfrac{qE}{mg}\right)$

25. Four point charges A, B, C and D are placed at the four corners of a square of side a. The energy required to take the charges C and D to infinity (they are also infinitely separated from each other) is



(a) $\dfrac{q^2}{4\pi\varepsilon_0 a}$

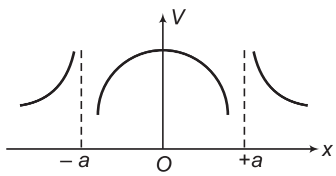
(b) $\dfrac{2q^2}{\pi\varepsilon_0 a}$

(c) $\dfrac{q^2}{4\pi\varepsilon_0 a}(\sqrt{2} + 1)$

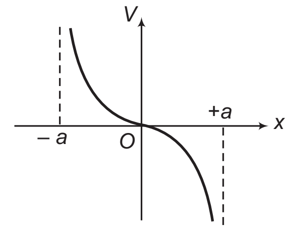
(d) $\dfrac{q^2}{4\pi\varepsilon_0 a}(\sqrt{2} - 1)$

26. Two identical positive charges are placed at $x = -a$ and $x = a$. The correct variation of potential V along the x-axis is given by

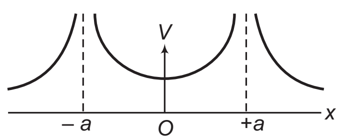
(a)



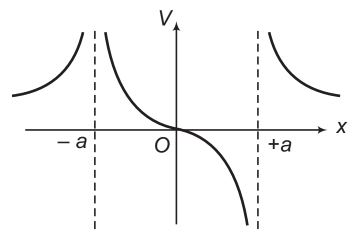
(b)



(c)



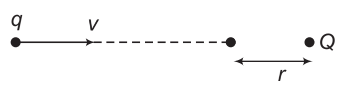
(d)



27. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is

(a) U (b) $2U$
(c) $3U$ (d) $4U$

28. A charged particle q is shot from a large distance towards another charged particle Q which is fixed, with a speed v. It approaches Q up to a closest distance r and then returns. If q were given a speed $2v$, the distance of approach would be

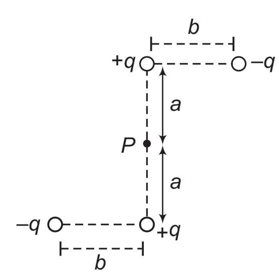


(a) r (b) $2r$
(c) $r/2$ (d) $r/4$

29. Two identical charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cc, the angle remains the same. The dielectric constant of the liquid is [density of the material of sphere is 1.6 g/cc]

(a) 2 (b) 4
(c) 2.5 (d) 3.5

30. The electrostatic potential due to the charge configuration at point P as shown in figure for $b << a$ is



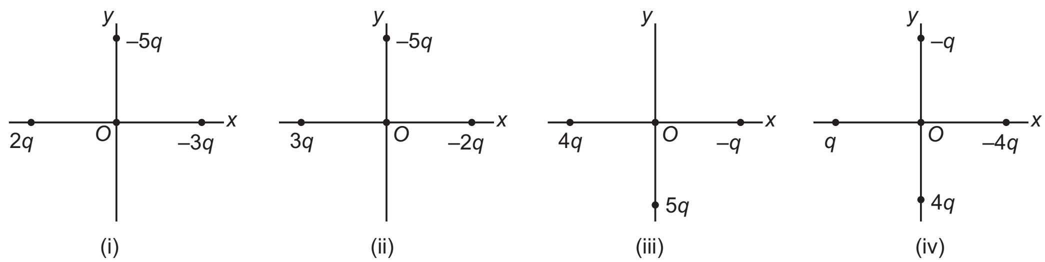
(a) $\dfrac{2q}{4\pi\varepsilon_0 a}$

(b) $\dfrac{2qb^2}{4\pi\varepsilon_0 a^3}$

(c) $\dfrac{qb^2}{4\pi\varepsilon_0 a^3}$

(d) zero

31. The figure shows four situations in which charged particles are at equal distances from the origin. If E_1, E_2, E_3 and E_4 be the magnitude of the net electric fields at the origin in four situations (i), (ii), (iii) and (iv) respectively, then



(a) $E_1 = E_2 = E_3 = E_4$ (b) $E_1 = E_2 > E_3 > E_4$
(c) $E_1 < E_2 < E_3 = E_4$ (d) $E_1 > E_2 = E_3 < E_4$

32. An isolated conducting sphere whose radius $R = 1$ m has a charge $q = \frac{1}{9}$ nC. The energy density at the surface of the sphere is

(a) $\frac{\varepsilon_0}{2}$ J/m^3 (b) ε_0 J/m^3
(c) $2\varepsilon_0$ J/m^3 (d) $\frac{\varepsilon_0}{3}$ J/m^3

33. Two conducting concentric, hollow spheres A and B have radii a and b respectively, with A inside B. Their common potentials is V. A is now given some charge such that its potential becomes zero. The potential of B will now be

(a) 0 (b) $V(1 - a/b)$
(c) Va/b (d) Vb/a

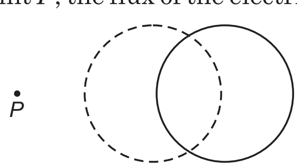
34. In a uniform electric field, the potential is 10 V at the origin of coordinates and 8 V at each of the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). The potential at the point (1, 1, 1) will be

(a) 0 (b) 4 V (c) 8 V (d) 10 V

35. There are two uncharged identical metallic spheres 1 and 2 of radius r separated by a distance d ($d >> r$). A charged metallic sphere of same radius having charge q is touched with one of the sphere. After some time it is moved away from the system. Now, the uncharged sphere is earthed. Charge on earthed sphere is

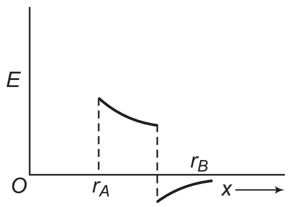
(a) $+\frac{q}{2}$ (b) $-\frac{q}{2}$
(c) $-\frac{qr}{2d}$ (d) $-\frac{qd}{2r}$

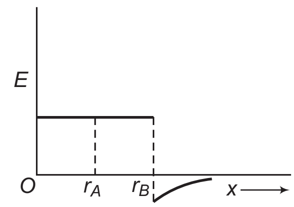
36. Figure shows a closed dotted surface which intersects a conducting uncharged sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface

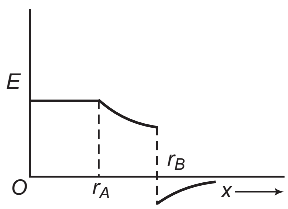


(a) will remain zero (b) will become positive
(c) will become negative (d) data insufficient

37. Two concentric conducting thin spherical shells A and B having radii r_A and $r_B (r_B > r_A)$ are charged to Q_A and $-Q_B (|Q_B| > |Q_A|)$. The electrical field along a line passing through the centre is

(a)


(b)


(c)


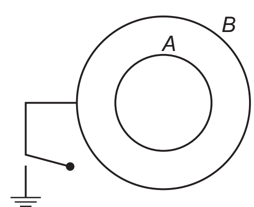
(d) None of these

38. The electric potential at a point (x, y) in the x-y plane is given by $V = -kxy$. The field intensity at a distance r in this plane, from the origin is proportional to

(a) r^2
(b) r
(c) $1/r$
(d) $1/r^2$

More than One Correct Options

1. Two concentric shells have radii R and $2R$ charges q_A and q_B and potentials $2\,V$ and $(3/2)\,V$ respectively. Now, shell B is earthed and let charges on them become q_A' and q_B'. Then,



(a) $q_A/q_B = 1/2$
(b) $q_A'/q_B' = 1$
(c) potential of A after earthing becomes $(3/2)\,V$
(d) potential difference between A and B after earthing becomes $V/2$

2. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity 10 ms^{-1}. There is a uniform horizontal electric field of 10^4 N/C, then

(a) the horizontal range of the particle is 10 m
(b) the time of flight of the particle is 2 s
(c) the maximum height reached is 5 m
(d) the horizontal range of the particle is 5 m

3. At a distance of 5 cm and 10 cm from surface of a uniformly charged solid sphere, the potentials are 100 V and 75 V respectively. Then,

(a) potential at its surface is 150 V
(b) the charge on the sphere is $\frac{50}{3} \times 10^{-10}$ C
(c) the electric field on the surface is 1500 V/m
(d) the electric potential at its centre is 25 V

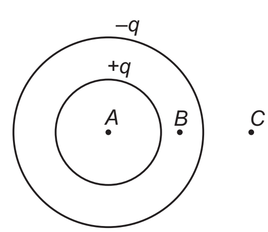
4. Three charged particles are in equilibrium under their electrostatic forces only. Then,
(a) the particles must be collinear
(b) all the charges cannot have the same magnitude
(c) all the charges cannot have the same sign
(d) the equilibrium is unstable

5. Charges Q_1 and Q_2 lie inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.
(a) If Q_1 changes, both E and ϕ will change
(b) If Q_2 changes, E will change but ϕ will not change
(c) If $Q_1 = 0$ and $Q_2 \neq 0$, then $E \neq 0$ but $\phi = 0$
(d) If $Q_1 \neq 0$ and $Q_2 = 0$, then $E = 0$ but $\phi \neq 0$

6. An electric dipole is placed at the centre of a sphere. Mark the correct options.
(a) The flux of the electric field through the sphere is zero
(b) The electric field is zero at every point of the sphere
(c) The electric field is not zero at anywhere on the sphere
(d) The electric field is zero on a circle on the sphere

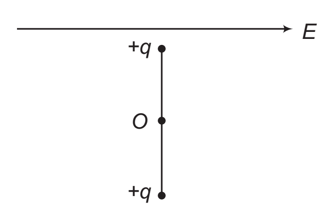
7. Mark the correct options.
(a) Gauss's law is valid only for uniform charge distributions
(b) Gauss's law is valid only for charges placed in vacuum
(c) The electric field calculated by Gauss's law is the field due to all the charges
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface

8. Two concentric spherical shells have charges $+q$ and $-q$ as shown in figure. Choose the correct options.



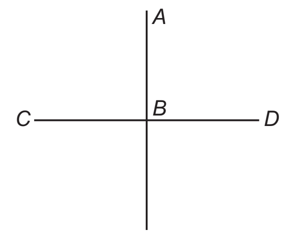
(a) At A electric field is zero, but electric potential is non-zero
(b) At B electric field and electric potential both are non-zero
(c) At C electric field is zero but electric potential is non-zero
(d) At C electric field and electric potential both are zero

9. A rod is hinged (free to rotate) at its centre O as shown in figure. Two point charges $+q$ and $+q$ are kept at its two ends. Rod is placed in uniform electric field E as shown. Space is gravity free. Choose the correct options.



(a) Net force from the hinge on the rod is zero
(b) Net force from the hinge on the rod is leftwards
(c) Equilibrium of rod is neutral
(d) Equilibrium of rod is stable

10. Two charges $+Q$ each are fixed at points C and D. Line AB is the bisector line of CD. A third charge $+q$ is moved from A to B, then from B to C.

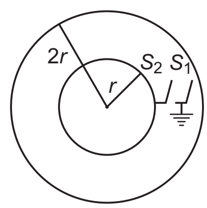


(a) From A to B electrostatic potential energy will decrease
(b) From A to B electrostatic potential energy will increase
(c) From B to C electrostatic potential energy will increase
(d) From B to C electrostatic potential energy will decrease

Comprehension Based Questions

Passage I (Q. No. 1 to 3)

There are two concentric spherical shell of radii r and $2r$. Initially, a charge Q is given to the inner shell and both the switches are open.



1. If switch S_1 is closed and then opened, charge on the outer shell will be
(a) Q
(b) $Q/2$
(c) $-Q$
(d) $-Q/2$

2. Now, S_2 is closed and opened. The charge flowing through the switch S_2 in the process is
(a) Q
(b) $Q/4$
(c) $Q/2$
(d) $2Q/3$

3. The two steps of the above two problems are repeated n times, the potential difference between the shells will be
(a) $\frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$
(b) $\frac{1}{2^{n}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$
(c) $\frac{1}{2^{n}}\left[\frac{Q}{2\pi\varepsilon_0 r}\right]$
(d) $\frac{1}{2^{n-1}}\left[\frac{Q}{2\pi\varepsilon_0 r}\right]$

Passage II (Q. No. 4 to 7)

A sphere of charge of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as $\rho = \rho_0\left(1-\frac{r}{R}\right)$, where ρ_0 is a constant. Assume ε as the permittivity of space.

4. The magnitude of electric field as a function of the distance r inside the sphere is given by
(a) $E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{3}-\frac{r^2}{4R}\right]$
(b) $E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{4}-\frac{r^2}{3R}\right]$
(c) $E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{3}+\frac{r^2}{4R}\right]$
(d) $E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{4}+\frac{r^2}{3R}\right]$

5. The magnitude of the electric field as a function of the distance r outside the ball is given by

(a) $E = \dfrac{\rho_0 R^3}{8\varepsilon r^2}$ (b) $E = \dfrac{\rho_0 R^3}{12\varepsilon r^2}$

(c) $E = \dfrac{\rho_0 R^2}{8\varepsilon r^3}$ (d) $E = \dfrac{\rho_0 R^2}{12\varepsilon r^3}$

6. The value of distance r_m at which electric field intensity is maximum is given by

(a) $r_m = \dfrac{R}{3}$ (b) $r_m = \dfrac{3R}{2}$

(c) $r_m = \dfrac{2R}{3}$ (d) $r_m = \dfrac{4R}{3}$

7. The maximum electric field intensity is

(a) $E_m = \dfrac{\rho_0 R}{9\varepsilon}$ (b) $E_m = \dfrac{\rho_0 \varepsilon}{9R}$

(c) $E_m = \dfrac{\rho_0 R}{3\varepsilon}$ (d) $E_m = \dfrac{\rho_0 R}{6\varepsilon}$

Passage III (Q. No. 8 to 10)

A solid metallic sphere of radius a is surrounded by a conducting spherical shell of radius $b\,(b > a)$. The solid sphere is given a charge Q. A student measures the potential at the surface of the solid sphere as V_a and the potential at the surface of spherical shell as V_b. After taking these readings, he decides to put charge of $-4Q$ on the shell. He then noted the readings of the potential of solid sphere and the shell and found that the potential difference is ΔV. He then connected the outer spherical shell to the earth by a conducting wire and found that the charge on the outer surface of the shell as q_1.

He then decides to remove the earthing connection from the shell and earthed the inner solid sphere. Connecting the inner sphere with the earth he observes the charge on the solid sphere as q_2. He then wanted to check what happens if the two are connected by the conducting wire. So he removed the earthing connection and connected a conducting wire between the solid sphere and the spherical shell. After the connections were made he found the charge on the outer shell as q_3.

Answer the following questions based on the readings taken by the student at various stages.

8. Potential difference (ΔV) measured by the student between the inner solid sphere and outer shell after putting a charge $-4Q$ is

(a) $V_a - 3V_b$ (b) $3(V_a - V_b)$

(c) V_a (d) $V_a - V_b$

9. q_2 is

(a) Q (b) $Q\left(\dfrac{a}{b}\right)$

(c) $-4Q$ (d) zero

10. q_3 is

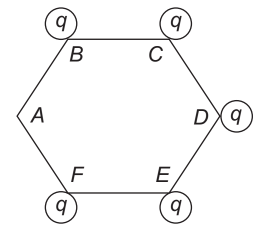
(a) $\dfrac{Q(a+b)}{a-b}$ (b) $\dfrac{Qa^2}{b}$

(c) $\dfrac{Q(a-b)}{b}$ (d) $-\dfrac{Qb}{a}$

Match the Columns

1. Five identical charges are kept at five vertices of a regular hexagon. Match the following two columns at centre of the hexagon. If in the given situation electric field at centre is E. Then,

Column I	Column II
(a) If charge at B is removed, then electric field will become	(p) $2E$
(b) If charge at C is removed, then electric field will become	(q) E
(c) If charge at D is removed then electric field will become	(r) zero
(d) If charges at B and C both are removed, then electric field will become	(s) $\sqrt{3}\,E$



Note *Only magnitudes of electric field are given.*

2. In an electric field $\mathbf{E} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N/C, electric potential at origin is 0 V. Match the following two columns.

Column I	Column II
(a) Potential at (4 m, 0)	(p) 8 V
(b) Potential at (–4 m, 0)	(q) – 8 V
(c) Potential at (0, 4 m)	(r) 16 V
(d) Potential at (0, – 4 m)	(s) – 16 V

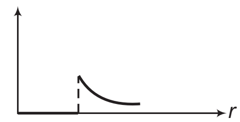
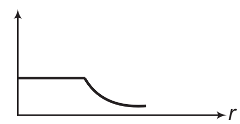
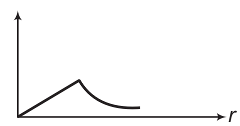
3. Electric potential on the surface of a solid sphere of charge is V. Radius of the sphere is 1m. Match the following two columns.

Column I	Column II
(a) Electric potential at $r = \frac{R}{2}$	(p) $\frac{V}{4}$
(b) Electric potential at $r = 2R$	(q) $\frac{V}{2}$
(c) Electric field at $r = \frac{R}{2}$	(r) $\frac{3V}{4}$
(d) Electric field at $r = 2R$	(s) None of these

4. Match the following two columns.

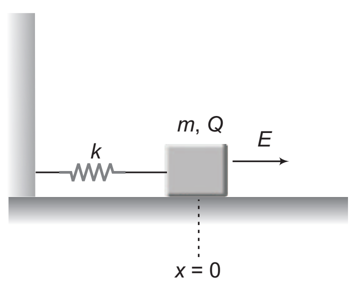
Column I	Column II
(a) Electric potential	(p) $[MLT^{-3}A^{-1}]$
(b) Electric field	(q) $[ML^3T^{-3}A^{-1}]$
(c) Electric flux	(r) $[ML^2T^{-3}A^{-1}]$
(d) Permittivity of free space	(s) None of these

5. Match the following two columns.

Column I	Column II
(a) Electric field due to charged spherical shell	(p) 
(b) Electric potential due to charged spherical shell	(q) 
(c) Electric field due to charged solid sphere	(r) 
(d) Electric potential due to charged solid sphere	(s) None of these

Subjective Questions

1. A 4.00 kg block carrying a charge $Q = 50.0\,\mu\text{C}$ is connected to a spring for which $k = 100\,\text{N/m}$. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude $E = 5.00 \times 10^5$ V/m, directed as shown in figure. If the block is released from rest when the spring is unstretched (at $x = 0$).



(a) By what maximum amount does the spring expand?
(b) What is the equilibrium position of the block?
(c) Show that the block's motion is simple harmonic and determine its period.
(d) Repeat part (a) if the coefficient of kinetic friction between block and surface is 0.2.

2. A particle of mass m and charge $-Q$ is constrained to move along the axis of a ring of radius a. The ring carries a uniform charge density $+\lambda$ along its length. Initially, the particle is in the centre of the ring where the force on it is zero. Show that the period of oscillation of the particle when it is displaced slightly from its equilibrium position is given by

$$T = 2\pi\sqrt{\frac{2\varepsilon_0 m a^2}{\lambda Q}}$$

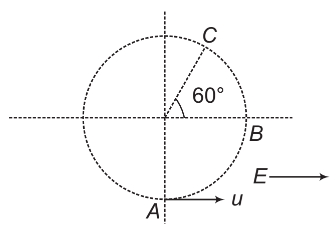
3. Three identical conducting plane parallel plates, each of area A are held with equal separation d between successive surfaces. Charges Q, $2Q$, and $3Q$ are placed on them. Neglecting edge effects, find the distribution of charges on the six surfaces.

4. A long non-conducting, massless rod of length L pivoted at its centre and balanced with a weight w at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$ respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q.
   (a) Find the distance x where the rod is horizontal and balanced.
   (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?

Note *Ignore the force between Q (beneath q) and 2q and the force between Q (beneath 2q) and q. Also the force between Q and Q.*

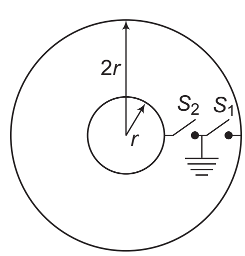
5. The electric potential varies in space according to the relation $V = 3x + 4y$. A particle of mass 10 kg starts from rest from point (2, 3.2) m under the influence of this field. Find the velocity of the particle when it crosses the x-axis. The charge on the particle is $+1\,\mu C$. Assume $V\,(x, y)$ are in SI units.

6. A simple pendulum with a bob of mass $m = 1$ kg, charge $q = 5\,\mu C$ and string length $l = 1$ m is given a horizontal velocity u in a uniform electric field $E = 2\times10^6$ V/m at its bottommost point A, as shown in figure. It is given that the speed u is such that the particle leaves the circle at point C. Find the speed u (Take $g = 10$ m/s^2)

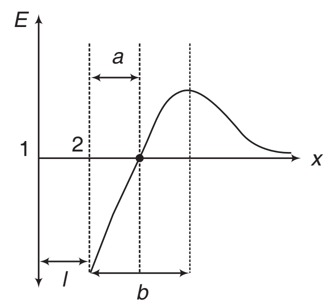


7. Eight point charges of magnitude Q are arranged to form the corners of a cube of side L. The arrangement is made in manner such that the nearest neighbour of any charge has the opposite sign. Initially, the charges are held at rest. If the system is let free to move, what happens to the arrangement? Does the cube-shape shrink or expand? Calculate the velocity of each charge when the side-length of the cube formation changes from L to nL. Assume that the mass of each point charge is m.

8. There are two concentric spherical shells of radii r and $2r$. Initially, a charge Q is given to the inner shell. Now, switch S_1 is closed and opened then S_2 is closed and opened and the process is repeated n times for both the keys alternatively. Find the final potential difference between the shells.



9. Two point charges Q_1 and Q_2 are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in the figure. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.



(a) Find the sign of each charge.

(b) Find the ratio of the absolute values of the charges $\left|\frac{Q_1}{Q_2}\right|$

(c) Find the value of b where the field intensity is maximum.

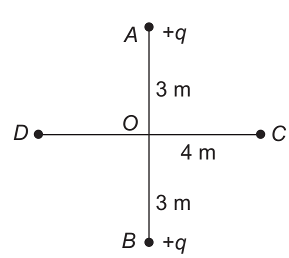
10. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius $R\,(> r)$ is mounted on an insulating stand, S_2 is initially uncharged. S_1 is given a charge Q. Brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

11. A proton of mass m and accelerated by a potential difference V gets into a uniform electric field of a parallel plate capacitor parallel to plates of length l at mid-point of its separation between plates. The field strength in it varies with time as $E = at$, where a is a positive constant. Find the angle of deviation of the proton as it comes out of the capacitor. (Assume that it does not collide with any of the plates.)

12. Two fixed, equal, positive charges, each of magnitude 5×10^{-5} C are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.

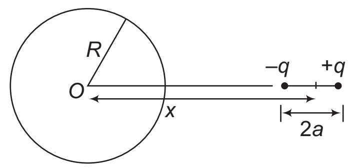


13. Positive charge Q is uniformly distributed throughout the volume of a sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity

v from a point A at distance $r(r > R)$ from the centre of the sphere. Find the minimum velocity v so that it can penetrate $R/2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

14. Two concentric rings placed in a gravity free region in yz-plane one of radius R carries a charge $+Q$ and second of radius $4R$ and charge $-8Q$ distributed uniformly over it. Find the minimum velocity with which a point charge of mass m and charge $+q$ should be projected from a point at a distance $3R$ from the centre of rings on its axis so that it will reach to the centre of the rings.

15. An electric dipole is placed at a distance x from centre O on the axis of a charged ring of radius R and charge Q uniformly distributed over it.



(a) Find the net force acting on the dipole.

(b) What is the work done in rotating the dipole through 180°?

16. A point charge $-q$ revolves around a fixed charge $+Q$ in elliptical orbit. The minimum and maximum distance of q from Q are r_1 and r_2, respectively. The mass of revolving particle is m. $Q > q$ and assume no gravitational effects. Find the velocity of q at positions when it is at r_1 and r_2 distance from Q.

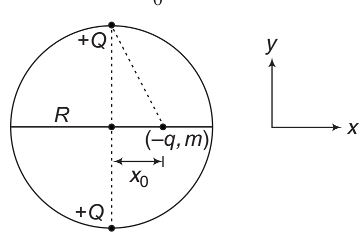
17. Three concentric, thin, spherical, metallic shells have radii 1, 2, and 4 cm and they are held at potentials 10, 0 and 40 V respectively. Taking the origin at the common centre, calculate the following:

(a) Potential at $r = 1.25$ cm

(b) Potential at $r = 2.5$ cm

(c) Electric field at $r = 1.25$ cm

18. A thin insulating wire is stretched along the diameter of an insulated circular hoop of radius R. A small bead of mass m and charge $-q$ is threaded onto the wire. Two small identical charges are tied to the hoop at points opposite to each other, so that the diameter passing through them is perpendicular to the thread (see figure). The bead is released at a point which is a distance x_0 from the centre of the hoop. Assume that $x_0 << R$.



(a) What is the resultant force acting on the charged bead?

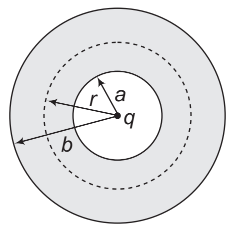
(b) Describe (qualitatively) the motion of the bead after it is released.

(c) Use the assumption that $\frac{x}{R} << 1$ to obtain an approximate equation of motion, and find the displacement and velocity of the bead as functions of time.

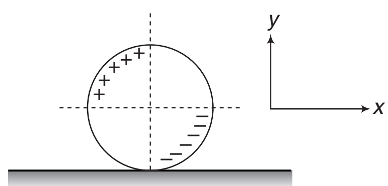
(d) When will the velocity of the bead will become zero for the first time?

19. The region between two concentric spheres of radii a and $b\,(> a)$ contains volume charge density $\rho(r) = \dfrac{C}{r}$, where C is a constant and r is the radial distance as shown in figure. A point charge q is placed at the origin, $r = 0$.

Find the value of C for which the electric field in the region between the spheres is constant (i.e. r independent).

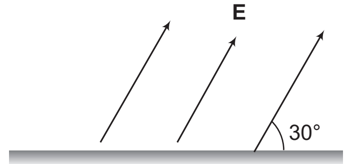


20. A non-conducting ring of mass m and radius R is charged as shown. The charge density, i.e. charge per unit length is λ. It is then placed on a rough non-conducting horizontal plane. At time $t = 0$, a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{i}}$ is switched on and the ring starts rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring when it starts moving.



21. A rectangular tank of mass m_0 and charge Q over it is placed on a smooth horizontal floor. A horizontal electric field E exists in the region. Rain drops are falling vertically in the tank at the constant rate of n drops per second. Mass of each drop is m. Find velocity of tank as function of time.

22. In a region, an electric field $E = 15$ N/C making an angle of 30° with the horizontal plane is present. A ball having charge $2C$, mass 3 kg and coefficient of restitution with ground 1/2 is projected at an angle of 30° with the horizontal in the direction of electric field with speed 20 m/s. Find the horizontal distance travelled by ball from first drop to the second drop.



Answers

Introductory Exercise 24.1

1. No, as attraction can take place between a charged and an uncharged body too.
2. Yes
3. Record gets charged when cleaned and then by induction, it attracts dust particles.
4. -2.89×10^5 C

Introductory Exercise 24.2

1. 2.27×10^{39} **2.** $[M^{-1} L^{-3} T^4 A^2]$, $\frac{C}{V\text{-}m}$ **3.** $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{a}\right)^2$ **4.** $\frac{1}{2\pi\varepsilon_0}\cdot\left(\frac{q}{a}\right)^2$

5. No **6.** Induction $\rightarrow$ Conduction $\rightarrow$ Repulsion **7.** Yes

8. No **9.** $(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N

Introductory Exercise 24.3

1. False **2.** At A **3.** False **4.** False

5. q_1 and q_3 are positive and q_2 is negative **6.** $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}$

7. $-(4.32\,\hat{\mathbf{i}} + 5.76\,\hat{\mathbf{j}}) \times 10^2$ N/C

Introductory Exercise 24.4

1. 18.97 m/s **2.** -9 mJ **3.** -10.6×10^{-8} J **4.** No, Yes

Introductory Exercise 24.5

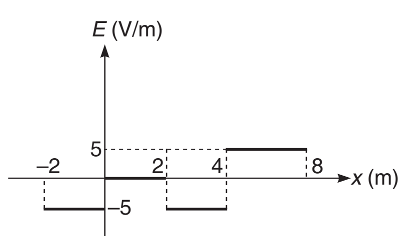
1. 1.2×10^3 V **2.** (a) $\frac{C}{m^2}$ (b) $\frac{1}{4\pi\varepsilon_0} \times \alpha\left[L - d\ln\left(1 + \frac{L}{d}\right)\right]$

3. $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2l}\ln\left(\frac{\sqrt{l^2 + d^2} + l}{\sqrt{l^2 + d^2} - l}\right)$ **4.** $\frac{Qq}{2\pi\varepsilon_0 L}$

Introductory Exercise 24.6

1. (a) $\mathbf{E} = -2a(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})$ (b) $\mathbf{E} = -a(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$

2.



3. False **4.** (a) Zero (b) 20 V (c) -20 V (d) -20 V

Introductory Exercise 24.7

1. (a) Zero (b) $\frac{q}{\varepsilon_0}$ (c) $\frac{q}{2\varepsilon_0}$ **2.** True **3.** (a) Zero (b) $\pi R^2 E$ **4.** Zero

Exercises

LEVEL 1

Assertion and Reason

1. (b) **2.** (a,b) **3.** (d) **4.** (b) **5.** (b) **6.** (d) **7.** (a,b) **8.** (d) **9.** (c) **10.** (b)

Objective Questions

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (d)
11. (a)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (d)
21. (a)	**22.** (d)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (d)
31. (b)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (d)	**37.** (d)	**38.** (d)	**39.** (a)	

Subjective Questions

1. $q = \frac{Q}{2}$ **2.** $\frac{F_e}{F_g} = 3.1 \times 10^{35}$ **3.** 5.31×10^{-11} C/m^2 **4.** $\frac{qx}{8\pi^2\varepsilon_0 R^3}$

5. $\pm 3\,\mu$C, $\mp 1\,\mu$C **8.** $\frac{q}{24\varepsilon_0}$, zero **9.** (a) $q_1 = 9q_2$ (b) $q_1 = -25q_2$

10. (a) Third charge is $-\frac{4q}{9}$ at a distance of $\frac{L}{3}$ from q between the two charges.

11. $\left(\frac{4\pi\,\varepsilon_0 mgR^2}{\sqrt{3}}\right)^{1/2}$ **12.** 3.3×10^{-8} C **13.** $(2\sqrt{2}+1)\frac{q}{8\pi\varepsilon_0 a^2}$

14. $(14.4\hat{\mathbf{j}} - 10.8\,\hat{\mathbf{i}})$ N/C **15.** $\frac{\lambda}{2\pi\,\varepsilon_0 R}$ **16.** $\frac{Q}{2\pi\,\varepsilon_0 a\sqrt{L^2+4a^2}}$

17. (a) Along positive y-axis (b) Along positive x-axis (c) Along positive y-axis

18. 9.30 **19.** 312.5 m **20.** (a) 37° and 53° (b) 1.66×10^{-7} s, 2.2×10^{-7} s

21. (a) $(-2.1 \times 10^{13}\,\hat{\mathbf{j}})$ m/s^2 (b) $(1.5\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \times 10^5$ m/s

22. At $x = 40$ cm and $x = -200$ cm **23.** $V = 2.634\frac{Q}{4\pi\,\varepsilon_0 a}$

24. (a) -6×10^{-4} J (b) 50 V **25.** 7.42 m/s, faster

26. (a) $\frac{-5Q}{4\pi\,\varepsilon_0 R}$ (b) $\frac{-5Q}{4\pi\,\varepsilon_0\sqrt{R^2+z^2}}$ **27.** -0.356 J

28. (a) -3.6×10^{-7} J (b) x = 0.0743 m **29.** $-q/2$

30. (a) zero (b) -20 kV **31.** (a) -80 V (b) 120 V (c) 0 V

32. (a) -80 V (b) -40 V

33. (a) [MT^{-3} A^{-1}] (b) $-A\{(y+z)\hat{\mathbf{i}} + (x+z)\hat{\mathbf{j}} + (x+y)\hat{\mathbf{k}}\}$ (c) $20\sqrt{3}$ N/C **34.** -100 V

35. (a) $E_x = -Ay + 2Bx$, $E_y = -Ax - C$, $E_z = 0$, (b) $x = -C/A$, $y = -2BC/A^2$, any value of z

36. 3.19 nC **37.** (a) $-4.07 \times 10^5\,\frac{\text{N-m}^2}{\text{C}}$ (b) 6.91 nC (c) No **38.** $240\,\frac{\text{N-m}^2}{\text{C}}$ **39.** 2.2×10^{-12} C

41. (a) $\phi_{s_1} = -CL^2$, $\phi_{s_2} = -DL^2$, $\phi_{s_3} = CL^2$, $\phi_{s_4} = DL^2$, $\phi_{s_5} = -BL^2$, $\phi_{s_6} = BL^2$ (b) zero

42. $\dfrac{q}{\varepsilon_0}\left[1-\dfrac{1}{\sqrt{1+(R/l)^2}}\right]$

43. $\dfrac{q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

44. $\dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{R+r}{R^2+r^2}\right)$

45. $\dfrac{q_0}{3\varepsilon_0}$

46. 2q, Zero

47. (a) $V_A=\dfrac{\sigma}{\varepsilon_0}(a-b+c), V_B=\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2}{b}-b+c\right), V_C=\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2}{c}-\dfrac{b^2}{c}+c\right)$ (b) $c=a+b$

48. (a) $\dfrac{-Q}{4\pi a^2},\dfrac{Q}{4\pi a^2}$ (b) $\dfrac{-Q}{4\pi a^2},\dfrac{Q+q}{4\pi a^2}$ (c) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$ in both situations

49. Inner surface of $B\to -q$, outer surface of $B\to\dfrac{b}{c}q$, inner surface of $C\to\left(\dfrac{-bq}{c}\right)$, outer surface of $C\to\left(\dfrac{b}{c}-1\right)q$

50.

	A	B	C
Inner Surface	0	$-\dfrac{6}{11}q$	$\dfrac{18}{11}q$
Outer Surface	$\dfrac{6}{11}q$	$-\dfrac{18}{11}q$	$\dfrac{9}{11}q$

51.

	A	B	C
Inner Surface	$-q$	$-2q$	$\dfrac{4}{3}q$
Outer Surface	$2q$	$-\dfrac{4}{3}q$	$\dfrac{2}{3}q$

52. (a) $\dfrac{Q}{2\pi\varepsilon_0 r^2}$ (b) $\dfrac{Q}{3\pi\varepsilon_0 R}$ (c) zero on inner and Q on outer (d) $\dfrac{Q}{3}$

53. (a) $\dfrac{Q}{4\pi\varepsilon_0 R},\dfrac{Q}{6\pi\varepsilon_0 R}$ (b) $\dfrac{-Q}{25\pi\varepsilon_0 R^2}\hat{\mathbf{r}}$ (c) $\dfrac{Q^2}{4\pi\varepsilon_0 R}$ (d) $Q_1=\dfrac{Q}{2}, Q_2=\dfrac{7Q}{2}$ (e) $\dfrac{-3Q}{50\pi\varepsilon_0 R^2}\hat{\mathbf{r}}$

LEVEL 2

Single Correct Option

1.(a) **2.**(a) **3.**(a) **4.**(b) **5.**(a) **6.**(c) **7.**(d) **8.**(a) **9.**(c) **10.**(a)
11.(c) **12.**(a) **13.**(d) **14.**(d) **15.**(c) **16.**(b) **17.**(c) **18.**(d) **19.**(a) **20.**(d)
21.(d) **22.**(a) **23.**(a) **24.**(d) **25.**(c) **26.**(c) **27.**(b) **28.**(d) **29.**(a) **30.**(c)
31.(a) **32.**(a) **33.**(b) **34.**(b) **35.**(c) **36.**(c) **37.**(a) **38.**(b)

More than One Correct Options

1.(a,d) **2.**(a,b,c) **3.**(a,b,c) **4.**(all) **5.**(a,b,c) **6.**(a,c) **7.**(c,d) **8.**(a,b,d) **9.**(b,c) **10.**(b,c)

Comprehension Based Questions

1.(c) **2.**(c) **3.**(a) **4.**(a) **5.**(b) **6.**(c) **7.**(a) **8.**(d) **9.**(b) **10.**(c)

Match the Columns

1. (a) → s (b) → q (c) → r (d) → p
2. (a) → q (b) → p (c) → s (d) → r
3. (a) → s (b) → q (c) → q (d) → p
4. (a) → r (b) → p (c) → q (d) → s
5. (a) → p (b) → q (c) → r (d) → s

Subjective Questions

1. (a) 0.5 m (b) 0.25 m (c) 1.26 s (d) 0.34 m

3. $3Q, -2Q, 2Q, 0, 0, 3Q$

4. (a) $\frac{L}{2}\left[1+\frac{Qq}{(4\pi\varepsilon_0)Wh^2}\right]$ (b) $\sqrt{\frac{3Qq}{(4\pi\varepsilon_0)W}}$

5. 2.0×10^{-3} m/s

6. 6 m/s **7.** shrink, $\sqrt{\frac{Q^2(1-n)(3\sqrt{6}+\sqrt{2}-3\sqrt{3})}{4nm\pi L\varepsilon_0\sqrt{6}}}$

8. $\frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$

9. (a) Q_2 is negative and Q_1 is positive (b) $\left(\frac{l+a}{a}\right)^2$ (c) $\frac{l}{\left(\frac{l+a}{a}\right)^{2/3}-1}$

10. (a) $\frac{q_n^2}{8\pi\varepsilon_0 R}$, where $q_n=\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]$ (b) $\frac{Q^2R}{8\pi\varepsilon_0 r^2}$

11. $\theta=\tan^{-1}\left\{\frac{al^2}{4V}\sqrt{\frac{m}{2eV}}\right\}$

12. $\sqrt{72}$ m

13. $\left[\frac{1}{2\pi\varepsilon_0}\frac{Qq}{Rm}\left(\frac{r-R}{r}+\frac{3}{8}\right)\right]^{\frac{1}{2}}$

14. $\sqrt{\frac{Qq}{2\pi\varepsilon_0 mR}\left(\frac{3\sqrt{10}-5}{5\sqrt{10}}\right)}$

15. (a) $\frac{aqQ}{2\pi\varepsilon_0}\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}$ (b) $\frac{aqQx}{\pi\varepsilon_0(R^2+x^2)^{3/2}}$

16. $\sqrt{\frac{Q\,q\,r_2}{2\pi\varepsilon_0 mr_1(r_1+r_2)}}$, $\sqrt{\frac{Qq\,r_1}{2\pi\varepsilon_0 mr_2(r_1+r_2)}}$

17. (a) 6 V (b) 16 V (c) 1280 V/m

18. (a) $F=-\frac{2kQqx}{(R^2+x^2)^{3/2}}$ (b) Periodic between $\pm x_0$

(c) $x=x_0\cos\omega t$, $v=-\omega x_0\sin\omega t$, where $\omega=\sqrt{\frac{2Qqk}{mR^3}}$ (d) $t=\sqrt{\frac{\pi^2mR^3}{2Qqk}}$, Here $k=\frac{1}{4\pi\varepsilon_0}$

19. $C=\frac{q}{2\pi a^2}$

20. $f=\lambda RE_0$ along positive x-axis

21. $v=QE\left(\frac{t}{m_0+mnt}\right)$

22. $70\sqrt{3}$ m

25

Capacitors

Chapter Contents

r

25.1 Capacitance

In practice, we cannot handle free point charges or hold them fixed at desired positions. A practical way to handle a charge would be to put it on a conductor. Thus, one use of a conductor is to store electric charge (or electric potential energy). But, every conductor has its maximum limit of storing the electric charge or potential energy. Beyond that limit, the dielectric in which the conductor is placed, becomes ionized. A capacitor is a device which can store more electric charge or potential energy compared to an isolated conductor.

Capacitors have a tremendous number of applications. In the flash light used by photographers, the energy and charge stored in a capacitor are recovered quickly. In other applications, the energy is released more slowly.

Capacitance of an Isolated Conductor

When a charge q is given to a conductor, it spreads over the outer surface of the conductor. The whole conductor comes to the same potential (say V). This potential V is directly proportional to the charge q, i.e.

$$V \propto q$$

When the proportionality sign is removed, a constant of proportionality $\frac{1}{C}$ comes in picture.

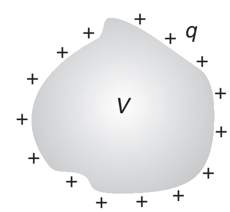


Fig. 25.1

Hence, $$V = \frac{q}{C}$$

or $$C = \frac{q}{V}$$

Here, C is called the **capacitance** of the conductor. The SI unit of capacitance is called **one farad** (1 F). One farad is equal to one coulomb per volt (1 C/V)

$\therefore$ $$1\text{ F} = 1\text{ farad} = 1\text{ C/V} = 1\text{ coulomb/volt}$$

Note *(i) An obvious question arises in mind that when a conductor stores electric charge and energy then why not the unit of capacitance is coulomb or joule. For example, the capacity of a storage tank is given in litres (the unit of volume) or gallons not in the name of some scientist. The reason is simple the capacity of tank does not depend on medium in which it is kept. While the capacity of a conductor to store charge (or energy) depends on the medium in which it is kept. It varies from medium to medium. So, it is difficult to express the capacity in terms of coulomb or joule. Because in that case we will have to mention the medium also.*

For example, we will say like this, capacity of this conductor in water is 1 C in oil it is 5 C, etc. On the other hand, the C discussed above gives us an idea about the dimensions of the conductor nothing about the charge which it can store because as we said earlier also it will vary from medium to medium. By knowing the C (or the dimensions of conductor) a physics student can easily find the maximum charge which it can store, provided the medium is also given.

(ii) Farad in itself is a large unit. Microfarad (μF) is used more frequently.

Method of Finding Capacitance of a Conductor

Give a charge q to the conductor. Find potential on it due to charge q. This potential V will be a function of q and finally find q/V, which is the desired capacitance C.

Capacitance of a Spherical Conductor

When a charge q is given to a spherical conductor of radius R, the potential on it is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

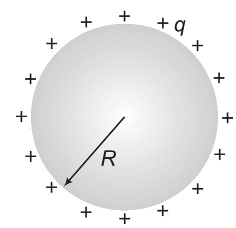


Fig. 25.2

From this expression, we find that $\frac{q}{V} = 4\pi\varepsilon_0 R = C$

Thus, capacitance of the spherical conductor is

$$\boxed{C = 4\pi\varepsilon_0 R}$$

From this expression, we can draw the following conclusions :

(i) $C \propto R$ or C depends on R only. Which we have already stated that C depends on the dimensions of the conductor. Moreover if two conductors have radii R_1 and R_2, then

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

(ii) Earth is also a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacity of earth is therefore,

$$C = \left(\frac{1}{9 \times 10^9}\right)(6.4 \times 10^6)$$

$$\approx 711 \times 10^{-6} \text{ F}$$

or $$C = 711 \, \mu\text{F}$$

From here, we can see that farad is a large unit. As capacity of such a huge conductor is only $711 \, \mu$F.

Extra Points to Remember

- **Dielectric strength of an insulator** In an insulator, most of the electrons are tightly bounded with the nucleus. If an electric field is applied on this insulator, an electrostatic force acts on these electrons in the opposite direction of electric field. As electric field increases, this force also increases. After a certain value of electric field, force becomes so large that these bounded electrons are knocked out or ionized. This maximum electric field is called dielectric strength of insulator. Its unit is N/C or V/m.
- With the help of capacitance (or dimensions of conductor) and dielectric strength of an insulator we can determine the maximum charge or energy which can be stored by this conductor.

Example 25.1 *Capacitance of a conductor is $1 \, \mu F$. What charge is required to raise its potential to $100 V$?*

Solution Using the equation

$$q = CV$$

We have, $$q = (1 \, \mu\text{F})(100 \, \text{V})$$

$$= 100 \, \mu\text{C}$$ **Ans.**

Example 25.2 *Radius of a spherical conductor is 2 m. This is kept in a dielectric medium of dielectric constant 10^6 N/C. Find*

(a) capacitance of the conductor

(b) maximum charge which can be stored on this conductor.

Solution (a)

$$C = 4\pi\varepsilon_0 R$$

$$= \left(\frac{1}{9\times 10^9}\right)(2)$$

$$= 2.2\times 10^{-10}\ \text{F}$$ **Ans.**

(b) Maximum electric field on the surface of spherical conductor is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

This should not exceed 10^6 N/C.

$\therefore$ $$E_{max} = \frac{1}{4\pi\varepsilon_0}\frac{q_{max}}{R^2} = 10^6$$

$\Rightarrow$ $$q_{max} = (4\pi\varepsilon_0)R^2(10^6)$$

$$= \left(\frac{1}{9\times 10^9}\right)(2)^2(10^6)$$

$$= 4.4\times 10^{-4}\ \text{C}$$ **Ans.**

25.2 Energy Stored in a Charged Capacitor

Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. This work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V_0 and let q_0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was $q\ (< q_0)$ is

$$V = \frac{q}{C}$$

Now, work done in bringing a small charge dq at this potential is

$$dW = Vdq = \left(\frac{q}{C}\right)dq$$

$\therefore$ Total work done in charging it from 0 to q_0 is

$$W = \int_0^{q_0} dW = \int_0^{q_0}\frac{q}{C}\,dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy,

$\therefore$ $$U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$, we can write this expression also as

$$U = \frac{1}{2} CV_0^2 = \frac{1}{2} q_0 V_0$$

In general, if a conductor of capacity C is charged to a potential V by giving it a charge q, then

$$\boxed{U = \frac{1}{2} CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} qV}$$

Redistribution of Charge

Let us take an analogous example. Some liquid is filled in two vessels of different sizes upto different heights. These are joined through a valve which was initially closed. When the valve is opened, the level in both the vessels becomes equal but the volume of liquid in the right side vessel is more than the liquid in the left side vessel. This is because the base area (or capacity) of this vessel is more.

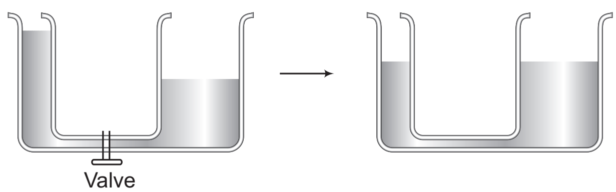


Fig. 25.3

Now, suppose two conductors of capacities C_1 and C_2 have charges q_1 and q_2 respectively when they are joined together by a conducting wire, charge redistributes in these conductors in the ratio of their capacities. Charge redistributes till potential of both the conductors becomes equal. Thus, let q_1' and q_2' be the final charges on them, then

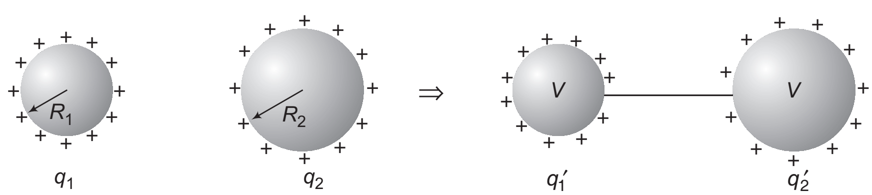


Fig. 25.4

$$q' \propto C \quad \text{or} \quad \frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

and if they are spherical conductors, then

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

$$\therefore \qquad \frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2}$$

Since, the total charge is $(q_1 + q_2)$. Therefore,

$$q_1' = \left(\frac{C_1}{C_1 + C_2}\right)(q_1 + q_2) = \left(\frac{R_1}{R_1 + R_2}\right)(q_1 + q_2)$$

and
$$q_2' = \left(\frac{C_2}{C_1 + C_2}\right)(q_1 + q_2) = \left(\frac{R_2}{R_1 + R_2}\right)(q_1 + q_2)$$

Note *The common potential is given by*

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{q_1 + q_2}{C_1 + C_2}$$

Loss of Energy During Redistribution of Charge

We can show that in redistribution of charge energy is always lost.

Initial potential energy, $\quad U_i = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$

Final potential energy, $\quad U_f = \frac{1}{2}\frac{(q_1 + q_2)^2}{C_1 + C_2}$

$$\Delta U = U_i - U_f = \frac{1}{2}\left[\frac{q_1^2}{C_1} + \frac{q_2^2}{C_2} - \frac{(q_1 + q_2)^2}{C_1 + C_2}\right]$$

or
$$\Delta U = \frac{1}{2C_1C_2(C_1 + C_2)}[q_1^2C_1C_2 + q_1^2C_2^2 + q_2^2C_1^2 + q_2^2C_1C_2 - q_1^2C_1C_2 - q_2^2C_1C_2 - 2q_1q_2C_1C_2]$$

$$= \frac{C_1^2C_2^2}{2C_1C_2(C_1 + C_2)}\left[\frac{q_1^2}{C_1^2} + \frac{q_2^2}{C_2^2} - \frac{2q_1q_2}{C_1C_2}\right]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}[V_1^2 + V_2^2 - 2V_1V_2]$$

or
$$\boxed{\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2}$$

Now, as C_1, C_2 and $(V_1 - V_2)^2$ are always positive. $U_i > U_f$, i.e. there is a decrease in energy. Hence, energy is always lost in redistribution of charge. Further,

$$\Delta U = 0 \quad \text{if} \quad V_1 = V_2$$

this is because no flow of charge takes place when both the conductors are at same potential.

Example 25.3 *Two isolated spherical conductors have radii 5 cm and 10 cm, respectively. They have charges of 12 μC and – 3 μC. Find the charges after they are connected by a conducting wire. Also find the common potential after redistribution.*

Solution

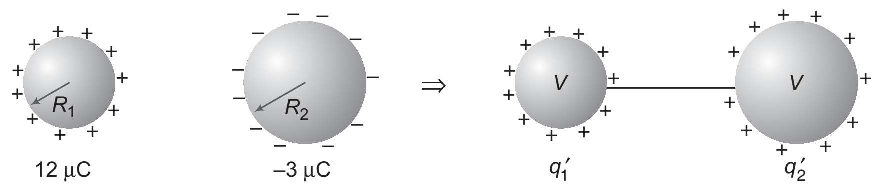


Fig. 25.5

Net charge = $(12-3)\,\mu C = 9\,\mu C$

Charge is distributed in the ratio of their capacities (or radii in case of spherical conductors), i.e.

$$\frac{q_1'}{q_2'} = \frac{R_1}{R_2} = \frac{5}{10} = \frac{1}{2}$$

$\therefore$
$$q_1' = \left(\frac{1}{1+2}\right)(9) = 3\,\mu C$$

and
$$q_2' = \left(\frac{2}{1+2}\right)(9) = 6\,\mu C$$

$$\text{Common potential, } V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{(9 \times 10^{-6})}{4\pi\varepsilon_0 (R_1 + R_2)}$$

$$= \frac{(9 \times 10^{-6})(9 \times 10^{9})}{(15 \times 10^{-2})}$$

$$= 5.4 \times 10^{5}\ V$$ **Ans.**

Example 25.4 *An insulated conductor initially free from charge is charged by repeated contacts with a plate which after each contact is replenished to a charge Q. If q is the charge on the conductor after first operation prove that the maximum charge which can be given to the conductor in this way is* $\dfrac{Qq}{Q-q}$.

Solution Let C_1 be the capacity of plate and C_2 that of the conductor. After first contact charge on conductor is q. Therefore, charge on plate will remain $Q - q$. As the charge redistributes in the ratio of capacities.

$$\frac{Q-q}{q} = \frac{C_1}{C_2} \qquad \text{...(i)}$$

Let q_m be the maximum charge which can be given to the conductor. Then, flow of charge from the plate to the conductor will stop when,

$$V_{\text{conductor}} = V_{\text{plate}}$$

$\therefore$
$$\frac{q_m}{C_2} = \frac{Q}{C_1} \Rightarrow q_m = \left(\frac{C_2}{C_1}\right) Q$$

Substituting $\dfrac{C_2}{C_1}$ from Eq. (i), we get

$$q_m = \frac{Qq}{Q-q}$$ **Hence Proved.**

Example 25.5 *A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times.* **(JEE 1998)**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

Solution Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2 is q_1. Therefore, charge on S_1 will be $Q - q_1$. Say it is q_1'

$$\therefore \qquad \frac{q_1}{q_1'} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$$

$$\therefore \qquad q_1 = Q\left(\frac{R}{R+r}\right) \qquad \text{...(i)}$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R+r}\right)$$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1 + \frac{R}{R+r}\right)\left(\frac{R}{R+r}\right)$$

$$= Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

Similarly,
$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

and
$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots + \left(\frac{R}{R+r}\right)^n\right]$$

or
$$q_n = Q\frac{R}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right] \qquad \text{...(ii)}$$

$$\left[S_n = \frac{a\,(1 - r^n)}{(1 - r)}\right]$$

Therefore, electrostatic energy of S_2 after n such contacts

$$=\frac{q_n^2}{2(4\pi\varepsilon_0 R)} \quad \text{or} \quad U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

where, q_n can be written from Eq. (ii).

(b) As $n \to \infty$

$$q_\infty = Q\frac{R}{r}$$

$$\therefore \qquad U_\infty = \frac{q_\infty^2}{2C} = \frac{Q^2 R^2/r^2}{8\pi\varepsilon_0 R}$$

$$\text{or} \qquad U_\infty = \frac{Q^2 R}{8\pi\varepsilon_0 r^2}$$

INTRODUCTORY EXERCISE 25.1

1. Find the dimensions of capacitance.
2. No charge will flow when two conductors having the same charge are connected to each other. Is this statement true or false?
3. Two conductors of capacitance $1\mu F$ and $2\mu F$ are charged to +10 V and –20 V. They are now connected by a conducting wire. Find
   (a) their common potential
   (b) the final charges on them
   (c) the loss of energy during redistribution of charges.

25.3 Capacitors

Any two conductors separated by an insulator (or a vacuum) form a capacitor.

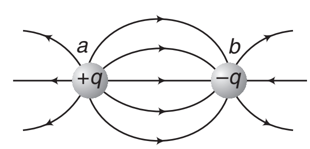


Fig. 25.6

In most practical applications, each conductor initially has zero net charge, and electrons are transferred from one conductor to the other. This is called charging of the conductor. Then, the two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero. When we say that a capacitor has charge q we mean that the conductor at higher potential has charge $+q$ and the conductor at lower potential has charge $-q$. In circuit diagram, a capacitor is represented by two parallel lines as shown in Fig. 25.7.

Fig. 25.7

One common way to charge a capacitor is to connect the two conductors to opposite terminals of a battery. This gives a fixed potential difference V_{ab} between the conductors, which is just equal to the voltage of the battery. The ratio $\frac{q}{V_{ab}}$ is called the capacitance of the capacitor. Hence,

$$\boxed{C = \frac{q}{V_{ab}}} \qquad \text{(capacitance of a capacitor)}$$

Calculation of Capacitance

Give a charge $+q$ to one plate and $-q$ to the other plate. Then, find potential difference V between the plates. Now,

$$\boxed{C = \frac{q}{V}}$$

Parallel Plate Capacitor

Two metallic parallel plates of any shape but of same size and separated by a small distance constitute parallel plate capacitor. Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum.

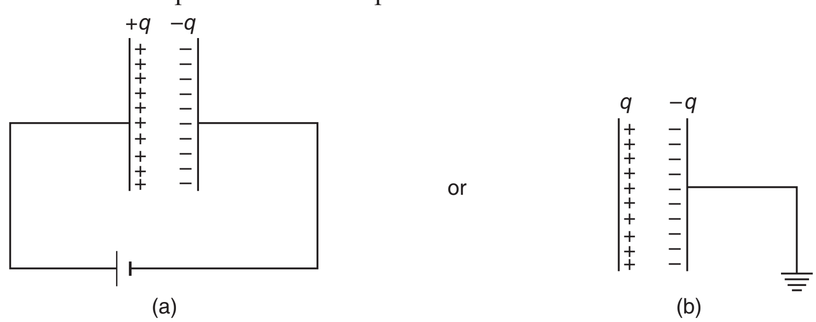


Fig. 25.8

We put a charge q on one plate and a charge $-q$ on the other. This can be done either by connecting one plate with the positive terminal and the other with negative plate of a battery [as shown in Fig. (a)] or by connecting one plate to the earth and by giving a charge $+q$ to the other plate only. This charge will induce a charge $-q$ on the earthed plate. The charges will appear on the facing surfaces. The charge density on each of these surfaces has a magnitude $\sigma = q/A$.

If the plates are large as compared to the separation between them, then the electric field between the plates (at point B) is uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated by using the fact that both positive and negative plates produce the electric field in the same direction (from positive plate towards negative plate) of magnitude $\sigma/2\varepsilon_0$ and therefore, the net electric field between the plates will be

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

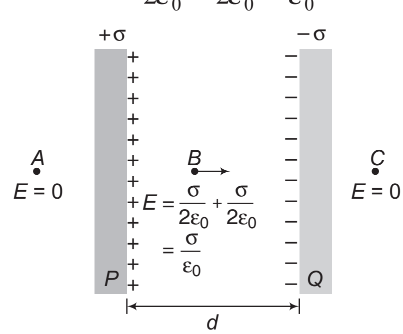


Fig. 25.9

Outside the plates (at points A and C) the field due to positive sheet of charge and negative sheet of charge are in opposite directions. Therefore, net field at these points is zero.

The potential difference between the plates is

$$\therefore \qquad V = E \cdot d = \left(\frac{\sigma}{\varepsilon_0}\right) d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d}$$

or

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Note *(i) Instead of two plates if there are n similar plates at equal distances from each other and the alternate plates are connected together, the capacitance of the arrangement is given by*

$$C = \frac{(n-1)\,\varepsilon_0 A}{d}$$

(ii) From the above relation, it is clear that the capacitance depends only on geometrical factors (A and d).

Effect of Dielectrics

Most capacitors have a dielectric between their conducting plates. Placing a solid dielectric between the plates of a capacitor serves the following three functions :

(i) It solves the problem of maintaining two large metal sheets at a very small separation without actual contact.

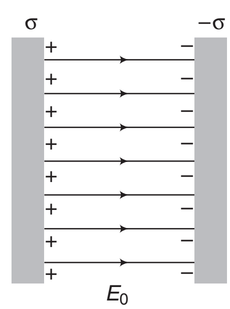


Fig. 25.10

(ii) It increases the maximum possible potential difference which can be applied between the plates of the capacitor without the dielectric breakdown. Many dielectric materials can tolerate stronger electric fields without breakdown than can air.

(iii) It increases the capacitance of the capacitor.

When a dielectric material is inserted between the plates (keeping the charge to be constant) the electric field and hence the potential difference decreases by a factor K (the dielectric constant of the dielectric).

$$\therefore \qquad \boxed{E = \frac{E_0}{K} \quad \text{and} \quad V = \frac{V_0}{K}} \qquad \text{(When } q \text{ is constant)}$$

Electric field is decreased because an induced charge of the opposite sign appears on each surface of the dielectric. This induced charge produces an electric field inside the dielectric in opposite directions and as a result net electric field is decreased. The induced charge in the dielectric can be calculated as under

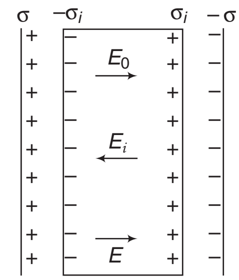


Fig. 25.11

$$E = E_0 - E_i \quad \text{or} \quad \frac{E_0}{K} = E_0 - E_i$$

∴ $$E_i = E_0\left(1 - \frac{1}{K}\right)$$

Therefore, $$\frac{\sigma_i}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(1 - \frac{1}{K}\right)$$

or $$\boxed{\sigma_i = \sigma\left(1 - \frac{1}{K}\right)}$$

or $$\boxed{q_i = q\left(1 - \frac{1}{K}\right)}$$

For a conductor $K = \infty$. Hence,

$$q_i = q, \ \sigma_i = \sigma \quad \text{and} \quad E = 0$$

and otherwise $$q_i < q$$

Thus, $$\boxed{q_i \le q, \quad \sigma_i \le \sigma}$$

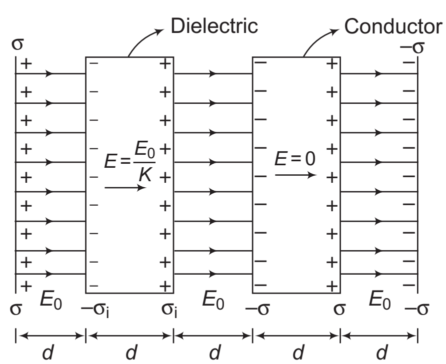


Fig. 25.12

Hence, we can conclude the above discussion as under:

(i) $E_{\text{vacuum}} = E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{q}{A\varepsilon_0}$

(ii) $E_{\text{dielectric}} = \dfrac{E_0}{K}$ (here, K = dielectric constant)

(iii) $E_{\text{conductor}} = 0$ (as $K = \infty$)

If we plot a graph between potential and distance from positive plate, it will be as shown in Fig. 25.13: Modulus of,

Slope of $AB =$ slope of $CD =$ slope of $EF = E_0$

Slope of $BC = \dfrac{E_0}{K}$

and slope of $DE = 0$

Further, the potential difference between positive and negative plate is

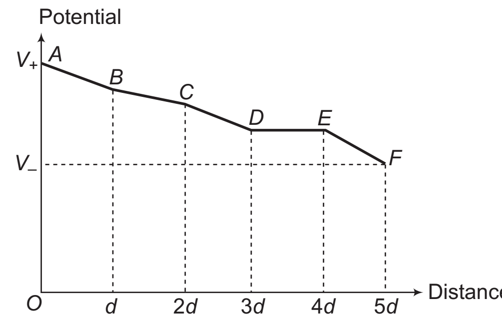


Fig. 25.13

$$V_+ - V_- = E_0 d + \frac{E_0}{K} \cdot d + E_0 d + 0 + E_0 d$$

$$= 3E_0 d + \frac{E_0}{K} \cdot d$$

Here we have used PD $= Ed$. (in uniform electric field)

Capacitance of a Capacitor Partially Filled with Dielectric

Suppose, a dielectric is partially filled with a dielectric (dielectric constant $= K$) as shown in figure. If a charge q is given to the capacitor, an induced charge q_i is developed on the dielectric.

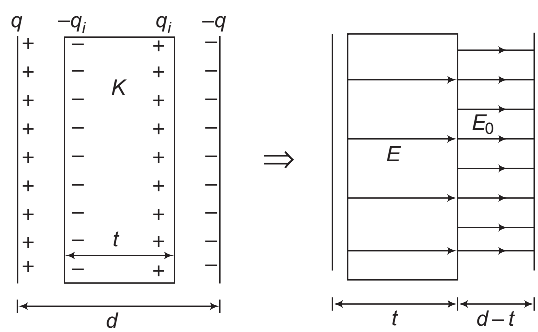


Fig. 25.14

where, $$q_i = q\left(1 - \frac{1}{K}\right)$$

Moreover, if E_0 is the electric field in the region where dielectric is absent, then electric field inside the dielectric will be $E = E_0/K$. The potential difference between the plates of the capacitor is

$$V = V_+ - V_- = Et + E_0 (d - t)$$

$$= \frac{E_0}{K} t + E_0 (d - t) = E_0 \left(d - t + \frac{t}{K}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Now, as per the definition of capacitance,

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \quad \text{or} \quad \boxed{C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}}$$

Different Cases

(i) If more than one dielectric slabs are placed between the capacitor, then

$$C = \frac{\varepsilon_0 A}{(d - t_1 - t_2 - \ldots - t_n) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots + \frac{t_n}{K_n}\right)}$$

(ii) If the slab completely filles the space between the plates, then $t = d$ and therefore,

$$C = \frac{\varepsilon_0 A}{d/K} = \frac{K\varepsilon_0 A}{d}$$

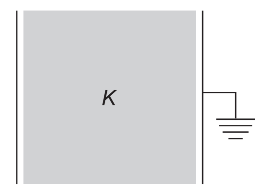


Fig. 25.15

(iii) If a conducting slab ($K = \infty$) is placed between the plates, then

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\infty}} = \frac{\varepsilon_0 A}{d - t}$$

This can be explained from the following figure:

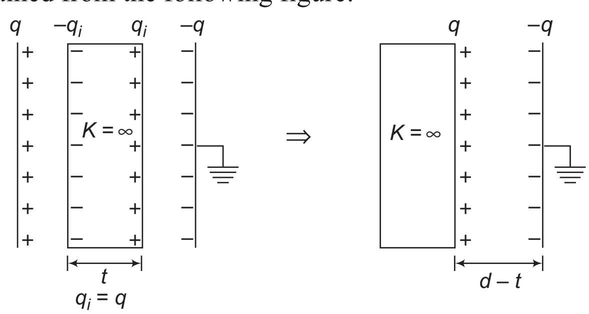


Fig. 25.16

(iv) If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

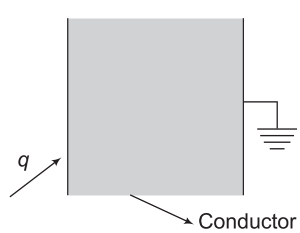


Fig. 25.17

Then,
$$C = \frac{\varepsilon_0 A}{d - d + \frac{d}{\infty}} = \infty$$

The significance of infinite capacitance can be explained as under:

If one of the plates of a capacitor is earthed and the second one is given a charge q, then the whole charge transfers to earth and as the capacity of earth is very large compared to the capacitor we can say that the capacitance has become infinite.

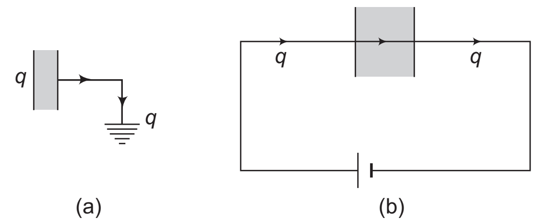


Fig. 25.18

Alternatively, if the plates of the capacitor are connected to a battery, the current starts flowing in the circuit. Thus, is as much charge enters the positive plate of the capacitor, the same charge leaves the negative plate. So, we can say, the positive plate can accept infinite amount of charge or its capacitance has become infinite.

Energy Stored in Charged Capacitor

A charged capacitor stores an electric potential energy in it, which is equal to the work required to charge it. This energy can be recovered if the capacitor is allowed to discharge. If the charging is done by a battery, electrical energy is stored at the expense of chemical energy of battery.

Suppose at time t, a charge q is present on the capacitor and V is the potential of the capacitor. If dq amount of charge is brought against the forces of the field due to the charge already present on the capacitor, the additional work needed will be

$$dW = (dq)V = \left(\frac{q}{C}\right)\cdot dq \qquad (\text{as } V = q/C)$$

$\therefore$ Total work to charge a capacitor to a charge q_0,

$$W = \int dW = \int_0^{q_0}\left(\frac{q}{C}\right)\cdot dq = \frac{q_0^2}{2C}$$

$\therefore$ Energy stored by a charged capacitor,

$$U = W = \frac{q_0^2}{2C} = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

Thus, if a capacitor is given a charge q, the potential energy stored in it is

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV}$$

The above relation shows that the charged capacitor is the electrical analog of a stretched spring whose elastic potential energy is $\frac{1}{2}Kx^2$. The charge q is analogous to the elongation x and $\frac{1}{C}$, i.e. the reciprocal of capacitance to the force constant k.

Extra Points to Remember

- **Capacitance of a spherical conductor enclosed by an earthed concentric spherical shell**

  If a charge q is given to the inner spherical conductor, it spreads over the outer surface of it and a charge $-q$ appears on the inner surface of the shell. The electric field is produced only between the two. From the principle of generator, the potential difference between the two will depend on the inner charge q only and is given by

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

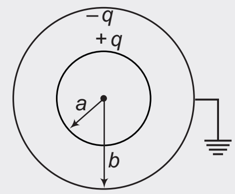


Fig. 25.19

  Hence, the capacitance of this system

$$C = \frac{q}{V}$$

  or

$$\boxed{C = 4\pi\varepsilon_0\left(\frac{1}{a} - \frac{1}{b}\right) = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)}$$

  From this expression we see that if $b = \infty$, $C = 4\pi\varepsilon_0 a$, which corresponds to that of an isolated sphere, i.e. the charged sphere may be regarded as a capacitor in which the outer surface has been removed to infinity.

- **Capacitance of a cylindrical capacitor** When a metallic cylinder of radius a is placed coaxially inside an earthed hollow metallic cylinder of radius b $(> a)$ we get cylindrical capacitor. If a charge q is given to the inner cylinder, induced charge $-q$ will reach to the inner surface of the outer cylinder. Assume that the capacitor is of very large length $(l >> b)$ so that the lines of force are radial. Using Gauss's law, we can prove that

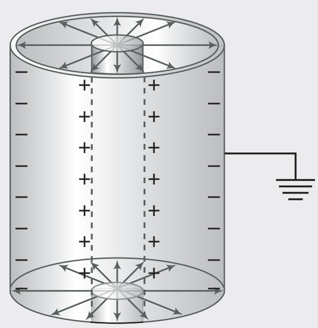


Fig. 25.20

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{for} \quad a \le r \le b$$

  Here, λ = charge per unit length

  Therefore, the potential difference between the cylinders

$$V = -\int_b^a \mathbf{E}\cdot d\mathbf{r} = -\int_b^a \frac{\lambda}{2\pi\varepsilon_0 r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$\therefore \qquad \frac{\lambda}{V} = \frac{\text{charge/length}}{\text{potential difference}} = \frac{\text{capacitance}}{\text{length}} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

  Hence,

$$\boxed{\text{capacitance per unit length} = \frac{2\pi\varepsilon_0}{\ln(b/a)}}$$

Example 25.6 *A parallel-plate capacitor has capacitance of* 1.0 *F. If the plates are* 1.0 *mm apart, what is the area of the plates?*

Solution $\because$
$$C = \frac{\varepsilon_0 A}{d}$$

$\therefore$
$$A = \frac{Cd}{\varepsilon_0} = \frac{(1)(10^{-3})}{8.86 \times 10^{-12}}$$
$$= 1.1 \times 10^8 \text{ m}^2$$

Example 25.7 *Two parallel plate vacuum capacitors have areas A_1 and A_2 and equal plate spacing d. Show that when the capacitors are connected in parallel, the equivalent capacitance is the same as for a single capacitor with plate area $A_1 + A_2$ and spacing d.* ***Note :*** *In parallel $C = C_1 + C_2$.*

Solution $C = C_1 + C_2$ (in parallel)

$\therefore$
$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

or
$$A = A_1 + A_2$$

Example 25.8 *(a) Two spheres have radii a and b and their centres are at a distance d apart. Show that the capacitance of this system is*
$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} \pm \frac{2}{d}}$$
provided that d is large compared with a and b.

(b) Show that as d approaches infinity the above result reduces to that of two isolated spheres in series. ***Note :*** *In series, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$.*

Solution (a) PD, $V = V_1 - V_2$

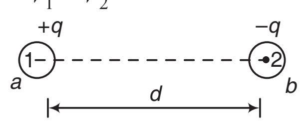


Fig. 25.21

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{q}{a} - \frac{q}{d}\right) - \left(\frac{-q}{b} + \frac{q}{d}\right)\right]$$

$\therefore$
$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

If $-q$ is given to first sphere and $+q$ to second sphere, then
$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} + \frac{2}{d}}$$

(b) If $d \to \infty$, then $C = \dfrac{4\pi\varepsilon_0}{\dfrac{1}{a} + \dfrac{1}{b}} = \dfrac{(4\pi\varepsilon_0)(ab)}{a+b}$

In series, $$C_{net} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4\pi\,\varepsilon_0\,a)(4\pi\,\varepsilon_0 b)}{(4\pi\,\varepsilon_0 a) + (4\pi\varepsilon_0 b)} = \frac{(4\pi\varepsilon_0)\,ab}{a+b}$$ **Hence Proved.**

INTRODUCTORY EXERCISE 25.2

1. A capacitor has a capacitance of 7.28 μF. What amount of charge must be placed on each of its plates to make the potential difference between its plates equal to 25.0 V?
2. A parallel plate air capacitor of capacitance 245 μF has a charge of magnitude 0.148 μC on each plate. The plates are 0.328 mm apart.
   (a) What is the potential difference between the plates?
   (b) What is the area of each plate?
   (c) What is the surface charge density on each plate?
3. Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the electric field is $E_0 = 3.20 \times 10^5$ V/m. When the space is filled with dielectric, the electric field is $E = 2.50 \times 10^5$ V/m.
   (a) What is the dielectric constant?
   (b) What is the charge density on each surface of the dielectric?

25.4 Mechanical Force on a Charged Conductor

We know that similar charges repel each other, hence the charge on any part of surface of the conductor is repelled by the charge on its remaining part. The surface of the conductor thus experiences a mechanical force.

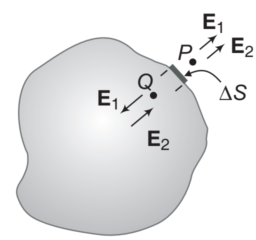


Fig. 25.22

The electric field at any point P near the conductor's surface can be assumed as due to a small part of the surface of area say ΔS immediately in the neighbourhood of the point under consideration and due to the rest of the surface. Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the field intensities due to these parts respectively.

Then, total electric field, $$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$\mathbf{E}$ has a magnitude σ/ε_0 at any point P just outside the conductor and is zero at point Q just inside the conductor. Thus,

$$E_1 + E_2 = \sigma/\varepsilon_0 \text{ at } P$$

and $$E_1 - E_2 = 0 \text{ at } Q$$

$\therefore$ $$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

Hence, the force experienced by small surface of area ΔS due to the charge on the rest of the surface is

$$F = qE_2 = (\sigma\Delta S)(E_2) = \frac{(\sigma^2)(\Delta S)}{2\varepsilon_0}$$

$$\therefore \qquad \frac{\text{Force}}{\text{Area}} = \frac{F}{\Delta S} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \qquad \left(\text{as } E = \frac{\sigma}{\varepsilon_0}\right)$$

$$\therefore \qquad \boxed{\frac{\text{Force}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E^2}$$

Force between the Plates of a Capacitor

Consider a parallel plate capacitor with plate area A. Suppose a positive charge q is given to one plate and a negative charge $-q$ to the other plate. The electric field on the negative plate due to positive charge is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}$$

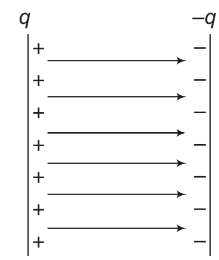


Fig. 25.23

The magnitude of force on the charge in negative plate is

$$F = qE = \frac{q^2}{2A\varepsilon_0}$$

This is the force with which both the plates attract each other. Thus,

$$\boxed{F = \frac{q^2}{2A\varepsilon_0}}$$

Example 25.9 *A capacitor is given a charge q. The distance between the plates of the capacitor is d. One of the plates is fixed and the other plate is moved away from the other till the distance between them becomes 2d. Find the work done by the external force.*

Solution When one plate is fixed, the other is attracted towards the first with a force

$$F = \frac{q^2}{2A\varepsilon_0} = \text{constant}$$

Hence, an external force of same magnitude will have to be applied in opposite direction to increase the separation between the plates.

$$\therefore \qquad W = F(2d - d) = \frac{q^2 d}{2A\varepsilon_0} \qquad \textbf{Ans.}$$

Alternate solution

$$W = \Delta U = U_f - U_i = \frac{q^2}{2C_f} - \frac{q^2}{2C_i} \qquad \ldots\text{(i)}$$

Here,

$$C_f = \frac{\varepsilon_0 A}{2d} \quad \text{and} \quad C_i = \frac{\varepsilon_0 A}{d}$$

Substituting in Eq. (i), we have

$$W = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{2d}\right)} - \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{q^2 d}{2\varepsilon_0 A} \qquad \textbf{Ans.}$$

25.5 Capacitors in Series and Parallel

In Series

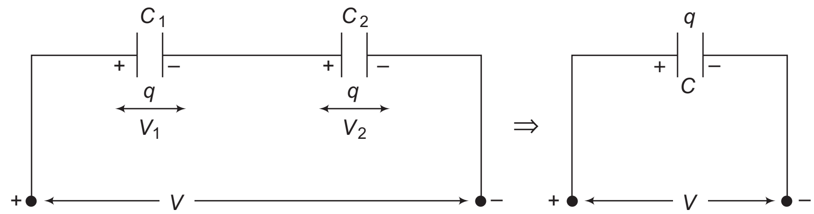


Fig. 25.24

In a series connection, the magnitude of charge on all plates is same. The potential is distributed in the inverse ratio of the capacity (as $V = q/C$ or $V \propto 1/C$). Thus, in the figure, if a potential difference V is applied across the two capacitors C_1 and C_2, then

$$\boxed{\frac{V_1}{V_2} = \frac{C_2}{C_1}}$$

or
$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V \quad \text{and} \quad V_2 = \left(\frac{C_1}{C_1 + C_2}\right)V$$

Further, in the figure,
$$V = V_1 + V_2 \quad \text{or} \quad \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2}$$

or
$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Here, C is the **equivalent capacitance**.

The equivalent capacitance of the series combination is defined as the capacitance of a single capacitor for which the charge q is the same as for the combination, when the same potential difference V is applied across it. In other words, the combination can be replaced by an equivalent capacitor of capacitance C. We can extend this analysis to any number of capacitors in series. We find the following result for the equivalent capacitance.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Following points are important in case of series combination of capacitors.

(i) In a series connection, the equivalent capacitance is always less than any individual capacitance.

(ii) For the equivalent capacitance of two capacitors it is better to remember the following form

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

For example, equivalent capacitance of two capacitors $C_1 = 6\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$ is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \left(\frac{6 \times 3}{6 + 3}\right)\mu\text{F} = 2\,\mu\text{F}$$

(iii) If n capacitors of equal capacity C are connected in series, then their equivalent capacitance is $\dfrac{C}{n}$.

Example 25.10 *In the circuit shown in figure, find*

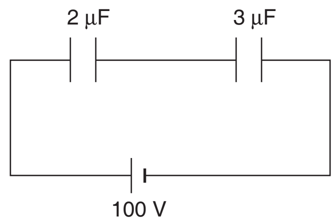


Fig. 25.25

(a) the equivalent capacitance,
(b) the charge stored in each capacitor and
(c) the potential difference across each capacitor.

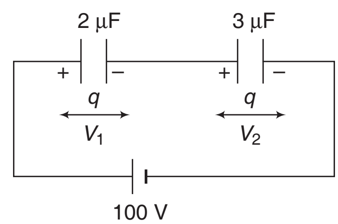


Fig. 25.26

Solution (a) The equivalent capacitance

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

or $$C = \frac{(2)(3)}{2+3} = 1.2\,\mu\text{F}$$ **Ans.**

(b) The charge q stored in each capacitor is

$$q = CV = (1.2 \times 10^{-6})(100)\text{ C}$$
$$= 120\ \mu\text{C}$$ **Ans.**

(c) In series combination, $V \propto \frac{1}{C}$ or $\frac{V_1}{V_2} = \frac{C_2}{C_1}$

$\therefore$ $$V_1 = \left(\frac{C_2}{C_1 + C_2}\right) V = \left(\frac{3}{2+3}\right)(100) = 60\text{ V}$$

and $$V_2 = V - V_1 = 100 - 60 = 40\text{ V}$$ **Ans.**

In Parallel

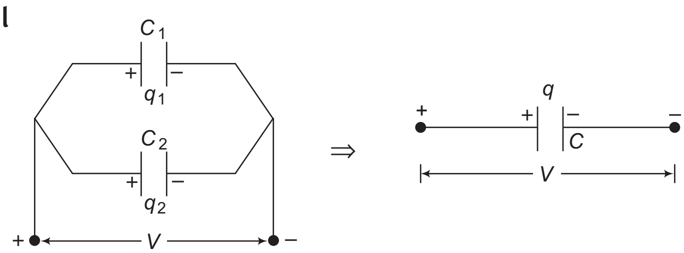


Fig. 25.27

The arrangement shown in figure is called a parallel connection. In a parallel combination, the potential difference for all individual capacitors is the same and the total charge q is distributed in the ratio of their capacities. (as $q = CV$ or $q \propto C$ for same potential difference). Thus,

$$\boxed{\frac{q_1}{q_2} = \frac{C_1}{C_2}}$$

or $$q_1 = \left(\frac{C_1}{C_1 + C_2}\right) q \quad \text{and} \quad q_2 = \left(\frac{C_2}{C_1 + C_2}\right) q$$

The parallel combination is equivalent to a single capacitor with the same total charge $q = q_1 + q_2$ and potential difference V.

Thus, $$q = q_1 + q_2$$

or $$CV = C_1V + C_2V$$

or $$\boxed{C = C_1 + C_2}$$

In the same way, we can show that for any number of capacitors in parallel,

$$\boxed{C = C_1 + C_2 + C_3 + \ldots}$$

In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.

Example 25.11 *In the circuit shown in figure, find*

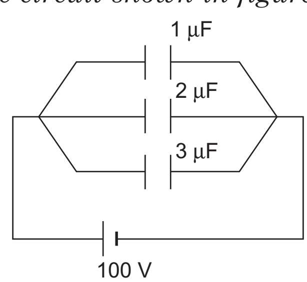


Fig. 25.28

(a) the equivalent capacitance and
(b) the charge stored in each capacitor.

Solution (a) The capacitors are in parallel. Hence, the equivalent capacitance is

$$C = C_1 + C_2 + C_3$$

or $$C = (1 + 2 + 3) = 6\,\mu\text{F}$$ **Ans.**

(b) Total charge drawn from the battery,

$$q = CV = 6 \times 100\,\mu\text{C} = 600\,\mu\text{C}$$

This charge will be distributed in the ratio of their capacities. Hence,

$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$$

$\therefore$ $$q_1 = \left(\frac{1}{1+2+3}\right) \times 600 = 100\,\mu\text{C}$$

$$q_2 = \left(\frac{2}{1+2+3}\right) \times 600 = 200\,\mu\text{C}$$

and $$q_3 = \left(\frac{3}{1+2+3}\right) \times 600 = 300\,\mu\text{C}.$$ **Ans.**

Alternate solution Since the capacitors are in parallel, the PD across each of them is 100 V. Therefore, from $q = CV$, the charge stored in $1\mu F$ capacitor is $100\mu C$, in $2\mu F$ capacitor is $200\mu C$ and that in $3\mu F$ capacitor is $300\mu C$.

INTRODUCTORY EXERCISE 25.3

1. Find charges on different capacitors.

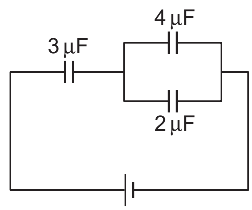


Fig. 25.29

2. Find charges on different capacitors.

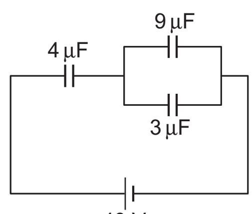


Fig. 25.30

25.6 Two Laws in Capacitors

Like an electric circuit having resistances and batteries in a complex circuit containing capacitors and the batteries charges on different capacitors can be obtained with the help of Kirchhoff 's laws.

First Law

This law is basically law of conservation of charge which is normally applied across a battery or in an isolated system.

(i) In case of a battery, both terminals of the battery supply equal amount of charge.

(ii) In an isolated system (not connected to any source of charge like terminal of a battery or earth) net charge remains constant.

For example, in the Fig. 25.31, the positive terminal of the battery supplies a positive charge $q_1 + q_2$. Similarly, the negative terminal supplies a negative charge of magnitude $q_3 + q_4$. Hence,

$$q_1 + q_2 = q_3 + q_4$$

Further, the plates enclosed by the dotted lines form an isolated system, as they are neither connected to a battery terminal nor to the earth. Initially, no charge was present in these plates. Hence, after charging net charge on these plates should also be zero. Or,

$$q_3 + q_5 - q_1 = 0 \quad \text{and} \quad q_4 - q_2 - q_5 = 0$$

So, these are the three equations which can be obtained from the first law.

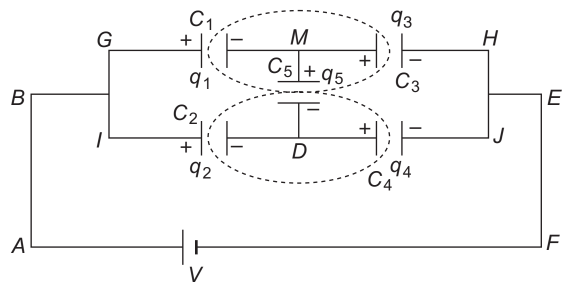


Fig. 25.31

Second Law

In a capacitor, potential drops by q/C when one moves from positive plate to the negative plate and in a battery it drops by an amount equal to the emf of the battery. Applying second law in loop *ABGHEFA*, we have

$$-\frac{q_1}{C_1}-\frac{q_3}{C_3}+V=0$$

Similarly, the second law in loop *GMDIG* gives the equation,

$$-\frac{q_1}{C_1}-\frac{q_5}{C_5}+\frac{q_2}{C_2}=0$$

Example 25.12 *Find the charges on the three capacitors shown in figure.*

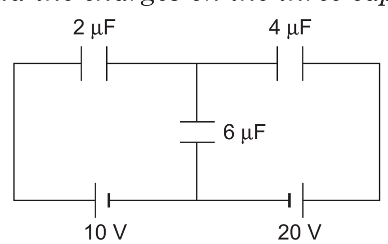


Fig. 25.32

Solution Let the charges in three capacitors be as shown in Fig. 25.33.

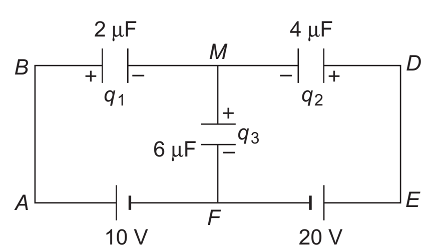


Fig. 25.33

Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2. Thus,

$$q_1+q_2=q_3 \quad \text{...(i)}$$

This relation can also be obtained by a different method. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries. Thus,

$$q_3-q_1-q_2=0$$

or

$$q_3=q_1+q_2$$

Applying second law in loops *BMFAB* and *MDEFM*, we have

$$-\frac{q_1}{2}-\frac{q_3}{6}+10=0$$

or $$q_3+3q_1=60 \qquad \text{...(ii)}$$

and $$\frac{q_2}{4}-20+\frac{q_3}{6}=0$$

or $$3q_2+2q_3=240 \qquad \text{...(iii)}$$

Solving the above three equations, we have

$$q_1=\frac{10}{3}\,\mu\text{C}$$

$$q_2=\frac{140}{3}\,\mu\text{C}$$

and $$q_3=50\,\mu\text{C}$$

Thus, charges on different capacitors are as shown in Fig. 25.34.

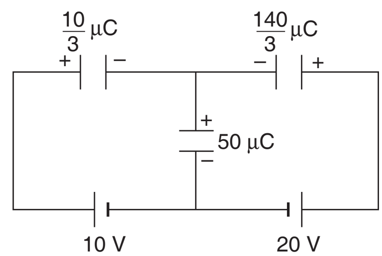


Fig. 25.34

Note *In the problem q_1, q_2 and q_3 are already in microcoulombs.*

25.7 Energy Density (*u*)

The potential energy of a charged conductor or a capacitor is stored in the electric field. The energy per unit volume is called the energy density (u). Energy density in a dielectric medium is given by

$$\boxed{u=\frac{1}{2}\varepsilon_0 KE^2}$$

This relation shows that the energy stored per unit volume depends on E^2. If E is the electric field in a space of volume dV, then the total stored energy in an electrostatic field is given by

$$\boxed{U=\frac{1}{2}\varepsilon_0 K\int E^2\,dV}$$

and if E is uniform throughout the volume (electric field between the plates of a capacitor is almost uniform), then the total stored energy can be given by

$$\boxed{U=u\,(\text{Total volume})=\frac{1}{2}K\varepsilon_0 E^2 V}$$

Example 25.13 *Using the concept of energy density, find the total energy stored in a*

(a) parallel plate capacitor

(b) charged spherical conductor.

Solution (a) Electric field is uniform between the plates of the capacitor. The magnitude of this field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

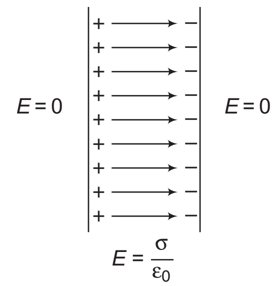


Fig. 25.35

Therefore, the energy density (u) should also be constant.

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{2A^2\varepsilon_0}$$

$\therefore$ Total stored energy,

$$U = (u)\,(\text{total volume})$$

$$= \left(\frac{q^2}{2A^2\varepsilon_0}\right)(A \cdot d) = \frac{q^2}{2\left(\frac{A\varepsilon_0}{d}\right)}$$

$$= \frac{q^2}{2C} \qquad \left(\text{as } C = \frac{A\varepsilon_0}{d}\right)$$

$\therefore$ $$U = \frac{q^2}{2C}$$ **Ans.**

(b) In case of a spherical conductor (of radius R) the excess charge resides on the outer surface of the conductor. The field inside the conductor is zero. It extends from surface to infinity. And since the potential energy is stored in the field only, it will be stored in the region extending from surface to infinity. But as the field is non-uniform, the energy density u is also non-uniform. So, the total energy will be calculated by integration. Electric field at a distance r from the centre is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

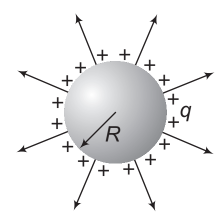


Fig. 25.36

$\therefore$ $$u(r) = \frac{1}{2}\varepsilon_0 E^2$$

$$= \frac{1}{2}\varepsilon_0 \left\{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\right\}^2$$

Energy stored in a volum $dV = (4\pi r^2)\,dr$ is

$$dU = u\,dV$$

$\therefore$ Total energy stored, $U = \int_{r=R}^{r=\infty} dU$

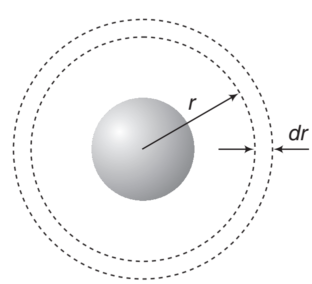


Fig. 25.37

Substituting the values, we get

$$U = \frac{q^2}{2\,(4\pi\varepsilon_0 R)}$$

or $$U = \frac{q^2}{2C}$$ **Ans.**

$$(\text{as } C = 4\pi\varepsilon_0 R)$$

25.8 *C-R* Circuits

Charging of a Capacitor in *C-R* Circuit

To understand the charging of a capacitor in *C-R* circuit, let us first consider the charging of a capacitor without resistance.

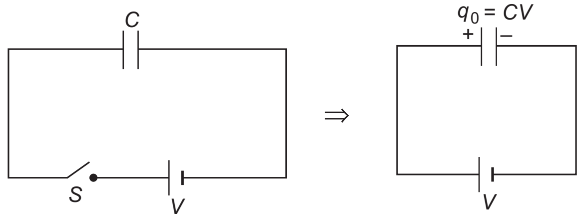


Fig. 25.38

Consider a capacitor connected to a battery of emf V through a switch S. When we close the switch the capacitor gets charged immediately. Charging takes no time. A charge $q_0 = CV$ appears in the capacitor as soon as switch is closed and the q-t graph in this case is a straight line parallel to t-axis as shown in Fig. 25.39

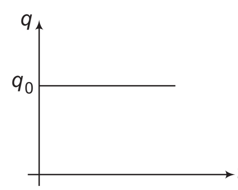


Fig. 25.39

If there is some resistance in the circuit charging takes some time. Because resistance opposes the charging (or current flow in the circuit). Final charge (called steady state charge) is still q_0 but it is acquired after a long period of time. The q-t equation in this case is

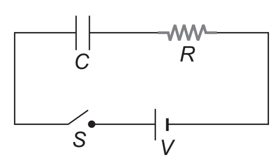


Fig. 25.40

$$\boxed{q = q_0\,(1 - e^{-t/\tau_C})}$$

Here, $q_0 = CV$ and $\tau_C = CR =$ time constant.

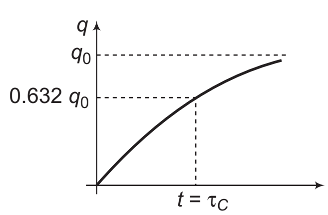


Fig. 25.41

q-t graph is an exponentially increasing graph. The charge q increases exponentially from 0 to q_0.

From the graph and equation, we see that

at $t = 0$, $q = 0$ and at $t = \infty$, $q = q_0$

Definition of τ_C

At $t = \tau_C$, $q = q_0\,(1 - e^{-1}) \approx 0.632\,q_0$

Hence, τ_C can be defined as the time in which 63.2% charging is over. Note that τ_C is the time.

Hence, $$[\tau_C] = [\text{time}]$$

or $$[CR] = [\text{M}^0\text{L}^0\,\text{T}]$$

Proof : Now, let us derive the q-t relation discussed above.

Suppose the switch is closed at time $t = 0$. At some instant of time, let charge in the capacitor is $q\ (< q_0)$ and it is still increasing and hence current is flowing in the circuit.

Applying loop law in $ABEDA$, we get

$$-\frac{q}{C} - iR + V = 0$$

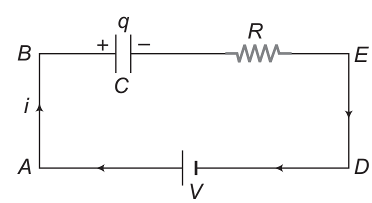


Fig. 25.42

Here, $$i = \frac{dq}{dt}$$

$\therefore$ $$-\frac{q}{C} - \left(\frac{dq}{dt}\right)R + V = 0$$

$\therefore$ $$\frac{dq}{V - \dfrac{q}{C}} = \frac{dt}{R}$$

or $$\int_0^q \frac{dq}{V - \dfrac{q}{C}} = \int_0^t \frac{dt}{R}$$

This gives $$q = CV\,(1 - e^{-\frac{t}{CR}})$$

Substituting $CV = q_0$ and $CR = \tau_C$, we have

$$q = q_0\,(1 - e^{-t/\tau_C}).$$

Charging Current

Current flows in a C-R circuit during charging of a capacitor. Once charging is over or the steady state condition is reached the current becomes zero. The current at any time t can be calculated by differentiating q with respect to t. Hence,

$$i = \frac{dq}{dt} = \frac{d}{dt}\{q_0\,(1 - e^{-t/\tau_C})\}$$

or $$i = \frac{q_0}{\tau_C}\,e^{-t/\tau_C}$$

Substituting $q_0 = CV$ and $\tau_C = CR$, we have

$$i = \frac{V}{R}\,e^{-t/\tau_C}$$

By letting, $\frac{V}{R} = i_0$ $\boxed{i = i_0 \, e^{-t/\tau_C}}$

i.e. current decreases exponentially with time.

The i-t graph is as shown in Fig. 25.43.

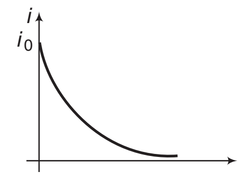


Fig. 25.43

Here, $i_0 = \frac{V}{R}$ is the current at time $t = 0$. This is the current which would had been in the absence of capacitor in the circuit.

Discharging of a Capacitor in *C-R* Circuit

To understand discharging through a C-R circuit again we first consider the discharging without resistance.

Suppose a capacitor has a charge q_0. The positive plate has a charge $+q_0$ and negative plate $-q_0$. It implies that the positive plate has deficiency of electrons and negative plate has excess of electrons. When the switch is closed, the extra electrons on negative plate immediately rush to the positive plate and net charge on both plates becomes zero. So, we can say that discharging takes place immediately.

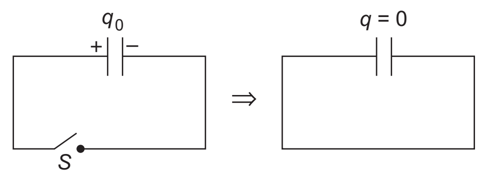


Fig. 25.44

In case of a C-R circuit, discharging also takes time. Final charge on the capacitor is still zero but after sufficiently long period of time. The q-t equation in this case is

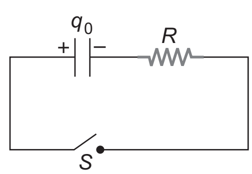


Fig. 25.45

$$\boxed{q = q_0 e^{-t/\tau_C}}$$

Thus, q decreases exponentially from q_0 to zero, as shown in Fig. 25.46.

From the graph and the equation, we see that

At $t = 0$, $q = q_0$

At $t = \infty$, $q = 0$.

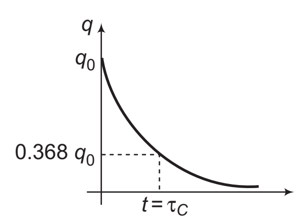


Fig. 25.46

Definition of Time Constant (τ_C)

In case of discharging, definition of τ_C is changed.

At time $t = \tau_C$,

$$q = q_0 e^{-1} = 0.368\ q_0$$

Hence, in this case τ_C can be defined as the time when charge reduces to 36.8% of its maximum value q_0.

Discharging Current

During discharging, current flows in the circuit till q becomes zero. This current can be found by differentiating q with respect to t but with negative sign because charge is decreasing with time. So,

$$i = \left(-\frac{dq}{dt}\right) = -\frac{d}{dt}(q_0 e^{-t/\tau_C})$$

$$= \frac{q_0}{\tau_C} e^{-t/\tau_C}$$

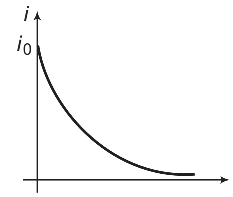


Fig. 25.47

By letting, $$\frac{q_0}{\tau_C} = i_0$$

We have, $$\boxed{i = i_0\, e^{-t/\tau_C}}$$

This is an exponentially decreasing equation. Thus, i-t graph decreases exponentially with time from i_0 to 0. The i-t graph is as shown in Fig. 25.47.

25.9 Methods of Finding Equivalent Resistance and Capacitance

We know that in series,

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

and $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

and in parallel, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

and $$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Sometimes there are circuits in which resistances/capacitors are in mixed grouping. To find R_{eq} or C_{eq} for such circuits few methods are suggested here which will help you in finding R_{eq} or C_{eq}.

Method of Same Potential

Give any arbitrary potentials ($V_1, V_2, \ldots$ etc.) to all terminals of capacitors/resistors. But notice that the points connected directly by a conducting wire will have at the same potential. The capacitors/resistors having the same PD are in parallel. Make a table corresponding to the figure. Now, corresponding to this table a simplified figure can be formed and from this figure C_{eq} and R_{eq} can be calculated.

Example 25.14 *Find equivalent capacitance between points A and B as shown in figure.*

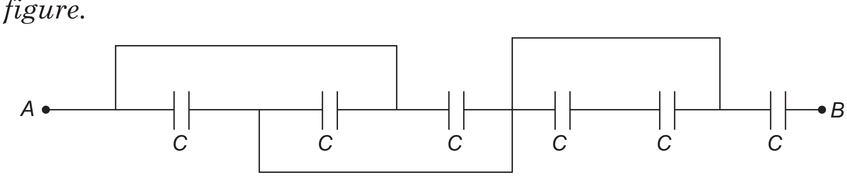


Fig. 25.48

Solution Three capacitors have PD, $V_1 - V_2$. So, they are in parallel. Their equivalent capacitance is $3C$.

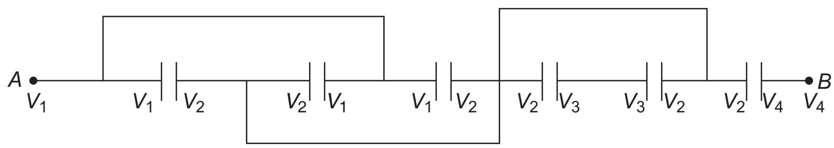


Fig. 25.49

Two capacitors have PD, $V_2 - V_3$. So, their equivalent capacitance is $2C$ and lastly there is one capacitor across which PD is $V_2 - V_4$. So, let us make a table corresponding to this information.

Table 25.1

PD	Capacitance
$V_1 - V_2$	$3C$
$V_2 - V_3$	$2C$
$V_2 - V_4$	C

Now, corresponding to this table, we make a simple figure as shown in Fig. 25.50.

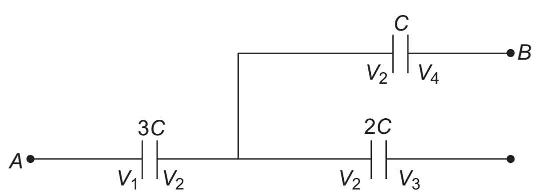


Fig. 25.50

As we have to find the equivalent capacitance between points A and B, across which PD is $V_1 - V_4$. From the simplified figure, we can see that the capacitor of capacitance $2C$ is out of the circuit and points A and B are as shown. Now, $3C$ and C are in series and their equivalent capacitance is

$$C_{eq} = \frac{(3C)(C)}{3C + C} = \frac{3}{4}C \qquad \textbf{Ans.}$$

EXERCISE *Find equivalent capacitance between points A and B.*

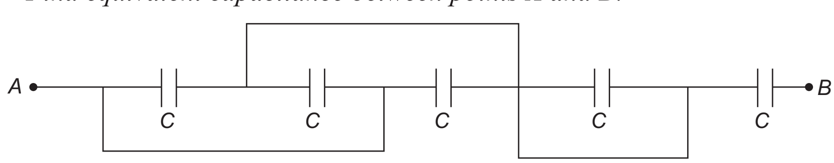


Fig. 25.51

HINT *In case PD across any capacitor comes out to be zero (i.e. the plates are short circuited), then this capacitor will not store charge. So ignore this capacitor.*

Ans. $\frac{3}{4}C$

EXERCISE *Identical metal plates are located in air at equal distance d from one another as shown in figure. The area of each plate is A. Find the capacitance of the system between points P and Q if plates are interconnected as shown.*

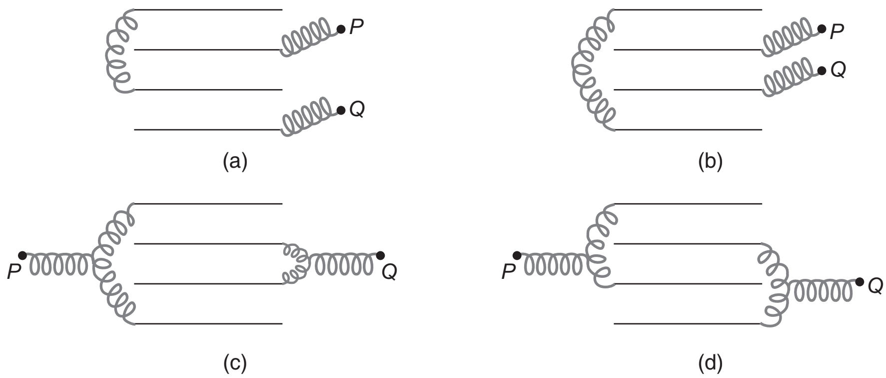


Fig. 25.52

Ans. (a) $\frac{2}{3}\frac{\varepsilon_0 A}{d}$ (b) $\frac{3}{2}\frac{\varepsilon_0 A}{d}$ (c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{3\varepsilon_0 A}{d}$

EXERCISE *Find equivalent resistance between A and B.*

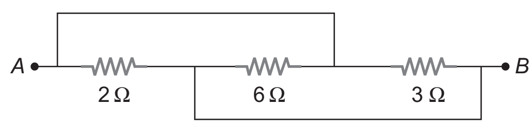


Fig. 25.53

Ans. $1\,\Omega$

EXERCISE *Find equivalent capacitance between points A and B.*

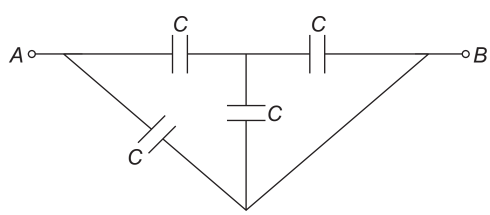


Fig. 25.54

Ans. $\frac{5}{3}C$.

Infinite Series Problems

This circuit consists of an infinite series of identical loops. To find C_{eq} or R_{eq} of such a series first we consider by ourself a value (say x) of C_{eq} or R_{eq}. Then, we break the chain in such a manner that only one loop is left with us and in place of the remaining portion we connect a capacitor or resistor x. Then, we find the C_{eq} or R_{eq} and put it equal to x. With this we get a quadratic equation in x. By solving this equation, we can find the desired value of x.

Example 25.15 *An infinite ladder network is constructed with 1 Ω and 2 Ω resistors as shown. Find the equivalent resistance between points A and B.*

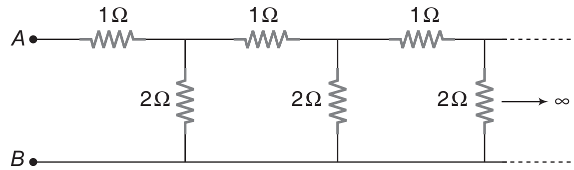


Fig. 25.55

Solution Let the equivalent resistance between A and B is x. We may consider the given circuit as shown in Fig. 25.56.

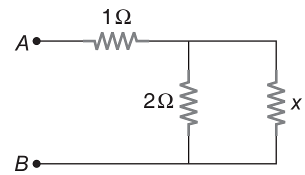


Fig. 25.56

In this diagram,

$$R_{AB} = \frac{2x}{2+x} + 1 \quad \text{or} \quad x = \frac{2x}{2+x} + 1 \qquad (\text{as } R_{AB} = x)$$

or $\quad x(2+x) = 2x + 2 + x \quad$ or $\quad x^2 - x - 2 = 0$

$$x = \frac{1 \pm \sqrt{1+8}}{2} = -1\,\Omega \text{ and } 2\,\Omega$$

Ignoring the negative value, we have $\quad R_{AB} = x = 2\,\Omega$ **Ans.**

Note *Care should be taken while breaking the chain. It should be broken from those points from where the broken chain resembles with the original chain.*

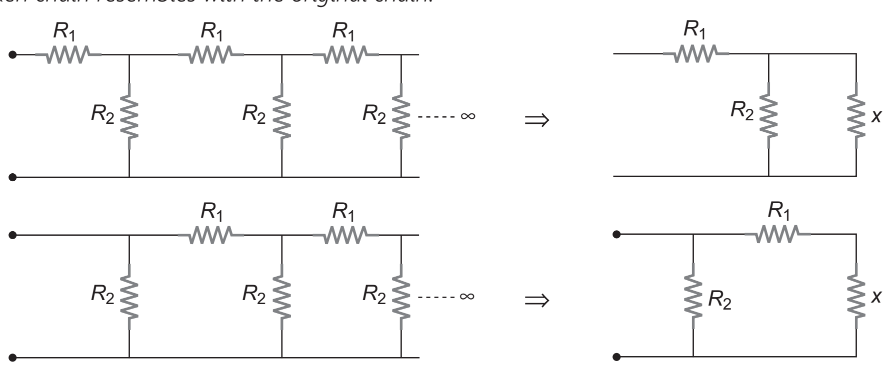


Fig. 25.57

EXERCISE *Find equivalent resistance between A and B.*

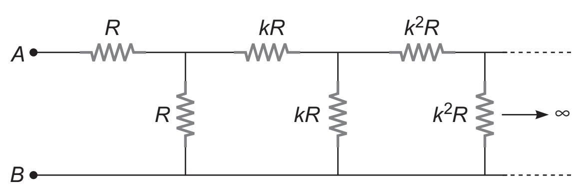


Fig. 25.58

HINT Let $R_{AB} = x$, then the resistance of the broken chain will be kx.

Ans. $R\,[(2k-1) + \sqrt{4k^2+1}\,]/2k$

Method of Symmetry

Symmetry of a circuit can be checked in the following four manners :

1. Points which are symmetrically located about the starting and last points are at same potentials. So, the resistances/capacitors between these points can be ignored. The following example will illustrate the theory.

Example 25.16 *Twelve resistors each of resistance r are connected together so that each lies along the edge of the cube as shown in figure. Find the equivalent resistance between*

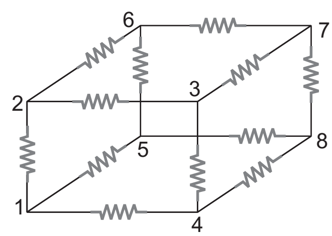


Fig. 25.59

(a) 1 *and* 4 *(b)* 1 *and* 3

Solution (a) **Between 1 and 4 :** Points 2 and 5 are symmetrically located w.r.t. points 1 and 4. So, they are at same potentials.

Similarly, points 3 and 8 are also symmetrically located w.r.t. points 1 and 4. So, they are again at same potential.

Now, we have 12 resistors each of resistance r connected across 1 and 2, 2 and 3,…, etc. So, redrawing them with the assumption that 2 and 5 are at same potential and 3 and 8 are at same potential. The new figure is as shown in Fig.25.60.

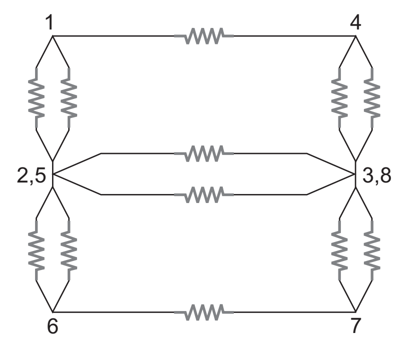


Fig. 25.60

Now, we had to find the equivalent resistance between 1 and 4. We can now simplify the circuit as

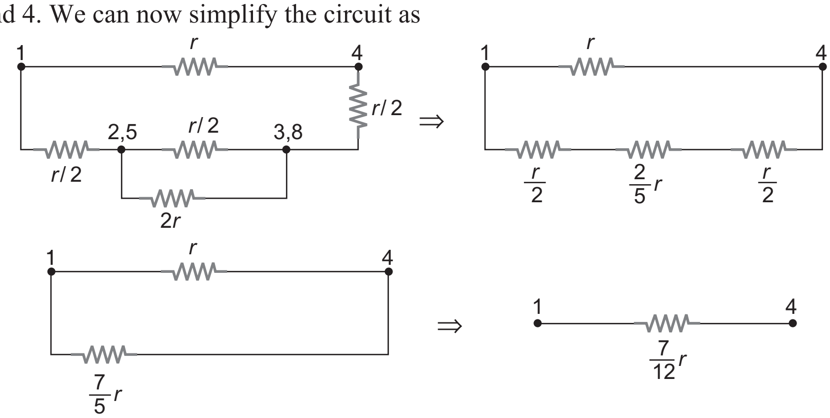


Fig. 25.61

Thus, the equivalent resistance between points 1 and 4 is $\frac{7}{12}r$. **Ans.**

(b) **Between 1 and 3 :** Points 6 and 8 are symmetrically located w.r.t. points 1 and 3. Similarly, points 2 and 4 are located symmetrically w.r.t. points 1 and 3. So, points 6 and 8 are at same potential. Similarly, 2 and 4 are at same potentials. Redrawing the simple circuit, we have Fig. 25.62.

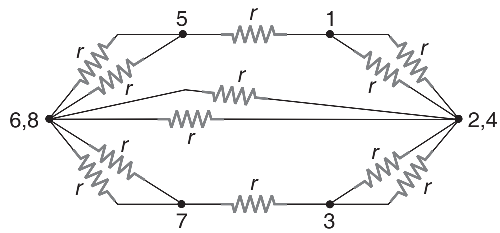


Fig. 25.62

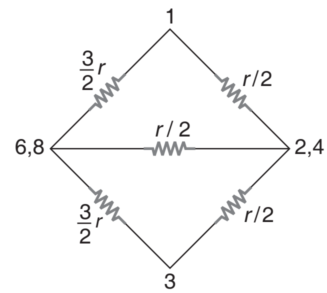


Fig. 25.63

Between 1 and 3, a balanced Wheatstone bridge is formed as shown in Fig. 25.63.

So, the resistance between 2 and 6 and between 4 and 8 can be removed.

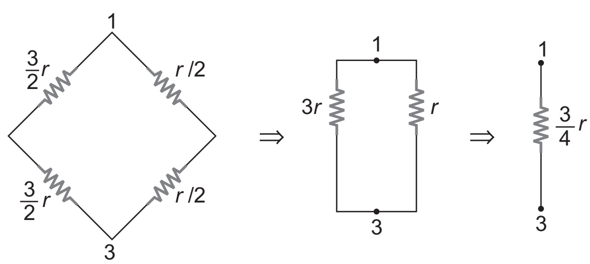


Fig. 25.64

Thus, the equivalent resistance between 1 and 3 is $\frac{3}{4}r$. **Ans.**

EXERCISE *Fourteen identical resistors each of resistance r are connected as shown. Calculate equivalent resistance between A and B.*

Ans. $1.2r$

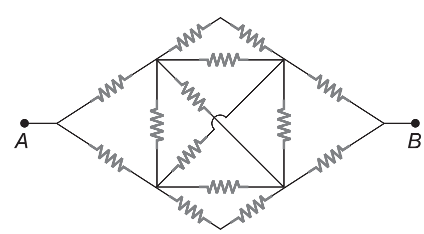


Fig. 25.65

EXERCISE *Eight identical resistances r each are connected along edges of a pyramid having square base ABCD as shown. Calculate equivalent resistance between A and O.*

Ans. $\frac{7r}{15}$

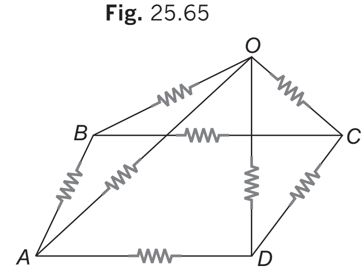


Fig. 25.66

2. If points A and B are connected to a battery and AB is a line of symmetry, then all points lying on perpendiculars drawn to AB are at the same potential. For example,

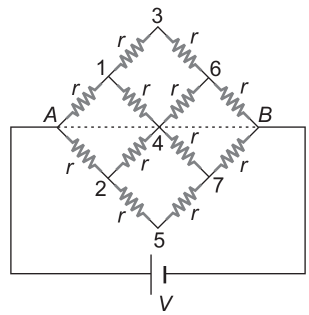


Fig. 25.67

In Fig. 25.67, points (1, 2), (3, 4, 5) and (6, 7) are at same potential. So, we can join these points and draw a simple circuit as shown in Fig. 25.68.

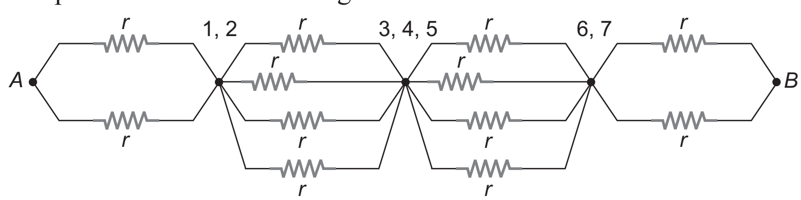


Fig. 25.68

Now, the equivalent of this series combination is

$$R_{\text{eq}} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$

EXERCISE *Solve the same problem by connection removal method (will be discussed later).*

3. Even if AB is not a line of symmetry but its perpendicular bisector is, then all the points on this perpendicular bisector are at the same potential.

For example,

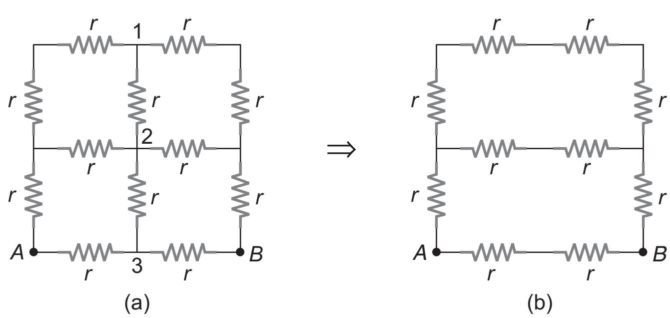


Fig. 25.69

In Fig. (a), AB is not a line of symmetry but, 1, 2 and 3 are line of symmetry. Hence, they are at same potential (if A and B are connected to a battery). This makes the resistors between 1 & 2 and 2 and 3 redundant because no current flows through them. So, the resistance between them can be removed [as shown in Fig. (b)]. The equivalent resistance between A and B can now be easily determined as $\frac{5r}{4}$.

4. Each wire in the cube has a resistance r. We are interested in calculating the equivalent resistance between A and B.

   This is a three-dimensional case and in place of a line of symmetry involving points A and B we locate a plane of symmetry involving A and B.

   Such a plane is the plane $ABce$ and for this plane points d and f and g and h have the same potential.

   The equivalent resistance between A and B can now be easily worked out (Using Wheatstone's bridge principle) as

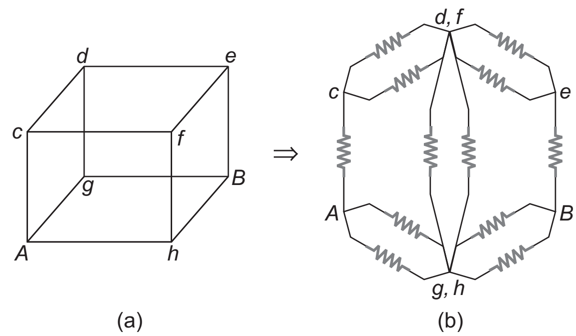


Fig. 25.70

$$R_{eq} = \frac{3r}{4}$$

Connection Removal Method

This method is useful when the circuit diagram is symmetric except for the fact that the input and output are reversed. That is the flow of current is a mirror image between input and output above a particular axis. In such cases, some junctions are unnecessarily made. Even if we remove that junction there is no difference in the remaining circuit or current distribution. But after removing the junction, the problem becomes very simple. The following example illustrates the theory.

Example 25.17 *Find the equivalent resistance between points A and B.*

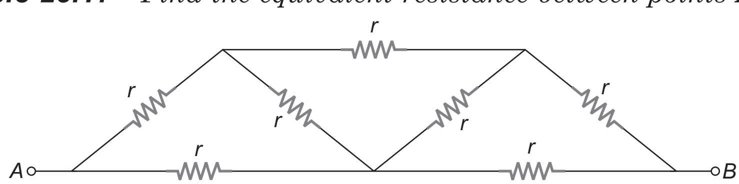


Fig. 25.71

Solution

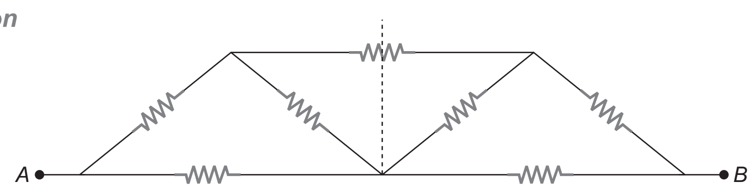


Fig. 25.72

Input and output circuits are mirror images of each other about the dotted line as shown. So, if a current i enters from A and leaves from B, it will distribute as shown below.

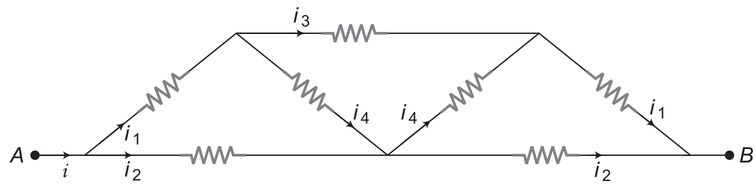


Fig. 25.73

Now, we can see in figure that the junction where i_2 and i_4 are meeting can be removed easily and then the circuit becomes simple.

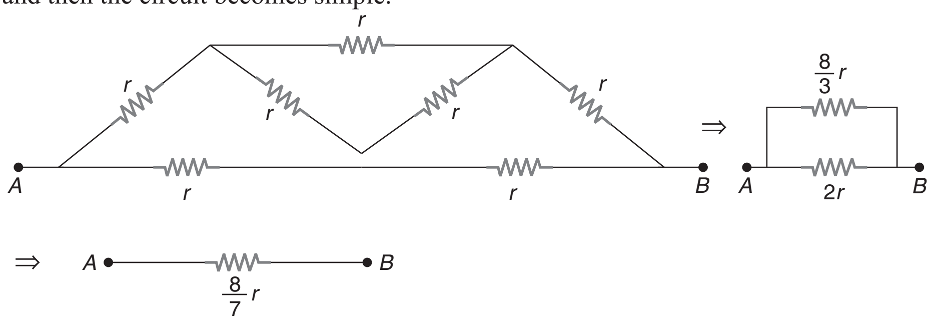


Fig. 25.74

Hence, the equivalent resistance between A and B is $\frac{8}{7}r$. **Ans.**

EXERCISE *Eight identical resistances r each are connected as shown. Find equivalent resistance between A and D.*

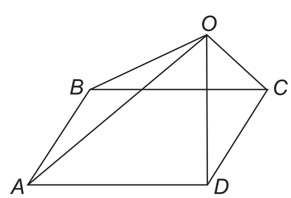


Fig. 25.75

Ans. $\dfrac{8r}{15}$

EXERCISE *Twelve resistors each of resistance r are connected as shown. Find equivalent resistance between A and B.*

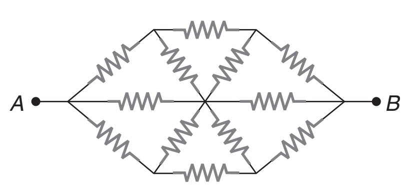


Fig. 25.76

Ans. $(4/5)\,r$

EXERCISE *Find equivalent resistance between A and B.*

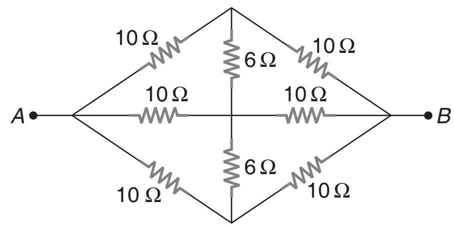


Fig. 25.77

Ans. $\frac{20}{3}\ \Omega$.

Wheatstone Bridge Circuits

Wheatstone bridge in case of resistors has already been discussed in the chapter of current electricity.

For capacitor, theory is same.

If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, bridge is said to be balanced and in that case

$$V_E = V_D \quad \text{or} \quad V_E - V_D \quad \text{or} \quad V_{ED} = 0$$

i.e. no charge is stored in C_5. Hence, it can be removed from the circuit.

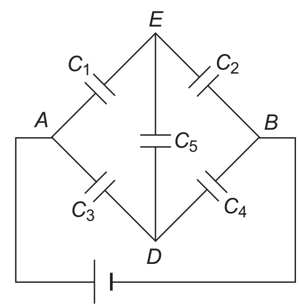


Fig. 25.78

EXERCISE In the circuit shown in figure, prove that $V_{AB} = 0$ if $\frac{R_1}{R_2} = \frac{C_2}{C_1}$.

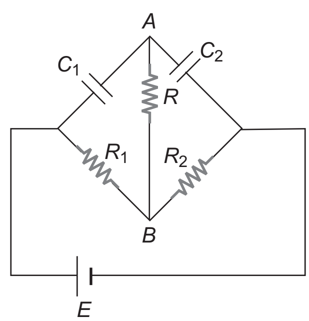


Fig. 25.79

By Distributing Current/Charge

Sometimes none of the above five methods is applicable. So, this one is the last and final method which can be applied everywhere. Of course this method is a little bit lengthy but is applicable everywhere, under all conditions. In this method, we assume a main current/charge, i or q. Distribute it in different resistors/capacitors as $i_1, i_2 \ldots$ (or $q_1, q_2, \ldots$, etc.). Using Kirchhoff's laws, we find $i_1, i_2, \ldots$ etc., (or $q_1, q_2, \ldots$, etc.) in terms of i (or q). Then, find the potential difference between starting and end points through any path and equate it with iR_{net} or q/C_{net}. By doing so, we can calculate R_{net} or C_{net}.

The following example is in support of the theory.

Example 25.18 *Find the equivalent capacitance between A and B.*

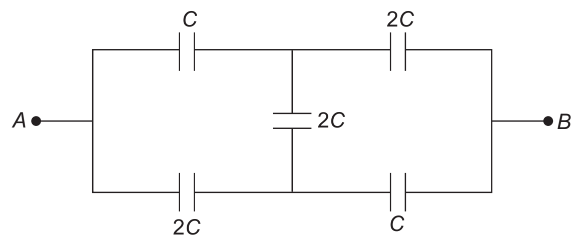


Fig. 25.80

Solution The given circuit forms a Wheatstone bridge. But the bridge is not balanced. Let us suppose point A is connected to the positive terminal of a battery and B to the negative terminal of the same battery; so that a total charge q is stored in the capacitors. Just by seeing input and output symmetry, we can say that charges will be distributed as shown below.

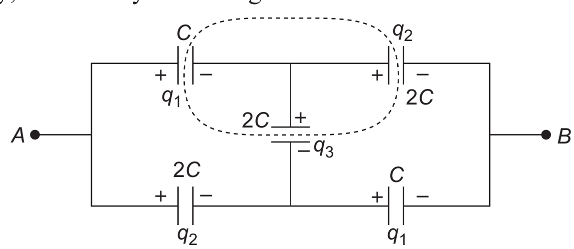


Fig. 25.81

$$q_1 + q_2 = q \quad \ldots\text{(i)}$$

Applying second law, we have

$$-\frac{q_1}{C} - \frac{q_3}{2C} + \frac{q_2}{2C} = 0$$

or

$$q_2 - q_3 - 2q_1 = 0 \quad \ldots\text{(ii)}$$

Plates inside the dotted line form an isolated system. Hence,

$$q_2 + q_3 - q_1 = 0 \quad \ldots\text{(iii)}$$

Solving these three equations, we have

$$q_1 = \frac{2}{5}q, \quad q_2 = \frac{3}{5}q \quad \text{and} \quad q_3 = -\frac{q}{5}$$

Now, let C_{eq} be the equivalent capacitance between A and B. Then,

$$V_A - V_B = \frac{q}{C_{eq}} = \frac{q_1}{C} + \frac{q_2}{2C}$$

$$\therefore \quad \frac{q}{C_{eq}} = \frac{2q}{5C} + \frac{3q}{10C} = \frac{7q}{10C}$$

$$\therefore \quad C_{eq} = \frac{10}{7}C$$ **Ans.**

Final Touch Points

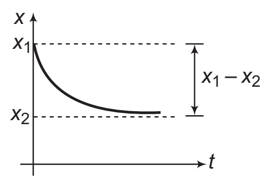
1. Now, onwards we will come across the following integration very frequently. So, remember the result as such.

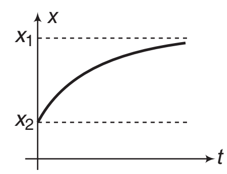
   If $$\int_0^x \frac{dx}{a - bx} = \int_0^t c\,dt, \quad \text{then} \quad x = \frac{a}{b}(1 - e^{-bct})$$

   and if $$\int_{x_0}^x \frac{dx}{a - bx} = \int_0^t c\,dt, \quad \text{then} \quad x = \frac{a}{b} - \left(\frac{a}{b} - x_0\right) e^{-bct}$$

   Here, *a, b* and *c* are constants.

2. Sometimes a physical quantity x decreases from x_1 to x_2, exponentially, then x-t equation is like



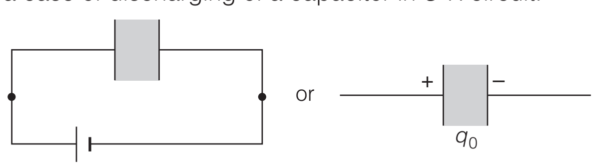


$$x = x_2 + (x_1 - x_2)\,e^{-Kt}$$

   Here, K is a constant.

   Similarly, if x increases from x_2 to x_1 exponentially, then x-t equation is
$$x = x_2 + (x_1 - x_2)(1 - e^{-Kt})$$

3. **Leakage Current Through a Capacitor** The space between the capacitor's plates is filled with a dielectric and we assume that no current flows through it when the capacitor is connected to a battery as in figure (a) or if the capacitor is charged, the charge on its plates remains forever. But every insulator has some conductivity. On account of which some current flows through the capacitor if connected to a battery. This small current is known as the leakage current. Similarly, when it is charged, the charge does not remain as it is for a long period of time. But it starts discharging. Or we can say it becomes a case of discharging of a capacitor in *C-R* circuit.

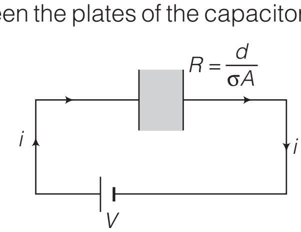


(a) (b)

   In both the cases, we will first find the resistance of the dielectric.
$$R = \frac{l}{\sigma A} \qquad (\sigma = \text{specific conductance})$$

   Here, $l = d$ (the distance between the plates of the capacitor)

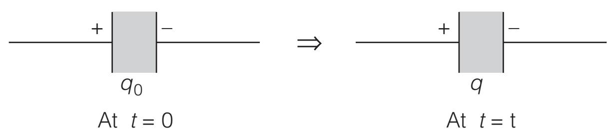
   $\therefore$ $$\boxed{R = \frac{d}{\sigma A}}$$



   Thus, the leakage current in the circuit shown in the figure is
$$i = \frac{V}{R}$$

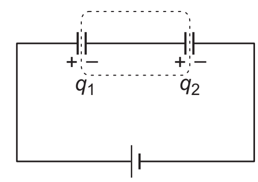
Similarly, if the capacitor is given a charge q_0 at time $t = 0$, then after time t, q charge will remain on it, where

$$\boxed{q = q_0 e^{-t/\tau_C}} \qquad \text{(Discharging of a capacitor)}$$



Here, $$\tau_C = CR = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{d}{\sigma A}\right) \quad \text{or} \quad \boxed{\tau_C = \frac{K\varepsilon_0}{\sigma}}$$

4. If capacitors are in series, then charges on them are equal, provided they are initially uncharged. This can be proved by the following illustration :



Let us suppose that charges on two capacitors are q_1 and q_2. The two plates encircled by dotted lines form an isolated system. So, net charge on them will remain constant.

$$\Sigma q_f = \Sigma q_i$$

If initially they are uncharged, then

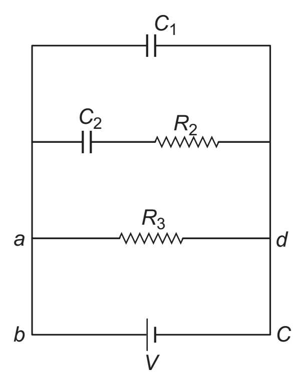
$$\Sigma q_i = 0$$

$$\Sigma q_f \text{ is also zero}$$

or $$-q_1 + q_2 = 0 \quad \text{or} \quad q_1 = q_2$$

So, this proves that charges are equal if initially they are uncharged.

5. Independent parallel circuit



Three circuits shown in figure are independently connected in parallel with the battery. Potential difference across each of the circuit is V. By this potential difference, capacitor C_1 is immediately charged. Capacitor C_2 is exponentially charged and current grows immediately in R_3. Thus,

$$q_{C_1} = C_1 V \qquad \text{(immediately)}$$

$$q_{C_2} = C_2 V\,(1 - e^{-\frac{t}{C_2 R_2}})$$

$$I_{R_3} = \frac{V}{R_3} \qquad \text{(immediately)}$$

Note *If any resistance or capacitance is connected between abcd, then it no longer remains an independent parallel circuit.*

Solved Examples

TYPED PROBLEMS

Type 1. *In a complex capacitor circuit method of finding values of q and V across different capacitors if values across one capacitor are known*

Concept

In series, q is same and V distributes in inverse ratio of capacity.

As, $$V = \frac{q}{C} \quad \Rightarrow \quad V \propto \frac{1}{C} \qquad (q \text{ is same})$$

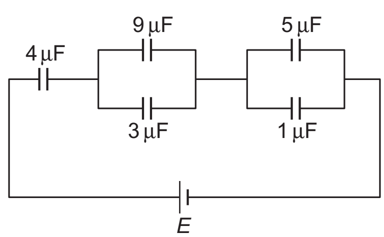
If capacitance is double, then V will be half.

In parallel, V is same and q distributes in direct ratio of capacity.

As, $$q = CV \quad \Rightarrow \quad q \propto C \qquad (V \text{ is same})$$

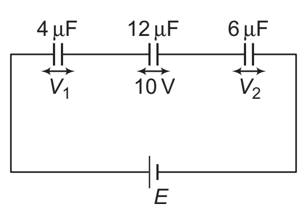
If C is double, then q is also double.

Example 1



In the circuit shown in figure potential difference across $3\,\mu F$ is $10\,V$. Find potential difference and charge stored in different capacitors. Also find emf of the battery E.

Solution The given circuit can be simplified as under



V_1: $4\,\mu$F is $\frac{1}{3}$rd of $12\,\mu$F. Therefore, V_1 is thrice of 10 V or 30 V.

V_2: $6\,\mu$F is half of $12\,\mu$F. Therefore, V_2 is twice of 10 V or 20 V.

$$E = V_1 + 10 + V_2$$
$$= 30 + 10 + 20$$
$$= 60\text{ V}$$

Ans.

Potential difference and charge on different capacitors in tabular form are given below.

Table 25.2

Capacitance	Potential difference	Charge $q = CV$
$4\,\mu F$	30 V	$120\,\mu C$
$9\,\mu F$	10 V	$90\,\mu C$
$3\,\mu F$	10 V	$30\,\mu C$
$5\,\mu F$	20 V	$100\,\mu C$
$1\,\mu F$	20 V	$20\,\mu C$

Type 2. *Configuration of capacitor is changed and change in five quantities q, C, V, U and E is asked*

Concept

Some problems are asked when a capacitor is charged through a battery and then the configuration of capacitor is changed :

(i) either by inserting a dielectric slab or removing the slab (if it already exists) or

(ii) by changing the distance between the plates of capacitor or

(iii) by both.

The questions will be based on the change in electric field, potential, etc. In such problems, two cases are possible.

Case 1 **When battery is removed after charging**

If the battery is removed after charging, then the charge stored in the capacitor remains constant.

$$q = \text{constant}$$

First of all, find the change in capacitance and according to the formula find the change in other quantities.

Example 2 *An air capacitor is first charged through a battery. The charging battery is then removed and a dielectric slab of dielectric constant $K = 4$ is inserted between the plates. Simultaneously, the distance between the plates is reduced to half, then find change in C, E, V and U.*

Solution Change in capacitance, $C = \dfrac{K\varepsilon_0 A}{d} \propto \dfrac{K}{d}$

$\therefore$ Capacitance will become 8 times $\left(K = 4,\ d' = \dfrac{d}{2}\right)$

Change in electric field

$$\therefore \qquad E = E_0/K$$

$$\text{or} \qquad E \propto \frac{1}{K} \qquad \text{(if } q = \text{constant)}$$

or the electric field will become $\dfrac{1}{4}$ times its initial value.

Change in potential difference,

$$V = \frac{q}{C}$$

or $$V \propto \frac{1}{C} \qquad \text{(if } q = \text{constant)}$$

Therefore, potential difference becomes $\frac{1}{8}$ times of its initial value.

Alternate method

$$V = Ed$$

Electric field has become $\frac{1}{4}$ times its initial value and d is reduced to half. Hence, V becomes $\frac{1}{8}$ times.

Change in stored potential energy,

$$U = \frac{1}{2}\frac{q^2}{C}$$

or $$U \propto \frac{1}{C} \qquad \text{(if } q = \text{constant)}$$

Capacitance has become 8 times. Therefore, the stored potential energy U will become $\frac{1}{8}$ times.

Case 2 **When battery remains connected**

If the battery remains connected, the potential difference V becomes constant. So, in the above example, capacitance will become 8 times.

The charge stored ($q = CV$ *or* $q \propto C$) will also increase to 8 times. The electric field $\left(E = \frac{V}{d} \text{ or } E \propto \frac{1}{d}\right)$ becomes twice and the stored PE $\left(U = \frac{1}{2}CV^2 \text{ or } U \propto C\right)$ is 8 times.

Type 3. *To find self energy of a system of charges*

Concept

The self energy of a system of charges is

$$\boxed{U_s = \int_0^q V dq}$$

This comes out to be equal to $\frac{q^2}{2C}$ in case of a capacitor or conductor.

A point charge does not have any self energy.

Example 3 *Find the electric potential energy of a uniformly charged sphere.*

Solution Consider a uniformly charged sphere of radius R having a total charge q_0. The volume charge density is

$$\rho = \frac{q_0}{\frac{4}{3}\pi R^3} = \frac{3q_0}{4\pi R^3}$$

When the radius of the sphere is r, the charge contained in it is

$$q = \left(\frac{4}{3}\pi r^3\right)\rho = \left(\frac{q_0}{R^3}\right) r^3$$

The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{q_0}{4\pi\varepsilon_0 R^3} r^2$$

The charge needed to increase the radius from r to $r + dr$ is

$$dq = (4\pi r^2)\, dr\, \rho$$
$$= \frac{3q_0}{R^3} \cdot r^2\, dr$$

∴ The self energy of the sphere is

$$U_s = \int_0^R V\, dq$$
$$= \int_0^R \left(\frac{q_0}{4\pi\varepsilon_0 R^3} r^2\right)\left(\frac{3q_0}{R^3} \cdot r^2\, dr\right)$$
$$= \frac{3q_0^2}{20\,\pi\varepsilon_0 R}$$ **Ans.**

Example 4 *Find the electric potential energy of a uniformly charged, thin spherical shell.*

Solution Consider a uniformly charged thin spherical shell of radius R having a total charge q_0. Suppose at some instant a charge q is placed on the shell. The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

∴ The self energy of the shell is

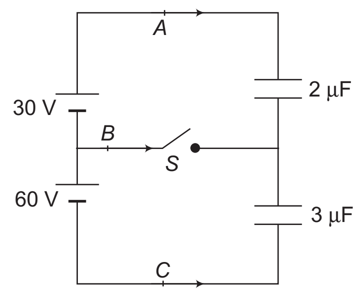
$$U_s = \int_0^{q_0} V\, dq$$
$$= \int_0^{q_0} \left(\frac{q}{4\pi\varepsilon_0 R}\right) dq$$
$$= \frac{q_0^2}{8\pi\varepsilon_0 R}$$ **Ans.**

Type 4. *Based on flow of charge when position of a switch is changed*

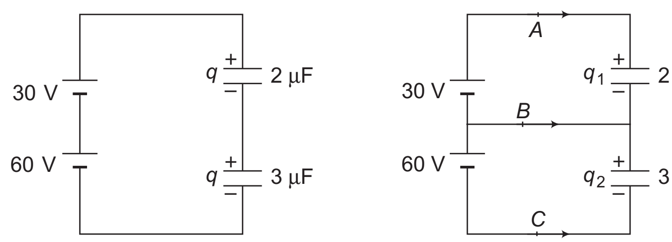
Concept

From the flow of charge we mean that when a switch in a circuit is either closed or opened or it is shifted from one position to the other, then how much charge will flow through certain points of the circuit. Such problems can be solved by finding charges on different capacitors at initial and final positions and then by the difference we can find the charge flowing through a certain point. The following example will illustrate the theory.

Example 5 *What charges will flow through A, B and C in the directions shown in the figure when switch S is closed?*



Solution Let us draw two figures and find the charge on both the capacitors before closing the switch and after closing the switch.



(a) (b)

Refer Fig. (a), when switch is open Both capacitors are in series. Hence, their equivalent capacitance is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2)(3)}{2+3} = \frac{6}{5}\,\mu\text{F}$$

Therefore, charge on both capacitors will be same. Hence, using $q = CV$, we get

$$q = (30 + 60)\left(\frac{6}{5}\right)\mu\text{C} = 108\,\mu\text{C}$$

Refer Fig. (b), when switch is closed Let q_1 and q_2 be the charges (in μC) on two capacitors. Then, applying second law in upper and lower loops, we have

$$30 - \frac{q_1}{2} = 0 \qquad \text{or} \qquad q_1 = 60\,\mu\text{C}$$

$$60 - \frac{q_2}{3} = 0 \qquad \text{or} \qquad q_2 = 180\,\mu\text{C}$$

Charges q_1 and q_2 can be calculated alternatively by seeing that upper plate of 2 μF capacitor is connected with positive terminal of 30 V battery. Therefore, they are at the same potential. Similarly, the lower plate of this capacitor is at the same potential as that of the negative terminal of 30 V battery. So, we can say that PD across 2 μF capacitor is also 30 V.

$$\therefore \qquad q_1 = (C)\,(\text{PD}) = (2)\,(30)\,\mu\text{C} = 60\,\mu\text{C}$$

Similarly, PD across 3 μF capacitor is same as that between 60 V battery. Hence,

$$q_2 = (3)\,(60)\,\mu\text{C} = 180\,\mu\text{C}$$

Now, let q_A charge flows from A in the direction shown. This charge goes to the upper plate of $2\,\mu F$ capacitor. Initially, it had a charge $+q$ and final charge on it is $+q_1$. Hence,

$$q_1 = q + q_A$$

or

$$q_A = q_1 - q = 60 - 108$$

$$= -48\,\mu C \qquad \textbf{Ans.}$$

Similarly, charge q_B goes to the upper plate of $3\,\mu F$ capacitor and lower plate of $2\,\mu F$ capacitor. Initially, both the plates had a charge $+q-q$ or zero. And finally they have a charge $(q_2 - q_1)$. Hence,

$$(q_2 - q_1) = q_B + 0$$

$\therefore$

$$q_B = q_2 - q_1 = 180 - 60$$

$$= 120\,\mu C \qquad \textbf{Ans.}$$

Charge q_C goes to the lower plate of $3\,\mu F$ capacitor. Initially, it had a charge $-q$ and finally $-q_2$. Hence,

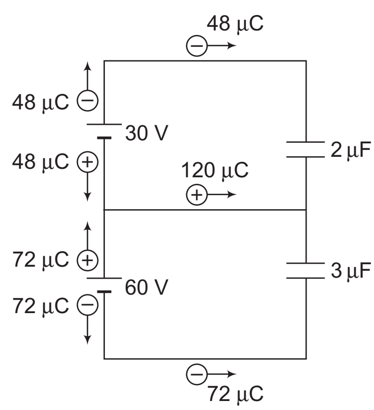
$$-q_2 = (-q) + q_C$$

$\therefore$

$$q_C = q - q_2 = 108 - 180$$

$$= -72\,\mu C \qquad \textbf{Ans.}$$

So, the charges will flow as shown below

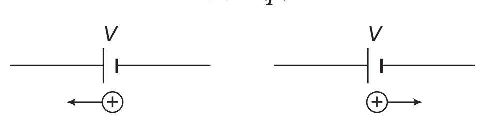


Type 5. *Based on heat generation or loss of energy during shifting of switch*

Concept

By heat generation (or loss of energy), we mean that when a switch is shifted from one position to the other, what amount of heat will be generated (or loss will be there) in the circuit. Such problems can be solved by simple energy conservation principle. For this, remember that when a charge $+q$ flows from negative terminal to the positive terminal inside a battery of emf V is supplied an energy,

$$E = qV$$



Energy supplied = qV Energy consumed = qV

and if opposite is the case, i.e. charge + q flows in opposite direction, then it consumes energy by the same amount.

Now, from energy conservation principle we can find the heat generated (or loss of energy) in the circuit in shifting the switch.

Heat generated or loss of energy = energy supplied by the battery/batteries − energy consumed by the battery/batteries + $\Sigma U_i - \Sigma U_f$

Here, ΣU_i = energy stored in all the capacitors initially and

ΣU_f = energy stored in all the capacitors finally

Example 6 *Find loss of energy in example 5.*

Solution In the above example, energy is supplied by 60 V battery and consumed by 30 V battery. Using $E = qV$, we have

$$\text{Energy supplied} = (72 \times 10^{-6})\,(60) = 4.32 \times 10^{-3}\ \text{J}$$

$$\text{Energy consumed} = (48 \times 10^{-6})\,(30) = 1.44 \times 10^{-3}\ \text{J}$$

$$\Sigma U_i = \frac{1}{2} \times \frac{6}{5} \times 10^{-6} \times (90)^2 = 4.86 \times 10^{-3}\ \text{J}$$

and

$$\Sigma U_f = \frac{1}{2} \times 2 \times 10^{-6} \times (30)^2 + \frac{1}{2} \times 3 \times 10^{-6} \times (60)^2 = 6.3 \times 10^{-3}\ \text{J}$$

$\therefore$

$$\text{Loss of energy} = (4.32 - 1.44 + 4.86 - 6.3) \times 10^{-3}\ \text{J} = 1.44 \times 10^{-3}\ \text{J}$$

Ans.

Example 7 *Prove that in charging a capacitor half of the energy supplied by the battery is stored in the capacitor and remaining half is lost during charging.*

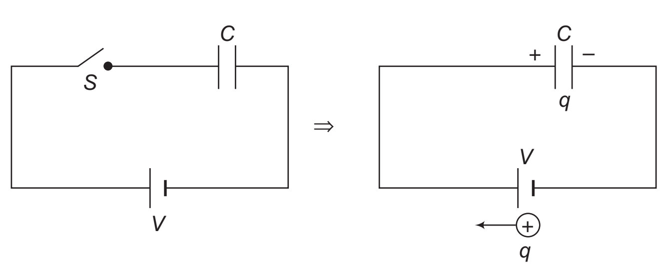
Solution When switch S is closed, $q = CV$ charge is stored in the capacitor.

Charge transferred from the battery is also q.

Hence,

$$\text{energy supplied by the battery} = qV = (CV)\,(V) = CV^2.$$

Half of its energy, i.e. $\frac{1}{2}CV^2$ is stored in the capacitor and the remaining 50 % or $\frac{1}{2}CV^2$ is lost.



Type 6. *Two or more than two capacitors are charged from different batteries and then connected in parallel*

Concept

In parallel, they come to a common potential given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

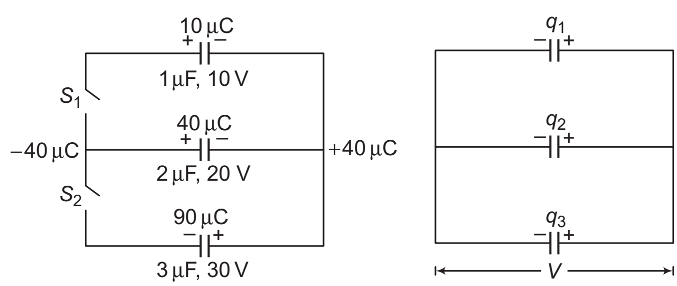
Moreover, total charge on them distributes in direct ratio of their capacity or we can also find the final charges on the capacitors by using the equation, $q = CV$

Example 8 *Three capacitors of capacities* $1\,\mu F$, $2\,\mu F$ *and* $3\,\mu F$ *are charged by* $10\,V$, $20\,V$ *and* $30\,V$ *respectively. Now, positive plates of first two capacitors are connected with the negative plate of third capacitor on one side and negative plates of first two capacitors are connected with positive plate of third capacitor on the other side. Find*

(a) common potential V

(b) final charges on different capacitors

Solution



Total charge on all three capacitors $= 40\,\mu C$

Total capacity $= (1 + 2 + 3)\,\mu F = 6\,\mu F$

(a) Common potential,

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{40\,\mu C}{6\,\mu F} = \frac{20}{3}\text{ volt}$$

(b)

$$q_1 = C_1 V = (1\,\mu F)\left(\frac{20}{3}\right) = \frac{20}{3}\,\mu C$$

$$q_2 = C_2 V = (2\,\mu F)\left(\frac{20}{3}\right) = \frac{40}{3}\,\mu C$$

$$q_3 = C_3 V = (3\,\mu F)\left(\frac{20}{3}\right) = 20\,\mu C$$

Type 7. *To find final charges on different capacitors when position of switch is changed (opened, closed or shifted from one position to other position)*

Concept

(i) To find current in a C-R circuit at any time t, a capacitor may be assumed a battery of emf E or $V = \frac{q}{C}$.

(ii) Difference between a normal battery and a capacitor battery is, emf of a normal battery remains constant while emf of a capacitor battery keeps on changing with q.

(iii) Before changing the position of switch, every loop of the circuit may not be balanced by Kirchhoff's second equation of potential. So, in a single loop problem rotate a charge q, either clockwise or anti-clockwise. From this charge q, some of the charges on capacitors may increase and others may decrease. To check this, always concentrate on positive plate of each capacitors. If positive charge comes towards this plate, then charge on this capacitor will increase.

(iv) With these charges, apply Kirchhoff's loop equation and find the final charges on them.

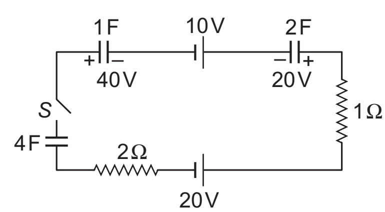
(v) In this redistribution of charges, there is some loss of energy as discussed in type 5.

(vi) If there are only capacitors in the circuit, then redistribution of charges is immediate and if there are resistors in the circuit, then redistribution is exponential.

(vii) If there are only capacitors, then loss is in the form of electromagnetic waves and if there are resistors in the circuit, then loss is in the form of heat.

Further, this loss is proportional to R if resistors are in series ($H = i^2Rt$ or $H \propto R$ as i is same in series) and this loss is inversely proportional to R if resistors are in parallel ($H = \frac{V^2t}{R}$ or $H \propto \frac{1}{R}$ as V is same in parallel).

Example 9 *In the circuit shown in figure, switch S is closed at time $t = 0$. Find*



(a) Initial current at $t = 0$ and final current at $t = \infty$ in the loop.

(b) Total charge q flown from the switch.

(c) Final charges on capacitors in steady state at time $t = \infty$.

(d) Loss of energy during redistribution of charges.

(e) Individual loss across $1\,\Omega$ and $2\,\Omega$ resistance.

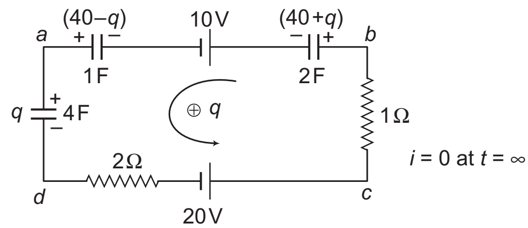
Solution (a) **At $t = 0$** Three capacitors may be assumed like batteries of emf 40 V, 20 V and 0 V.

$$\therefore \qquad i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{40 + 20 - 10 - 20}{1 + 2}$$

$$= 10\text{ A} \qquad \text{(anti-clockwise)}$$

At $t = \infty$ When charge redistribution is complete and loop is balanced by Kirchhoff's second equation of potential, current in the loop becomes zero, as insulator is filled between the capacitors.

(b) Redistribution current was anti-clockwise. So, we can assume that $+q$ charge rotates anti-clockwise in the loop (between time $t = 0$ and $t = \infty$). After this rotation of charges, final charges on different capacitors are as shown below.



Applying loop equation in loop *abcda*,

$$-\frac{(40-q)}{1} + 10 + \frac{(40+q)}{2} - 20 + \frac{q}{4} = 0$$

Solving this equation, we get $\qquad q = \frac{120}{7}\text{ C}$ **Ans.**

(c) **Final charges**

$$q_{1\text{F}} = 40 - q = 40 - \frac{120}{7} = \frac{160}{7}\text{ C}$$

$$q_{2\text{F}} = 40 + q = 40 + \frac{120}{7} = \frac{400}{7}\text{ C}$$

$$q_{4\text{F}} = q = \frac{120}{7}\text{ C}$$

(d) **Total loss of energy during redistribution**

$$\Sigma U_i = \frac{1}{2} \times (1)\,(40)^2 + \frac{1}{2} \times (2)\,(20)^2 = 1200\text{ J}$$

$$\Sigma U_f = \frac{1}{2} \times \frac{(160/7)^2}{1} + \frac{1}{2} \times \frac{(400/7)^2}{2} + \frac{1}{2} \times \frac{(120/7)^2}{4}$$

$$= 1114.3\text{ J}$$

$$\Delta U = \Sigma U_f - \Sigma U_i = -85.7\text{ J}$$

20 V battery will supply energy but 10 V battery will consume energy. So,

$$\text{Total energy supplied} = 20 \times \frac{120}{7} - 10 \times \frac{120}{7} = 171.4\text{ J}$$

$$\text{Total heat produced} = \text{Energy supplied} - \Delta U$$

$$= 171.4 - (-85.7) = 257.1\text{ J}$$ **Ans.**

(e) Resistors are in series. Hence,

$$H \propto R \quad \text{or} \quad \frac{H_1}{H_2} = \frac{R_1}{R_2} = \frac{1\,\Omega}{2\,\Omega} = \frac{1}{2}$$

$\therefore$
$$H_1 = \left(\frac{1}{1+2}\right)(257.1) = 85.7\text{ J}$$ **Ans.**

$$H_2 = \left(\frac{2}{1+2}\right)(257.1) = 171.4\text{ J}$$ **Ans.**

Note *(i) For making the calculations simple, we have taken capacities in Farad, otherwise Farad is a large unit.*
(ii) For two loop problems, we will rotate two charges q_1 and q_2.

Type 8. *Shortcut method of finding time varying functions in a C-R circuit like q or i etc.*

Concept

(i) At time $t = 0$, when capacitor is uncharged it offers maximum current passing through it. So, it may be assumed like a conducting wire of zero resistance. With this concept, find initial values of q or i etc.

(ii) At time $t = \infty$, when capacitor is fully charged it does not allow current through it, as insulator is filled between the plates. So, its resistance may be assumed as infinite. With this concept, find steady state values at time $t = \infty$ of q or i etc.

(iii) **Equivalent time constant** To find the equivalent time constant of a circuit, the following steps are followed :

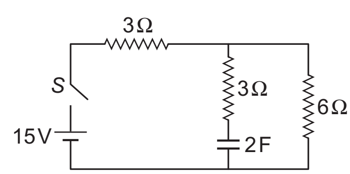
(a) Short-circuit the battery.

(b) Find net resistance across the capacitor (suppose it is R_{net})

(c) $\tau_C = (R_{\text{net}})C$

(iv) In C - R circuit, increase or decrease is always exponential. So, first make exponential graph and then write exponential equation corresponding to this graph with the time constant obtained by the method discussed above.

Example 10 *Switch S is closed at time $t = 0$ in the circuit shown in figure.*



(a) Find the time varying quantities in the circuit.

(b) Find their values at time $t = 0$.

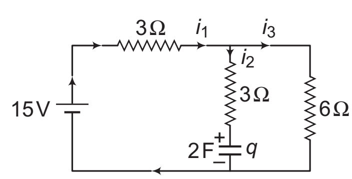
(c) Find their values at time $t = \infty$

(d) Find time constant of all time varying functions.

(e) Make their exponential graphs and write their exponential equations.

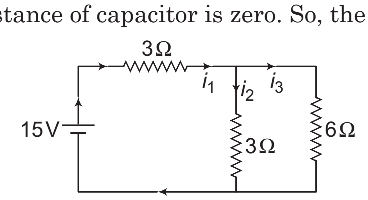
(f) Just write the equations to solve them to find different time varying functions.

Solution (a)



There are four times variable functions i_1, i_2, i_3 and q.

(b) At $t = 0$, equivalent resistance of capacitor is zero. So, the simple circuit is as shown below



$$R_{net} = 3 + \frac{3 \times 6}{3 + 6} = 5\,\Omega$$

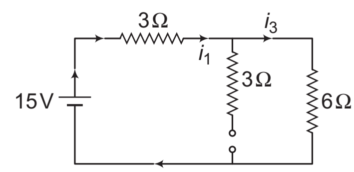
$$i_1 = \frac{15}{5} = 3\text{ A} \quad \Rightarrow \quad \frac{i_2}{i_3} = \frac{6}{3} = \frac{2}{1}$$

$\therefore$ $$i_2 = \frac{2}{2+1}\,(3\text{ A}) = 2\text{ A}$$

$$i_3 = \left(\frac{1}{2+1}\right)(3\text{ A}) = 1\text{ A} \quad \text{and} \quad q = 0$$

(c) At $t = \infty$, equivalent resistance of capacitor is infinite. So, equivalent circuit is as shown below

$$i_1 = i_3 = \frac{15}{3+6} = \frac{5}{3}\text{ A}$$

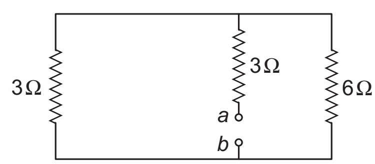


$$i_2 = 0$$

$$V_{2F} = V_{6\,\Omega} = iR = \left(\frac{5}{3}\right)(6) = 10\text{ volt}$$

$\therefore$ $$q = CV = (2)\,(10) = 20\text{ C}$$

(d) By short-circuiting the battery, the simplified circuit is as shown below

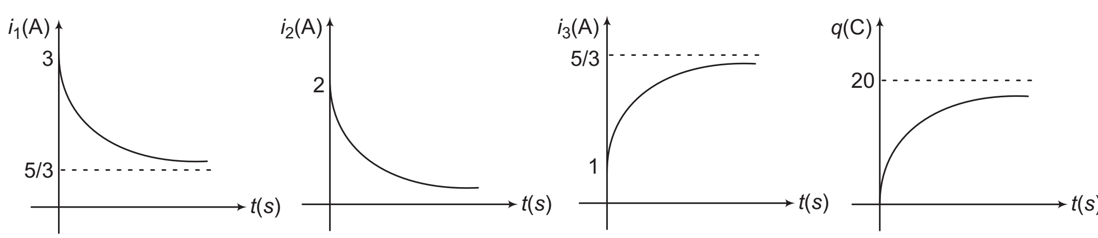


Net resistance across capacitor or ab is

$$R_{net} = 3 + \frac{3 \times 6}{3+6} = 5\,\Omega$$

$\therefore$ $$\tau_C = CR_{net} = (2)\,(5) = 10\text{ s}$$

(e) Exponential graphs and their exponential equations are as under.



$$i_1 = \frac{5}{3} + \left(3 - \frac{5}{3}\right) e^{\frac{-t}{\tau_C}} = \frac{5}{3} + \frac{4}{3}\,e^{-\frac{t}{10}}$$

$$i_2 = 2e^{-\frac{t}{\tau_C}} = 2e^{-\frac{t}{10}}$$

$$i_3 = 1 + \left(\frac{5}{3} - 1\right)(1 - e^{-\frac{t}{\tau_C}}) = 1 + \frac{2}{3}\,(1 - e^{-\frac{t}{10}})$$

$$q = 20\,(1 - e^{-\frac{t}{\tau_C}}) = 20\,(1 - e^{-\frac{t}{10}})$$

(f) Unknowns are four: i_1, i_2, i_3 and q. So, corresponding to the figure of part (a), four equations are

$$i_1 = i_2 + i_3 \quad \text{...(i)}$$

$$i_2 = \frac{dq}{dt} \quad \text{...(ii)}$$

Applying loop equation in left hand side loop,

$$+15 - 3i_1 - 3i_2 - \frac{q}{2} = 0 \quad \text{...(iii)}$$

Applying loop equation in right hand side loop,

$$+\frac{q}{2} + 3i_2 - 6i_3 = 0 \quad \text{...(iv)}$$

Solving these equations (with some integration), we can find same time functions as we have obtained in part (e).

Type 9. *To find current and hence potential difference between two points in a wire having a capacitor*

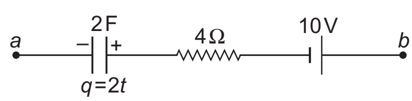
Concept

If charge on capacitor is constant, then current through capacitor wire is zero. If charge is variable, then current is non-zero. Magnitude of this current is

$$i = \left|\frac{dq}{dt}\right|$$

and direction of this current is towards the positive plate if charge is increasing and away from the positive plate if charge is decreasing.

Example 11 *In the circuit shown in figure, find V_{ab} at 1 s.*

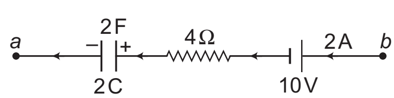


Solution Charge on capacitor is increasing. So, there is a current in the circuit from right to left. This current is given by

$$i = \frac{dq}{dt} = 2 \text{ A}$$

At 1 s, $q = 2$ C.

So, at 1 s, circuit is as shown in figure.



$$V_a + \frac{2}{2} + (2)(4) + 10 = V_b$$

$\therefore$ $$V_a - V_b$$

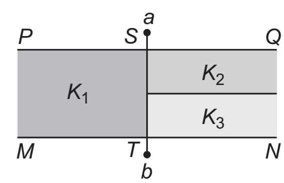
or $$V_{ab} = -19 \text{ volt}$$ **Ans.**

Type 10. *To find capacitance of a capacitor filled with two or more than two dielectrics*

Concept

PQ and MN are two metallic plates.

If we wish to find net capacitance between a and b, then



$$V_{PS} = V_{SQ} = V_1 \text{ (say)}$$

$$V_{MT} = V_{TN} = V_2 \text{ (say)}$$

Hence, $$V_{PS} - V_{MT} = V_{SQ} - V_{TN} = V_1 - V_2$$

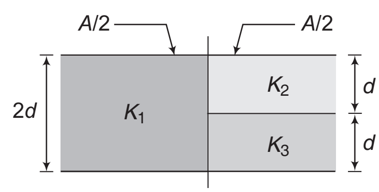
Therefore, there are two capacitors, one on right hand side and other on left hand side which are in parallel.

$\therefore$ $$C = C_{\text{RHS}} + C_{\text{LHS}}$$

For C_{RHS}, we can use the formula,

$$C = \frac{\varepsilon_0 A}{d - t_1 - t_2 + \dfrac{t_1}{K_1} + \dfrac{t_2}{K_2}}$$

Example 12 *What is capacitance of the capacitor shown in figure?*



Solution $C = C_{\text{LHS}} + C_{\text{RHS}}$

$$= \frac{K_1 \varepsilon_0 (A/2)}{2d} + \frac{\varepsilon_0 (A/2)}{(2d - d - d) + (d/K_2) + (d/K_3)}$$

$$= \frac{\varepsilon_0 A}{2d}\left[\frac{K_1}{2} + \frac{K_2K_3}{K_2 + K_3}\right]$$

Type 11. *To find charge on different capacitors in a C-R circuit*

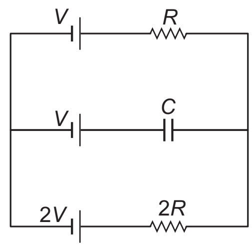
Concept

In a C - R circuit, charge on different capacitors is normally asked either at $t = 0$, $t = \infty$ or $t = t$. If nothing is given in the question, then we have to find charges on capacitors at $t = \infty$ or steady state charges.

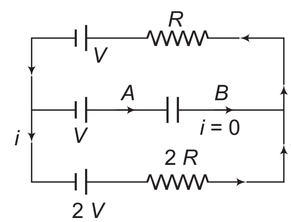
In steady state, no current flows through a wire having capacitor. But, if there is any other closed circuit then current can flow through that circuit. So, first find this current and then steady state potential difference (say V_0) across two plates of capacitor. Now,

$$q_0 = CV_0 = \text{steady state charge}$$

Example 13 *Find potential difference across the capacitor (obviously in steady state)*



Solution In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,



$$i = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

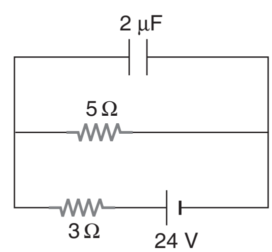
Potential difference between A and B,

$$V_A - V + V + iR = V_B$$

$$\therefore \qquad V_B - V_A = iR = \left(\frac{V}{3R}\right) R = \frac{V}{3}$$

Note *In this problem, charge stored in the capacitor can also be asked, which is equal to $q = C\frac{V}{3}$ with positive charge on B side and negative on A side because $V_B > V_A$.*

Example 14 *Find the charge stored in the capacitor.*



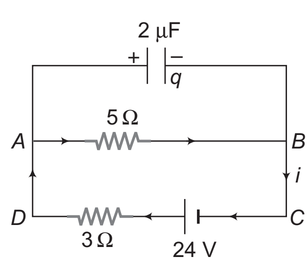
HOW TO PROCEED *Insulator is filled between the plates of the capacitor. Therefore, a capacitor does not allow current flow through it after charging is over. Hence, in the circuit current will flow through 3 Ω and 5 Ω resistances and it will not flow through the capacitor. To find the charge stored in the capacitor, we need the PD across it. So, first we will find PD across the capacitor and then apply,*

$$q = CV$$

where, V = PD across the capacitor.

Solution As we said earlier also, current will flow in loop $ABCDA$ when charging is over. And this current is

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{24}{5+3} = 3 \text{ A}$$



Now, PD across the capacitor is equal to the PD across the 5 Ω resistance.

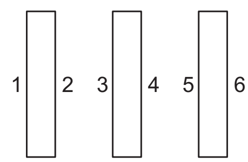
Hence, $V = V_A - V_B = iR = (3)(5) = 15$ V

∴ $q = CV = (2 \times 15)\,\mu C = 30\,\mu C$ **Ans.**

Note *$V_A - V_B = 15$ V, therefore $V_A > V_B$, i.e. the positive charge will be collected on the left plate of the capacitor and negative on the right plate.*

Type 12. *To find distribution of charges on different faces on parallel conducting plates*

Concept



(i) $q_1 = q_6 = \dfrac{q_{\text{Total}}}{2}$

(ii) $q_2 = -q_3$ and $q_4 = -q_5$

Note *The above two results are proved in example 15.*

(iii) Electric field between 2 and 3 is due to the charges q_2 and q_3. This electric field is given by

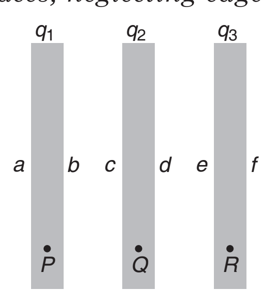
$$E = \frac{\sigma}{\varepsilon_0} = \frac{q/A}{\varepsilon_0} \qquad [\,|q_2| = |q_3| = q]$$

(iv) This electric field is uniform, so potential difference between any two points is given by

$$V = Ed$$

(v) If two plates are connected to each other, then distance between the plates is required, otherwise there is no requirement of that.

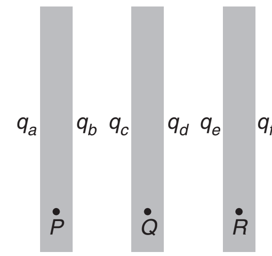
Example 15 *Three parallel metallic plates each of area A are kept as shown in figure and charges q_1, q_2 and q_3 are given to them. Find the resulting charge distribution on the six surfaces, neglecting edge effects as usual.*



Solution The plate separations do not affect the distribution of charge in this problem.

In the figure, $q_b = q_1 - q_a$, $q_d = q_2 - q_c$

and $q_f = q_3 - q_e$.



Electric field at point P is zero because this point is lying inside a conductor.

$$E_P = 0$$

At P, charge q_a will give an electric field towards right. All other charges q_b, $q_c \ldots$, etc., will give the electric field towards left. So,

$$\frac{1}{2A\varepsilon_0}[q_a - (q_1 - q_a) - q_c - (q_2 - q_c) - q_e - (q_3 - q_e)] = 0$$

or $$2q_a - q_1 - q_2 - q_3 = 0$$

or $$q_a = \frac{q_1 + q_2 + q_3}{2}$$

Similarly the condition, $$E_R = 0$$

will give the result,

$$q_f = \frac{q_1 + q_2 + q_3}{2}$$

From here we may conclude that, **half of the sum of all charges appears on each of the two outermost surfaces of the system of plates.**

Further we have a condition,

$$E_Q = 0$$

$$\frac{1}{2A\varepsilon_0}[q_a + (q_1 - q_a) + q_c - (q_2 - q_c) - q_e - (q_3 - q_e)] = 0$$

or $$q_1 + 2q_c - q_2 - q_3 = 0$$

$\therefore$ $$q_c = \frac{q_2 + q_3 - q_1}{2}$$

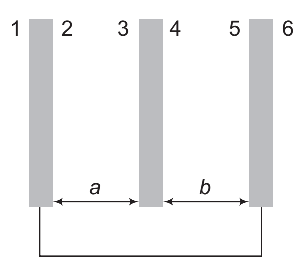
$$q_b = q_1 - q_a = \frac{q_1 - q_2 - q_3}{2} = -q_c$$

Similarly, we can show that

$$q_d = -q_e.$$

From here we can find another important result that **the pairs of opposite surfaces like *b, c* and *d,e* carry equal and opposite charges.**

Example 16 *Three identical metallic plates are kept parallel to one another at a separation of a and b. The outer plates are connected by a thin conducting wire and a charge Q is placed on the central plate. Find final charges on all the six plate's surfaces.*

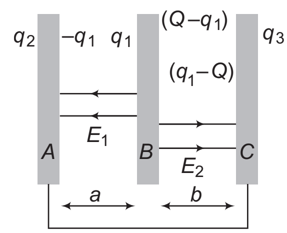


Solution Let the charge distribution in all the six faces be as shown in figure. While distributing the charge on different faces, we have used the fact that two opposite faces have equal and opposite charges on them.

Net charge on plates A and C is zero. Hence,

$$q_2 - q_1 + q_3 + q_1 - Q = 0$$

or $$q_2 + q_3 = Q \qquad \text{...(i)}$$



Further A and C are at same potentials. Hence,

$$V_B - V_A = V_B - V_C$$

or $$E_1 a = E_2 b$$

$\therefore$ $$\frac{q_1}{A\varepsilon_0} \cdot a = \frac{Q - q_1}{A\varepsilon_0} \cdot b \qquad (A = \text{Area of plates})$$

$\therefore$ $$q_1 a = (Q - q_1)\, b$$

$\therefore$ $$q_1 = \frac{Qb}{a + b} \qquad \text{...(ii)}$$

Electric field inside any conducting plate (say inside C) is zero. Therefore,

$$\frac{q_2}{2A\varepsilon_0} - \frac{q_1}{2A\varepsilon_0} + \frac{q_1}{2A\varepsilon_0} + \frac{Q - q_1}{2A\varepsilon_0} + \frac{q_1 - Q}{2A\varepsilon_0} - \frac{q_3}{2A\varepsilon_0} = 0$$

$\therefore$ $$q_2 - q_3 = 0 \qquad \text{...(iii)}$$

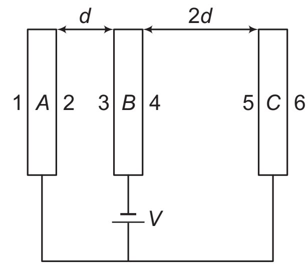
Solving these three equations, we get $q_1 = \dfrac{Qb}{a + b}$, $q_2 = q_3 = \dfrac{Q}{2}$

Hence, charge on different faces are as follows.

Table 25.3

Face	Charge
1	$q_2 = \frac{Q}{2}$
2	$-q_1 = -\frac{Qb}{a+b}$
3	$q_1 = \frac{Qb}{a+b}$
4	$Q - q_1 = \frac{Qa}{a+b}$
5	$q_1 - Q = -\frac{Qa}{a+b}$
6	$q_3 = \frac{Q}{2}$

Example 17 *Area of each plate is A. The conducting plates are connected to a battery of emf V volts. Find charges q_1 to q_6.*



Solution Net charge drawn from the battery is zero or

$$q_{\text{Total}} = 0$$

$\therefore$
$$q_1 = q_6 = \frac{q_{\text{Total}}}{2} = 0$$

$$V_{AB} = V \text{ with } V_A > V_B$$

$\therefore$
$$|q_2| = |q_3| = CV = \left(\frac{\varepsilon_0 A}{d}\right) V$$

$$q_2 = +\frac{\varepsilon_0 AV}{d} \text{ and } q_3 = -\frac{\varepsilon_0 AV}{d}$$

Similarly,

$$V_{BC} = V \text{ with } V_C > V_B$$

$\therefore$
$$|q_4| = |q_5| = CV = \left(\frac{\varepsilon_0 A}{2d}\right) V$$

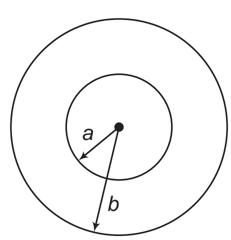
with
$$q_5 = +\frac{\varepsilon_0 AV}{2d} \text{ and } q_4 = -\frac{\varepsilon_0 AV}{2d}$$

Type 13. *To find total electrostatic potential energy due to spherical charged shells*

Concept

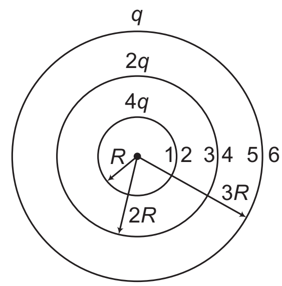
(i) Capacity of a spherical capacitor is given by

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$



(ii) $U = \dfrac{1}{2}\dfrac{q^2}{C}$

Example 18 *In the figure shown,*



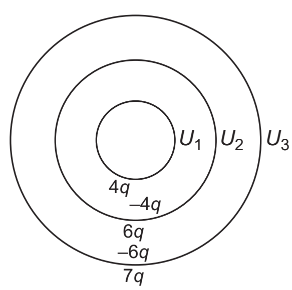
(a) *Find q_1 to q_6.*

(b) *Total electrostatic potential energy.*

Solution (a)

$$q_1 = 0$$
$$q_2 = 4q$$
$$q_3 = -q_2 = -4q$$
$$q_4 = 2q - q_3 = 6q$$
$$q_5 = -q_4 = -6q$$
$$q_6 = q - q_5 = 7q$$

(b) $U_{\text{Total}} = U_1 + U_2 + U_3$



Here,
$$U_1 = \frac{1}{2}\frac{(4q)^2}{C_1}$$

where,
$$C_1 = \frac{4\pi\varepsilon_0}{\frac{1}{R} - \frac{1}{2R}}$$

$$U_2 = \frac{1}{2}\frac{(6q)^2}{C_2}$$

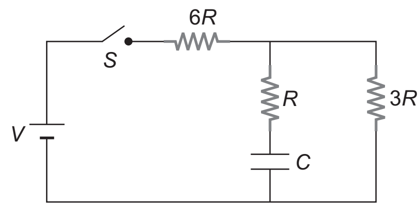
where,
$$C_2 = \frac{4\pi\varepsilon_0}{\frac{1}{2R} - \frac{1}{3R}}$$

and
$$U_3 = \frac{1}{2}\frac{(7q)^2}{C_3}$$

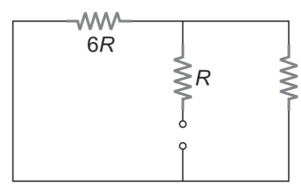
where,
$$C_3 = \frac{4\pi\varepsilon_0}{\frac{1}{3R} - \frac{1}{\infty}} = 4\pi\varepsilon_0(3R)$$

Miscellaneous Examples

Example 19 *In the circuit shown in figure switch S is closed at time $t = 0$. Find the current through different wires and charge stored on the capacitor at any time t.*



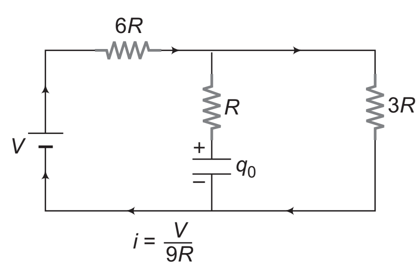
Solution **Calculation of τ_C** Equivalent resistance across capacitor after short-circuiting the battery is



$$R_{net} = R + \frac{(6R)(3R)}{6R + 3R} = 3R$$

$$\therefore \qquad \tau_C = (C)R_{net} = 3RC$$

Calculation of steady state charge q_0 At $t = \infty$, capacitor is fully charged and no current flows through it.



PD across capacitor = PD across $3R$

$$= \left(\frac{V}{9R}\right)(3R)$$

$$= \frac{V}{3}$$

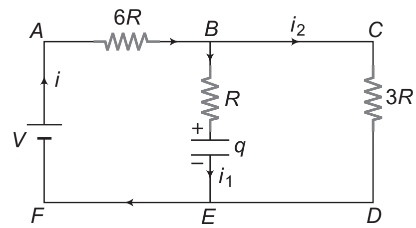
$$\therefore \qquad q_0 = \frac{CV}{3}$$

Now, let charge on the capacitor at any time t be q and current through it is i_1. Then,

$$q = q_0\,(1 - e^{-t/\tau_C})$$

and
$$i_1 = \frac{dq}{dt} = \frac{q_0}{\tau_C}\,e^{-t/\tau_C} \qquad \text{...(i)}$$

Applying Kirchhoff's second law in loop $ACDFA$, we have



$$-6iR - 3i_2R + V = 0$$

or
$$2i + i_2 = \frac{V}{3R} \qquad \text{...(ii)}$$

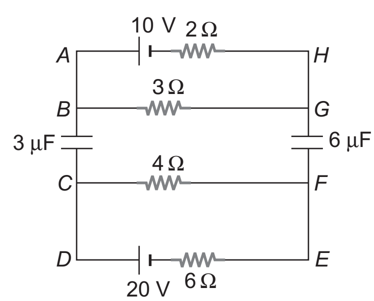
Applying Kirchhoff's junction law at B, we have

$$i = i_1 + i_2 \qquad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we have

$$i_2 = \frac{V}{9R} - \frac{2}{3}i_1 = \frac{V}{9R} - \frac{2q_0}{3\tau_C}e^{-t/\tau_C} \quad \text{and} \quad i = \frac{V}{9R} + \frac{q_0}{3\tau_C}e^{-t/\tau_C}$$

Example 20 *In the circuit shown in figure, find the steady state charges on both the capacitors.*



HOW TO PROCEED *In steady state a capacitor offers an infinite resistance. Therefore, the two circuits ABGHA and CDEFC have no relation with each other. Hence, the battery of emf 10 V is not going to contribute any current in the lower circuit. Similarly, the battery of emf 20 V will not contribute to the current in the upper circuit. So, first we will calculate the current in the two circuits, then find the potential difference V_{BG} and V_{CF} and finally we can connect two batteries of emf V_{BG} and V_{CF} across the capacitors to find the charges stored in them.*

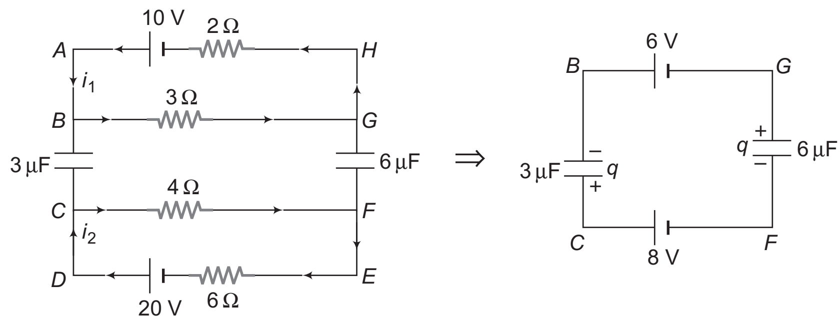
Solution Current in the upper circuit, $i_1 = \dfrac{10}{3+2} = 2\ \text{A}$

$\therefore$
$$V_{BG} = V_B - V_G = 3i_1 = 3 \times 2 = 6\ \text{V}$$

Current in the lower circuit, $i_2 = \dfrac{20}{4+6} = 2\text{ A}$

$\therefore$ $$V_{CF} = V_C - V_F = 4i_2 = 4 \times 2 = 8\text{ V}$$

Charge on both the capacitors will be same. Let it be q. Applying Kirchhoff's second law in loop *BGFCB*,



$$-6 - \frac{q}{6 \times 10^{-6}} + 8 - \frac{q}{3 \times 10^{-6}} = 0$$

or $$\frac{(10^6)\, q}{2} = 2$$

or $$q = 4 \times 10^{-6}\text{ C}$$

or $$q = 4\ \mu\text{C}$$ **Ans.**

Example 21 *An isolated parallel plate capacitor has circular plates of radius* 4.0 *cm. If the gap is filled with a partially conducting material of dielectric constant K and conductivity* $5.0 \times 10^{-14}\ \Omega^{-1} m^{-1}$. *When the capacitor is charged to a surface charge density of* $15\ \mu C/cm^2$, *the initial current between the plates is* $1.0\ \mu A$?

(a) *Determine the value of dielectric constant K.*

(b) *If the total joule heating produced is* 7500 *J, determine the separation of the capacitor plates.*

Solution (a) This is basically a problem of discharging of a capacitor from inside the capacitor. Charge at any time t is

$$q = q_0 e^{-t/\tau_C}$$

Here, q_0 = (area of plates) (surface charge density)

and discharging current, $$i = \left(\frac{-dq}{dt}\right) = \frac{q_0}{\tau_C} \cdot e^{-t/\tau_C} = i_0 e^{-t/\tau_C}$$

Here, $$i_0 = \frac{q_0}{\tau_C} = \frac{q_0}{CR}$$

$$C = \frac{K\varepsilon_0 A}{d} \quad \text{and} \quad R = \frac{d}{\sigma A}$$

$\therefore$ $$CR = \frac{K\varepsilon_0}{\sigma}$$

Therefore, $$i_0 = \frac{q_0}{\dfrac{K\varepsilon_0}{\sigma}} = \frac{\sigma q_0}{K\varepsilon_0} \quad \Rightarrow \quad K = \frac{\sigma q_0}{i_0 \varepsilon_0}$$

Substituting the values, we have

$$K = \frac{(5.0 \times 10^{-14})\,(\pi)\,(4.0)^2\,(15 \times 10^{-6})}{(1.0 \times 10^{-6})\,(8.86 \times 10^{-12})} = 4.25 \qquad \textbf{Ans.}$$

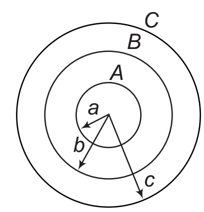
(b) $$U = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}\frac{q_0^2}{\dfrac{K\varepsilon_0 A}{d}}$$

$\therefore$ $$d = \frac{2K\varepsilon_0 AU}{q_0^2}$$

$$= \frac{2 \times 4.25 \times 8.86 \times 10^{-12} \times \pi \times (4.0 \times 10^{-2})^2 \times 7500}{(15 \times 10^{-6} \times \pi \times 4.0 \times 4.0)^2}$$

$$= 5.0 \times 10^{-3} \text{ m} = 5.0 \text{ mm} \qquad \textbf{Ans.}$$

Example 22 *Three concentric conducting shells A, B and C of radii a, b and c are as shown in figure. A dielectric of dielectric constant K is filled between A and B. Find the capacitance between A and C.*



HOW TO PROCEED *When the dielectric is filled between A and B, the electric field will change in this region. Therefore, the potential difference and hence the capacitance of the system will change. So, first find the electric field E(r) in the region $a \le r \le c$. Then, find the PD (V) between A and C and finally the capacitance of the system will be*

$$C = \frac{q}{V}$$

Here, q = charge on A

Solution $$E(r) = \frac{q}{4\pi\varepsilon_0 K r^2} \quad \text{for} \quad a \le r \le b$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \quad \text{for} \quad b \le r \le c$$

Using, $dV = -\int \mathbf{E} \cdot d\mathbf{r}$

the PD between A and C is

$\therefore$ $$V = V_A - V_C = -\int_a^b \frac{q}{4\pi\varepsilon_0 K r^2} \cdot dr - \int_b^c \frac{q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{K}\left(\frac{1}{a} - \frac{1}{b}\right) + \left(\frac{1}{b} - \frac{1}{c}\right)\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(b-a)}{Kab} + \frac{(c-b)}{bc}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 Kabc}\,[c\,(b-a) + Ka\,(c-b)]$$

$\therefore$ The desired capacitance is

$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0 Kabc}{Ka\,(c-b) + c\,(b-a)} \qquad \textbf{Ans.}$$

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(*c*) *If **Assertion** is true, but the **Reason** is false.*

(*d*) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : From the relation $C = \frac{q}{V}$. We can say that, if more charge q is given to a conductor, its capacitance should increase.

Reason : Ratio $\frac{q}{V}$ will remain constant for a given conductor.

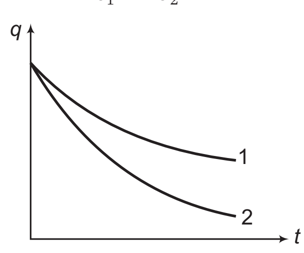
2. Assertion : A parallel plate capacitor is first charged and then distance between the plates is increased. In this process, electric field between the plates remains the same, while potential difference gets decreased.

Reason : $E = \frac{q}{A\,\varepsilon_0}$ and $V = \frac{q}{Ad\,\varepsilon_0}$. Since, q remains same, E will remain same while V will decrease.

3. Assertion : When an uncharged capacitor is charged by a battery, only 50% of the energy supplied by a battery is stored in the capacitor.

Reason : Rest 50% is lost.

4. Assertion : Discharging graphs of two C-R circuits having the same value of C is shown in figure. From the graph we can say that $\tau_{C_1} > \tau_{C_2}$.

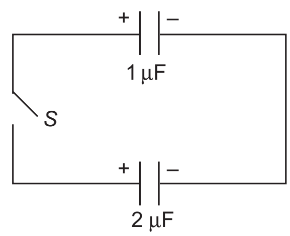


Reason : $R_1 > R_2$.

5. Assertion : In series combination, charges on two capacitors are always equal.

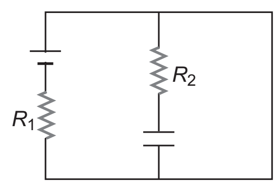
Reason : If charges are same, the total potential difference applied across two capacitors will be distributed in inverse ratio of capacities.

6. Assertion : Two capacitors are charged from the same battery and then connected as shown. A current will flow in anti-clockwise direction as soon as switch is closed.



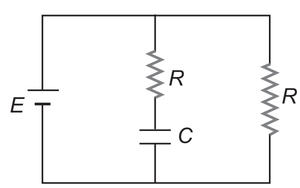
Reason : In steady state charges on two capacitors are in the ratio 1 : 2..

7. Assertion : In the circuit shown in figure no charge will be stored in the capacitor.



Reason : Current through R_2 will be zero.

8. Assertion : In the circuit shown in figure, time constant of charging of capacitor is $\frac{CR}{2}$.

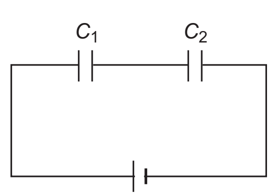


Reason : In the absence of capacitor in the circuit, two resistors are in parallel with the battery.

9. Assertion : Two capacitors are connected in series with a battery. Energy stored across them is in inverse ratio of their capacity.

Reason : $U = \frac{1}{2}qV$ or $U \propto qV$.

10. Assertion : In the circuit shown in figure, when a dielectric slab is inserted in C_2, the potential difference across C_2 will decrease.



Reason : By inserting the slab a current will flow in the circuit in clockwise direction.

Objective Questions

1. The separation between the plates of a charged parallel- plate capacitor is increased. The force between the plates

(a) increases
(b) decreases
(c) remains same
(d) first increases then decreases

2. If the plates of a capacitor are joined together by a conducting wire, then its capacitance

(a) remains unchanged
(b) decreases
(c) becomes zero
(d) becomes infinite

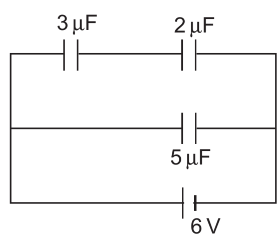
3. Two metal spheres of radii a and b are connected by a thin wire. Their separation is very large compared to their dimensions. The capacitance of this system is

(a) $4\pi\varepsilon_0(ab)$
(b) $2\pi\varepsilon_0(a+b)$
(c) $4\pi\varepsilon_0(a+b)$
(d) $4\pi\varepsilon_0\left(\frac{a^2+b^2}{2}\right)$

4. n identical capacitors are connected in parallel to a potential difference V. These capacitors are then reconnected in series, their charges being left undisturbed. The potential difference obtained is

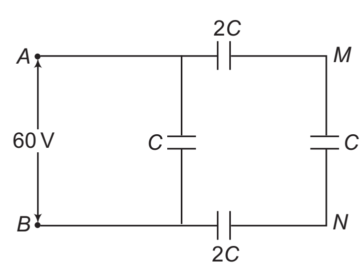
(a) zero
(b) $(n-1)V$
(c) nV
(d) n^2V

5. In the circuit shown in figure, the ratio of charge on $5\,\mu F$ and $2\,\mu F$ capacitor is



(a) 5/4
(b) 5/3
(c) 3/8
(d) None of these

6. In the circuit shown, a potential difference of 60 V is applied across AB. The potential difference between the points M and N is

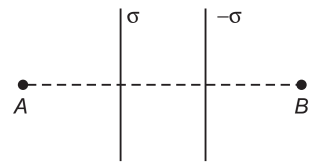


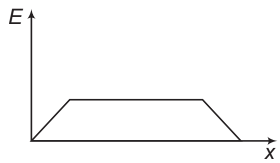
(a) 10 V
(b) 15 V
(c) 20 V
(d) 30 V

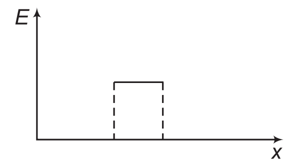
7. In Milikan's oil drop experiment, an oil drop of radius r and charge q is held in equilibrium between the plates of a charged parallel-plate capacitor when the potential difference is V. To keep a drop of radius $2r$ and with a charge $2q$ in equilibrium between the plates the potential difference V required is

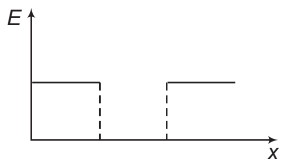
(a) V
(b) $2\,V$
(c) $4\,V$
(d) $8\,V$

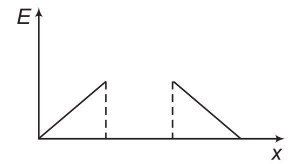
8. Two large parallel sheets charged uniformly with surface charge density σ and $-\sigma$ are located as shown in the figure. Which one of the following graphs shows the variation of electric field along a line perpendicular to the sheets as one moves from A to B?



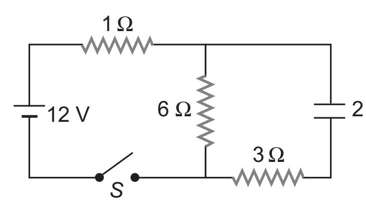
(a)


(b)


(c)


(d)


9. When the switch is closed, the initial current through the 1 Ω resistor is

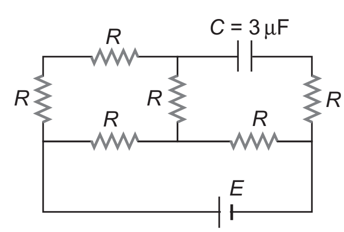


(a) 2 A (b) 4 A (c) 3 A (d) 6 A

10. A capacitor of capacitance C carrying charge Q is connected to a source of emf E. Finally, the charge on capacitor would be

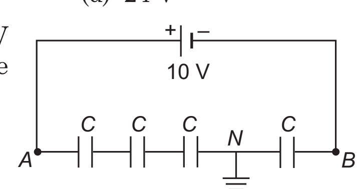
(a) Q (b) $Q + CE$ (c) CE (d) None of these

11. In the circuit, the potential difference across the capacitor is 10 V. Each resistance is of 3 Ω. The cell is ideal. The emf of the cell is



(a) 14 V (b) 16 V (c) 18 V (d) 24 V

12. Four identical capacitors are connected in series with a 10 V battery as shown in the figure. The point N is earthed. The potentials of points A and B are



(a) 10 V, 0 V
(b) 7.5 V, – 2.5 V
(c) 5 V, – 5 V
(d) 7.5 V, 2.5 V

13. A capacitor of capacity 2 μF is charged to 100 V. What is the heat generated when this capacitor is connected in parallel to an another capacitor of same capacity?

(a) 2.5 mJ (b) 5.0 mJ
(c) 10 mJ (d) 4 mJ

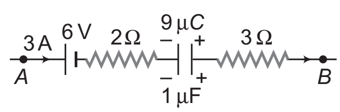
14. A charged capacitor is discharged through a resistance. The time constant of the circuit is η. Then, the value of time constant for the power dissipated through the resistance will be

(a) η (b) 2η
(c) $\eta/2$ (d) zero

15. A capacitor is charged by a cell of emf E and the charging battery is then removed. If an identical capacitor is now inserted in the circuit in parallel with the previous capacitor, the potential difference across the new capacitor is

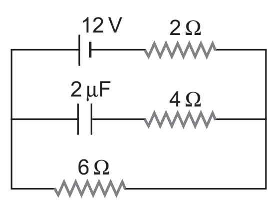
(a) $2E$ (b) E
(c) $E/2$ (d) zero

16. The potential difference $V_A - V_B$ between points A and B for the circuit segment shown in figure at the given instant is



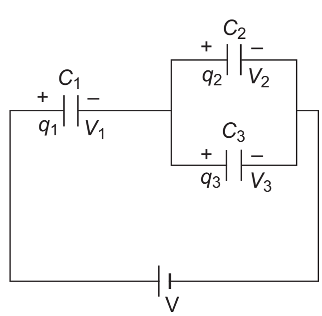
(a) 12 V (b) – 12 V
(c) 6 V (d) – 6 V

17. For the circuit arrangement shown in figure, in the steady state condition charge on the capacitor is



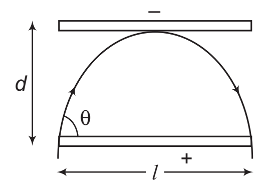
(a) 12 μC (b) 14 μC
(c) 20 μC (d) 18 μC

18. In the circuit as shown in figure if all the symbols have their usual meanings, then identify the correct statement,



(a) $q_2 = q_3$; $V_2 = V_3$ (b) $q_1 = q_2 + q_3$; $V_2 = V_3$
(c) $q_1 = q_2 + q_3$; $V = V_1 + V_2 + V_3$ (d) $q_1 + q_2 + q_3 = 0$; $V_2 = V_3 = V - V_1$

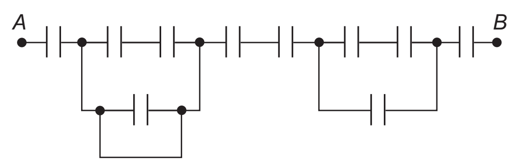
19. An electron enters the region between the plates of a parallel-plate capacitor at an angle θ to the plates. The plate width is l. The plate separation is d. The electron follows the path shown, just missing the upper plate. Neglect gravity. Then,



(a) $\tan\theta = 2d/l$
(b) $\tan\theta = 4d/l$
(c) $\tan\theta = 8d/l$
(d) The data given is insufficient to find a relation between d, l and θ

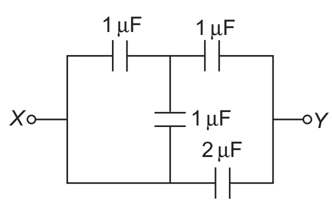
20. An infinite sheet of charge has a surface charge density of $10^{-7}\,\mathrm{C/m^2}$. The separation between two equipotential surfaces whose potentials differ by 5 V is
(a) 0.64 cm
(b) 0.88 mm
(c) 0.32 cm
(d) 5×10^{-7} m

21. Find the equivalent capacitance across A and B for the arrangement shown in figure. All the capacitors are of capacitance C



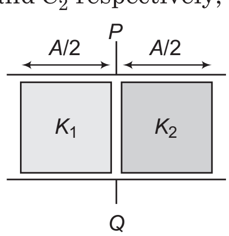
(a) $\dfrac{3C}{14}$
(b) $\dfrac{C}{8}$
(c) $\dfrac{3C}{16}$
(d) None of these

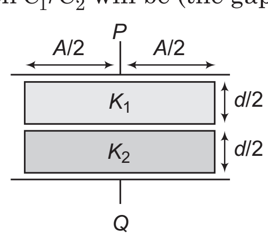
22. The equivalent capacitance between X and Y is



(a) $5/6\,\mu$F
(b) $7/6\,\mu$F
(c) $8/3\,\mu$F
(d) $1\,\mu$F

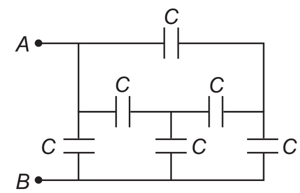
23. In the arrangement shown in figure, dielectric constant $K_1 = 2$ and $K_2 = 3$. If the capacitance across P and Q are C_1 and C_2 respectively, then C_1/C_2 will be (the gaps shown are negligible)





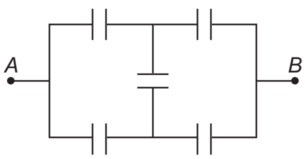
(a) 1 : 1
(b) 2 : 3
(c) 9 : 5
(d) 25 : 24

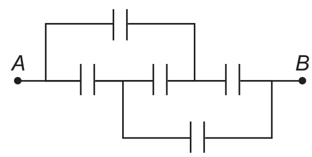
24. Six equal capacitors each of capacitance C are connected as shown in the figure. The equivalent capacitance between points A and B, is

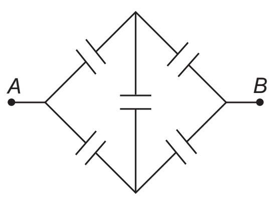


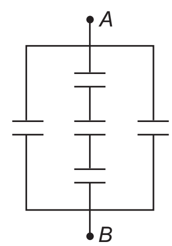
(a) 1.5 C
(b) C
(c) $2C$
(d) 0.5 C

25. Four ways of making a network of five capacitors of the same value are shown in four choices. Three out of four are identical. The one which is different is

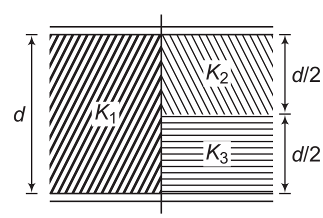
(a)


(b)


(c)


(d)


26. The equivalent capacitance of the arrangement shown in figure, if A is the area of each plate, is



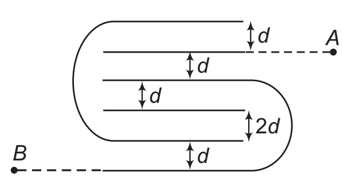
(a) $C = \frac{\varepsilon_0 A}{d}\left[\frac{K_1}{2} + \frac{K_2 + K_3}{K_2 K_3}\right]$

(b) $C = \frac{\varepsilon_0 A}{d}\left[\frac{K_1}{2} + \frac{K_2 K_3}{K_2 + K_3}\right]$

(c) $C = \frac{\varepsilon_0 A}{2d}\left[K_1 + \frac{K_2 K_3}{K_2 + K_3}\right]$

(d) $C = \frac{\varepsilon_0 A}{d}\left[K_1 + \frac{K_2 K_3}{K_2 + K_3}\right]$

27. Find equivalent capacitance between points A and B. [Assume each conducting plate is having same dimensions and neglect the thickness of the plate, $\frac{\varepsilon_0 A}{d} = 7\,\mu\text{F}$, where A is area of plates]

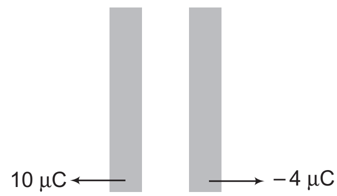


(a) 7 μF
(b) 11 μF
(c) 12 μF
(d) 15 μF

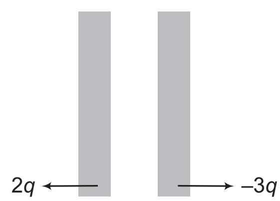
Subjective Questions

Note *You can take approximations in the answers.*

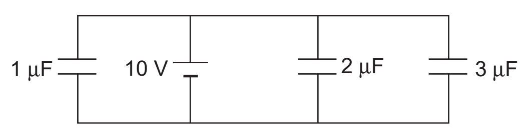
1. Two metallic plates are kept parallel to one another and charges are given to them as shown in figure. Find the charge on all the four faces.



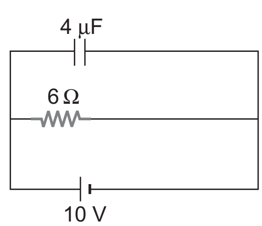
2. Charges $2q$ and $-3q$ are given to two identical metal plates of area of cross-section A. The distance between the plates is d. Find the capacitance and potential difference between the plates.



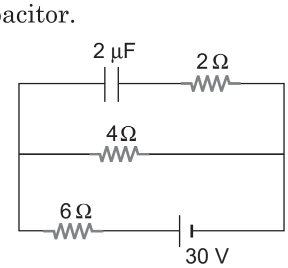
3. Find the charge stored in all the capacitors.



4. Find the charge stored in the capacitor.



5. Find the charge stored in the capacitor.



6. A 1 μF capacitor and a 2 μF capacitor are connected in series across a 1200 V supply line.
(a) Find the charge on each capacitor and the voltage across them.
(b) The charged capacitors are disconnected from the line and from each other and reconnected with terminals of like sign together. Find the final charge on each and the voltage across them.

7. A 100 μF capacitor is charged to 100 V. After the charging, battery is disconnected. The capacitor is then connected in parallel to another capacitor. The final voltage is 20 V. Calculate the capacity of second capacitor.

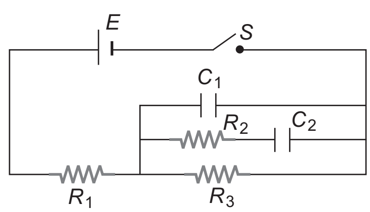
8. An uncharged capacitor C is connected to a battery through a resistance R. Show that by the time the capacitor gets fully charged, the energy dissipated in R is the same as the energy stored in C.

9. How many time constants will elapse before the current in a charging R-C circuit drops to half of its initial value?

10. A capacitor of capacitance C is given a charge q_0. At time $t = 0$ it is connected to an uncharged capacitor of equal capacitance through a resistance R. Find the charge on the first capacitor and the second capacitor as a function of time t. Also plot the corresponding q-t graphs.

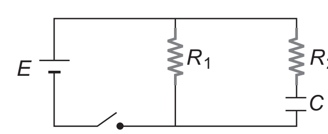
11. A capacitor of capacitance C is given a charge q_0. At time $t = 0$, it is connected to a battery of emf E through a resistance R. Find the charge on the capacitor at time t.

12. Determine the current through the battery in the circuit shown in figure.

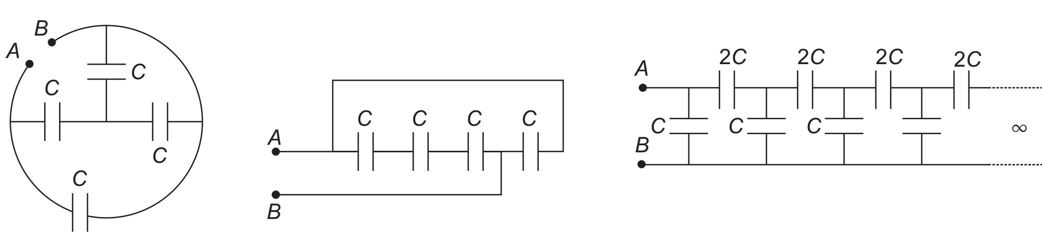


(a) immediately after the switch S is closed
(b) after a long time.

13. For the circuit shown in figure, find
(a) the initial current through each resistor
(b) steady state current through each resistor
(c) final energy stored in the capacitor
(d) time constant of the circuit when switch is opened.



14. Find equivalent capacitance between points A and B,



(a) (b) (c)

15. A 4.00 μF capacitor and a 6.00 μF capacitor are connected in parallel across a 660 V supply line.

(a) Find the charge on each capacitor and the voltage across each.

(b) The charged capacitors are disconnected from the line and from each other, and then reconnected to each other with terminals of unlike sign together. Find the final charge on each and the voltage across each.

16. A 5.80 μF parallel-plate air capacitor has a plate separation of 5.00 mm and is charged to a potential difference of 400 V. Calculate the energy density in the region between the plates, in J/m^3.

17. The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 3.60 and a dielectric strength of 1.60×10^7 V/m. The capacitor is to have a capacitance of 1.25×10^{-9} F and must be able to withstand a maximum potential difference of 5500 V. What is the minimum area the plates of the capacitor may have?

18. Two condensers are in parallel and the energy of the combination is 0.1 J, when the difference of potential between terminals is 2 V. With the same two condensers in series, the energy is 1.6×10^{-2} J for the same difference of potential across the series combination. What are the capacities?

19. A circuit has section AB as shown in figure. The emf of the source equals $E = 10$ V, the capacitor capacitances are equal to $C_1 = 1.0\,\mu$F and $C_2 = 2.0\,\mu$F, and the potential difference $V_A - V_B = 5.0$ V. Find the voltage across each capacitor.

20. Several 10 pF capacitors are given, each capable of withstanding 100 V. How would you construct :

(a) a unit possessing a capacitance of 2 pF and capable of withstanding 500 V?

(b) a unit possessing a capacitance of 20 pF and capable of withstanding 300 V?

21. Two, capacitors A and B are connected in series across a 100 V supply and it is observed that the potential difference across them are 60 V and 40 V. A capacitor of 2 μF capacitance is now connected in parallel with A and the potential difference across B rises to 90 V. Determine the capacitance of A and B.

22. A 10.0 μF parallel-plate capacitor with circular plates is connected to a 12.0 V battery.

(a) What is the charge on each plate?

(b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery?

(c) How much charge would be on the plates if the capacitor were connected to the 12.0 V battery after the radius of each plate was doubled without changing their separation?

23. A 450 μF capacitor is charged to 295 V. Then, a wire is connected between the plates. How many joule of thermal energy are produced as the capacitor discharges if all of the energy that was stored goes into heating the wire?

24. The plates of a parallel-plate capacitor in vacuum are 5.00 mm apart and 2.00 m^2 in area. A potential difference of 10,000 V is applied across the capacitor. Compute

(a) the capacitance

(b) the charge on each plate, and

(c) the magnitude of the electric field in the space between them.

25. Three capacitors having capacitances of 8.4 μF, 8.2 μF and 4.2 μF are connected in series across a 36 V potential difference.

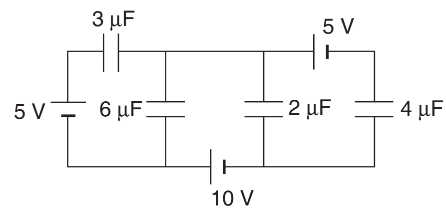
(a) What is the charge on 4.2 μF capacitor?

(b) What is the total energy stored in all three capacitors?

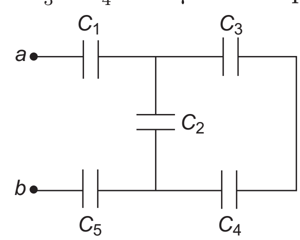
(c) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other, with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination?

(d) What is the total energy now stored in the capacitors?

26. Find the charges on 6 μF and 4 μF capacitors.



27. In figure, $C_1 = C_5 = 8.4\,\mu\text{F}$ and $C_2 = C_3 = C_4 = 4.2\,\mu\text{F}$. The applied potential is $V_{ab} = 220$ V.



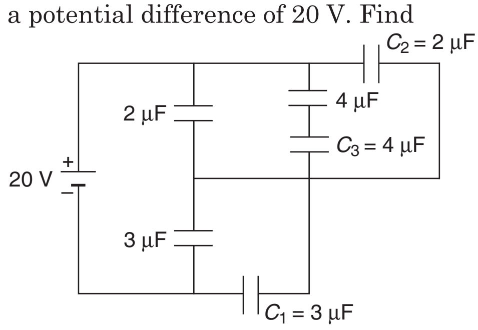
(a) What is the equivalent capacitance of the network between points a and b?

(b) Calculate the charge on each capacitor and the potential difference across each capacitor.

28. Two condensers A and B each having slabs of dielectric constant $K = 2$ are connected in series. When they are connected across 230 V supply, potential difference across A is 130 V and that across B is 100 V. If the dielectric in the condenser of smaller capacitance is replaced by one for which $K = 5$, what will be the values of potential difference across them?

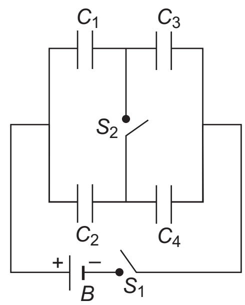
29. A capacitor of capacitance $C_1 = 1.0\,\mu\text{F}$ charged upto a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing the capacitance $C_2 = 2.0\,\mu\text{F}$ and $C_3 = 3.0\,\mu\text{F}$. What charge will flow through the connecting wires?

30. In figure, the battery has a potential difference of 20 V. Find



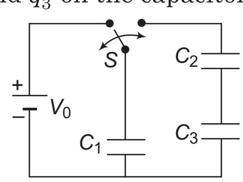
(a) the equivalent capacitance of all the capacitors across the battery and
(b) the charge stored on that equivalent capacitance.
Find the charge on
(c) capacitor 1,
(d) capacitor 2, and
(e) capacitor 3.

31. In figure, battery B supplies 12 V. Find the charge on each capacitor



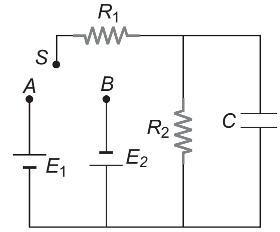
(a) first when only switch S_1 is closed and
(b) later when S_2 is also closed.
(Take $C_1 = 1.0\,\mu\text{F}, C_2 = 2.0\,\mu\text{F}, C_3 = 3.0\,\mu\text{F}$ and $C_4 = 4.0\,\mu\text{F}$)

32. When switch S is thrown to the left in figure, the plates of capacitor 1 acquire a potential difference V_0. Capacitors 2 and 3 are initially uncharged. The switch is now thrown to the right. What are the final charges q_1, q_2 and q_3 on the capacitors?



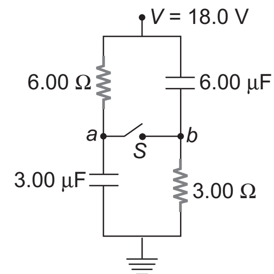
33. A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected, and the plates are pulled apart until their separation is $2d$. Derive expression in terms of A, d and V for
(a) the new potential difference
(b) the initial and final stored energies, U_i and U_f and
(c) the work required to increase the separation of plates from d to $2d$.

34. In the circuit shown in figure $E_1 = 2E_2 = 20\text{ V}, R_1 = R_2 = 10\text{ k}\Omega$ and $C = 1\,\mu\text{F}$. Find the current through R_1, R_2 and C when



(a) S has been kept connected to A for a long time.
(b) The switch is suddenly shifted to B.

35. (a) What is the steady state potential of point a with respect to point b in figure when switch S is open?

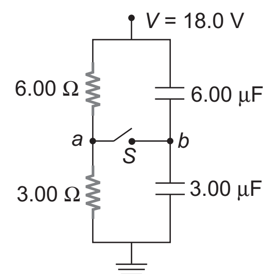


(b) Which point, a or b, is at the higher potential?

(c) What is the final potential of point b with respect to ground when switch S is closed?

(d) How much does the charge on each capacitor change when S is closed?

36. (a) What is the potential of point a with respect to point b in figure, when switch S is open?

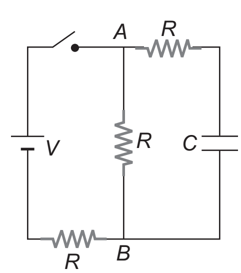


(b) Which point, a or b, is at the higher potential?

(c) What is the final potential of point b with respect to ground when switch S is closed?

(d) How much charge flows through switch S when it is closed?

37. In the circuit shown in figure, the battery is an ideal one with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.



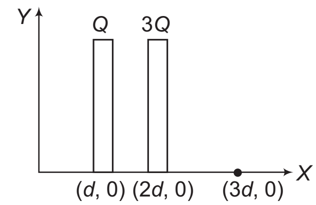
(a) Find the charge Q on the capacitor at time t.

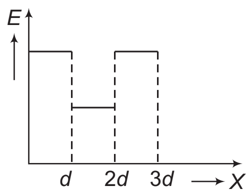
(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

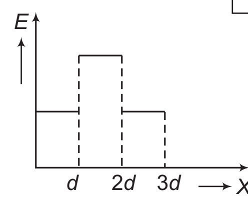
LEVEL 2

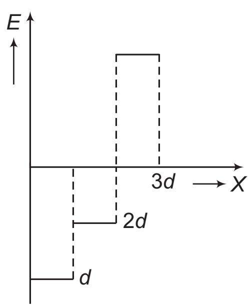
Single Correct Option

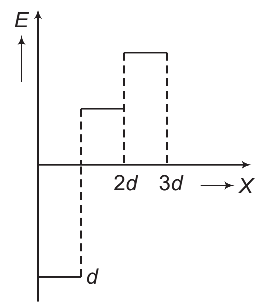
1. Two very large thin conducting plates having same cross-sectional area are placed as shown in figure. They are carrying charges Q and $3Q$, respectively. The variation of electric field as a function at x (for $x = 0$ to $x = 3d$) will be best represented by



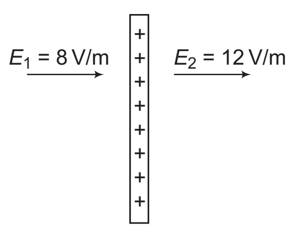
(a)


(b)


(c)


(d)


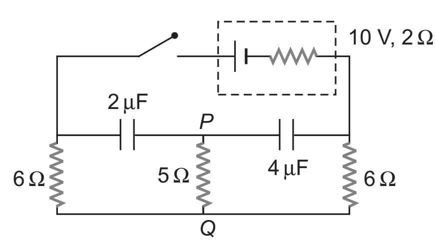
2. The electric field on two sides of a thin sheet of charge is shown in the figure. The charge density on the sheet is



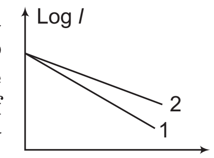
(a) $2\,\varepsilon_0$
(b) $4\,\varepsilon_0$
(c) $10\,\varepsilon_0$
(d) zero

3. In the circuit shown in figure, the capacitors are initially uncharged. The current through resistor PQ just after closing the switch is

(a) 2 A from P to Q
(b) 2 A from Q to P
(c) 6 A from P to Q
(d) zero



4. A graph between current and time during charging of a capacitor by a battery in series with a resistor is shown. The graphs are drawn for two circuits. R_1, R_2, C_1, C_2 and V_1, V_2 are the values of resistance, capacitance and EMF of the cell in the two circuits. If only two parameters (out of resistance, capacitance, EMF) are different in the two circuits. What may be the correct option(s)?

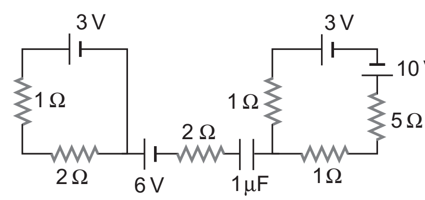


(a) $V_1 = V_2, R_1 > R_2, C_1 > C_2$
(b) $V_1 > V_2, R_1 > R_2, C_1 = C_2$
(c) $V_1 < V_2, R_1 < R_2, C_1 = C_2$
(d) $V_1 < V_2, R_1 = R_2, C_1 < C_2$

5. A capacitor of capacitance C is charged by a battery of emf E and internal resistance r. A resistance $2r$ is also connected in series with the capacitor. The amount of heat liberated inside the battery by the time capacitor is 50% charged is

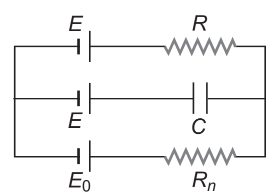
(a) $\frac{3}{8}E^2C$
(b) $\frac{E^2C}{6}$
(c) $\frac{E^2C}{12}$
(d) $\frac{E^2C}{24}$

6. For the circuit shown in the figure, determine the charge on capacitor in steady state.



(a) 4 μC
(b) 6 μC
(c) 1 μC
(d) Zero

7. For the circuit shown in the figure, find the charge stored on capacitor in steady state.

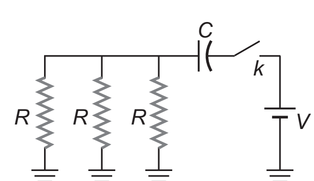


(a) $\frac{RC}{R+R_0}E$
(b) $\frac{RC}{R_0}(E-E_0)$
(c) zero
(d) $\frac{RC}{R+R_0}(E-E_0)$

8. Two similar parallel-plate capacitors each of capacity C_0 are connected in series. The combination is connected with a voltage source of V_0. Now, separation between the plates of one capacitor is increased by a distance d and the separation between the plates of another capacitor is decreased by the distance $d/2$. The distance between the plates of each capacitor was d before the change in separation. Then, select the correct choice.

(a) The new capacity of the system will increase
(b) The new capacity of the system will decrease
(c) The new capacity of the system will remain same
(d) data insufficient

9. The switch shown in the figure is closed at $t = 0$. The charge on the capacitor as a function of time is given by

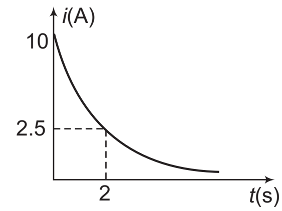


(a) $CV\ (1 - e^{-t/RC})$
(b) $3CV\ (1 - e^{-t/RC})$
(c) $CV\ (1 - e^{-3t/RC})$
(d) $CV\ (1 - e^{-t/3RC})$

10. A $2\,\mu F$ capacitor C_1 is charged to a voltage 100 V and a $4\,\mu F$ capacitor C_2 is charged to a voltage 50 V. The capacitors are then connected in parallel. What is the loss of energy due to parallel connection?

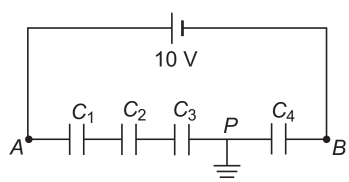
(a) 1.7 J
(b) 0.17 J
(c) 1.7×10^{-2} J
(d) 1.7×10^{-3} J

11. The figure shows a graph of the current in a discharging circuit of a capacitor through a resistor of resistance $10\,\Omega$.



(a) The initial potential difference across the capacitor is 100 V
(b) The capacitance of the capacitor is $\dfrac{1}{10 \ln 2}$ F
(c) The total heat produced in the circuit will be $\dfrac{500}{\ln 2}$ J
(d) All of the above

12. Four capacitors are connected in series with a battery of emf 10 V as shown in the figure. The point P is earthed. The potential of point A is equal in magnitude to potential of point B but opposite in sign if

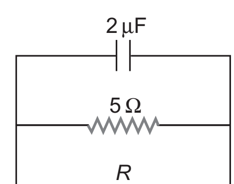


(a) $C_1 + C_2 + C_3 = C_4$
(b) $\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{1}{C_4}$
(c) $\dfrac{C_1 C_2 C_3}{C_1^2 + C_2^2 + C_3^2} = C_4$
(d) It is never possible

13. A capacitor of capacity C is charged to a potential difference V and another capacitor of capacity $2C$ is charged to a potential difference $4V$. The charging batteries are disconnected and the two capacitors are connected with reverse polarity (i.e. positive plate of first capacitor is connected to negative plate of second capacitor). The heat produced during the redistribution of charge between the capacitors will be

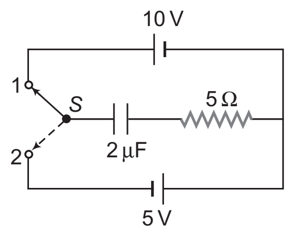
(a) $\dfrac{125\,CV^2}{3}$
(b) $\dfrac{50\,CV^2}{3}$
(c) $2CV^2$
(d) $\dfrac{25\,CV^2}{3}$

14. A capacitor of capacitance $2\,\mu F$ is charged to a potential difference of 5 V. Now, the charging battery is disconnected and the capacitor is connected in parallel to a resistor of $5\,\Omega$ and another unknown resistor of resistance R as shown in figure. If the total heat produced in $5\,\Omega$ resistance is $10\,\mu J$, then the unknown resistance R is equal to



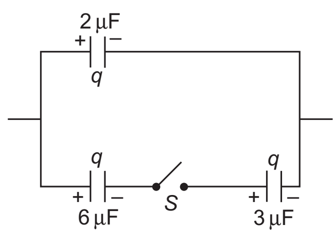
(a) $10\,\Omega$
(b) $15\,\Omega$
(c) $\dfrac{10}{3}\,\Omega$
(d) $7.5\,\Omega$

15. In the circuit shown in figure switch S is thrown to position 1 at $t = 0$. When the current in the resistor is 1 A, it is shifted to position 2. The total heat generated in the circuit after shifting to position 2 is



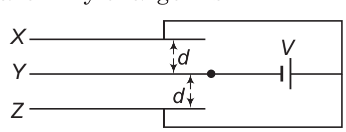
(a) zero (b) 625 μJ (c) 100 μJ (d) None of these

16. The flow of charge through switch S if it is closed is



(a) zero (b) $q/4$ (c) $2q/3$ (d) $q/3$

17. Consider the arrangement of three plates X, Y and Z each of the area A and separation d. The energy stored when the plates are fully charged is

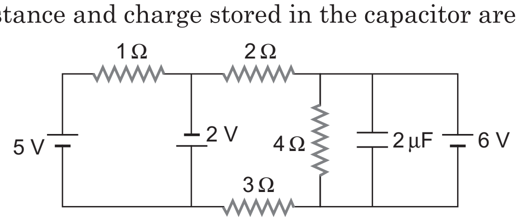


(a) $\varepsilon_0 AV^2/2d$
(b) $\varepsilon_0 AV^2/d$
(c) $2\varepsilon_0 AV^2/d$
(d) $3\varepsilon_0 AV^2/d$

18. Consider a capacitor – charging circuit. Let Q_1 be the charge given to the capacitor in time interval of 20 ms and Q_2 be the charge given in the next time interval of 20 ms. Let 10 μC charge be deposited in a time interval t_1 and the next 10 μC charge is deposited in the next time interval t_2. Then,

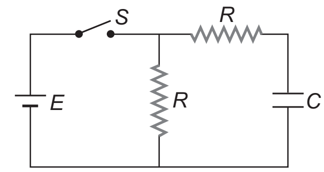
(a) $Q_1 > Q_2, t_1 > t_2$
(b) $Q_1 > Q_2, t_1 < t_2$
(c) $Q_1 < Q_2, t_1 > t_2$
(d) $Q_1 < Q_2, t_1 < t_2$

19. The current in 1 Ω resistance and charge stored in the capacitor are



(a) 4 A , 6 μC (b) 7 A, 12 μC (c) 4 A , 12 μC (d) 7 A, 6 μC

20. A capacitor C is connected to two equal resistances as shown in the figure. Consider the following statements.



(i) At the time of charging of capacitor time constant of the circuit is $2CR$

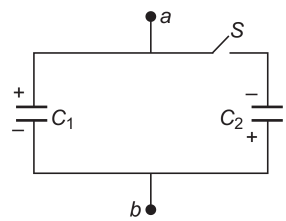
(ii) At the time of discharging of the capacitor the time constant of the circuit is CR

(iii) At the time of discharging of the capacitor the time constant of the circuit is $2CR$

(iv) At the time of charging of capacitor the time constant of the circuit is CR

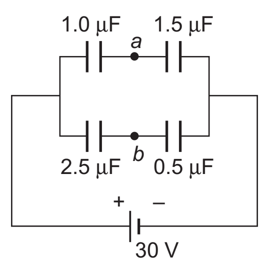
(a) Statements (i) and (ii) only are correct
(b) Statements (ii) and (iii) only are correct
(c) Statements (iii) and (iv) only are correct
(d) Statements (i) and (iii) only are correct

21. Two capacitors $C_1 = 1\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$ each are charged to a potential difference of 100 V but with opposite polarity as shown in the figure. When the switch S is closed, the new potential difference between the points a and b is



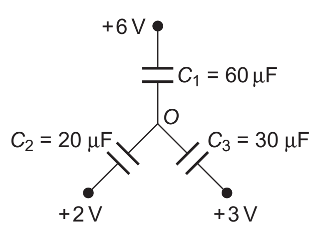
(a) 200 V (b) 100 V (c) 50 V (d) 25 V

22. Four capacitors are connected as shown in figure to a 30 V battery. The potential difference between points a and b is



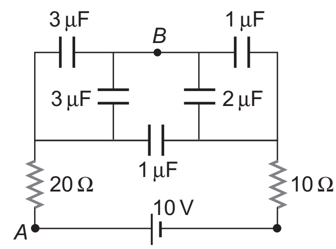
(a) 5 V
(b) 9 V
(c) 10 V
(d) 13 V

23. Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected to one another as shown in figure. The potential at O will be



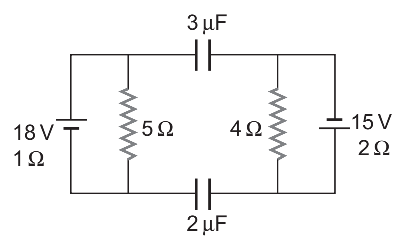
(a) 3 V (b) $\frac{49}{11}$ V (c) 4 V (d) $\frac{3}{11}$ V

24. In the circuit shown in figure, the potential difference between the points A and B in the steady state is



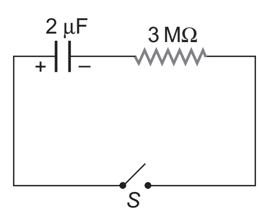
(a) zero (b) 6 V (c) 4 V (d) $\frac{10}{3}$ V

25. Two cells, two resistors and two capacitors are connected as shown in figure. The charge on 2 μF capacitor is



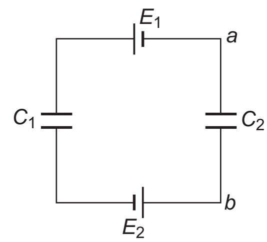
(a) 30 μC (b) 20 μC (c) 25 μC (d) 48 μC

26. In the circuit shown in figure, the capacitor is charged with a cell of 5 V. If the switch is closed at $t = 0$, then at $t = 12$ s, charge on the capacitor is



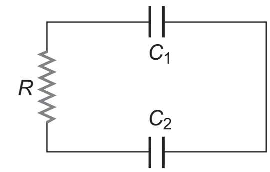
(a) (0.37) 10 μC (b) $(0.37)^2$ 10 μC (c) (0.63) 10 μC (d) $(0.63)^2$ 10 μC

27. The potential difference between points a and b of circuits shown in the figure is



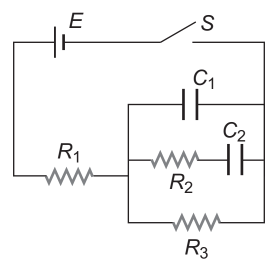
(a) $\left(\frac{E_1 + E_2}{C_1 + C_2}\right) C_2$ (b) $\left(\frac{E_1 - E_2}{C_1 + C_2}\right) C_2$ (c) $\left(\frac{E_1 + E_2}{C_1 + C_2}\right) C_1$ (d) $\left(\frac{E_1 - E_2}{C_1 + C_2}\right) C_1$

28. A capacitor C_1 is charged to a potential V and connected to another capacitor in series with a resistor R as shown. It is observed that heat H_1 is dissipated across resistance R, till the circuit reaches steady state. Same process is repeated using resistance of $2R$. If H_2 is heat dissipated in this case, then



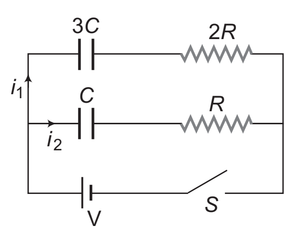
(a) $\frac{H_2}{H_1} = 1$
(b) $\frac{H_2}{H_1} = 4$
(c) $\frac{H_2}{H_1} = \frac{1}{4}$
(d) $\frac{H_2}{H_1} = 2$

29. In the circuit diagram, the current through the battery immediately after the switch S is closed is



(a) zero
(b) $\frac{E}{R_1}$
(c) $\frac{E}{R_1 + R_2}$
(d) $\frac{E}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$

30. In the circuit shown, switch S is closed at $t = 0$. Let i_1 and i_2 be the current at any finite time t, then the ratio i_1 / i_2

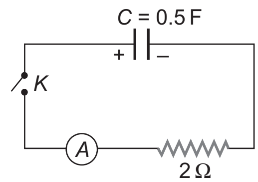


(a) is constant
(b) increases with time
(c) decreases with time
(d) first increases and then decreases

31. A leaky parallel capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12} \Omega^{-1}\, m^{-1}$. Charge on the plate at instant $t = 0$ is $q = 8.885 \mu C$. Then, time constant of leaky capacitor is

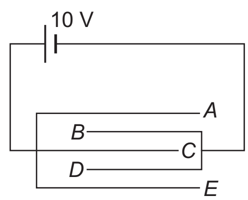
(a) 3 s
(b) 4 s
(c) 5 s
(d) 6 s

32. A charged capacitor is allowed to discharge through a resistor by closing the key at the instant $t = 0$. At the instant $t = (\ln 4)\,\mu s$, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to



(a) $0.5\,\Omega$
(b) $1\,\Omega$
(c) $2\,\Omega$
(d) $4\,\Omega$

33. Five identical capacitor plates are arranged such that they make four capacitors each of $2\,\mu F$. The plates are connected to a source of emf 10 V. The charge on plate C is



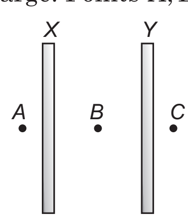
(a) $+20\,\mu C$
(b) $+40\,\mu C$
(c) $+60\,\mu C$
(d) $+80\,\mu C$

34. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now

(a) V
(b) $V + \dfrac{Q}{C}$
(c) $V + \dfrac{Q}{2C}$
(d) $V - \dfrac{Q}{C}$, if $Q < CV$

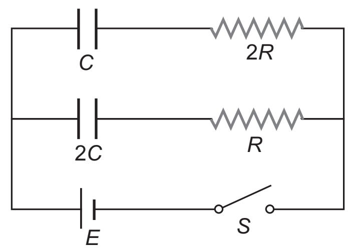
More than One Correct Options

1. X and Y are large, parallel conducting plates close to each other. Each face has an area A. X is given a charge Q. Y is without any charge. Points A, B and C are as shown in the figure.



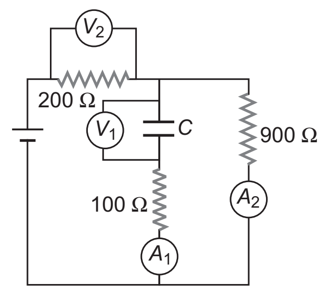
(a) The field at B is $\dfrac{Q}{2\varepsilon_0 A}$
(b) The field at B is $\dfrac{Q}{\varepsilon_0 A}$
(c) The fields at A, B and C are of the same magnitude
(d) The fields at A and C are of the same magnitude, but in opposite directions

2. In the circuit shown in the figure, switch S is closed at time $t = 0$. Select the correct statements.



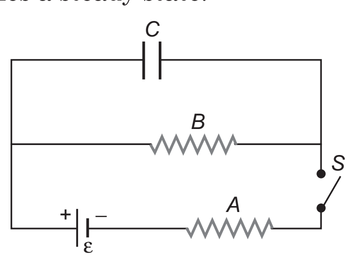
(a) Rate of increase of charge is same in both the capacitors
(b) Ratio of charge stored in capacitors C and $2C$ at any time t would be 1 : 2
(c) Time constants of both the capacitors are equal
(d) Steady state charges on capacitors C and $2C$ are in the ratio of 1 : 2

3. An electrical circuit is shown in the given figure. The resistance of each voltmeter is infinite and each ammeter is 100 Ω. The charge on the capacitor of 100 μF in steady state is 4 mC. Choose correct statement(s) regarding the given circuit.



(a) Reading of voltmeter V_2 is 16 V
(b) Reading of ammeter A_1 is zero and A_2 is 1/25 A
(c) Reading of voltmeter V_1 is 40 V
(d) Emf of the ideal cell is 66 V

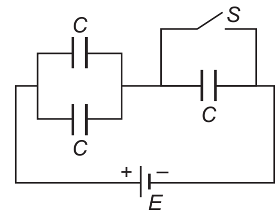
4. In the circuit shown, A and B are equal resistances. When S is closed, the capacitor C charges from the cell of emf ε and reaches a steady state.



(a) During charging, more heat is produced in A than in B
(b) In steady state, heat is produced at the same rate in A and B
(c) In the steady state, energy stored in C is $\frac{1}{4}C\varepsilon^2$
(d) In the steady state energy stored in C is $\frac{1}{8}C\varepsilon^2$

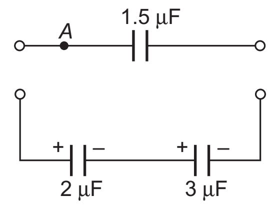
5. A parallel-plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased
(a) The force of attraction between the plates will decrease
(b) The field in the region between the plates will not change
(c) The energy stored in the capacitor will increase
(d) The potential difference between the plates will decrease

6. In the circuit shown, each capacitor has a capacitance C. The emf of the cell is E. If the switch S is closed, then



(a) positive charge will flow out of the positive terminal of the cell
(b) positive charge will enter the positive terminal of the cell
(c) the amount of the charge flowing through the cell will be $\frac{1}{3}CE$
(d) the amount of charge flowing through the cell is $\left(\frac{4}{3}\right)CE$

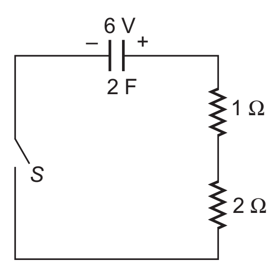
7. Two capacitors of 2 μF and 3 μF are charged to 150 V and 120 V, respectively. The plates of capacitor are connected as shown in the figure. An uncharged capacitor of capacity 1.5 μF falls to the free end of the wire. Then,



(a) charge on 1.5 μF capacitor is 180 μC
(b) charge on 2 μF capacitor is 120 μC
(c) positive charge flows through A from right to left
(d) positive charge flows through A from left to right

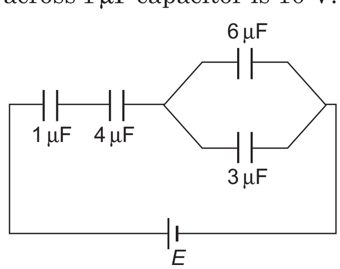
8. A parallel plate capacitor is charged and then the battery is disconnected. When the plates of the capacitor are brought closer, then
(a) energy stored in the capacitor decreases
(b) the potential difference between the plates decreases
(c) the capacitance increases
(d) the electric field between the plates decreases

9. A capacitor of 2 F (practically not possible to have a capacity of 2 F) is charged by a battery of 6 V. The battery is removed and circuit is made as shown. Switch is closed at time $t = 0$. Choose the correct options.



(a) At time $t = 0$ current in the circuit is 2 A
(b) At time $t = (6 \ln 2)$ second, potential difference across capacitor is 3 V
(c) At time $t = (6 \ln 2)$ second, potential difference across 1 Ω resistance is 1 V
(d) At time $t = (6 \ln 2)$ second, potential difference across 2 Ω resistance is 2 V.

10. Given that potential difference across 1 μF capacitor is 10 V. Then,

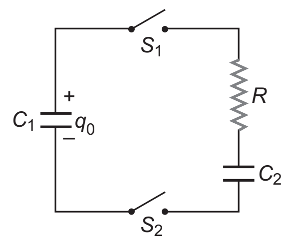


(a) potential difference across 4 μF capacitor is 40 V
(b) potential difference across 4 μF capacitor is 2.5 V
(c) potential difference across 3 μF capacitor is 5 V
(d) value of E is 50 V

Comprehension Based Questions

Passage I (Q. No. 1 and 2)

The capacitor C_1 in the figure shown initially carries a charge q_0. When the switches S_1 and S_2 are closed, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which is initially uncharged.



1. The charge flown through wires as a function of time t is

(a) $q_0 e^{-t/RC} + \dfrac{C}{C_2} q_0$

(b) $\dfrac{q_0 C}{C_1} \times [1 - e^{-t/RC}]$

(c) $q_0 \dfrac{C}{C_1} e^{-t/CR}$

(d) $q_0 e^{-t/RC}$

where, $C = \dfrac{C_1 C_2}{C_1 + C_2}$

2. The total heat dissipated in the circuit during the discharging process of C_1 is

(a) $\dfrac{q_0^2}{2C_1^2} \times C$

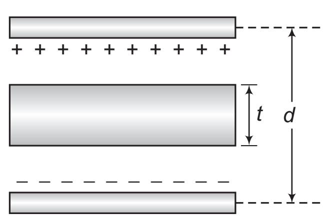
(b) $\dfrac{q_0^2}{2C}$

(c) $\dfrac{q_0^2 C_2}{2C_1^2}$

(d) $\dfrac{q_0^2}{2C_1 C_2}$

Passage II (Q. No. 3 and 4)

Figure shows a parallel plate capacitor with plate area A and plate separation d. A potential difference is being applied between the plates. The battery is then disconnected and a dielectric slab of dielectric constant K is placed in between the plates of the capacitor as shown.



Now, answer the following questions based on above information.

3. The electric field in the gaps between the plates and the dielectric slab will be

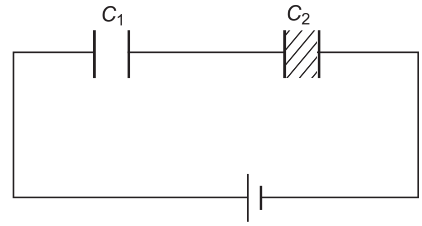
(a) $\dfrac{\varepsilon_0 AV}{d}$ (b) $\dfrac{V}{d}$ (c) $\dfrac{KV}{d}$ (d) $\dfrac{V}{d-t}$

4. The electric field in the dielectric slab is

(a) $\dfrac{V}{Kd}$ (b) $\dfrac{KV}{d}$ (c) $\dfrac{V}{d}$ (d) $\dfrac{KV}{t}$

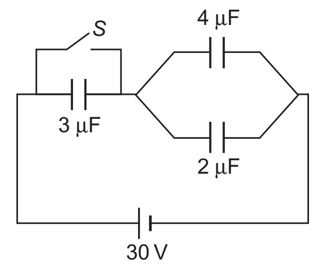
Match the Columns

1. In the figure shown, $C_1 = 4\,\mu\text{F}$ (without dielectric) and $C_2 = 4\,\mu\text{F}$ (with a dielectric slab of dielectric constant $K = 2$) . Now, the same slab after removing from C_2 is filled in C_1. Then, match the following two columns.



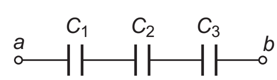
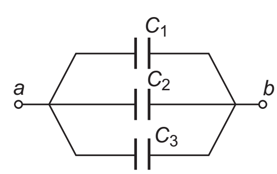
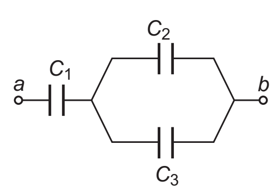
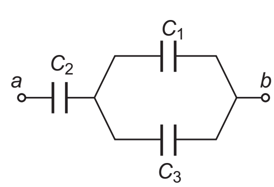
Column I	Column II
(a) Charge on C_2	(p) will increase
(b) Energy stored in C_2	(q) will decrease
(c) Potential difference across C_2	(r) will remain same
(d) Electric field between the plates of C_2	(s) data insufficient

2. In the circuit shown in figure, match the following two columns for the flow of charge when switch is closed.

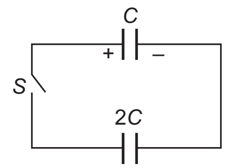


Column I	Column II
(a) From the battery	(p) $40\,\mu\text{C}$
(b) From $2\,\mu\text{F}$ capacitor	(q) $100\,\mu\text{C}$
(c) From $3\,\mu\text{F}$	(r) $60\,\mu\text{C}$
(d) From $4\,\mu\text{F}$ capacitor	(s) None of these

3. Three identical capacitors are connected in three different configurations as shown in Column II. Points a and b are connected with a battery. Match the two columns.

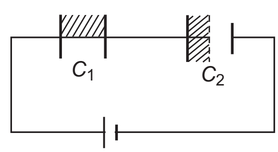
Column I	Column II
(a) Maximum charge on C_1	(p) 
(b) Minimum charge on C_2	(q) 
(c) Maximum potential difference across C_1	(r) 
(d) Minimum potential difference across C_1	(s) 

4. A capacitor C is charged by a battery of V volts. Then, it is connected to an uncharged capacitor of capacity $2C$ as shown in figure. Now, match the following two columns.



Column I	Column II
(a) After closing the switch energy stored in C.	(p) $\frac{1}{9}CV^2$
(b) After closing the switch energy stored in $2C$.	(q) $\frac{1}{6}CV^2$
(c) After closing the switch loss of energy during redistribution of charge.	(r) $\frac{1}{18}CV^2$
	(s) None of these

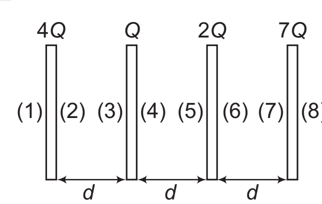
5. Two identical sized capacitors C_1 and C_2 are connected with a battery as shown in figure. Capacitor plates are square plates. A dielectric slab of dielectric constant $K = 2$, is filled in half the region of the two capacitors as shown :



$C \rightarrow$ capacity, $q \rightarrow$ charge stored, $U \rightarrow$ energy stored. Match the following two columns.

Column I	Column II
(a) C_1/C_2	(p) $9/4$
(b) q_1/q_2	(q) $4/9$
(c) U_1/U_2	(r) $4/3$
	(s) None of these

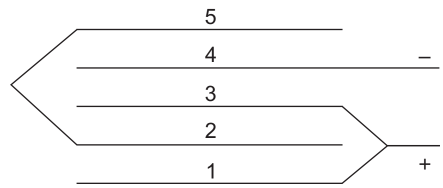
6. Four large parallel identical conducting plates are arranged as shown.



Column I	Column II
(a) Surfaces having charges of same magnitude and sign	(p) 1 and 8
(b) Surfaces having positive charges	(q) 3 and 5
(c) Uncharged surfaces	(r) 2 and 3
(d) Charged surfaces	(s) 6 and 7

Subjective Questions

1. Five identical conducting plates, 1, 2, 3,4 and 5 are fixed parallel plates equidistant from each other (see figure). A conductor connects plates 2 and 5 while another conductor joins 1 and 3. The junction of 1 and 3 and the plate 4 are connected to a source of constant emf V_0. Find

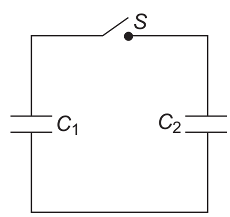


(a) the effective capacity of the system between the terminals of source.

(b) the charges on the plates 3 and 5.

Given, d = distance between any two successive plates and A = area of either face of each plate.

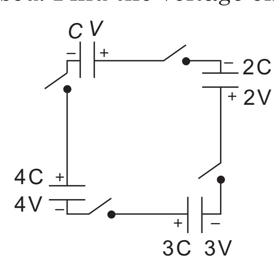
2. A $8\,\mu F$ capacitor C_1 is charged to $V_0 = 120$ V. The charging battery is then removed and the capacitor is connected in parallel to an uncharged $+\,4\mu F$ capacitor C_2.



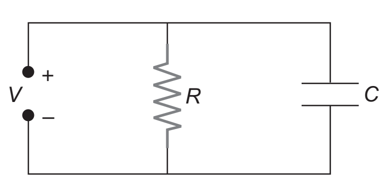
(a) what is the potential difference V across the combination?

(b) what is the stored energy before and after the switch S is closed?

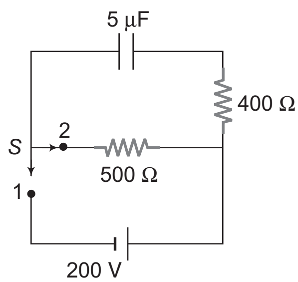
3. Condensers with capacities $C, 2C, 3C$ and $4C$ are charged to the voltage, $V, 2\,V, 3\,V$ and $4\,V$ correspondingly. The circuit is closed. Find the voltage on all condensers in the equilibrium.



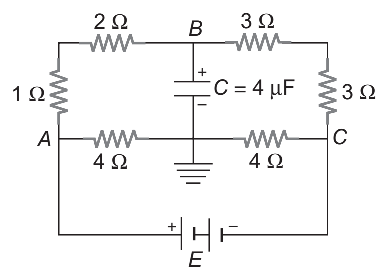
4. In the circuit shown, a time varying voltage $V = 2000t$ volt is applied where t is in second. At time $t = 5$ ms, determine the current through the resistor $R = 4\,\Omega$ and through the capacitor $C = 300\,\mu F$.



5. A capacitor of capacitance $5\,\mu F$ is connected to a source of constant emf of 200 V. Then, the switch was shifted to contact 2 from contact 1. Find the amount of heat generated in the $400\,\Omega$ resistance.

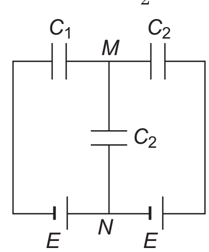


6. Analyze the given circuit in the steady state condition. Charge on the capacitor is $q_0 = 16\,\mu C$.

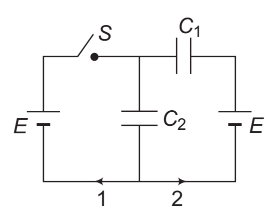


(a) Find the current in each branch
(b) Find the emf of the battery.
(c) If now the battery is removed and the points A and C are shorted. Find the time during which charge on the capacitor becomes $8\,\mu C$.

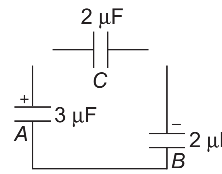
7. Find the potential difference between points M and N of the system shown in figure, if the emf is equal to $E = 110$ V and the capacitance ratio $\dfrac{C_1}{C_2}$ is 2.



8. In the given circuit diagram, find the charges which flow through directions 1 and 2 when switch S is closed.



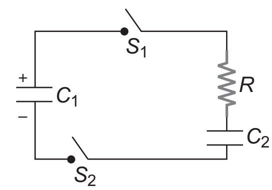
9. Two capacitors A and B with capacities $3\,\mu\text{F}$ and $2\,\mu\text{F}$ are charged to a potential difference of 100 V and 180 V, respectively. The plates of the capacitors are connected as shown in figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\,\mu\text{F}$ capacitor C with lead wires falls on the free ends to complete the circuit. Calculate



(i) the final charge on the three capacitors,

(ii) the amount of electrostatic energy stored in the system before and after completion of the circuit.

10. The capacitor C_1 in the figure initially carries a charge q_0. When the switch S_1 and S_2 are closed, capacitor C_1 is connected to a resistor R and a second capacitor C_2, which initially does not carry any charge.



(a) Find the charges deposited on the capacitors in steady state and the current through R as a function of time.

(b) What is heat lost in the resistor after a long time of closing the switch?

11. A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\text{m}^{-1}$. If the charge on the capacitor at the instant $t = 0$ is $q_0 = 8.55\,\mu\text{C}$, then calculate the leakage current at the instant $t = 12$ s.

12. A parallel plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed.

(a) What is the total energy stored in the capacitor?

(b) The plates are pulled apart an additional distance dx. What is the change in the stored energy?

(c) If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = Fdx$ done in pulling the plates apart. Find an expression for F.

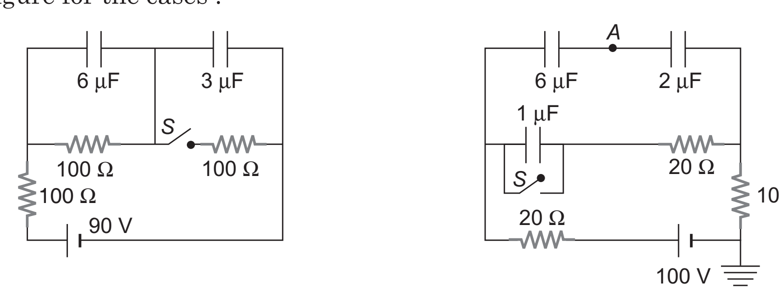
(d) Explain why F is not equal to QE, where E is the electric field between the plates.

13. A spherical capacitor has the inner sphere of radius 2 cm and the outer one of 4 cm. If the inner sphere is earthed and the outer one is charged with a charge of $2\,\mu\text{C}$ and isolated. Calculate

(a) the potential to which the outer sphere is raised.

(b) the charge retained on the outer surface of the outer sphere.

14. Calculate the charge on each capacitor and the potential difference across it in the circuits shown in figure for the cases :



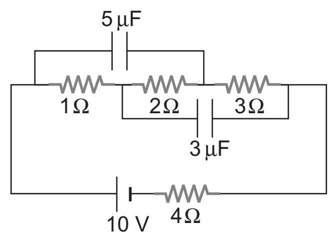
(a) (b)

(i) switch S is closed and

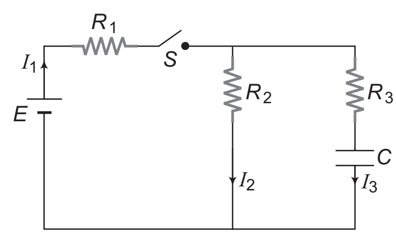
(ii) switch S is open.

(iii) In figure (b), what is the potential of point A when S is open?

15. In the shown network, find the charges on capacitors of capacitances $5\,\mu F$ and $3\,\mu F$, in steady state.

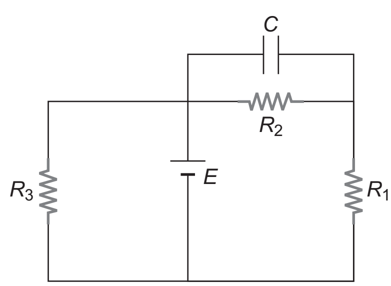


16. In the circuit shown, $E = 18\,kV$, $C = 10\,\mu F$, $R_1 = 4\,M\Omega$, $R_2 = 6\,M\Omega$, $R_3 = 3\,M\Omega$. With C completely uncharged, switch S is suddenly closed (at $t = 0$).

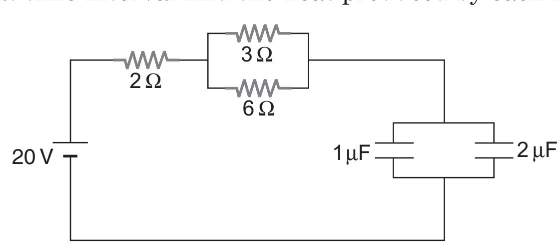


(a) Determine the current through each resistor for $t = 0$ and $t = \infty$.
(b) What are the values of V_2 (potential difference across R_2) at $t = 0$ and $t = \infty$?
(c) Plot a graph of the potential difference V_2 *versus* t and determine the instantaneous value of V_2.

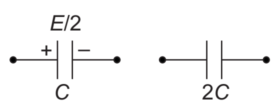
17. The charge on the capacitor is initially zero. Find the charge on the capacitor as a function of time t. All resistors are of equal value R.

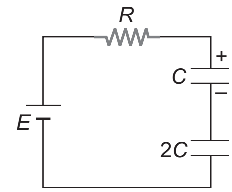


18. The capacitors are initially uncharged. In a certain time the capacitor of capacitance $2\,\mu F$ gets a charge of $20\,\mu C$. In that time interval find the heat produced by each resistor individually.



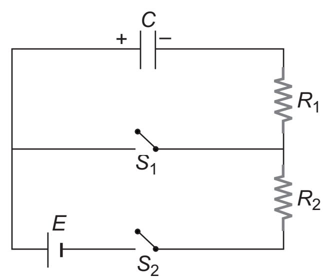
19. A capacitor of capacitance C has potential difference $E/2$ and another capacitor of capacitance $2C$ is uncharged. They are joined to form a closed circuit as shown in the figure.



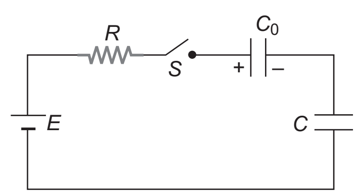


(a) Find the current in the circuit at $t = 0$.
(b) Find the charge on C as a function of time.

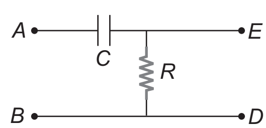
20. The capacitor shown in figure has been charged to a potential difference of V volt, so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at $t = 0$. At $t = R_1C$ switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at $t = 2R_1C + R_2C$.

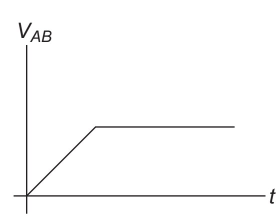


21. The switch S is closed at $t = 0$. The capacitor C is uncharged but C_0 has a charge $Q_0 = 2\,\mu\text{C}$ at $t = 0$. If $R = 100\,\Omega, C = 2\,\mu\text{F}, C_0 = 2\,\mu\text{F}, E = 4$ V. Calculate $i(t)$ in the circuit.



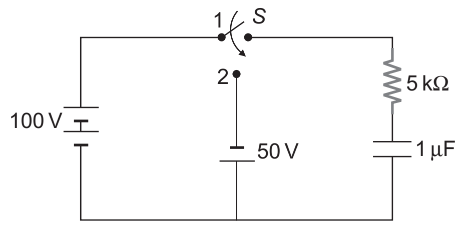
22. A time varying voltage is applied to the clamps A and B such that voltage across the capacitor plates is as shown in the figure. Plot the time dependence of voltage across the terminals of the resistance E and D.



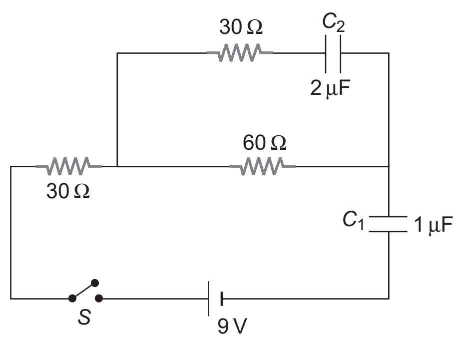


23. In the above problem if given graph is between V_{AB} and time. Then, plot graph between V_{ED} and time.

24. Initially, the switch is in position 1 for a long time. At $t = 0$, the switch is moved from 1 to 2. Obtain expressions for V_C and V_R for $t > 0$.



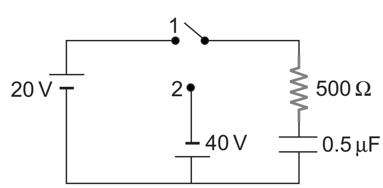
25. For the arrangement shown in the figure, the switch is closed at $t = 0$. C_2 is initially uncharged while C_1 has a charge of $2\,\mu C$.



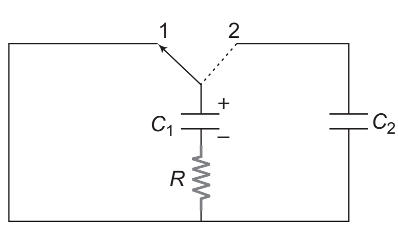
(a) Find the current coming out of the battery just after the switch is closed.

(b) Find the charge on the capacitors in the steady state condition.

26. In the given circuit, the switch is closed in the position 1 at $t = 0$ and then moved to 2 after $250\,\mu s$. Derive an expression for current as a function of time for $t > 0$. Also plot the variation of current with time.



27. A charged capacitor C_1 is discharged through a resistance R by putting switch S in position 1 of circuit shown in figure. When discharge current reduces to I_0, the switch is suddenly shifted to position 2. Calculate the amount of heat liberated in resistance R starting from this instant.



Answers

Introductory Exercise 25.1

1. $[M^{-1} L^{-2} T^4 A^2]$ **2.** False **3.** (a) –10 V (b) –10 μC , –20 μC (c) 3.0×10^{-4} J

Introductory Exercise 25.2

1. ±182 μC **2.** (a) 604 V (b) 90.8 cm^2 (c) 16.3 μC/m^2 **3.** (a) 1.28 (b) 6.2×10^{-7} C/m^2

Introductory Exercise 25.3

1. $q_{3\mu F} = 30\,\mu C$, $q_{4\mu F} = 20\,\mu C$, $q_{2\mu F} = 10\,\mu C$ **2.** $q_{4\mu F} = 120\,\mu C$, $q_{9\mu F} = 90\,\mu C$, $q_{3\mu F} = 30\,\mu C$

Exercises

LEVEL 1

Assertion and Reason

1. (d) **2.** (a) **3.** (a,b) **4.** (a,b) **5.** (d) **6.** (d) **7.** (b) **8.** (d) **9.** (b) **10.** (b)

Objective Questions

1. (c) **2.** (d) **3.** (c) **4.** (c) **5.** (d) **6.** (d) **7.** (c) **8.** (b) **9.** (b) **10.** (c)
11. (a) **12.** (b) **13.** (b) **14.** (c) **15.** (c) **16.** (a) **17.** (d) **18.** (b) **19.** (b) **20.** (b)
21. (a) **22.** (c) **23.** (d) **24.** (c) **25.** (d) **26.** (b) **27.** (b)

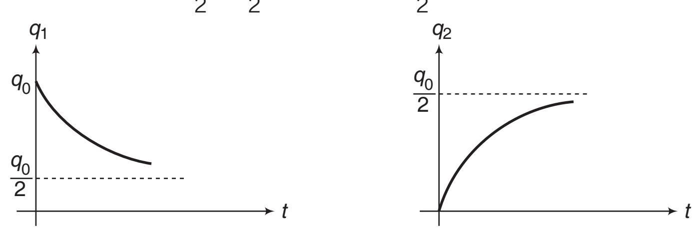
Subjective Questions

1. Starting from the left face the charges are, 3 μC, 7 μC, –7 μC, 3 μC **2.** $\frac{\varepsilon_0 A}{d}, \frac{5dq}{2A\varepsilon_0}$

3. 10 μC, 20 μC, 30 μC **4.** 40 μC **5.** 24 μC

6. (a) 800 μC, 800 V, 800 μC, 400 V (b) $\frac{1600}{3}$ μC, $\frac{3200}{3}$ μC, $\frac{1600}{3}$ V **7.** 400 μF

9. 0.69 **10.** $q_1 = \frac{q_0}{2} + \frac{q_0}{2} e^{-2t/RC}$, $q_2 = \frac{q_0}{2}(1 - e^{-2t/RC})$



11. $CE(1 - e^{-t/CR}) + q_0 \; e^{-t/CR}$

12. (a) E/R_1 (b) $E/(R_1 + R_3)$ **13.** (a) $i_1 = E/R_1$, $i_2 = E/R_2$ (b) $i_1 = E/R_1$, $i_2 = 0$ (c) $\frac{1}{2}CE^2$ (d) $C(R_1 + R_2)$

14. (a) $\frac{5C}{3}$ (b) $\frac{4C}{3}$ (c) $2C$

15. (a) 4.0 μF : 2.64×10^{-3}C, 660 V, 6.0 μF : 3.96×10^{-3} C, 660 V

(b) 4.0 μF : 5.28×10^{-4}C, 132 V, 6.0 μF : 7.92×10^{-4}C, 132 V

16. 2.83×10^{-2} J/m^3 **17.** 0.0135 m^2 **18.** 40 mF, 10 mF **19.** 10 V, 5 V

20. (a) Five capacitors in series (b) Six rows of three capacitors in each row.

21. 0.16 μF, 0.24 μF **22.** (a) 120μC (b) 60μC (c) 480μC **23.** 19.6 J

24. (a) 3.54×10^{-9} F (b) ±35.4 μC (c) 2.0×10^{6} N/C

25. (a) 76 μC (b) 1.4 mJ (c) 11 V (d) 1.2 mJ **26.** $10\mu C, \frac{40}{3}\mu C$

27. (a) 2.5μF (b) $Q_1 = 5.5\times 10^{-4}$ C, $V_1 = 66$ V, $Q_2 = 3.7\times 10^{-4}$C, $V_2 = 88$ V, $Q_3 = Q_4 = 1.8\times 10^{-4}$ C, $V_3 = V_4 = 44$ V, $Q_5 = Q_1, V_5 = V_1$

28. 78.68 V, 151.32 V **29.** 60μC **30.** (a) 3μF (b) 60μC (c) 30μC (d) 20μC (e) 20μC

31. (a) $q_1 = q_3 = 9\,\mu C$, $q_2 = q_4 = 16\,\mu C$ (b) $q_1 = 8.64\,\mu C$, $q_2 = 17.28\,\mu C$, $q_3 = 10.08\,\mu C$, $q_4 = 13.44\,\mu C$

32. $q_2 = q_3 = \dfrac{C_1V_0}{1+\dfrac{C_1}{C_2}+\dfrac{C_1}{C_3}}$, $q_1 = C_1V_0 - \dfrac{C_1V_0}{1+\dfrac{C_1}{C_2}+\dfrac{C_1}{C_3}}$

33. (a) 2 V (b) $U_i = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2, U_f = \left(\frac{\varepsilon_0 A}{d}\right)V^2$ (c) $W = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$

34. (a) 1 mA, 1 mA, 0 (b) 2 mA, 1 mA, 3 mA **35.** (a) 18 V (b) a is at higher potential (c) 6 V (d) –36 μC on both the capacitors

36. (a) –6.0 V (b) b (c) 6.0 V (d) –54.0 μC

37. (a) $\frac{CV}{2}(1-e^{-\alpha t})$ (b) $\frac{V}{2R} - \frac{V}{6R}e^{-\alpha t}, \frac{V}{2R}$ Here $\alpha = \frac{2}{3RC}$

LEVEL 2

Single Correct Option

1.(c)	**2.**(b)	**3.**(d)	**4.**(c)	**5.**(d)	**6.**(d)	**7.**(d)	**8.**(b)	**9.**(c)	**10.**(d)
11.(d)	**12.**(b)	**13.**(d)	**14.**(c)	**15.**(c)	**16.**(a)	**17.**(b)	**18.**(b)	**19.**(b)	**20.**(c)
21.(c)	**22.**(d)	**23.**(b)	**24.**(d)	**25.**(a)	**26.**(b)	**27.**(c)	**28.**(a)	**29.**(b)	**30.**(b)
31.(d)	**32.**(c)	**33.**(b)	**34.**(c)						

More than One Correct Options

1. (a,c,d) **2.** (b,c,d) **3.** (b,c) **4.** (a,b,d) **5.** (b,c) **6.** (a,d) **7.** (a,b,d) **8.** (a,b,c) **9.** (a,b,c,d) **10.** (b)

Comprehension Based Questions

1.(b) **2.**(a) **3.**(b) **4.**(a)

Match the Columns

1.	(a) → q	(b) → p	(c) → p	(d) → p
2.	(a) → s	(b) → p	(c) → r	(d) → s
3.	(a) → q	(b) → p,r	(c) → q	(d) → p,s
4.	(a) → r	(b) → p	(c) → s	
5.	(a) → s	(b) → s	(c) → s	
6.	(a) → p	(b) → p,q	(c) → s	(d) → p,q,r

Subjective Questions

1. (a) $\frac{5}{3}\left(\frac{\varepsilon_0 A}{d}\right)$ (b) $q_3 = \frac{4}{3}\left(\frac{\varepsilon_0 A V_0}{d}\right), q_5 = \frac{2}{3}\left(\frac{\varepsilon_0 A V_0}{d}\right)$ **2.** (a) 80 V (b) 57.6 mJ, 38.4 mJ

3. $-\frac{19}{5}$ V, $-\frac{2}{5}$ V, $\frac{7}{5}$ V, $\frac{14}{5}$ V **4.** 2.5 A, 0.6 A **5.** 44.4 mJ

6. (a) 3 A, 2.67A (b) 24 V (c) 11.1 μs **7.** $V_N - V_M = \frac{110}{3}$ volt **8.** $q_1 = EC_2, q_2 = \frac{-EC_1C_2}{(C_1 + C_2)}$

9. (i) 90μC, 210μC, 150μC (ii) (a) 47.4 mJ (b) 18 mJ

10. (a) $q_1 = \left(\frac{C_1}{C_1 + C_2}\right) q_0$ and $q_2 = \left(\frac{C_2}{C_1 + C_2}\right) q_0$, $i = \frac{q_0}{RC_1} e^{-t/RC}$ (b) $\Delta H = \frac{q_0^2 C_2}{2C_1(C_1 + C_2)}$, here $C = \frac{C_1 C_2}{C_1 + C_2}$

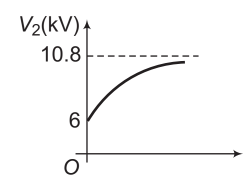
11. 0.193 μA **12.** (a) $\frac{Q^2 x}{2\varepsilon_0 A}$ (b) $\left(\frac{Q^2}{2\varepsilon_0 A}\right) \cdot dx$ (c) $\frac{Q^2}{2\varepsilon_0 A}$ **13.** (a) 2.25×10^5 V (b) +1 μC

14.

		Fig. (a)		Fig. (b)		
		6 μF	**3** μF	**1** μF	**6** μF	**2** μF
(i)	PD (volts)	30	30	0	10	30
	charge (μC)	180	90	0	60	60
(ii)	PD (volts)	0	90	100	25	75
	charge (μC)	0	270	100	150	150
(iii)	$V_A = 75$ volt					

15. 15 μC, 15 μC

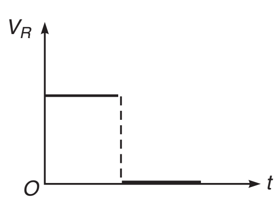
16. (a) At $t = 0$, $i_1 = 3$ mA, $i_2 = 1$ mA, $i_3 = 2$ mA At $t = \infty$,
$i_1 = i_2 = 1.8$ mA, $i_3 = 0$ (b) At $t = 0$, $V_2 = 6$ kV At $t = \infty$, $V_2 = 10.8$ kV
(c) $V_2 = (10.8 - 4.8\, e^{-t/54})$ kV



17. $q = \frac{CE}{2}\left(1 - e^{-\frac{2t}{CR}}\right)$ **18.** $H_2 = 0.075$ mJ, $H_3 = 0.05$ mJ, $H_6 = 0.025$ mJ

19. (a) $\frac{E}{2R}$ (b) $\frac{CE}{6}[5 - 2e^{-3t/2RC}]$ **20.** $EC\left(1 - \frac{1}{e}\right) + \frac{VC}{e^2}$ **21.** $(0.03\, e^{-10^4 t})$ A

22.



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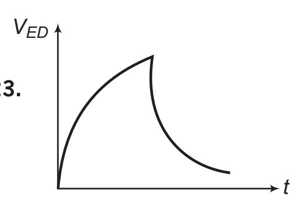


24. $V_C = 50\,(3\, e^{-200t} - 1)$, $V_R = 150\, e^{-200t}$ **25.** (a) $\frac{7}{50}$ A or $\frac{11}{50}$ A (b) $Q_1 = 9\,\mu C$, $Q_2 = 0$

26. $i = (0.04 e^{-4000t})$ A for $t \le 250$ μs, $= -(0.11\, e^{-4000t})$ A for $t \ge 250$ μs
For i - t graph, see the hints.

27. $\frac{(I_0 R)^2 C_1 C_2}{2(C_1 + C_2)}$

26

Magnetics

Chapter Contents

26.1 Introduction

The fascinating attractive properties of magnets have been known since ancient times. The word magnet comes from ancient Greek place name Magnesia (the modern town Manisa in Western Turkey), where the natural magnets called lodestones were found. The fundamental nature of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges and current carrying wires.

We will describe magnetic forces using the concept of a field. A magnetic field is established by a permanent magnet, by an electric current or by other moving charges. This magnetic field, in turn, exerts forces on other moving charges and current carrying conductors. In this chapter, first we study the magnetic forces and torques exerted on moving charges and currents by magnetic fields, then we will see how to calculate the magnetic fields produced by currents and moving charges.

26.2 Magnetic Force on a Moving Charge ($\mathbf{F}_m$)

An unknown electric field can be determined by magnitude and direction of the force on a test charge q_0 at rest. To explore an unknown magnetic field (denoted by **B**), we must measure the magnitude and direction of the force on a moving test charge.

The magnetic force ($\mathbf{F}_m$) on a charge q moving with velocity **v** in a magnetic field **B** is given, both in magnitude and direction, by

$$\boxed{\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})} \qquad \text{...(i)}$$

Following points are worthnoting regarding the above expression.

(i) The magnitude of $\mathbf{F}_m$ is

$$\boxed{F_m = Bqv \sin\theta}$$

where, θ is the angle between **v** and **B**.

(ii) $\mathbf{F}_m$ is zero when,

(a) $B = 0$, i.e. no magnetic field is present.

(b) $q = 0$, i.e. particle is neutral.

(c) $v = 0$, i.e. charged particle is at rest or

(d) $\theta = 0°$ or $180°$, i.e. $\mathbf{v} \uparrow\uparrow \mathbf{B}$ or $\mathbf{v} \uparrow\downarrow \mathbf{B}$

(iii) $\mathbf{F}_m$ is maximum at $\theta = 90°$ and this maximum value is Bqv.

(iv) The units of **B** must be the same as the units of F/qv. Therefore, the SI unit of B is equivalent to $\frac{\text{N-s}}{\text{C-m}}$. This unit is called the **tesla** (abbreviated as T), in honour of Nikola Tesla, the prominent Serbian-American scientist and inventor.

Thus,

$$\boxed{1 \text{ tesla} = 1\text{T} = \frac{1\,\text{N-s}}{\text{C-m}} = \frac{1\,\text{N}}{\text{A-m}}}$$

The CGS unit of **B**, the **gauss** ($1\text{G} = 10^{-4}$ T) is also in common use.

(v) In equation number (i) q is to be substituted with sign. If q is positive, magnetic force is along $\mathbf{v} \times \mathbf{B}$ and if q is negative, magnetic force is in a direction opposite to $\mathbf{v} \times \mathbf{B}$.

(vi) **Direction of $\mathbf{F}_m$** From the property of cross product we can infer that $\mathbf{F}_m$ is perpendicular to both **v** and **B** or it is perpendicular to the plane formed by **v** and **B**. The exact direction of $\mathbf{F}_m$ can be given by any of the following methods:

(a) Direction of $\mathbf{F}_m$ = (sign of q) (direction of $\mathbf{v} \times \mathbf{B}$) or, as we stated earlier also,

$$\mathbf{F}_m \uparrow\uparrow \mathbf{v} \times \mathbf{B} \quad \text{if } q \text{ is positive and}$$

$$\mathbf{F}_m \uparrow\downarrow \mathbf{v} \times \mathbf{B} \quad \text{if } q \text{ is negative.}$$

(b) Fleming's left hand rule According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge ($\mathbf{v}_{+q}$) and forefinger shows the direction of magnetic field (**B**), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$). If instead of positive charge we have the negative charge, then $\mathbf{F}_m$ is in opposite direction.

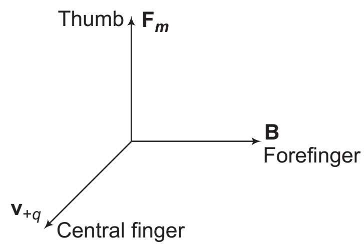


Fig. 26.1

(c) Right hand rule Wrap the fingers of your right hand around the line perpendicular to the plane of **v** and **B** as shown in figure, so that they curl around with the sense of rotation from **v** to **B** through the smaller angle between them. Your thumb then points in the direction of the force $\mathbf{F}_m$ on a positive charge. (Alternatively, the direction of the force $\mathbf{F}_m$ on a positive charge is the direction in which a right hand thread screw would advance if turned the same way).

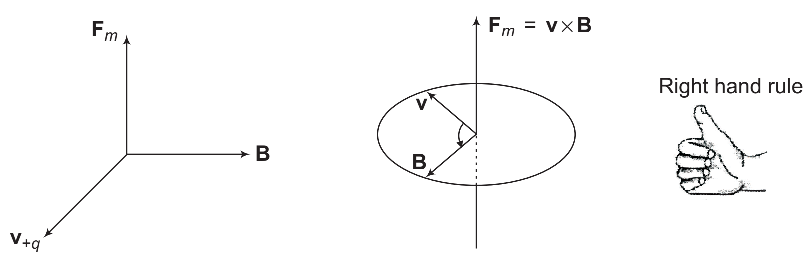


Fig. 26.2

(vii) $\mathbf{F}_m \perp \mathbf{v}$ or $\mathbf{F}_m \perp \dfrac{d\mathbf{s}}{dt}$. Therefore, $\mathbf{F}_m \perp d\mathbf{s}$ or the work done by the magnetic force in a static magnetic field is zero.

$$W_{\mathbf{F}_m} = 0$$

So, from work energy theorem KE and hence the speed of the charged particle remains constant in magnetic field. The magnetic force can change the direction only. It cannot increase or decrease the speed or kinetic energy of the particle.

Note *By convention the direction of magnetic field* **B** *perpendicular to the paper going inwards is shown by ⊗ and the direction perpendicular to the paper coming out is shown by ⊙.*

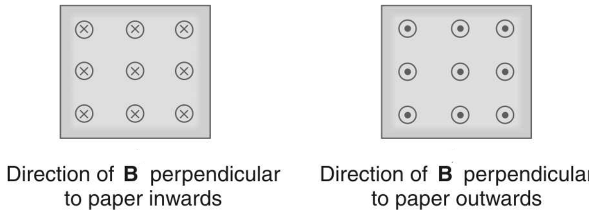


Fig. 26.3

Example 26.1 *A charged particle is projected in a magnetic field*

$$\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 10^{-2}\ T$$

The acceleration of the particle is found to be

$$\mathbf{a} = (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ m/s^2$$

Find the value of x.

Solution As we have read, $\mathbf{F}_m \perp \mathbf{B}$

i.e. the acceleration $\quad \mathbf{a} \perp \mathbf{B} \quad$ or $\quad \mathbf{a} \cdot \mathbf{B} = 0$

or $\quad (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \cdot (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 10^{-2} = 0$

or $\quad (3x + 8) \times 10^{-2} = 0$

$\therefore \quad x = -\dfrac{8}{3}\ \text{m/s}^2$ **Ans.**

Example 26.2 *When a proton has a velocity* $\mathbf{v} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^6\ m/s$, *it experiences a force* $\mathbf{F} = -(1.28 \times 10^{-13}\ \hat{\mathbf{k}})\ N$. *When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?*

HOW TO PROCEED *In the second part of the question, it is given that magnetic force is along x-axis when velocity is along z-axis. Hence, magnetic field should be along negative y-direction. As in case of positive charge (here proton)*

$$\boxed{\mathbf{F}_m \uparrow\uparrow \mathbf{v} \times \mathbf{B}}$$

So, let $\quad \mathbf{B} = -B_0\hat{\mathbf{j}}$

where, B_0 = *positive constant.*

Now, applying $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$ *we can find value of* B_0 *from the first part of the question.*

Solution Substituting proper values in, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

We have, $\quad -(1.28 \times 10^{-13}\ \hat{\mathbf{k}}) = (1.6 \times 10^{-19})[(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-B_0\hat{\mathbf{j}})] \times 10^6$

$\therefore \quad 1.28 = 1.6 \times 2 \times B_0$

or $\quad B_0 = \dfrac{1.28}{3.2} = 0.4$

Therefore, the magnetic field is $\quad \mathbf{B} = (-0.4\ \hat{\mathbf{j}})\ \mathrm{T}$ **Ans.**

Example 26.3 *A magnetic field of* $(4.0 \times 10^{-3}\ \hat{\mathbf{k}})$ *T exerts a force* $(4.0\ \hat{\mathbf{i}} + 3.0\ \hat{\mathbf{j}}) \times 10^{-10}$ *N on a particle having a charge* 10^{-9} *C and moving in the x-y plane. Find the velocity of the particle.*

Solution Given, $\mathbf{B} = (4 \times 10^{-3}\ \hat{\mathbf{k}})$ T, $q = 10^{-9}$ C

and magnetic force $\mathbf{F}_m = (4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}) \times 10^{-10}$ N

Let velocity of the particle in x-y plane be

$$\mathbf{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}$$

Then, from the relation

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$$

We have

$$(4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}) \times 10^{-10} = 10^{-9}\,[(v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}) \times (4 \times 10^{-3}\,\hat{\mathbf{k}})]$$
$$= (4v_y \times 10^{-12}\ \hat{\mathbf{i}} - 4v_x \times 10^{-12}\ \hat{\mathbf{j}})$$

Comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we have

$$4 \times 10^{-10} = 4v_y \times 10^{-12}$$

$\therefore$ $$v_y = 10^2\ \text{m/s} = 100\ \text{m/s}$$

and $$3.0 \times 10^{-10} = -\,4v_x \times 10^{-12}$$

$\therefore$ $$v_x = -\,75\ \text{m/s}$$

$\therefore$ $$\mathbf{v} = (-\,75\,\hat{\mathbf{i}} + 100\,\hat{\mathbf{j}})\ \text{m/s}$$ **Ans.**

INTRODUCTORY EXERCISE 26.1

1. Write the dimensions of E/B. Here, E is the electric field and B the magnetic field.
2. In the relation $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$, which pairs are always perpendicular to each other.
3. If a beam of electrons travels in a straight line in a certain region. Can we say there is no magnetic field?
4. A charge $q = -4\ \mu$C has an instantaneous velocity $\mathbf{v} = (2\,\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times 10^6$ m/s in a uniform magnetic field $\mathbf{B} = (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\,\hat{\mathbf{k}}) \times 10^{-2}$ T. What is the force on the charge?
5. A particle initially moving towards south in a vertically downward magnetic field is deflected toward the east. What is the sign of the charge on the particle?
6. An electron experiences a magnetic force of magnitude 4.60×10^{-15} N, when moving at an angle of 60° with respect to a magnetic field of magnitude 3.50×10^{-3} T. Find the speed of the electron.
7. He^{2+} ion travels at right angles to a magnetic field of 0.80 T with a velocity of 10^5 m/s. Find the magnitude of the magnetic force on the ion.

26.3 Path of a Charged Particle in Uniform Magnetic Field

The path of a charged particle in uniform magnetic field depends on the angle θ (the angle between $\mathbf{v}$ and $\mathbf{B}$). Depending on the different values of θ, the following three cases are possible.

Case 1 **When θ is 0° or 180°**

As we have seen in Art. 26.2, $\mathbf{F}_m = 0$, when θ is either $0°$ or $180°$. Hence, path of the charged particle is a straight line (undeviated) when it enters parallel or antiparallel to magnetic field.

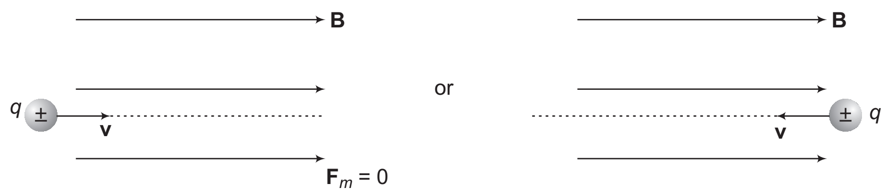


Fig. 26.4

Case 2 **When $\theta = 90°$**

When $\theta = 90°$, the magnetic force is $F_m = Bqv \sin 90° = Bqv$. This magnetic force is perpendicular to the velocity at every instant. Hence, path is a **circle**. The necessary centripetal force is provided by the magnetic force. Hence, if r be the radius of the circle, then

$$\frac{mv^2}{r} = Bqv$$

or $$r = \frac{mv}{Bq}$$

This expression of r can be written in the following different ways

$$\boxed{r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2qVm}}{Bq}}$$

Here, p = momentum of particle

$$K = \text{KE of particle } = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2Km}$$

We also know that if the charged particle is accelerated by a potential difference of V volts, it acquires a KE given by

$$K = qV$$

Further, time period of the circular path will be

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\dfrac{mv}{Bq}\right)}{v} = \frac{2\pi m}{Bq}$$

or $$\boxed{T = \frac{2\pi m}{Bq}}$$

or The angular speed (ω) of the particle is $\omega = \frac{2\pi}{T} = \frac{Bq}{m}$

$\therefore$ $$\boxed{\omega = \frac{Bq}{m}}$$

Frequency of rotation is $f = \frac{1}{T}$

or $$\boxed{f = \frac{Bq}{2\pi m}}$$

The following points are worthnoting regarding a circular path:

(i) The plane of the circle is perpendicular to magnetic field. If the magnetic field is along z-direction, the circular path is in x-y plane. The speed of the particle does not change in magnetic field.

Hence, if v_0 be the speed of the particle, then velocity of particle at any instant of time will be

$$\mathbf{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}$$

where, $$v_x^2 + v_y^2 = v_0^2$$

(ii) T, f and ω are independent of v while the radius is directly proportional to v.

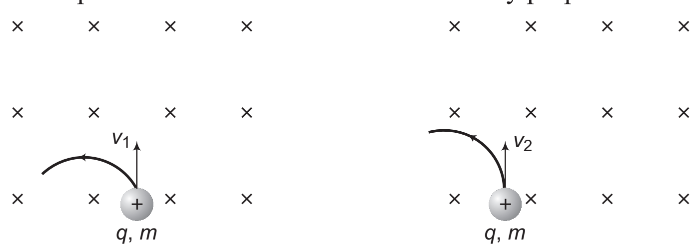


Fig. 26.5

Hence, if two charged particles of equal mass and charge enter in a magnetic field **B** with different speeds v_1 and v_2 $(> v_1)$ at right angles, then

$$\boxed{T_1 = T_2}$$

but $$\boxed{r_2 > r_1}$$

as shown in figure.

Note *Charge per unit mass* $\frac{q}{m}$ *is known as specific charge. It is sometimes denoted by* α. *So, in terms of* α, *the above formulae can be written as*

$$r = \frac{v}{B\alpha}, \quad T = \frac{2\pi}{B\alpha}, \quad f = \frac{B\alpha}{2\pi} \quad \text{and} \quad \omega = B\alpha$$

Case 3 **When θ is other than 0°, 180° or 90°**

In this case velocity can be resolved into two components, one along **B** and another perpendicular to **B**. Let the two components be $v_{||}$ and $v_{\perp}$.

Then,

$$v_{||} = v\cos\theta$$

and

$$v_{\perp} = v\sin\theta$$

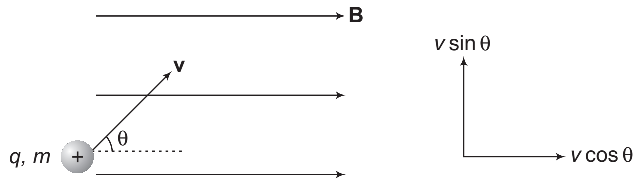


Fig. 26.6

The component perpendicular to field ($v_{\perp}$) gives a circular path and the component parallel to field ($v_{||}$) gives a straight line path. The resultant path is a helix as shown in figure.

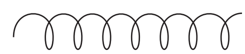


Fig. 26.7

The radius of this helical path is

$$\boxed{r = \frac{mv_{\perp}}{Bq} = \frac{mv\sin\theta}{Bq}}$$

Time period and frequency do not depend on velocity and so they are given by

$$\boxed{T = \frac{2\pi m}{Bq}} \quad \text{and} \quad \boxed{f = \frac{Bq}{2\pi m}}$$

There is one more term associated with a helical path, that is **pitch *(p)*** of the helical path. Pitch is defined as the distance travelled along magnetic field in one complete cycle.

i.e.

$$p = v_{||}T$$

or

$$p = (v\cos\theta)\frac{2\pi m}{Bq}$$

∴

$$\boxed{p = \frac{2\pi mv\cos\theta}{Bq}}$$

Example 26.4 *Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in the figure. Then,* **(JEE 2001)**

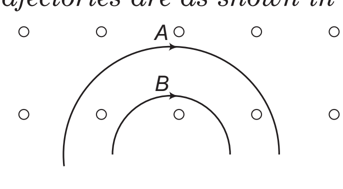


Fig. 26.8

(a) $m_A v_A < m_B v_B$
(b) $m_A v_A > m_B v_B$
(c) $m_A < m_B$ and $v_A < v_B$
(d) $m_A = m_B$ and $v_A = v_B$

Solution Radius of the circle $= \dfrac{mv}{Bq}$

or radius $\propto mv$ if B and q are same.

$$(\text{Radius})_A > (\text{Radius})_B$$

$$\therefore \qquad m_A v_A > m_B v_B$$

$\therefore$ Correct option is (b).

Example 26.5 *A proton, a deuteron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then* **(JEE 1997)**

(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$

(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

Solution Radius of the circular path is given by

$$r = \frac{mv}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, K is the kinetic energy to the particle.

Therefore, $r \propto \dfrac{\sqrt{m}}{q}$ if K and B are same.

$$\therefore \qquad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

Hence, $$r_\alpha = r_p < r_d$$

$\therefore$ Correct option is (a).

Example 26.6 *Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of the mass of X to that of Y is* **(JEE 1988)**

(a) $(R_1/R_2)^{1/2}$ (b) R_2/R_1

(c) $(R_1/R_2)^2$ (d) R_1/R_2

Solution $R = \dfrac{\sqrt{2qVm}}{Bq}$

or $$R \propto \sqrt{m}$$

or $$\frac{R_1}{R_2} = \sqrt{\frac{m_X}{m_Y}}$$

or $$\frac{m_X}{m_Y} = \left(\frac{R_1}{R_2}\right)^2$$

$\therefore$ Correct option is (c).

INTRODUCTORY EXERCISE 26.2

1. A neutron, a proton, an electron and an α - particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labeled in figure. The electron follows track…… and the α - particle follows track…… **(JEE 1984)**

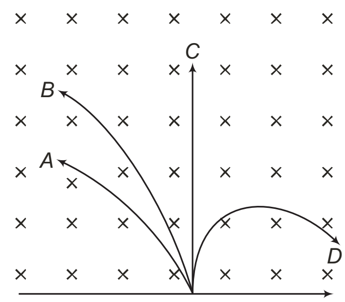


Fig. 26.9

2. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular path of the same radius. Is this statement true or false? **(JEE 1985)**
3. A charged particle enters a region of uniform magnetic field at an angle of 85° to the magnetic line of force. The path of the particle is a circle. Is this statement true or false? **(JEE 1983)**
4. Can a charged particle be accelerated by a magnetic field. Can its speed be increased?
5. An electron beam projected along positive x-axis deflects along the positive y-axis. If this deflection is caused by a magnetic field, what is the direction of the field?
6. An electron and a proton are projected with same velocity perpendicular to a magnetic field.
   (a) Which particle will describe the smaller circle?
   (b) Which particle will have greater frequency?
7. An electron is accelerated through a PD of 100 V and then enters a region where it is moving perpendicular to a magnetic field $B = 0.2$ T. Find the radius of the circular path. Repeat this problem for a proton.

26.4 Magnetic Force on a Current Carrying Conductor

A charged particle in motion experiences a magnetic force in a magnetic field. Similarly, a current carrying wire also experiences a force when placed in a magnetic field. This follows from the fact that the current is a collection of many charged particles in motion. Hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.

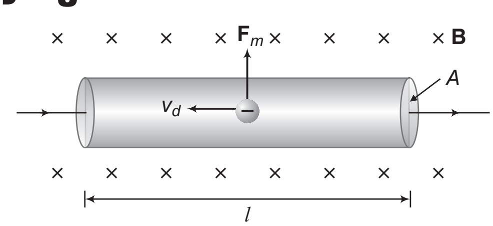


Fig. 26.10

Suppose a conducting wire carrying a current i is placed in a magnetic field **B**. The length of the wire is l and area of cross-section is A. The free electrons drift with a speed v_d opposite to the direction of current. The magnetic force exerted on the electron is

$$d\mathbf{F}_m = -e(\mathbf{v}_d \times \mathbf{B})$$

If n be the number of free electrons per unit volume of the wire, then total number of electrons in volume Al of the wire are, nAl. Therefore, total force on the wire is

$$\mathbf{F}_m = -e(nAl)(\mathbf{v}_d \times \mathbf{B})$$

If we denote the length l along the direction of the current by **l**, then the above equation becomes

$$\boxed{\mathbf{F}_m = i(\mathbf{l} \times \mathbf{B})} \qquad \text{...(i)}$$

where, $neAv_d = i$

The following points are worthnoting regarding the above expression :

(i) Magnitude of $\mathbf{F}_m$ is, $F_m = ilB \sin\theta$, here θ is the angle between **l** and **B**. F_m is zero for $\theta = 0°$ or $180°$ and maximum for $\theta = 90°$.

(ii) Here, **l** is a vector that points in the direction of the current i and has a magnitude equal to the length.

(iii) The above expression applies only to a straight segment of wire in a uniform magnetic field.

(iv) For the magnetic force on an arbitrarily shaped wire segment, let us consider the magnetic force exerted on a small segment of vector length $d\mathbf{l}$.

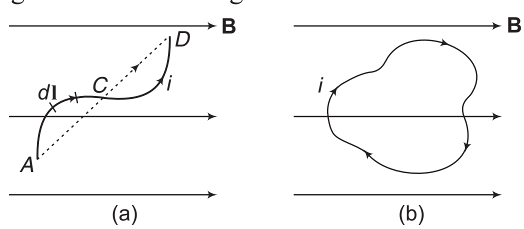


Fig. 26.11

$$d\mathbf{F}_m = i(d\mathbf{l} \times \mathbf{B}) \qquad \text{...(ii)}$$

To calculate the total force $\mathbf{F}_m$ acting on the wire shown in figure, we integrate Eq. (ii) over the length of the wire.

$$\mathbf{F}_m = i\int_A^D (d\mathbf{l} \times \mathbf{B}) \qquad \text{...(iii)}$$

Now, let us consider two special cases involving Eq. (iii). In both cases, the magnetic field is taken to be constant in magnitude and direction.

Case 1 A curved wire ACD as shown in Fig. (a) carries a current i and is located in a uniform magnetic field **B**. Because the field is uniform, we can take **B** outside the integral in Eq. (iii) and we obtain,

$$\mathbf{F}_m = i\left(\int_A^D d\mathbf{l}\right) \times \mathbf{B} \qquad \text{...(iv)}$$

But, the quantity $\int_A^D d\mathbf{l}$ represents the vector sum of all length elements from A to D. From the polygon law of vector addition, the sum equals the vector **l** directed from A to D. Thus,

$$\mathbf{F}_m = i\,(\mathbf{l} \times \mathbf{B})$$

or we can write

$$\boxed{\mathbf{F}_{ACD} = \mathbf{F}_{AD} = i\,(\mathbf{AD} \times \mathbf{B})} \text{ in uniform field.}$$

Case 2 An arbitrarily shaped closed loop carrying a current i is placed in a uniform magnetic field as shown in Fig. (b). We can again express the force acting on the loop in the form of Eq. (iv), but this time we must take the vector sum of the length elements $d\mathbf{l}$ over the entire loop,

$$\mathbf{F}_m = i\,(\oint d\mathbf{l}) \times \mathbf{B}$$

Because the set of length elements forms a closed polygon, the vector sum must be zero.

$$\therefore \qquad \mathbf{F}_m = 0$$

Thus, **the net magnetic force acting on any closed current carrying loop in a uniform magnetic field is zero**.

(v) The direction of $\mathbf{F}_m$ can be given by Fleming's left hand rule as discussed in Art. 26.2. According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of current (or $\mathbf{l}$) and forefinger shows the direction of magnetic field ($\mathbf{B}$), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$).

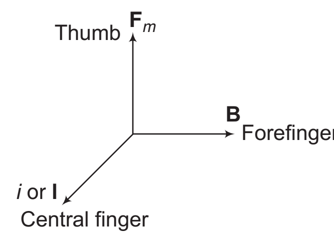


Fig. 26.12

Example 26.7 *A horizontal rod 0.2 m long is mounted on a balance and carries a current. At the location of the rod a uniform horizontal magnetic field has magnitude 0.067 T and direction perpendicular to the rod. The magnetic force on the rod is measured by the balance and is found to be 0.13 N. What is the current?*

Solution

$$F = ilB \sin 90°$$

$$\therefore \qquad i = \frac{F}{lB} = \frac{0.13}{0.2 \times 0.067}$$

$$= 9.7 \text{ A}$$

Ans.

Example 26.8 *A square of side 2.0 m is placed in a uniform magnetic field $\mathbf{B} = 2.0\,T$ in a direction perpendicular to the plane of the square inwards. Equal current $i = 3.0\,A$ is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.*

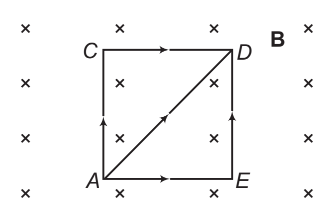


Fig. 26.13

Solution Force on wire ACD = Force on AD = Force on AED

$\therefore$ Net force on the loop $= 3\,(\mathbf{F}_{AD})$

or

$$F_{\text{net}} = 3\,(i)\,(AD)\,(B)$$

$$= (3)\,(3.0)\,(2\sqrt{2})\,(2.0)\text{ N} = 36\sqrt{2}\text{ N}$$

Direction of this force is towards EC.

Example 26.9 *In the figure shown a semicircular wire loop is placed in a uniform magnetic field $B = 1.0\,T$. The plane of the loop is perpendicular to the magnetic field. Current $i = 2\,A$ flows in the loop in the directions shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is $1.0\,m$*

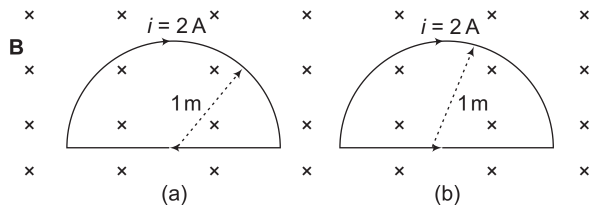


Fig. 26.14

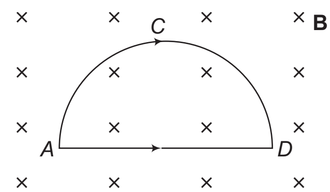


Fig. 26.15

Solution **Refer figure (a)** It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero. **Ans.**

Refer figure (b) In this case although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.

$$\mathbf{F}_{ACD} = \mathbf{F}_{AD}$$

$$\therefore \quad \mathbf{F}_{\text{loop}} = \mathbf{F}_{ACD} + \mathbf{F}_{AD} = 2\mathbf{F}_{AD}$$

$$\therefore \quad |\mathbf{F}_{\text{loop}}| = 2|\mathbf{F}_{AD}|$$

$$= 2ilB\sin\theta \qquad [l = 2r = 2.0\text{ m}]$$

$$= (2)(2)(2)(1)\sin 90° = 8\text{ N}$$ **Ans.**

INTRODUCTORY EXERCISE 26.3

1. A wire of length l carries a current i along the x-axis. A magnetic field $\mathbf{B} = B_0(\hat{\mathbf{j}} + \hat{\mathbf{k}})$ exists in the space. Find the magnitude of the magnetic force acting on the wire.
2. In the above problem will the answer change if magnetic field becomes $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$.
3. A wire along the x-axis carries a current of 3.50 A in the negative direction. Calculate the force (expressed in terms of unit vectors) on a 1.00 cm section of the wire exerted by these magnetic fields
   (a) $\mathbf{B} = -(0.65\text{ T})\hat{\mathbf{j}}$ (b) $\mathbf{B} = +(0.56\text{ T})\hat{\mathbf{k}}$ (c) $\mathbf{B} = -(0.31\text{ T})\hat{\mathbf{i}}$
   (d) $\mathbf{B} = +(0.33\text{ T})\hat{\mathbf{i}} - (0.28\text{ T})\hat{\mathbf{k}}$ (e) $\mathbf{B} = +(0.74\text{ T})\hat{\mathbf{j}} - (0.36\text{ T})\hat{\mathbf{k}}$
4. Find net force on the equilateral loop of side 4 m carrying a current of 2 A kept in a uniform magnetic field of 2 T as shown in figure.

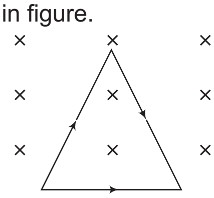


Fig. 26.16

26.5 Magnetic Dipole

Every current carrying loop is a magnetic dipole. It has two poles: south (S) and north (N). This is similar to a bar magnet. Magnetic field lines emanate from the north pole and after forming a closed path terminate on south pole. Each magnetic dipole has some magnetic moment (**M**). The magnitude of **M** is

$$\boxed{|\mathbf{M}| = NiA}$$

Here, N = number of turns in the loop

i = current in the loop and

A = area of cross-section of the loop.

For the direction of **M** any one of the following methods can be used:

(i) As in case of an electric dipole, the dipole moment **p** has a direction from negative charge to positive charge. In the similar manner, direction of **M** is from **south to north** pole. The south and north poles can be identified by the sense of current. The side from where the current seems to be clockwise becomes south pole and the opposite side from where it seems anti-clockwise becomes north pole.

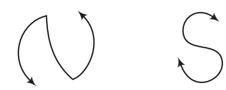


Fig. 26.17

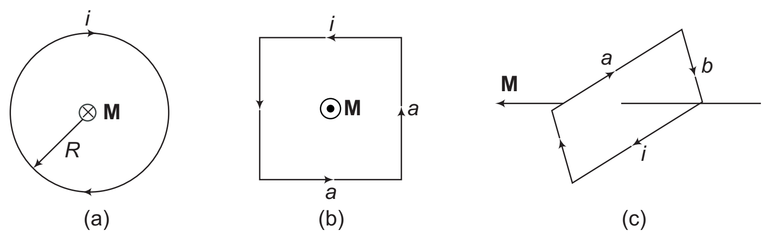


Fig. 26.18

Now, let us find the direction and magnitude of **M** in the three loops shown in Fig. 26.18.

Refer figure (a) In this case, current appears to be clockwise from outside the paper, so this side becomes the south pole. From the back of the paper it seems anti-clockwise. Hence, this side becomes the north pole. As the magnetic moment is from south to north pole. It is directed perpendicular to paper inwards. Further,

$$|\mathbf{M}| = NiA = \pi R^2 i$$

Refer figure (b) Here, opposite is the case. South pole is into the paper and north pole is outside the paper. Therefore, magnetic moment is perpendicular to paper in outward direction. The magnitude of **M** is

$$|\mathbf{M}| = a^2 i$$

Refer figure (c) In this case, south pole is on the right side of the loop and north pole on the left side. Hence, **M** is directed from right to left. The magnitude of magnetic moment is

$$|\mathbf{M}| = abi$$

(ii) Vector **M** is along the normal to the plane of the loop. The orientation (up or down along the normal) is given by the right hand rule. Wrap your fingers of the right hand around the perimeter of the loop in the direction of current as shown in figure. Then, extend your thumb so that it is perpendicular to the plane of the loop. The thumb points in the direction of **M**.

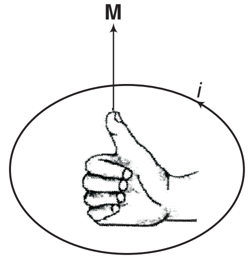


Fig. 26.19

Extra Points to Remember

- In addition to the method discussed above for finding **M** here are two more methods for calculating **M**.

  Method 1 This method is useful for calculating **M** for a rectangular or square loop.

  The magnetic moment (**M**) of the rectangular loop shown in figure is

  $$\mathbf{M} = i\,(\mathbf{AB} \times \mathbf{BC}) = i\,(\mathbf{BC} \times \mathbf{CD}) = i\,(\mathbf{CD} \times \mathbf{DA}) = i\,(\mathbf{DA} \times \mathbf{AB})$$

  Here, the cross product of any two consecutive sides (taken in order) gives the area as well as the correct direction of **M** also.

Fig 26.20

Note *If coordinates of vertices are known. Then, vector of any side can be written in terms of coordinates, e.g.*

$$\boxed{\mathbf{AB} = (x_B - x_A)\,\hat{\mathbf{i}} + (y_B - y_A)\,\hat{\mathbf{j}} + (z_B - z_A)\,\hat{\mathbf{k}}}$$

Method 2 Sometimes, a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now, the net magnetic moment of the given loop is the vector sum of individual loops.

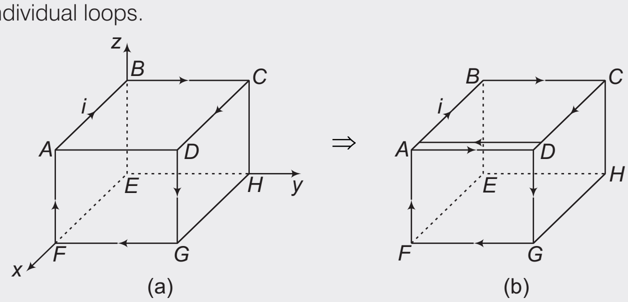


Fig. 26.21

For example, in Fig. (a), six sides of a cube of side l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.

$$\mathbf{M}_{ABCDA} = -il^2\hat{\mathbf{k}}$$

$$\mathbf{M}_{ADGFA} = -il^2\,\hat{\mathbf{i}}$$

$$\therefore \qquad \mathbf{M}_{\text{net}} = -il^2\,(\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

Example 26.10 *A square loop OABCO of side l carries a current i. It is placed as shown in figure. Find the magnetic moment of the loop.*

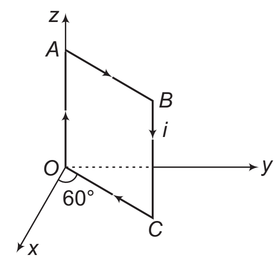


Fig. 26.22

Solution As discussed above, magnetic moment of the loop can be written as

$$\mathbf{M} = i\,(\mathbf{BC} \times \mathbf{CO})$$

Here, $\mathbf{BC} = -l\,\hat{\mathbf{k}}$, $\quad \mathbf{CO} = -l\cos 60°\,\hat{\mathbf{i}} - l\sin 60°\,\hat{\mathbf{j}} = -\frac{l}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}l}{2}\hat{\mathbf{j}}$

$\therefore$ $$\mathbf{M} = i\left[(-l\hat{\mathbf{k}}) \times \left(-\frac{l}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}l}{2}\hat{\mathbf{j}}\right)\right]$$

or $$\mathbf{M} = \frac{il^2}{2}(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$$ **Ans.**

Example 26.11 *Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is* 10 *cm long and current in the loop is i* = 2.0 *A.*

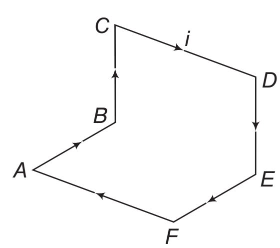


Fig. 26.23

Solution By assuming two equal and opposite currents in *BE*, two current carrying loops (*ABEFA* and *BCDEB*) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

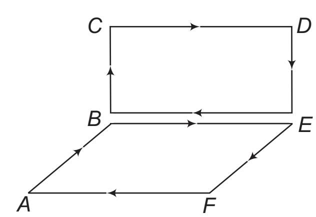


Fig. 26.24

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}\,M$$

where, $$M = iA = (2.0)(0.1)(0.1) = 0.02\ \text{A-m}^2$$

$\therefore$ $$M_{\text{net}} = (\sqrt{2})(0.02)\ \text{A-m}^2$$

$$= 0.028\ \text{A-m}^2$$ **Ans.**

26.6 Magnetic Dipole in Uniform Magnetic Field

Let us consider a rectangular ($a \times b$) current carrying loop $OACDO$ placed in xy - plane. A uniform magnetic field

$$\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

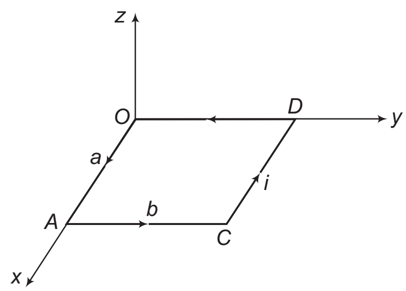


Fig. 26.25

exists in space.

We are interested in finding the net force and torque in the loop.

Force : Net force on the loop is

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_{OA} + \mathbf{F}_{AC} + \mathbf{F}_{CD} + \mathbf{F}_{DO} \\ &= i\,[(\mathbf{OA} \times \mathbf{B}) + (\mathbf{AC} \times \mathbf{B}) + (\mathbf{CD} \times \mathbf{B}) + (\mathbf{DO} \times \mathbf{B})] \\ &= i\,[(\mathbf{OA} + \mathbf{AC} + \mathbf{CD} + \mathbf{DO}) \times \mathbf{B}] \\ &= \text{null vector} \end{aligned}$$

or $|\mathbf{F}| = 0$, as $\mathbf{OA} + \mathbf{AC} + \mathbf{CD} + \mathbf{DO}$ forms a null vector.

Torque : Using $\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})$, we have

$$\begin{aligned} \mathbf{F}_{OA} &= i\,(\mathbf{OA} \times \mathbf{B}) = i\,[(a\hat{\mathbf{i}}) \times (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}})] = ia\,[B_y \hat{\mathbf{k}} - B_z \hat{\mathbf{j}}] \\ \mathbf{F}_{AC} &= i\,(\mathbf{AC} \times \mathbf{B}) = i\,[(b\hat{\mathbf{j}}) \times (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}})] = ib\,[-B_x \hat{\mathbf{k}} + B_z \hat{\mathbf{i}}] \\ \mathbf{F}_{CD} &= i\,(\mathbf{CD} \times \mathbf{B}) = i\,[(-a\hat{\mathbf{i}}) \times (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}})] = ia\,[-B_y \hat{\mathbf{k}} + B_z \hat{\mathbf{j}}] \\ \mathbf{F}_{DO} &= i\,(\mathbf{DO} \times \mathbf{B}) = i\,[(-b\hat{\mathbf{j}}) \times (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}})] = ib\,[B_x \hat{\mathbf{k}} - B_z \hat{\mathbf{i}}] \end{aligned}$$

All these forces are acting at the centre of the wires. For example, $\mathbf{F}_{OA}$ will act at the centre of OA. When the forces are in equilibrium, net torque about any point remains the same. Let us find the torque about O.

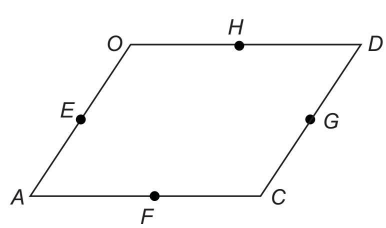


Fig. 26.26

E, F, G and H are the mid-points of OA, AC, CD and DO, respectively.

Using $\quad \tau = \mathbf{r} \times \mathbf{F}$, we have

$$\tau_O = (\mathbf{OE} \times \mathbf{F}_{OA}) + (\mathbf{OF} \times \mathbf{F}_{AC}) + (\mathbf{OG} \times \mathbf{F}_{CD}) + (\mathbf{OH} \times \mathbf{F}_{DO})$$

$$= \left[\left(\frac{a}{2}\hat{\mathbf{i}}\right) \times \{ia\,(B_y\hat{\mathbf{k}} - B_z\hat{\mathbf{j}})\}\right] + \left[\left(a\hat{\mathbf{i}} + \frac{b}{2}\hat{\mathbf{j}}\right) \times \{ib\,(-B_x\hat{\mathbf{k}} + B_z\hat{\mathbf{i}})\}\right]$$

$$+ \left[\left(\frac{a}{2}\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right) \times \{ia\,(-B_y\hat{\mathbf{k}} + B_z\hat{\mathbf{j}})\}\right] + \left[\left(\frac{b}{2}\hat{\mathbf{j}}\right) \times ib\,(B_x\hat{\mathbf{k}} - B_z\hat{\mathbf{i}})\right]$$

$$= iab\,B_x\hat{\mathbf{j}} - iabB_y\hat{\mathbf{i}}$$

This can also be written as

$$\tau_O = (iab\hat{\mathbf{k}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$$

Here, $\quad iab\hat{\mathbf{k}} =$ magnetic moment of the dipole $\mathbf{M}$

and $\quad B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = \mathbf{B}$

$\therefore$
$$\boxed{\tau = \mathbf{M} \times \mathbf{B}}$$

Note that although this formula has been derived for a rectangular loop, it comes out to be true for any shape of loop. The following points are worthnoting regarding the torque acting on the loop in uniform magnetic field.

(i) Magnitude of τ is $MB\sin\theta$ or $NiAB\sin\theta$. Here, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$. Torque is zero when $\theta = 0°$ or $180°$ and it is maximum at $\theta = 90°$.

(ii) If the loop is free to rotate in a magnetic field, the axis of rotation becomes an axis parallel to τ passing through the centre of mass of the loop.

The above equation for the torque is very similar to that of an electric dipole in an electric field. The similarity between electric and magnetic dipoles extends even further as illustrated in the table below.

Table 26.1

S.No.	Field of similarity	Electric dipole	Magnetic dipole
1.	Magnitude	$\lvert\mathbf{p}\rvert = q\,(2d)$	$\lvert\mathbf{M}\rvert = NiA$
2.	Direction	from $-q$ to $+q$	from S to N
3.	Net force in uniform field	zero	zero
4.	Torque	$\tau = \mathbf{p} \times \mathbf{E}$	$\tau = \mathbf{M} \times \mathbf{B}$
5.	Potential energy	$U = -\mathbf{p} \cdot \mathbf{E}$	$U = -\mathbf{M} \cdot \mathbf{B}$
6.	Work done in rotating the dipole	$W_{\theta_1 - \theta_2} = pE\,(\cos\theta_1 - \cos\theta_2)$	$W_{\theta_1 - \theta_2} = MB\,(\cos\theta_1 - \cos\theta_2)$
7	Field along axis	$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}}{r^3}$	$\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\mathbf{M}}{r^3}$
8.	Field perpendicular to axis	$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}}{r^3}$	$\mathbf{B} = -\frac{\mu_0}{4\pi} \cdot \frac{\mathbf{M}}{r^3}$

Note *In last two points $r >>$ size of loop.*

Note that the expressions for the magnetic dipole can be obtained from the expressions for the electric dipole by replacing **p** by **M** and ε_0 by $\frac{1}{\mu_0}$. Here, μ_0 is called the permeability of free space. It is related with ε_0 and speed of light c as

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

and it has the value,

$$\boxed{\mu_0 = 4\pi \times 10^{-7} \text{ T-m/A}}$$

Dimensions of $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ are that of speed or $[\text{LT}^{-1}]$.

Hence,
$$\boxed{\left[\frac{1}{\sqrt{\varepsilon_0 \mu_0}}\right] = [\text{LT}^{-1}]}$$

Example 26.12 *A circular loop of radius $R = 20$ cm is placed in a uniform magnetic field $\mathbf{B} = 2\,T$ in xy-plane as shown in figure. The loop carries a current $i = 1.0$ A in the direction shown in figure. Find the magnitude of torque acting on the loop.*

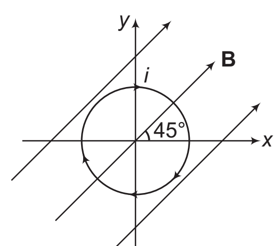


Fig. 26.27

Solution Magnitude of torque is given by

$$|\tau| = MB \sin \theta$$

Here, $M = NiA = (1)(1.0)(\pi)(0.2)^2$

$$= (0.04\,\pi) \text{ A-m}^2$$

$$B = 2\text{T}$$

and θ = angle between **M** and **B** = 90°

$\therefore$ $|\tau| = (0.04\,\pi)(2) \sin 90°$

$$= 0.25 \text{ N-m}$$ **Ans.**

Note *$\mathbf{M}$ is along negative z-direction (perpendicular to paper inwards) while $\mathbf{B}$ is in xy-plane. So, the angle between $\mathbf{M}$ and $\mathbf{B}$ is 90° not 45°. If the direction of torque is also desired, then we can write*

$$\mathbf{B} = 2 \cos 45° \,\hat{\mathbf{i}} + 2 \sin 45° \,\hat{\mathbf{j}} = \sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})\,T$$

and $\qquad \mathbf{M} = -(0.04\,\pi)\,\hat{\mathbf{k}}\ A\text{-}m^2$

$\therefore \qquad \tau = \mathbf{M} \times \mathbf{B} = (0.04\,\sqrt{2}\pi)\,(-\hat{\mathbf{j}} + \hat{\mathbf{i}})$

or $\qquad \tau = 0.18\,(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ ***Ans.***

INTRODUCTORY EXERCISE 26.4

1. A charge q is uniformly distributed on a non-conducting disc of radius R. It is rotated with an angular speed ω about an axis passing through the centre of mass of the disc and perpendicular to its plane. Find the magnetic moment of the disc.

   [**Hint :** For any charge distribution : Magnetic moment $= \left(\dfrac{q}{2m}\right)$ (angular momentum)]

2. A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A. A vector of unit length and parallel to the dipole moment **M** of the loop is given by $0.60\,\hat{\mathbf{i}} - 0.80\,\hat{\mathbf{j}}$. If the loop is located in uniform magnetic field given by $\mathbf{B} = (0.25\text{ T})\,\hat{\mathbf{i}} + (0.30\text{ T})\,\hat{\mathbf{k}}$ find,

   (a) the torque on the loop and

   (b) the magnetic potential energy of the loop.

3. A length L of wire carries a current i. Show that if the wire is formed into a circular coil, then the maximum torque in a given magnetic field is developed when the coil has one turn only, and that maximum torque has the magnitude $\tau = L^2 iB/4\pi$.

4. A coil with magnetic moment 1.45 A-m^2 is oriented initially with its magnetic moment antiparallel to a uniform 0.835 T magnetic field. What is the change in potential energy of the coil when it is rotated 180° so that its magnetic moment is parallel to the field?

26.7 Biot Savart Law

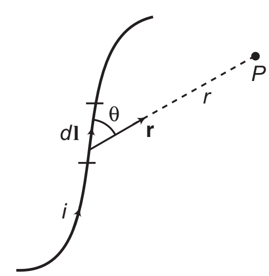


Fig. 26.28

In the preceding articles, we discussed the magnetic force exerted on a charged particle and current carrying conductor in a magnetic field. To complete the description of the magnetic interaction, this and the next article deals with the origin of the magnetic field. As in electrostatics, there are two methods of calculating the electric field at some point. One is Coulomb's law which gives the electric field due to a point charge and the another is Gauss's law which is useful in calculating the electric field of a highly symmetric configuration of charge. Similarly, in magnetics, there are basically two methods of calculating magnetic field at some point. One is **Biot Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere's law**, which is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

We begin by showing how to use the law of Biot and Savart to calculate the magnetic field produced at some point in space by a small current element. Using this formalism and the principle of superposition, we then calculate the total magnetic field due to various current distributions.

From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{l}$ of a wire carrying a steady current i.

(i) The vector $d\mathbf{B}$ is perpendicular to both $d\mathbf{l}$ (which points in the direction of the current) and the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{l}$ to P.

(ii) The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{l}$ to P.

(iii) The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude dl of the length element $d\mathbf{l}$.

(iv) The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$ where θ is the angle between $d\mathbf{l}$ and $\hat{\mathbf{r}}$. These observations are summarized in mathematical formula known today as **Biot Savart law**

$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\,(d\mathbf{l}\times\hat{\mathbf{r}})}{r^2}} \qquad \ldots\text{(i)}$$

Here, $$\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{\text{T-m}}{\text{A}}$$

It is important to note that $d\mathbf{B}$ in Eq. (i) is the field created by the current in only a small length element $d\mathbf{l}$ of the conductor. To find the total magnetic field $\mathbf{B}$ created at some point by a current of finite size, we must sum up contributions from all current elements that make up the current. That is, we must evaluate $\mathbf{B}$ by integrating Eq. (i).

$$\boxed{\mathbf{B} = \frac{\mu_0 i}{4\pi}\int\frac{d\mathbf{l}\times\hat{\mathbf{r}}}{r^2}}$$

where, the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore, a vector quantity.

The following points are worthnoting regarding the Biot Savart law.

(i) Magnitude of $d\mathbf{B}$ is given by

$$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

$|d\mathbf{B}|$ is zero at $\theta = 0°$ or $180°$ and maximum at $\theta = 90°$.

(ii) For the direction of $d\mathbf{B}$ either of the following methods can be employed.

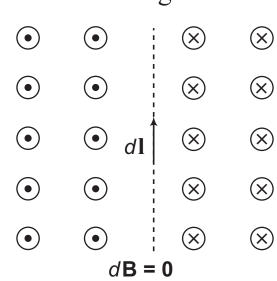


Fig. 26.29

(a) $d\mathbf{B} \uparrow\uparrow d\mathbf{l}\times\hat{\mathbf{r}}$. So, $d\mathbf{B}$ is along $d\mathbf{l}\times\mathbf{r}$.

(b) If $d\mathbf{l}$ is in the plane of paper. $d\mathbf{B} = 0$ at all points lying on the straight line passing through $d\mathbf{l}$. The magnetic field to the right of this line is in ⊗ direction and to the left of this line is in ⊙ direction.

26.8 Applications of Biot Savart Law

Let us now consider few applications of Biot Savart law.

Magnetic Field Surrounding a Thin, Straight Conductor

According to Biot Savart law,

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{i d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \qquad \ldots(\text{i})$$

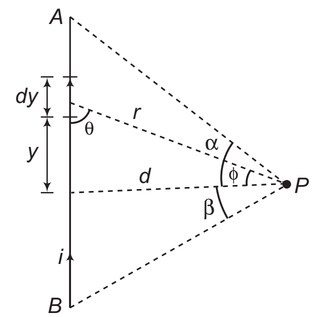


Fig. 26.30

As here every element of the wire contributes to **B** in the same direction (which is here ⊗).

Eq. (i) for this case becomes,

$$B = \frac{\mu_0}{4\pi}\int \frac{i dl \sin\theta}{r^2} = \frac{\mu_0 i}{4\pi}\int \frac{dy \sin\theta}{r^2}$$

$$y = d\tan\phi \quad \text{or} \quad dy = (d\sec^2\phi) d\phi$$

$$r = d\sec\phi \quad \text{and} \quad \theta = 90° - \phi$$

$$B = \frac{\mu_0 i}{4\pi}\int_{\phi=-\beta}^{\phi=\alpha} \frac{\{(d\sec^2\phi)\, d\phi\}\sin(90° - \phi)}{(d\sec\phi)^2}$$

or $$B = \frac{\mu_0 i}{4\pi d}\int_{-\beta}^{\alpha} \cos\phi \cdot d\phi \quad \text{or} \quad \boxed{B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)}$$

Note down the following points regarding the above equation.

(i) For an infinitely long straight wire, $\alpha = \beta = 90°$

$\therefore$ $$\sin\alpha + \sin\beta = 2 \quad \text{or} \quad \boxed{B = \frac{\mu_0}{2\pi}\frac{i}{d}}$$

(ii) The direction of magnetic field at a point P due to a long straight wire can be found by the right hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through P, the direction of the fingers at P gives the direction of magnetic field there.

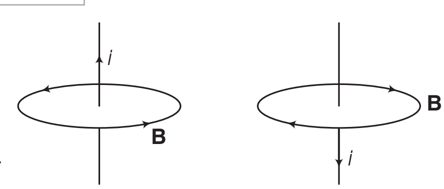


Fig. 26.31

(iii) $B \propto \frac{1}{d}$, i.e. B-d graph for an infinitely long straight wire is a rectangular hyperbola as shown in the figure.

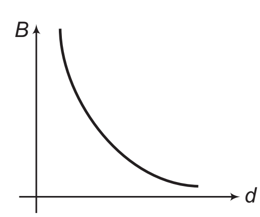


Fig. 26.32

Magnetic Field on the Axis of a Circular Coil

Suppose a current carrying circular loop has a radius R. Current in the loop is i. We want to find the magnetic field at a point P on the axis of the loop a distance z from the centre.

We can take the loop in xy-plane with its centre at origin and point P on the z-axis.

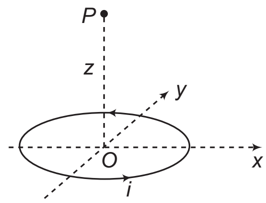


Fig. 26.33

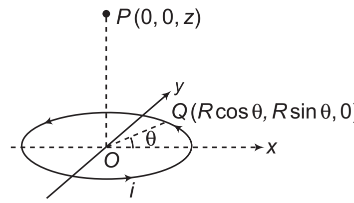


Fig. 26.34

Let us take a small current element at angle θ as shown.

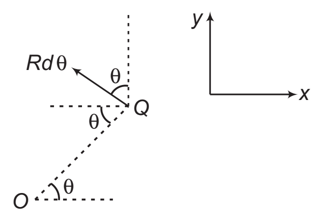


Fig. 26.35

$$P \equiv (0, 0, z)$$

$$Q \equiv (R\cos\theta, R\sin\theta, 0)$$

$$d\mathbf{l} = -(Rd\theta)\sin\theta\,\hat{\mathbf{i}} + (Rd\theta)\cos\theta\,\hat{\mathbf{j}}$$

$$\hat{\mathbf{r}} = \text{unit vector along } QP$$

$$= \frac{(-R\cos\theta\,\hat{\mathbf{i}} - R\sin\theta\,\hat{\mathbf{j}} + z\hat{\mathbf{k}})}{r}$$

Here, $$r = \text{distance } QP = \sqrt{R^2 + z^2}$$

Now, magnetic field at point P, due to current element $d\,\mathbf{l}$ at Q is

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i}{r^2}(d\mathbf{l}\times\hat{\mathbf{r}})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{r^3}[(-R\sin\theta\,d\theta\,\hat{\mathbf{i}} + R\cos\theta\,d\theta\,\hat{\mathbf{j}})\times(-R\cos\theta\,\hat{\mathbf{i}} - R\sin\theta\,\hat{\mathbf{j}} + z\hat{\mathbf{k}})]$$

or $$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i}{r^3}[(zR\cos\theta\,d\theta)\hat{\mathbf{i}} + (zR\sin\theta\,d\theta)\hat{\mathbf{j}} + (R^2 d\theta)\hat{\mathbf{k}}]$$

$$= dB_x\hat{\mathbf{i}} + dB_y\hat{\mathbf{j}} + dB_z\hat{\mathbf{k}}$$

Here, $$dB_x = \frac{\mu_0}{4\pi}\frac{i}{r^3}(zR\cos\theta d\theta)$$

$$dB_y = \frac{\mu_0}{4\pi}\frac{i}{r^3}(zR\sin\theta d\theta)$$

and $$dB_z = \frac{\mu_0}{4\pi}\frac{i}{r^3}(R^2 d\theta)$$

Integrating these differentials from $\theta = 0°$ to $\theta = 2\pi$ for the complete loop, we get

$$B_x = \frac{\mu_0}{4\pi}\frac{ziR}{r^3}\int_0^{2\pi}\cos\theta\, d\theta = 0$$

$$B_y = \frac{\mu_0}{4\pi}\frac{ziR}{r^3}\int_0^{2\pi}\sin\theta\, d\theta = 0$$

and $$B_z = \frac{\mu_0}{4\pi}\frac{iR^2}{r^3}\int_0^{2\pi} d\theta = \frac{\mu_0}{2}\frac{iR^2}{r^3}$$

Substituting $r = (R^2 + z^2)^{1/2}$, we get $B_P = B_z = \dfrac{\mu_0 iR^2}{2(R^2 + z^2)^{3/2}}$

For N number of loops, $$\boxed{B = \frac{\mu_0 NiR^2}{2(R^2 + z^2)^{3/2}}}$$

Note down the following points regarding a circular current carrying loop.

(i) At the centre of the loop, $z = 0$

and $$\boxed{B\text{ (centre)} = \frac{\mu_0 Ni}{2R}}$$

(ii) For $z >> R$, $z^2 + R^2 \approx z^2$

$\therefore$ $$B = \frac{\mu_0 NiR^2}{2z^3} = \left(\frac{\mu_0}{4\pi}\right)\frac{(2Ni\pi R^2)}{z^3} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2M}{z^3}\right)$$

Here, M = magnetic moment of the loop = $NiA = Ni\pi R^2$.

This result was expected as the magnetic field on the axis of a dipole is $\dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$.

(iii) Direction of magnetic field on the axis of a circular loop can be obtained using the right hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.

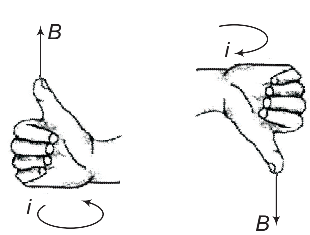


Fig. 26.36

(iv) The magnetic field at a point not on the axis is mathematically difficult to calculate. We have shown qualitatively in figure the magnetic field lines due to a circular current.

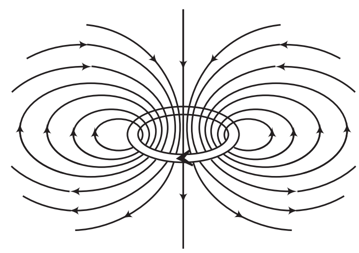


Fig. 26.37

(v) Magnetic field is maximum at the centre and decreases as we move away from the centre (on the axis of the loop). The B-z graph is somewhat like shown in figure.

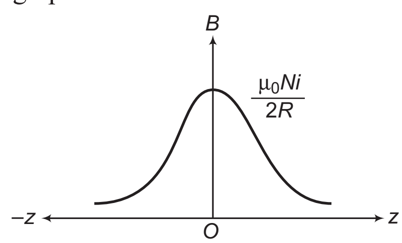


Fig. 26.38

(vi) Magnetic field due to an arc of a circle at the centre is

$$B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 i}{2R} = \frac{\mu_0}{4\pi}\left(\frac{i}{R}\right)\theta$$

or

$$\boxed{B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\theta}$$

Here, θ is to be substituted in radians.

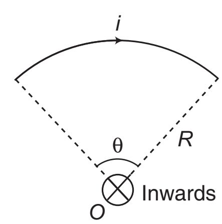


Fig. 26.39

Field Along the Axis of a Solenoid

The name solenoid was first given by Ampere to a wire wound in a closely spaced spiral over a hollow cylindrical non-conducting core. If n is the number of turns per unit length, each carries a current i uniformly wound round a cylinder of radius R, the number of turns in length dx are ndx. Thus, the magnetic field at the axial point O due to this element dx is

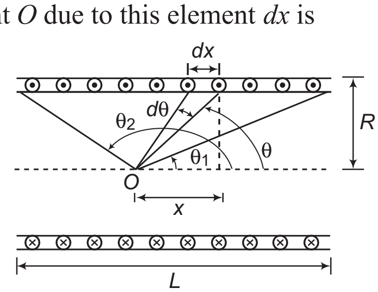


Fig. 26.40

$$dB = \frac{\mu_0}{2} \frac{(indx)R^2}{(R^2 + x^2)^{3/2}} \qquad \text{...(i)}$$

Its direction is along the axis of the solenoid. From the geometry, we know

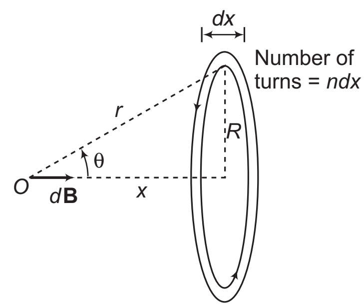


Fig. 26.41

$$x = R \cot\theta$$

$$dx = -R \operatorname{cosec}^2 \theta \cdot d\theta$$

Substituting these values in Eq. (i), we get

$$dB = -\frac{1}{2}\mu_0 ni \sin\theta \cdot d\theta$$

Total field B due to the entire solenoid is

$$B = \frac{1}{2}\mu_0 ni \int_{\theta_1}^{\theta_2} (-\sin\theta)\, d\theta$$

$\therefore$

$$\boxed{B = \frac{\mu_0 ni}{2}(\cos\theta_2 - \cos\theta_1)}$$

If the solenoid is very long ($L >> R$) and the point O is chosen at the middle, i.e. if $\theta_1 = 180°$ and $\theta_2 = 0°$, then we get

$$\boxed{B\ (\text{centre}) = \mu_0 ni} \qquad [\text{For } L >> R]$$

At the end of the solenoid,

$\theta_2 = 0°, \quad \theta_1 = 90°$ and we get

$$\boxed{B\ (\text{end}) = \frac{1}{2}\mu_0 ni} \qquad [\text{For } L >> R]$$

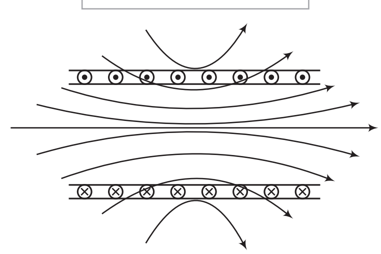


Fig. 26.42

Thus, the field at the end of a solenoid is just one half at the centre. The field lines are as shown in Fig. 26.42.

Example 26.13 *In a high tension wire electric current runs from east to west. Find the direction of magnetic field at points above and below the wire.*

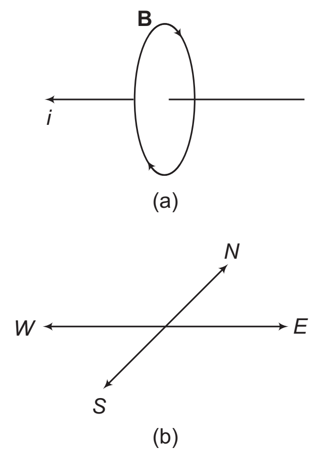


Fig. 26.43

Solution When the current flows from east to west, magnetic field lines are circular round it as shown in figure (a). And so, the magnetic field above the wire is towards north and below the wire towards south.

Example 26.14 *A current path shaped as shown in figure produces a magnetic field at P, the centre of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6 m, what are the magnitude and direction of the field produced at P if the current is 3.0 A.*

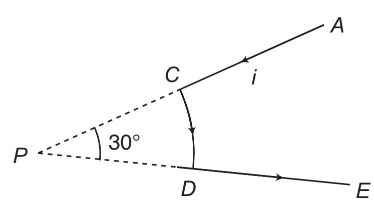


Fig. 26.44

Solution The magnetic field at P due to the straight segments AC and DE is zero.

CD is arc of circle.

$$\therefore \quad B = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2R}\right) \qquad (N = 1)$$

or

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\theta$$

$$\therefore \quad B = (10^{-7})\left(\frac{3.0}{0.6}\right)\left(\frac{\pi}{6}\right)$$

$$= 2.62 \times 10^{-7}\ \text{T}$$ **Ans.**

Example 26.15 *Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O.*

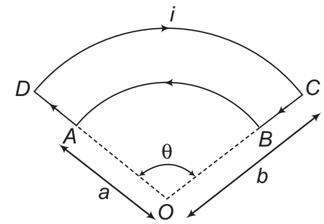


Fig. 26.45

Solution Magnetic field at point O, due to wires CB and AD will be zero.

Magnetic field due to wire BA will be

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$$

Direction of field $\mathbf{B}_1$ is coming out of the plane of the figure.

Similarly, field at O due to arc DC will be

$$B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$$

Direction of field $\mathbf{B}_2$ is going into the plane of the figure. The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 i\theta(b-a)}{4\pi ab} \qquad \textbf{Ans.}$$

Coming out of the plane.

Example 26.16 *The magnetic field B due to a current carrying circular loop of radius* 12 *cm at its centre is* 0.5×10^{-4} *T. Find the magnetic field due to this loop at a point on the axis at a distance of* 5.0 *cm from the centre.*

Solution Magnetic field at the centre of a circular loop is

$$B_1 = \frac{\mu_0 i}{2R}$$

and that at an axial point,

$$B_2 = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$$

Thus,

$$\frac{B_2}{B_1} = \frac{R^3}{(R^2 + x^2)^{3/2}}$$

or

$$B_2 = B_1\left[\frac{R^3}{(R^2 + x^2)^{3/2}}\right]$$

Substituting the values, we have

$$B_2 = (0.5 \times 10^{-4})\left[\frac{(12)^3}{(144 + 25)^{3/2}}\right]$$

$$= 3.9 \times 10^{-5} \text{ T} \qquad \textbf{Ans.}$$

INTRODUCTORY EXERCISE 26.5

1. (a) A conductor in the shape of a square of edge length $l = 0.4$ m carries a current $i = 10.0$ A. Calculate the magnitude and direction of magnetic field at the centre of the square.
   (b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the centre.

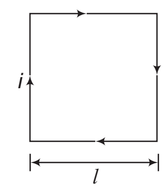


Fig. 26.46

2. Determine the magnetic field at point P located a distance x from the corner of an infinitely long wire bent at right angle as shown in figure. The wire carries a steady current i.

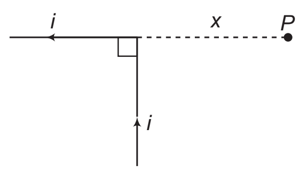


Fig. 26.47

3. A conductor consists of a circular loop of radius $R = 10$ cm and two straight, long sections as shown in figure. The wire lies in the plane of the paper and carries a current of $i = 7.00$ A. Determine the magnitude and direction of the magnetic field at the centre of the loop.

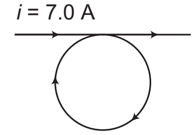


Fig. 26.48

4. The segment of wire shown in figure carries a current of $i = 5.0$ A, where the radius of the circular arc is $R = 3.0$ cm. Determine the magnitude and direction of the magnetic field at the origin. (Fig. 26.49)

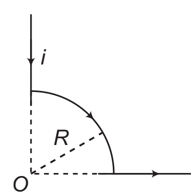


Fig. 26.49

5. Consider the current carrying loop shown in figure formed of radial lines and segments of circles whose centres are at point P. Find the magnitude and direction of **B** at point P. (Fig. 26.50)

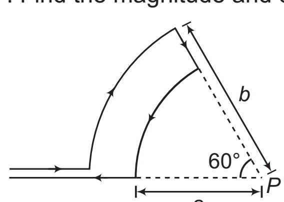


Fig. 26.50

26.9 Ampere's Circuital Law

The electrical force on a charge is related to the electric field (caused by other charges) by the equation,

$$\mathbf{F}_e = q\mathbf{E}$$

Just like the gravitational force, the static electrical force is a conservative force. This means that the work done by the static electric force around any closed path is zero.

$$q \oint \mathbf{E} \cdot d\mathbf{l} = 0 \text{ J}$$

Hence, we have

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \text{ V}$$

In other words, the integral of the static (time independent) electric field around a closed path is zero.

What about the integral of the magnetic field around a closed path? That is, we want to determine the value of

$$\oint \mathbf{B} \cdot d\mathbf{l}$$

Here, we have to be careful. The quantity $\mathbf{B} \cdot d\mathbf{l}$ does not represent some physical quantity, and certainly not work. Although the static magnetic force does no work on a moving charge, we cannot conclude that the path integral of the magnetic field around a closed path is zero. We are just curious about what this analogous line integral amounts to.

The line integral $\oint \mathbf{B} \cdot d\mathbf{l}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \, (i_{\text{net}}) \qquad \text{...(i)}$$

This is known as **Ampere's circuital law**.

Eq. (i) in simplified form can be written as

$$\boxed{Bl = \mu_0 \, (i_{\text{net}})} \qquad \text{...(ii)}$$

But this equation can be used only under the following conditions.

(i) At every point of the closed path $\mathbf{B} \parallel d\mathbf{l}$.

(ii) Magnetic field has the same magnitude B at all places on the closed path.

If this is not the case, then Eq. (i) is written as

$$B_1 dl_1 \cos\theta_1 + B_2 dl_2 \cos\theta_2 + \ldots = \mu_0 \, (i_{\text{net}})$$

Here, θ_1 is the angle between $\mathbf{B}_1$ and $d\mathbf{l}_1$, θ_2 the angle between $\mathbf{B}_2$ and $d\mathbf{l}_2$ and so on. Besides the Biot Savart law, Ampere's law gives another method to calculate the magnetic field due to a given current distribution. Ampere's law may be derived from the Biot Savart law and Bio Savart law may be derived from the Ampere's law. However, Ampere's law is more useful under certain symmetrical conditions. To illustrate the theory now let us take few applications of Ampere's circuital law.

Magnetic Field Created by a Long Current Carrying Wire

A long straight wire of radius R carries a steady current i that is uniformly distributed through the cross-section of the wire.

For finding the behaviour of magnetic field due to this wire, let us divide the whole region into two parts. One is $r \geq R$ and the another is $r < R$. Here, r is the distance from the centre of the wire.

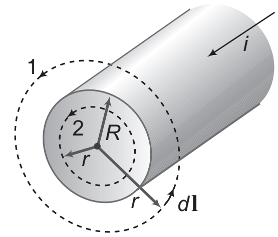


Fig. 26.51

For $r \geq R$: Let us choose for our path of integration circle 1. From symmetry **B** must be constant in magnitude and parallel to $d\mathbf{l}$ at every point on this circle. Because the total current passing through the plane of the circle is i. Ampere's law gives

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$$

or $$Bl = \mu_0 i \qquad \text{[simplified form]}$$

or $$B(2\pi r) = \mu_0 i$$

$\therefore$ $$\boxed{B = \frac{\mu_0}{2\pi}\frac{i}{r}} \qquad [\text{for } r \geq R] \quad \ldots\text{(iii)}$$

For $r < R$: Here, the current i' passing through the plane of circle 2 is less than the total current i. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire.

$$\frac{i'}{i} = \frac{\pi r^2}{\pi R^2} \quad \Rightarrow \quad i' = \left(\frac{r^2}{R^2}\right) i$$

Then, the following procedure same as for circle 1, we apply Ampere's law to circle 2.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$$

$$Bl = \mu_0 i' \qquad \text{[simplified form]}$$

$\therefore$ $$B\,(2\pi r) = \mu_0 \left(\frac{r^2}{R^2}\right) i$$

$\therefore$ $$\boxed{B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r} \qquad [\text{For } r < R] \quad \ldots\text{(iv)}$$

This result is similar in the form to the expression for the electric field inside a uniformly charged sphere. The magnitude of the magnetic field *versus* r for this configuration is plotted in figure. Note that inside the wire $B \to 0$ as $r \to 0$. Note also that Eqs. (iii) and (iv) give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.

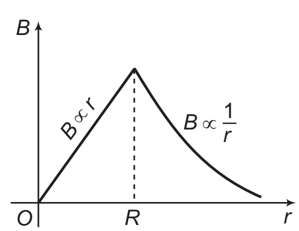


Fig. 26.52

Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire, which we shall call the interior of the solenoid, when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns (as done in Art. 26.9). If the turns are closely spaced and the solenoid is of infinite length, the magnetic field lines are as shown in Fig. 26.53.

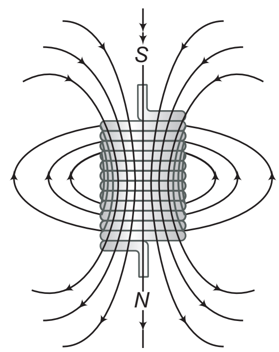


Fig. 26.53

One end of the solenoid behaves like the north pole (N) and the opposite end behaves like the south pole (S). As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker. An ideal solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns.

In this case, the external field is zero, and the interior field is uniform over a great volume.

We can use Ampere's law to obtain an expression for the interior magnetic field in an ideal solenoid. Fig. 26.54 shows a longitudinal cross-section of part of such a solenoid carrying a current *i*. Because the solenoid is ideal, **B** in the interior space is uniform and parallel to the axis, and **B** in the exterior space is zero.

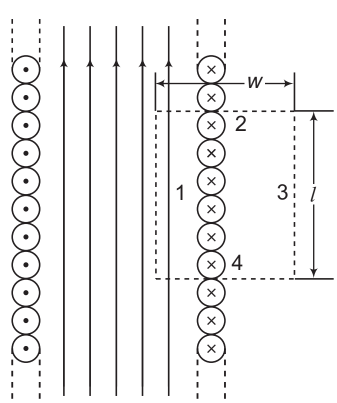


Fig. 26.54

Consider the rectangular path of length l and width w as shown in figure. We can apply Ampere's law to this path by evaluating the line integral $\mathbf{B} \cdot d\mathbf{l}$ over each side of the rectangle.

$$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 3}} = 0 \qquad \text{as} \qquad B = 0$$

$$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 2 and 4}} = 0 \qquad \text{as} \qquad \mathbf{B} \perp d\mathbf{l} \text{ or } B = 0 \text{ along these paths}$$

$$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 1}} = Bl \qquad \text{as} \qquad \mathbf{B} \text{ is uniform and parallel to } d\mathbf{l}$$

The integral over the closed rectangular path is therefore,

$$\oint \mathbf{B} \cdot d\mathbf{l} = Bl$$

The right side of Ampere's law involves the total current passing through the area bounded by the path of integration.

In this case,

$$i_{\text{net}} = \text{(number of turns inside the area) (current through each turn)}$$

$$= (nl)\,(i) \qquad (n = \text{number of turns per unit length})$$

Using Ampere's law, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$

or $Bl = (\mu_0)\,(nli)$ or $\boxed{B = \mu_0 ni}$ ...(v)

This result is same as obtained in Art. 26.9. Eq. (v) is valid only for points near the centre (that is far from the ends) of a very long solenoid. The field near each end is half the value given by Eq. (v).

Example 26.17 *A closed curve encircles several conductors. The line integral $\int \mathbf{B} \cdot d\mathbf{l}$ around this curve is 3.83×10^{-7} T-m.*

(a) What is the net current in the conductors?

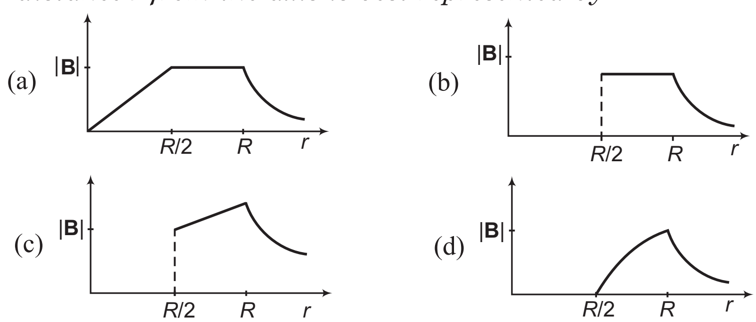
(b) If you were to integrate around the curve in the opposite directions, what would be the value of the line integral?

Solution (a) $\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$

$$\therefore \qquad i_{\text{net}} = \frac{\int \mathbf{B} \cdot d\mathbf{l}}{\mu_0} = \frac{3.83 \times 10^{-7}}{4\pi \times 10^{-7}} = 0.3\,\text{A}$$

(b) In opposite direction, line integral will be negative.

Example 26.18 *An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance r from the axis is best represented by* **(JEE 2012)**



Solution

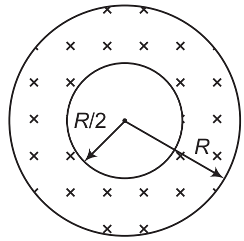


Fig. 26.55

r = distance of a point from centre

For $r \le R/2$ Using Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$$

or $$Bl = \mu_0 \, (I_{\text{in}})$$

or $$B\,(2\pi r) = \mu_0 \, (I_{\text{in}})$$

or $$B = \frac{\mu_0}{2\pi} \frac{I_{\text{in}}}{r} \quad \text{...(i)}$$

Since, $$I_{\text{in}} = 0$$

$\therefore$ $$B = 0$$

For $\frac{R}{2} \le r \le R$ $$I_{\text{in}} = \left[\pi r^2 - \pi \left(\frac{R}{2}\right)^2\right] \sigma$$

Here, σ = current per unit area

Substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \frac{\left[\pi r^2 - \pi \frac{R^2}{4}\right]\sigma}{r}$$

$$= \frac{\mu_0 \sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $r = \frac{R}{2}$, $B = 0$

At $r = R$, $B = \frac{3\mu_0 \sigma R}{8}$

For $r \ge R$ $I_{\text{in}} = I_{\text{Total}} = I$ (say)

Therefore, substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \text{or} \quad B \propto \frac{1}{r}$$

$\therefore$ The correct graph is (d).

Example 26.19 *A device called a toroid (figure) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a torus) made of a non-conducting material. For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the centre.*

Solution To calculate this field, we must evaluate $\oint \mathbf{B} \cdot d\mathbf{l}$ over the circle of radius r. By symmetry we see that the magnitude of the field is constant on this circle and tangent to it.

So, $$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = B\,(2\pi r)$$

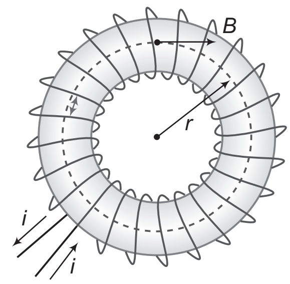


Fig. 26.56

Furthermore, the circular closed path surrounds N loops of wire, each of which carries a current i. Therefore, right side of Eq. (i) is $\mu_0 Ni$ in this case.

$\therefore$ $$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{net}}$$

or $$B(2\pi r) = \mu_0 Ni$$

or $$B = \frac{\mu_0 Ni}{2\pi r}$$

This result shows that $B \propto \frac{1}{r}$ and hence is non-uniform in the region occupied by torus. However, if r is very large compared with the cross-sectional radius of the torus, then the field is approximately uniform inside the torus. In that case,

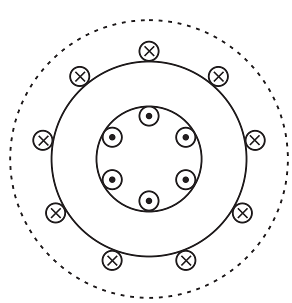


Fig. 26.57

$\frac{N}{2\pi r} = n =$ number of turns per unit length of torus

$\therefore$ $$B = \mu_0 ni$$

For an ideal toroid, in which turns are closely spaced, the external magnetic field is zero. This is because the net current passing through any circular path lying outside the toroid is zero. Therefore, from Ampere's law we find that $B = 0$, in the regions exterior to the torus.

INTRODUCTORY EXERCISE 26.6

1. Figure given in the question is a cross-sectional view of a coaxial cable. The centre conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. The current in the inner conductor is 1.0 A out of the page, and the current in the outer conductor is 3.0 A into the page. Determine the magnitude and direction of the magnetic field at points a and b.

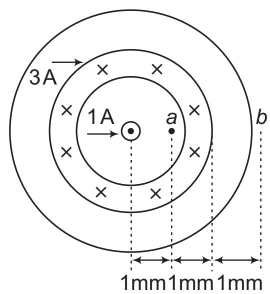


Fig. 26.58

2. Figure shows, in cross-section, several conductors that carry currents through the plane of the figure. The currents have the magnitudes $I_1 = 4.0$ A, $I_2 = 6.0$ A, and $I_3 = 2.0$ A, in the directions shown. Four paths labelled a to d, are shown. What is the line integral $\oint \mathbf{B} \cdot d\mathbf{l}$ for each path? Each integral involves going around the path in the counter-clockwise direction.

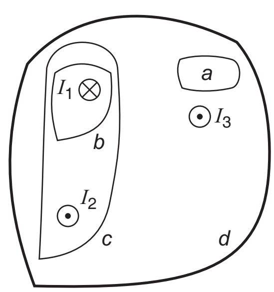


Fig. 26.59

3. A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then, **(JEE 1993)**
   (a) the magnetic field at all points inside the pipe is the same, but not zero
   (b) the magnetic field at any point inside the pipe is zero
   (c) the magnetic field is zero only on the axis of the pipe
   (d) the magnetic field is different at different points inside the pipe

26.10 Force Between Parallel Current Carrying Wires

Consider two long wires 1 and 2 kept parallel to each other at a distance r and carrying currents i_1 and i_2 respectively in the same direction.

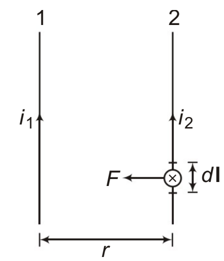


Fig. 26.60

Magnetic field on wire 2 due to current in wire 1 is, $B = \frac{\mu_0}{2\pi} \cdot \frac{i_1}{r}$ [in ⊗ direction]

Magnetic force on a small element dl of wire 2 due to this magnetic field is

$$d\mathbf{F} = i_2 (d\mathbf{l} \times \mathbf{B})$$

Magnitude of this force is $dF = i_2 [(dl)(B) \sin 90°]$

$$= i_2 (dl) \left(\frac{\mu_0}{2\pi} \frac{i_1}{r} \right) = \frac{\mu_0}{2\pi} \cdot \frac{i_1 i_2}{r} \cdot dl$$

Direction of this force is along $d\mathbf{l} \times \mathbf{B}$ or towards the wire 1.

The force per unit length of wire 2 due to wire 1 is

$$\boxed{\frac{dF}{dl} = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{r}}$$

The same force acts on wire 1 due to wire 2. The wires attract each other if currents in the wires are flowing in the same direction and they repel each other if the currents are in opposite directions.

Example 26.20 *Two long parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.00×10^{-5} N/m, and the wires repel each other. The current in one wire is 0.600 A.*

(a) What is the current in the second wire?

(b) Are the two currents in the same direction or in opposite directions?

Solution (a) $\frac{F}{l} = \left(\frac{\mu_0}{2\pi} \right) \frac{i_1 i_2}{r}$

$$\therefore \quad 4 \times 10^{-5} = \frac{(2 \times 10^{-7})(0.6) i_2}{2.5 \times 10^{-2}}$$

$$\therefore \quad i_2 = 8.33 \text{ A}$$

(b) Wires repel each other if currents are in opposite directions.

Example 26.21 *Consider three long straight parallel wires as shown in figure. Find the force experienced by a 25 cm length of wire C.*

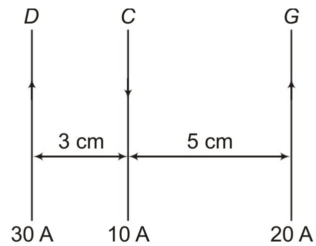


Fig. 26.61

Solution Repulsion by wire D, [towards right]

$$F_1 = \frac{\mu_0}{2\pi}\frac{i_1 i_2 l}{r}$$

$$= \frac{(2\times 10^{-7})(30\times 10)}{3\times 10^{-2}}(0.25)$$

$$= 5\times 10^{-4}\ \text{N}$$

Repulsion by wire G, [towards left]

$$F_2 = \frac{(2\times 10^{-7})(20\times 10)}{5\times 10^{-2}}(0.25)$$

$$= 2\times 10^{-4}\ \text{N}$$

$\therefore$ $$F_{\text{net}} = F_1 - F_2$$

$$= 3\times 10^{-4}\ \text{N}$$ [towards right]

26.11 Magnetic Poles and Bar Magnets

In electricity, the isolated charge q is the simplest structure that can exist. If two such charges of opposite sign are placed near each other, they form an electric dipole characterized by an electric dipole moment **p**. In magnetism isolated magnetic 'poles' which would correspond to isolated electric charges do not exist. The simplest magnetic structure is the **magnetic dipole**, characterized by a magnetic dipole moment **M**. A current loop, a bar magnet and a solenoid of finite length are examples of magnetic dipoles.

When a magnetic dipole is placed in an external magnetic field **B**, a magnetic torque τ acts on it, which is given by

$$\tau = \mathbf{M}\times\mathbf{B}$$

Alternatively, we can measure **B** due to the dipole at a point along its axis a (large) distance r from its centre by the expression,

$$\boxed{B = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}}$$

A bar magnet might be viewed as two poles (North and South) separated by some distance. However, all attempts to isolate these poles fail. If a magnet is broken, the fragments prove to be dipoles and not isolated poles. If we break up a magnet into the electrons and nuclei that make up its atoms, it will be found that even these elementary particles are magnetic dipoles.

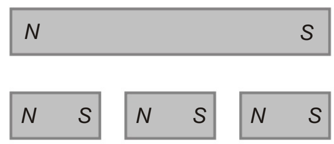


Fig. 26.62 If a bar magnet is broken, each fragment becomes a small dipole.

Each current carrying loop is just like a magnetic dipole, whose magnetic dipole moment is given by

$$\boxed{\mathbf{M} = ni\mathbf{A}}$$

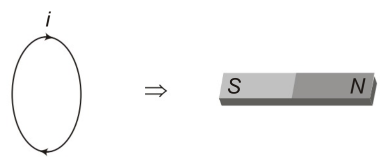


Fig. 26.63

Here, n is the number of turns in the loop, i is the current and $\mathbf{A}$ represents the area vector of the current loop.

The behaviour of a current loop can be described by the following hypothetical model:

(i) There are two magnetic charges; positive magnetic charge and negative magnetic charge. We call the positive magnetic charge a north pole and the negative magnetic charge as the south pole. Every pole has a pole strength m. The unit of pole strength is A-m.

(ii) A magnetic charge placed in a magnetic field experiences a force,

$$\boxed{\mathbf{F} = m\mathbf{B}}$$

The force on positive magnetic charge is along the field and a force on a negative magnetic charge is opposite to the field.

(iii) A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation d. The magnetic dipole moment is

$$\boxed{M = md}$$

The direction of $\mathbf{M}$ is from $-m$ to $+m$.

Geometrical Length and Magnetic Length

In case of a bar magnet, the poles appear at points which are slightly inside the two ends. The distance between the locations of the assumed poles is called the **magnetic length** of the magnet. The distance between the ends is called the **geometrical length.**

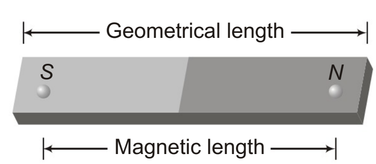


Fig. 26.64

The magnetic length of a bar magnet is written as $2l$. If m be the pole strength and $2l$ the magnetic length of a bar magnet, then its magnetic moment is

$$\boxed{M = 2ml}$$

Extra Points to Remember

- Current carrying loop, solenoid etc. are just like magnetic dipoles, whose dipole moment M is equal to NiA. Direction of **M** is from south pole (S) to north pole (N).
- The behaviour of a magnetic dipole (may be a bar magnet also) is similar to the behaviour of an electric dipole.
  The only difference is that the electric dipole moment **p** is replaced by magnetic dipole moment **M** and the constant $\frac{1}{4\pi\varepsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.
- Table given below makes a comparison between an electric dipole and a magnetic dipole.

Table 26.2

S.No.	Physical quality to be compared	Electric dipole	Magnetic dipole
1.	Dipole moment	$p = q\,(2l)$	$M = m\,(2l)$
2.	Direction of dipole moment	From negative charge to the positive charge	From south to north pole
3.	Net force in uniform field	0	0
4.	Net torque in uniform field	$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$	$\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}$
5.	Field at far away point on the axis	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$ (along **p**)	$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$ (along **M**)
6.	Field at far away point on perpendicular bisector	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$ (opposite to **p**)	$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$ (opposite to **M**)
7.	Potential energy	$U_\theta = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$	$U_\theta = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$
8.	Work done in rotating the dipole	$W_{\theta_1 - \theta_2} = pE(\cos\theta_1 - \cos\theta_2)$	$W_{\theta_1 - \theta_2} = MB(\cos\theta_1 - \cos\theta_2)$

Note *In the above table, θ is the angle between field (**E** or **B**) and dipole moment (**p** or **M**).*

Example 26.22 *Calculate the magnetic induction (or magnetic field) at a point 1 Å away from a proton, measured along its axis of spin. The magnetic moment of the proton is 1.4×10^{-26} A-m^2.*

Solution On the axis of a magnetic dipole, magnetic induction is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Substituting the values, we get

$$B = \frac{(10^{-7})\,(2)\,(1.4 \times 10^{-26})}{(10^{-10})^3}$$

$$= 2.8 \times 10^{-3}\ \mathrm{T}$$

$$= 2.8\ \mathrm{mT}$$ **Ans.**

Example 26.23 *A bar magnet of magnetic moment 2.0 $A\text{-}m^2$ is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east-west position. Find the kinetic energy of the magnet as it takes the north-south position. The horizontal component of the earth's magnetic field is $B = 25\ \mu T$. Earth's magnetic field is from south to north.*

Solution Gain in kinetic energy = loss in potential energy

Thus,
$$\text{KE} = U_i - U_f$$
As,
$$U = -MB\cos\theta$$
$\therefore$
$$\text{KE} = -MB\cos\left(\frac{\pi}{2}\right) - (-MB\cos 0^\circ)$$
$$= MB$$
Substituting the values, we have
$$\text{KE} = (2.0)(25\times 10^{-6})\ \text{J}$$
$$= 50\ \mu\text{J}$$
Ans.

26.12 Earth's Magnetism

Our earth behaves as it has a powerful magnet within it. The value of magnetic field on the surface of earth is a few tenths of a gauss ($1\,\text{G} = 10^{-4}$ T). The earth's south magnetic pole is located near the north geographic pole and the earth's north magnetic pole is located near the south geographic pole. In fact, the configuration of the earth's magnetic field is very much like the one that would be achieved by burying a gigantic bar magnet deep in the interior of the earth.

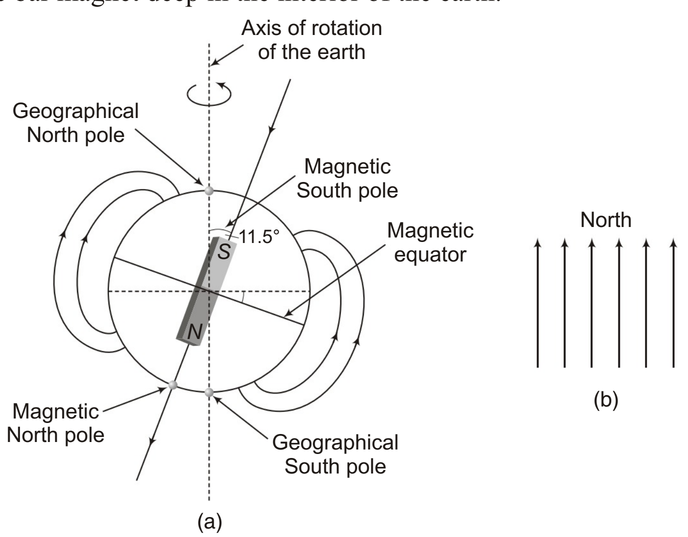


Fig. 26.65

The axis of earth's magnet makes an angle of 11.5° with the earth's rotational axis.

Theories Regarding the Origin of Earth's Magnetism

First Theory : Gilbert for the first time in 1600, gave the idea that there is a powerful magnet within the earth at its centre. Later on this theory was denied because the temperature in the interior of the earth is so high that it is impossible to retain its magnetism.

Second Theory : The second theory was put forward by Grover in 1849. He put the view that the earth magnetism is due to electric currents flowing near the outer surface of the earth. Hot air, rising from the region near equator, goes towards north and south hemispheres and become electrified. These currents magnetise the ferromagnetic material near the outer surface of the earth.

Third Theory : There are many conducting materials including iron and nickel in the molten state within the central core of the earth. Conventional currents are generated in this semifluid core due to earth's rotation about its axis. Due to these currents, magnetism is generated within the earth.

Till date not a single theory can explain all events regarding earth's magnetism.

Elements of Earth's Magnetism

There are three elements of earth's magnetism.

(i) **Angle of Declination (α)** At any point (say P) on earth's surface the longitude determines the north-south direction. The vertical plane in the direction of longitude or the vertical plane passing through the line joining the geographical north and south poles is called the **'geographical meridian'**. At point P, there also exists the magnetic field **B**. A vertical plane in the direction of **B** is called **'magnetic meridian'**.

At any place the acute angle between the magnetic meridian and the geographical meridian is called **'angle of declination 'α'.**

(ii) **Angle of Dip (θ)** 'The angle of dip (θ) at a place is the angle between the direction of earth's magnetic field and the horizontal at that place.'

Angle of dip at some place can be measured from a magnetic needle free to rotate in a vertical plane about a horizontal axis passing through centre of gravity of the needle. At earth's magnetic poles the magnetic field of earth is vertical, i.e. angle of dip is 90°, the freely suspended magnetic needle is vertical there. At magnetic equator field is horizontal, or angle of dip is 0°. The needle is horizontal. In northern hemisphere, the north pole of the magnetic needle inclines downwards, whereas in the southern hemisphere the south pole of the needle inclines downwards.

(iii) **Horizontal Component of Earth's Magnetic Field** Let B_e be the net magnetic field at some point. H and V be the horizontal and vertical components of B_e. Let θ is the angle of dip at the same place, then we can see that

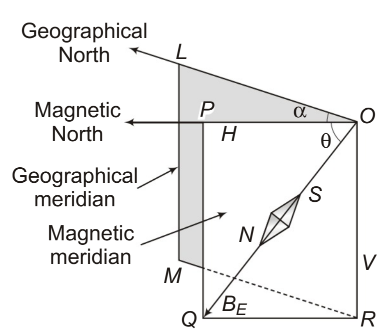


Fig. 26.66

$$H = B_e \cos\theta \qquad \ldots\text{(i)}$$

and

$$V = B_e \sin\theta \qquad \ldots\text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$B_e = \sqrt{H^2 + V^2}$$

Further, dividing Eq. (ii) by Eq. (i), we get

$$\theta = \tan^{-1}\left(\frac{V}{H}\right)$$

By knowing H and θ at some place we can find B_e and V at that place.

Neutral Points

When a magnet is placed at some point on earth's surface, there are points where horizontal component of earth's magnetic field is just equal and opposite to the field due to the magnet. Such points are called neutral points. If a magnetic compass is placed at a neutral point, no force acts on it and it may set in any direction.

Suppose a small bar magnet is placed such that north pole of the magnet is towards the magnetic south pole of the earth then neutral points are obtained both sides on the axis of the magnet. If distance of each neutral point from the middle point of a magnet be r, and the magnitude of the magnetic moment of the magnet be M, then

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = H$$

When north pole of bar magnet is towards the magnetic north pole of the earth, the neutral points are obtained on perpendicular bisectors of the magnet. Let r be the distance of neutral points from centre, then

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = H$$

Example 26.24 *In the magnetic meridian of a certain place, the horizontal component of earth's magnetic field is 0.26 G and the dip angle is 60°. Find*

(a) vertical component of earth's magnetic field.

(b) the net magnetic field at this place.

Solution Given, $H = 0.26\,\text{G}$ and $\theta = 60°$

(a) $\tan\theta = \dfrac{V}{H}$

$\therefore$

$$V = H\tan\theta = (0.26)\tan 60°$$
$$= 0.45\,\text{G}$$

Ans.

(b) $H = B_e \cos\theta$

$\therefore$

$$B_e = \frac{H}{\cos\theta} = \frac{0.26}{\cos 60°}$$
$$= 0.52\,\text{G}$$

Ans.

Example 26.25 *A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.*

Solution In a vertical plane at 30° from the magnetic meridian, the horizontal component is

$$H' = H \cos 30°$$

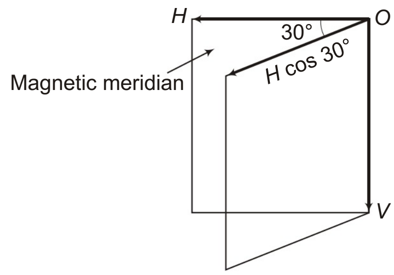


Fig. 26.67

While vertical component is still V. Therefore, apparent dip will be given by

$$\tan \theta' = \frac{V}{H'} = \frac{V}{H \cos 30°}$$

But, $$\frac{V}{H} = \tan \theta \qquad \text{(where, } \theta = \text{true angle of dip)}$$

$\therefore$ $$\tan \theta' = \frac{\tan \theta}{\cos 30°}$$

$\therefore$ $$\theta = \tan^{-1} [\tan \theta' \cos 30°]$$

$$= \tan^{-1} [(\tan 45°)(\cos 30°)]$$

$$\approx 41°$$ **Ans.**

26.13 Vibration Magnetometer

Vibration magnetometer is an instrument which is used for the following two purposes:

(i) To find magnetic moment of a bar magnet.

(ii) To compare magnetic fields of two magnets.

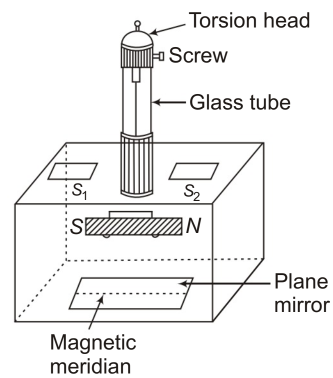


Fig. 26.68

The construction of a vibration magnetometer is as shown in figure.The magnet shown in figure is free to rotate in a horizontal plane. The magnet stays parallel to the horizontal component of earth's magnetic field. If the magnet is now displaced through an angle θ, a restoring torque of magnitude $MH \sin \theta$ acts on it and the magnet starts oscillating. From the theory of simple harmonic motion, we can find the time period of oscillations of the magnet.

Restoring torque in displaced position is

$$\tau = - MH \sin \theta \qquad \ldots(i)$$

Here, M = Magnetic moment of the magnet

and H = Horizontal component of earth's magnetic field.

Negative sign shows the restoring nature of torque. Now since, $\tau = I\alpha$ and $\sin\theta \approx \theta$ for small angular displacement.

Thus, Eq. (i) can be written as

$$I\alpha = -MH\theta$$

Since, α is proportional to $-\theta$. Therefore, motion is simple harmonic in nature, time period of which will be given by

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{I}{MH}}$$

$$\therefore \qquad \boxed{T = 2\pi\sqrt{\frac{I}{MH}}} \qquad \ldots\text{(ii)}$$

In the expression of T, I is the moment of inertia of the magnet about its axis of vibration.

(i) Measurement of Magnetic Moment : By finding time period T of vibrations of the given magnet, we can calculate magnetic moment M by the relation,

$$M = \frac{4\pi^2 I}{T^2 H}$$

(ii) Comparison of Two Magnetic Fields : Suppose we wish to compare the magnetic fields B_1 and B_2 at some point P due to two magnets. For this, vibration magnetometer is so placed that the centre of its magnet lies on P. Now, one of the given magnets is placed at some known distance from P in the magnetic meridian, such that point P lies on its axial line and its north pole points north. In this position, the field B_1 at P produced by the magnet will be in the direction of H. Hence, the magnet suspended in the magnetometer will vibrate in the resultant magnetic field $(H + B_1)$. Its period of vibration is noted, say it is T_1, then

$$T_1 = 2\pi\sqrt{\frac{I}{M\,(H + B_1)}}$$

Now, the first magnet is replaced by the second magnet and the second magnet is placed in the same position and again the time period is noted. If the field produced at P due to this magnet be B_2 and the new time period be T_2, then

$$T_2 = 2\pi\sqrt{\frac{I}{M(H + B_2)}}$$

Finally, the time period of the magnetometer under the influence of the earth's magnetic field alone is determined. Let it be T, then

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Solving above three equations for T, T_1 and T_2, we can show that

$$\boxed{\frac{B_1}{B_2} = \frac{(T^2 - T_1^2)\,T_2^2}{(T^2 - T_2^2)\,T_1^2}}$$

Example 26.26 *A short bar magnet is placed with its north pole pointing north. The neutral point is* 10 *cm away from the centre of the magnet. If* $H = 0.4$ *G, calculate the magnetic moment of the magnet.*

Solution When north pole of the magnet points towards magnetic north, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of earth's magnetic field H.

Thus, $$H = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \quad \text{or} \quad M = \frac{Hr^3}{(\mu_0/4\pi)}$$

Substituting the values, we have

$$M = \frac{(0.4 \times 10^{-4})(10 \times 10^{-2})^3}{10^{-7}} = 0.4 \text{ A-m}^2$$ **Ans.**

Example 26.27 *A magnetic needle performs* 20 *oscillations per minute in a horizontal plane. If the angle of dip be* 30°, *then how many oscillations per minute will this needle perform in vertical north-south plane and in vertical east-west plane?*

Solution In horizontal plane, the magnetic needle oscillates in horizontal component H.

$\therefore$ $$T = 2\pi\sqrt{\frac{I}{MH}}$$

In the vertical north-south plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e, and in vertical east-west plane (plane perpendicular to the magnetic meridian) it oscillates only in earth's vertical component V. If its time period be T_1 and T_2, then

$$T_1 = 2\pi\sqrt{\frac{I}{MB_e}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{I}{MV}}$$

From above equations, we can find

$$\frac{T_1^2}{T^2} = \frac{H}{B_e} \quad \text{or} \quad \frac{n_1^2}{n^2} = \frac{B_e}{H}$$

Similarly, $$\frac{n_2^2}{n^2} = \frac{V}{H}$$

Further, $$\frac{B_e}{H} = \sec\theta = \sec 30° = \frac{2}{\sqrt{3}}$$

and $$\frac{V}{H} = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$$

$\therefore$ $$n_1^2 = (n)^2\left(\frac{B_e}{H}\right) = (20)^2\left(\frac{2}{\sqrt{3}}\right)$$

or $$n_1 = 21.5 \text{ oscillations/min}$$ **Ans.**

and $$n_2^2 = (n)^2\left(\frac{V}{H}\right) = (20)^2\left(\frac{1}{\sqrt{3}}\right)$$

$\therefore$ $$n_2 = 15.2 \text{ oscillations/min}$$ **Ans.**

26.14 Magnetic Induction and Magnetic Materials

We know that the electric lines of force change when a dielectric is placed between the parallel plates of a capacitor. Experiments show that magnetic lines also get modified due to the presence of certain materials in the magnetic field.

Few substances such as O_2, air, platinum, aluminium etc., show a very small increase in the magnetic flux passing through them, when placed in a magnetic field. Such substances are called **paramagnetic substances**. Few other substances such as H_2, H_2O, Cu, Zn, Sb etc. show a very small decrease in flux and are said to be **diamagnetic**. There are other substances like Fe, Co etc. through which the flux increases to a larger value and are known as **ferromagnetic substances**.

Magnetisation of Matter

A material body is consisting of large number of atoms and thus large number of electrons. Each electron produces orbital and spin magnetic moments and can be assumed as magnetic dipoles. In the absence of any external magnetic field, the dipoles of individual atoms are randomly oriented and the magnetic moments thus, cancel.

When we apply an external magnetic field to a substance, two processes may occur.

(i) All atoms which have non-zero magnetic moment are aligned along the magnetic field.

(ii) If the atom has a zero magnetic moment, the applied magnetic field distorts the electron orbit and thus, induces magnetic moment in opposite directions.

In diatomic substances, the individual atoms do not have a magnetic moment by its own. When an external field is applied, the second process occurs. The induced magnetic moment is thus set up in the direction opposite to **B**. In this case, the magnetic flux density in the interior of the body will be less than that of the external field **B**.

In paramagnetic substances, the constituent atoms have intrinsic magnetic moments. When an external magnetic field is applied, both of the above processes occur and the resultant magnetic moment is always in the direction of magnetic field **B** as the first effect predominates over the second.

26.15 Some Important Terms Used in Magnetism

Magnetic Induction (B)

When a piece of any substance is placed in an external magnetic field, the substance becomes magnetised. If an iron bar is placed in a uniform magnetic field, the magnetised bar produces its own magnetic field in the same direction as those of the original field inside the bar, but in opposite direction outside the bar. This results in a concentration of the lines of force within the bar. The magnetic flux density within the bar is increased whereas it becomes weak at certain places outside the bar.

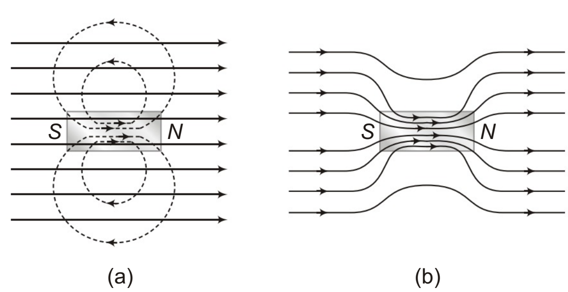


Fig. 26.69

"The number of magnetic lines of induction inside a magnetic substance crossing unit area normal to their direction is called the magnitude of magnetic induction, or magnetic flux density inside the substance. It is denoted by **B**. The SI unit of **B** is tesla (T) or weber/metre^2 (Wb/m^2). The CGS unit is gauss (G).

$$\boxed{1\ \mathrm{Wb/m^2} = 1\ \mathrm{T} = 10^4\ \mathrm{G}}$$

Intensity of Magnetisation (*I*)

"Intensity of magnetisation (***I***) is defined as the magnetic moment per unit volume of the magnetised substance." This basically represents the extent to which the substance is magnetised. Thus,

$$\boxed{I = \frac{M}{V}}$$

The SI unit of *I* is ampere/metre (A/m).

Magnetic Intensity or Magnetic Field Strength (H)

When a substance is placed in an external magnetic field, the actual magnetic field inside the substance is the sum of the external field and the field due to its magnetisation.

The capability of the magnetising field to magnetise the substance is expressed by means of a vector **H**, called the 'magnetic intensity' of the field. It is defined through the vector relation,

$$\boxed{\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}}$$

The SI unit of **H** is same as that of **I**, i.e. ampere/metre (A/m). The CGS unit is oersted.

Magnetic Permeability (μ)

"It is defined as the ratio of the magnetic induction **B** inside the magnetised substance to the magnetic intensity **H** of the magnetising field, i.e.

$$\boxed{\mu = \frac{\mathbf{B}}{\mathbf{H}}}$$

It is basically a measure of conduction of magnetic lines of force through it. The SI unit of magnetic permeability is weber/ampere-metre (Wb/A-m).

Relative Magnetic Permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

Thus,

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

μ_r is a pure ratio, hence, dimensionless. For vacuum its value is 1.

μ_r can also be defined as the ratio of the magnetic field B in the substance when placed in magnetic field B_0. Thus,

$$\boxed{\mu_r = \frac{B}{B_0}}$$

For paramagnetic substance, $\mu_r > 1$,

For diamagnetic substance, $\mu_r < 1$ and

For ferromagnetic substance, $\mu_r >> 1$.

Magnetic Susceptibility (χ_m)

We know that both diamagnetic and paramagnetic substances develop a magnetic moment depending on the applied field. Magnetic susceptibility is a measure of how easily a substance is magnetised in a magnetising field. For paramagnetic and diamagnetic substances, **I**, **H** and χ_m are related by the equation,

$$\mathbf{I} = \chi_m \mathbf{H}$$

or

$$\chi_m = \frac{I}{H}$$

Thus, the magnetic susceptibility χ_m may be defined as the ratio of the intensity of magnetisation to the magnetic intensity of the magnetising field.

Since, I and H have the same units, χ_m is unitless. It is a pure number.

By doing simple calculation, we can prove that μ_r and χ_m are related by

$$\mu_r = 1 + \chi_m$$

For paramagnetic substances χ_m is slightly positive. For diamagnetic substances, it is slightly negative and for ferromagnetic substances, χ_m is positive and very large.

26.16 Properties of Magnetic Materials

As discussed earlier, all substances (whether solid, liquid or gaseous) may be classified into three categories in terms of their magnetic properties. (i) paramagnetic, (ii) diamagnetic and (iii) ferromagnetic.

Paramagnetic Substances

Examples of such substances are platinum, aluminium, chromium, manganese, $CuSO_4$ solution etc. They have the following properties:

(i) The substances when placed in a magnetic field, acquire a feeble magnetisation in the same sense as the applied field. Thus, the magnetic inductance inside the substance is slightly greater than outside to it.

(ii) In a uniform magnetic field, these substances rotate until their longest axes are parallel to the field.

(iii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

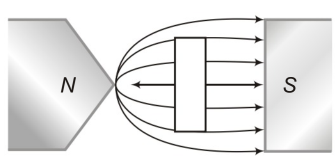


Fig. 26.70

Figure shows a strong electromagnet in which one of the pole pieces is sharply pointed while the other is flat. Magnetic field is much stronger near the pointed pole than near the flat pole. If a small piece of paramagnetic material is suspended in this region, a force can be observed in the direction of arrow.

(iv) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched on.

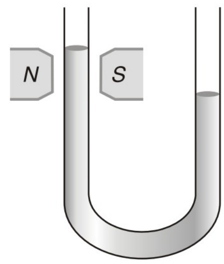


Fig. 26.71

(v) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.

(vi) At a given temperature the magnetic susceptibility χ_m does not change with the magnetising field. However, it varies inversely as the absolute temperature. As temperature increases, χ_m decreases. At some higher temperature, χ_m becomes negative and the substance becomes diamagnetic.

Diamagnetic Substances

Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol etc. They have the following properties:

(i) These substances when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to that of the applied field. Thus, the lines of induction inside the substance is smaller than that outside to it.

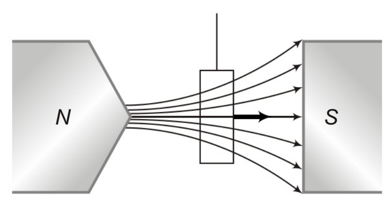


Fig. 26.72

(ii) In a uniform field, these substances rotate until their longest axes are normal to the field.

(iii) In a non-uniform field, these substances move from stronger to weaker parts of the field.

(iv) If a diamagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole of an electromagnet, the level of liquid depresses when the field is switched on.

(v) The relative permeability μ_r is slightly less than 1.

(vi) The susceptibility χ_m of such substances is always negative. It is constant and does not vary with field or the temperature.

Ferromagnetic Substances

Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree with paramagnetic substances as regard their behaviour. They have the following additional properties:

(i) These substances are strongly magnetised by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of hundreds and thousands.

(iii) The susceptibility is positive and very large.

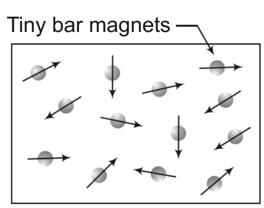
(iv) Susceptibility remains constant for very small values of **H**, increases for larger values of **H** and then decreases for very large values of **H**.

(v) Susceptibility decreases steadily with the rise of temperature. Above a certain temperature known as **Curie Temperature**, the ferromagnetic substances become paramagnetic. It is 1000°C for iron, 770°C for steel, 360°C for nickel and 1150°C for cobalt.

26.17 Explanation of Paramagnetism, Diamagnetism and Ferromagnetism

There are three properties of atoms that give rise to magnetic dipole moment.

1. The electrons moving around the nucleus in the orbits act as small current loops and contribute magnetic moments.
2. The spinning electron has an intrinsic magnetic dipole moment.
3. The nucleus contribute to magnetic moment due to the motion of charge within the nucleus. The magnitude of nuclear moments is about 10^{-3} times that of electronic moments or the spin magnetic moments, as the later two are of the same order. Still most of the magnetic moment of an atom is produced by electron spin, the net contribution of the orbital revolution is very small. This is because most of the electrons pair off in such a way that they produce equal and opposite orbital magnetic moment and they cancel out. Although, the electrons also try to pair up with their opposite spins but in case of spin motion of an electron it is not always possible to form equal and opposite pairs.



In the absence of external magnetic field

Fig. 26.73

Paramagnetism

The property of paramagnetism is found in those substances whose atoms or molecules have an excess of electrons spinning in the same direction.

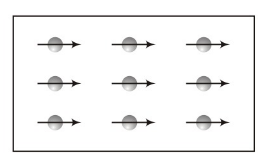


Fig. 26.74

Hence, atoms of paramagnetic substances have a permanent magnetic moment and behave like tiny bar magnets. In the absence of external magnetic field, the atomic magnets are randomly oriented and net magnetic moment is thus, zero.

When paramagnetic substance is placed in an external magnetic field, then each atomic magnet experiences a torque which tends to turn the magnet in the direction of the field. The atomic magnets are thus, aligned in the direction of the field. Thus, the whole substance is magnetised in the direction of the external magnetic field.

As the temperature of substance is increased, the thermal agitation disturbs the magnetic alignment of the atoms. Thus, we can say that paramagnetism is temperature dependent.

Curie's law

According to Curie's law, magnetic susceptibility of a paramagnetic substance is inversely proportional to absolute temperature T.

$$\boxed{\chi_m \propto \frac{1}{T}}$$

The exact law is beyond the scope of our course.

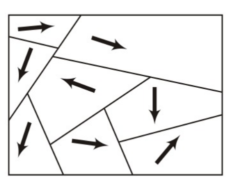
Diamagnetism

The property of diamagnetism is generally found in those substances whose atoms (or molecules) have even number of electrons which form pairs. "The net magnetic moment of an atom of a diamagnetic substance is thus zero." When a diamagnetic substance is placed in an external magnetic field, the spin motion of electrons is so modified that the electrons which produce the magnetic moments in the direction of external field slow down while the electrons which produce magnetic moments in opposite direction get accelerated. Thus, a net magnetic moment is induced in the opposite directions of applied magnetic field. Hence, the substance is magnetised opposite to the external field.

Note *That diamagnetism is temperature independent.*

Ferromagnetism

Iron like elements and their alloys are known as ferromagnetic substances. The susceptibility of these substances is in several thousands. Like paramagnetic substances, atoms of ferromagnetic substances have a permanent magnetic moment and behave like tiny magnets. But in ferromagnetic substances the atoms form innumerable small effective regions called 'domains'.



Unmagnetised

Fig. 26.75

The size of the domain vary from about 10^{-6}cm^3 to 10^{-2}cm^3. Each domain has 10^{17} to 10^{21} atoms whose magnetic moments are aligned in the same direction. In an unmagnetised ferromagnetic specimen, the domains are oriented randomly, so that their resultant magnetic moment is zero.

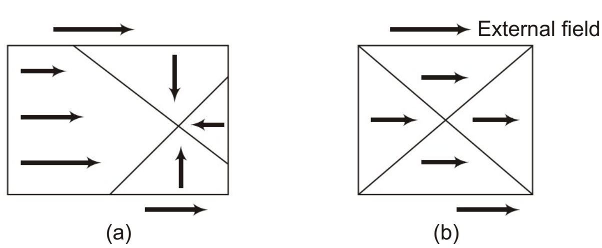


Fig. 26.76

When the specimen is placed in a magnetic field, the resultant magnetisation may increase in two different ways.

(a) The domains which are oriented favourably with respect to the field increase in size. Whereas those oriented opposite to the external field are reduced.

(b) The domains rotate towards the field direction.

Note *That if the external field is weak, specimen gets magnetised by the first method and if the field is strong they get magnetised by the second method.*

Hysteresis : Retentivity and Coercivity

The distinguishing characteristics of a ferromagnetic material is not that it can be strongly magnetised but that the intensity of magnetisation **I** is not directly proportional to the magnetising field **H**. If a gradually increasing magnetic field **H** is applied to an unmagnetised piece of iron, its magnetisation increases non-linearly until it reaches a maximum.

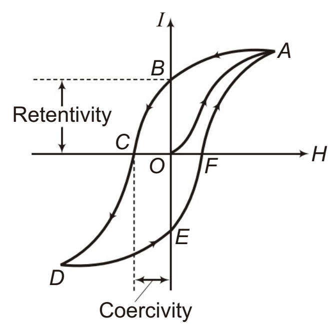


Fig. 26.77

If **I** is plotted against **H**, a curve like *OA* is obtained. This curve is known as magnetisation curve. At this stage all the dipoles are aligned and **I** has reached to a maximum or saturated value. If the magnetic field **H** is now decreased, the **I** does not return along magnetisation curve but follows path *AB*. At $\mathbf{H} = 0$, **I** does not come to its zero value but its value is still near the saturated value. The value of **I** at this point (i.e. *OB*) is known as **remanence, remanent magnetisation or retentivity**. The value of **I** at this point is known as residual induction. On applying a reverse field the value of **I** finally becomes zero. The abscissa *OC* represents the reversed magnetic field needed to demagnetise the specimen. This is known as **coercivity** of the material.

If the reverse field is further increased, a reverse magnetisation is set up which quickly reaches the saturation value. This is shown as *CD*. If **H** is now taken back from its negative saturation value to its original positive saturation value, a similar curve *DEFA* will be traced. The whole graph *ABCDEFA* thus, forms a closed loop, usually known as **hysteresis loop**. The whole process described above and the property of the iron characterized by it are called **hysteresis**. The energy lost per unit volume of a substance in a complete cycle is equal to the area. Thus, we can conclude the following three points from the above discussion:

(i) The retentivity of a substance is a measure of the magnetisation remaining in the substance when the magnetising field is removed.

(ii) The coercivity of a substance is a measure of the reverse magnetising field required to destroy the residual magnetism of the substance.

(iii) The energy loss per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

Demagnetisation

It is clear from the hysteresis loop that the intensity of magnetisation **I** does not reduce to zero on removing the magnetising field **H**. Further, **I** is zero when the magnetising field **H** is equal to the coercive field.

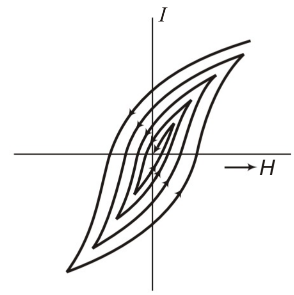


Fig. 26.78

At these points the magnetic induction is not zero, and the specimen is not demagnetised. To demagnetise a substance, it is subjected to several cycles is magnetisation, each time with decreasing magnetising field and finally the field is reduced to zero. In this way, the size of the hysteresis curve goes on decreasing and the area finally reduces to zero.

Demagnetisation is obtained by placing the specimen in an alternating field of continuously diminishing amplitude. It is also obtained by heating. Ferromagnetic materials become practically non-magnetic at sufficiently high temperatures.

Magnetic Properties of Soft Iron and Steel

A comparison of the magnetic properties of ferromagnetic substances can be made by the comparison of the shapes and sizes of their hysteresis loops.

Following three conclusions can be drawn from their hysteresis loops:

(i) Retentivity of soft iron is more than the retentivity of steel.

(ii) Coercivity of soft iron is less than the coercivity of steel.

(iii) Area of hysteresis loop (i.e. hysteresis loss) in soft iron is smaller than that in steel.

Choice of Magnetic Materials

The choice of a magnetic material for different uses is decided from the hysteresis curve of a specimen of the material.

(i) **Permanent Magnets** The materials for a permanent magnet should have

(a) high retentivity (so that the magnet is strong) and

(b) high coercivity (so that the magnetising is not wiped out by stray magnetic fields). As the material in this case is never put to cyclic changes of magnetisation, hence, hysteresis is immaterial. From the point of view of these facts **steel** is more suitable for the construction of permanent magnets than soft iron.

Modern permanent magnets are made of ‘cobalt-steel’, alloys ‘ticonal’.

(ii) **Electromagnets** The materials for the construction of electromagnets should have

(a) high initial permeability

(b) low hysteresis loss

From the view point of these facts, **soft iron** is an ideal material for this purpose.

(iii) **Transformer Cores and Telephone Diaphragms** As the magnetic material used in these cases is subjected to cyclic changes. Thus, the essential requirements for the selection of the material are

(a) high initial permeability

(b) low hysteresis loss to prevent the breakdown

Electromagnet

As we know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an 'electromagnet'. It is a temporary magnet.

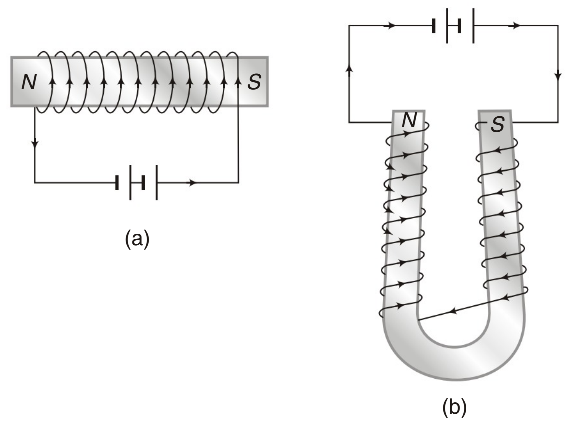


Fig. 26.79

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a horse shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.

Applications of Electromagnets

(i) Electromagnets are used in electric bell, transformer, telephone diaphragms etc.
(ii) In medical field, they are used in extracting bullets from the human body.
(iii) Large electromagnets are used in cranes for lifting and transferring big machines and parts.

26.18 Moving Coil Galvanometer

The moving coil galvanometer is a device used to measure an electric current.

Principle

Action of a moving coil galvanometer is based upon the principle that when a current carrying coil is placed in a magnetic field, it experiences a torque whose magnitude depends on the magnitude of current.

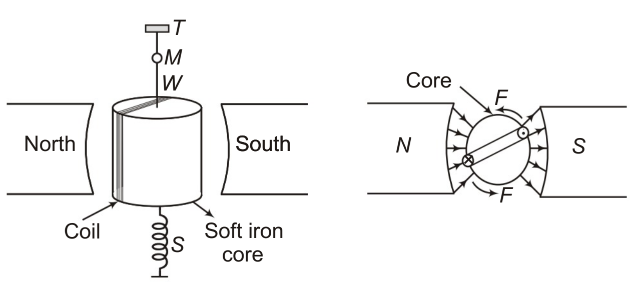


Fig. 26.80

Construction

The main parts of a moving coil galvanometer are shown in figure. The galvanometer consists of a coil, with many turns free to rotate about a fixed vertical axis in a uniform radial magnetic field. There is a cylindrical soft iron core, which not only makes the field radial but also increases the strength of magnetic field.

Theory

The current to be measured is passed through the galvanometer. As the coil is in the magnetic field (of constant magnitude) it experiences a torque given by

$$\tau = MB\sin\theta$$

$$= (NiA)\,B\sin\theta \qquad \text{...(i)}$$

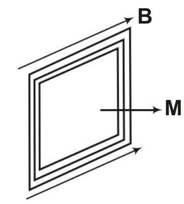


Fig. 26.81

As shown in the figure, the pole pieces are made cylindrical, the magnetic field always remains parallel to the plane of the coil. Or angle between **B** and **M** always remains 90°. Therefore, Eq. (i) can be written as

$$\tau = NiAB \qquad (\text{as } \sin\theta = \sin 90^\circ = 1)$$

Here, N = total number of turns of the coil

i = current passing through the coil

A = area of cross-section of the coil and

B = magnitude of radial magnetic field.

This torque rotates the coil. The spring S shown in figure provides a counter torque $k\phi$ that balances the above torque $NiAB$. In equilibrium,

$$k\phi = NiAB \qquad \text{...(ii)}$$

Here, k is the torsional constant of the spring. With rotation of coil a small light mirror M (attached with phosphor bronze wire W) also rotates and equilibrium deflection ϕ can be measured by a lamp and scale arrangement.

The above Eq. (ii) can be written as

$$\boxed{i = \left(\frac{k}{NAB}\right)\phi} \qquad \text{...(iii)}$$

Hence, the current i is proportional to the deflection ϕ.

Galvanometer Constant

The constant $\frac{k}{NAB}$ in Eq. (iii) is called galvanometer constant.

Hence,

$$\boxed{\text{Galvanometer constant} = \frac{k}{NAB}} \qquad \text{...(iv)}$$

This constant may be found by passing a known current through the coil. Measuring the deflection ϕ and putting these values in Eq. (iii), we can find galvanometer constant.

Sensitivity of Galvanometer

Deflection per unit current (ϕ/i) is called sensitivity of galvanometer. From Eq. (iii), we can see that $\phi/i = NAB/k$. Hence,

$$\boxed{\text{Sensitivity} = \frac{\phi}{i} = \frac{NAB}{k}} \qquad \text{...(v)}$$

The sensitivity of a galvanometer can be increased by

(i) increasing the number of turns in the coil N or

(ii) increasing the magnitude of magnetic field.

Example 26.28 *A rectangular coil of area* 5.0×10^{-4} m^2 *and* 60 *turns is pivoted about one of its vertical sides. The coil is in a radial horizontal magnetic field of* 9×10^{-3} *T. What is the torsional constant of the spring connected to the coil if a current of* 0.20 *mA produces an angular deflection of* $18°$?

Solution From the equation,

$$i = \left(\frac{k}{NAB}\right)\phi$$

We find that torsional constant of the spring is given by

$$k = \frac{NABi}{\phi}$$

Substituting the values in SI units, we have

$$k = \frac{(60)(5.0 \times 10^{-4})(9 \times 10^{-3})(0.2 \times 10^{-3})}{18}$$

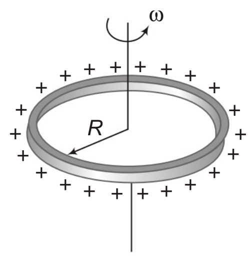
$$= 3 \times 10^{-9} \text{ N-m/degree}$$ **Ans.**

INTRODUCTORY EXERCISE 26.7

1. A coil of a moving coil galvanometer twists through $90°$ when a current of one microampere is passed through it. If the area of the coil is 10^{-4} m^2 and it has 100 turns, calculate the magnetic field of the magnet of the galvanometer. Given, $k = 10^{-8}$ N-m /degree.
2. A galvanometer coil 5 cm $\times$ 2 cm with 200 turns is suspended vertically in a field of 5×10^{-2} T. The suspension fibre needs a torque of 0.125×10^{-7} N-m to twist it through one radian. Calculate the strength of the current required to be maintained in the coil if we require a deflection of 6°.

Final Touch Points

1. Sometimes, a non-conducting charged body is rotated with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m the mass of the body.



e.g. In case of a ring of mass m, radius R and charge q distributed on its circumference.

$$\text{Angular momentum,} \quad L = I\omega = (mR^2)(\omega) \qquad \ldots(i)$$

$$\text{Magnetic moment,} \quad M = iA = (qf)(\pi R^2)$$

Here, f = frequency $= \dfrac{\omega}{2\pi}$

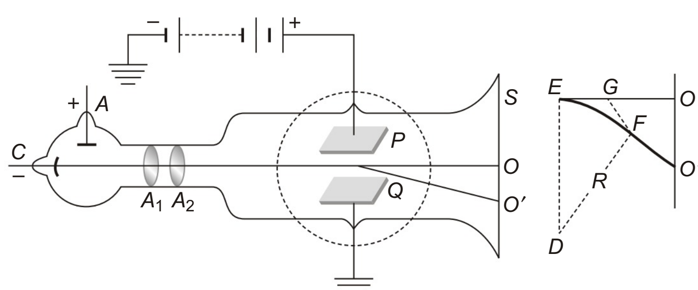
$$\therefore \qquad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\,\frac{\omega R^2}{2} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\boxed{\frac{M}{L} = \frac{q}{2m}}$$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere.

2. Determination of e/m of an Electron (Thomson Method) JJ Thomson in 1897, devised an experiment for the determination of e/m (specific charge) of the electron by using electric and magnetic fields in mutually perpendicular directions.



The discharge is maintained by the application of high PD between the cathode C and anode A of a discharge tube containing air at a very low pressure ($\sim 10^{-2}$ mm of Hg). The electrons so produced are allowed to pass through slits A_1 and A_2 also kept at the potential of A. The beam then passes along the axis of the tube and produces a spot of light at O on the fluorescent screen S. The electric field E is applied between two horizontal plates P and Q. The magnetic field B is applied in the direction perpendicular to the paper plane by passing the current through coils, in the region within the dotted circle. It is clear from Fleming's left hand rule, that F due to E is in upward direction, while due to B in downward direction.

Hence, fields E and B can be adjusted so that the electrons suffer no deflection and strike at point O on the screen. In this case,

or $$eE = evB \quad \text{or} \quad v = E/B \qquad \text{...(i)}$$

Now, electric field is switched off, the electrons thus, describe the circular arc and fall at O' on the screen. In this case, the force $F = Bev$ bends the electron beam in a circular arc, such that it is balanced by the centripetal force mv^2/R.

$\therefore$ $$Bev = mv^2/R$$

or $$v = BRe/m \qquad \text{...(ii)}$$

Combining Eqs. (i) and (ii), we get

$$e/m = E/RB^2 \qquad \text{...(iii)}$$

As E and B are known. To find R, consider arc EF of the circular path in the magnetic field region. From the geometry, we get

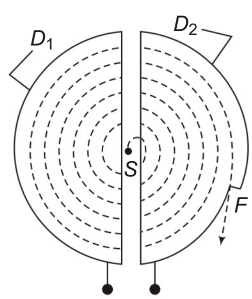
$$OO'/GO = EF/R$$

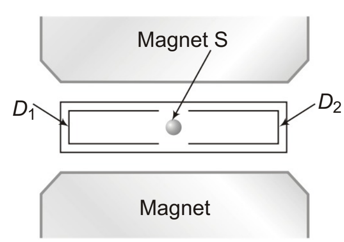
or $$R = EF \times GO/OO' \qquad \text{...(iv)}$$

Practically, EF is replaced by the width of the magnetic flux region and G is taken at the middle of the region. Thomson's value for e/m was 1.7×10^{11} C/kg, which is in excellent agreement with the modern value of 1.75890×10^{11} C/kg.

3. **Cyclotron** In 1932, Lawrence developed a machine named cyclotron, for the acceleration of charged particles, such as protons or deuterons. These particles (ions) are caused to move in circular orbits by magnetic field and are accelerated by the electric field.

In its simplest form, it consists of two flat semicircular metal boxes, called dees because of their shape. These hollow chambers have their diametric edges parallel and slightly separated from each other. An alternating potential (with frequency of the order of megacycles per second) is applied between the dees. The dees are placed between the poles of a strong electromagnet which provides a magnetic field perpendicular to the plane of the dees.





Suppose that at any particular instant the alternating potential is in the direction which makes D_1 positive and D_2 negative. A positive ion of mass m, charge q starting from the source S (of positive ion) will be attracted by the dee D_2. Let its velocity while entering in dee D_2 is v. Due to magnetic field B, it will move in a circular path of radius r inside the dee D_2, where

$$r = \frac{mv}{Bq}$$

or $$\boxed{v = \frac{Bqr}{m}}$$

In the interior of the dee, the speed of the ion remains constant. After it has traversed half a cycle, the ion comes to the edge of D_2. If in the meantime, the potential difference between D_1 and D_2 has changed direction so that D_2 is now positive and D_1 negative, the positive ion will receive an additional acceleration, while going across the gap between the dees and speed of ion will increase. Then, it travels in a circular path of larger radius inside D_1 under the influence of magnetic field (because $r \propto v$). After traversing a half cycle in D_1, it will reach the edge of D_1 and receive an additional acceleration between the gaps because in the meantime the direction of potential difference between the dees has changed. The ion will continue travelling in a semicircle of increasing radii, the direction of potential difference changes every time the ion goes from D_1 to D_2 and from D_2 to D_1. The time taken by the charged particle to traverse the semicircular path in the dee is given by

$$t = \pi r/v = \frac{\pi m}{Bq} \quad \text{...(i)}$$

This relation indicates that time t is independent of the velocity of the particle and of the radius. For any given value of m/q, it is determined by the magnetic field intensity. By adjusting the magnetic field intensity the time can be made the same as that required to change the potentials. On the other hand, the oscillator frequency (of alternating potential) can also be adjusted to the nature of a given ion and to the strength of the magnetic field. The frequency of the oscillations required to keep the ion in phase is given by the relation

$$\boxed{f = \frac{1}{T} = \frac{1}{2t} = \frac{Bq}{2\pi m}} \quad \text{...(ii)}$$

If the oscillation frequency is adjusted to keep the charged ion always in phase, each time the ion crosses the gap it receives an additional energy and at the same time it describes a flat spiral of increasing radius. Eventually, the ion reaches the periphery of the dee, where it can be brought out of the chamber by means of a deflecting plate charged to a high negative potential. This attractive force draws the ion out of its spiral path and thus can be used easily. If R is the radius of the dee, kinetic energy of the ion emerging from the cyclotron is thus given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\,(BqR/m)^2$$

$$K = B^2R^2q^2/2m \quad \text{...(iii)}$$

This relation indicates that the maximum energy attained by the ion is limited by the radius R, magnetic field B or the frequency of the alternating potential f. It is independent of the alternating voltage. It can be explained by the fact that when the voltage is low, the ion makes a large number of turns before reaching the periphery, but when the voltage is high the number of turns is small. The total energy remains same in both the cases provided B and R are unchanged.

Note *The cyclotron is used to bombard nuclei with energetic particles and study the resulting nuclear reactions. It is also used in hospitals to produce radioactive substances which can be used in diagnosis and treatment.*

Cyclotron is suitable only for accelerating heavy particles like proton, deuteron, α-particle etc. Electrons cannot be accelerated by the cyclotron because the mass of the electron is small and a small increase in energy of the electron makes the electrons move with a very high speed.

The uncharged particles (e.g., neutrons) cannot be accelerated by cyclotron.

Solved Examples

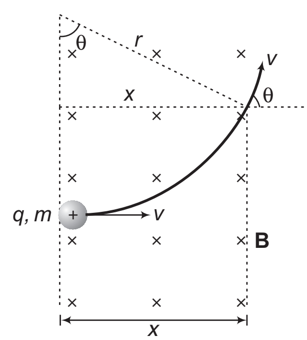
TYPED PROBLEMS

Type 1. *Based on deviation of charged particle in uniform magnetic field when $\theta = 90°$ or path is uniform circular*

Concept

Suppose a charged particle (q, m) enters a uniform magnetic field $\mathbf{B}$ at right angles with speed v as shown in figure. The magnetic field extends upto a length x. The path of the particle is a circle of radius r, where

$$r = \frac{mv}{Bq}$$



The speed of the particle in magnetic field does not change. But, it gets deviated in the magnetic field. The deviation θ can be found in two ways

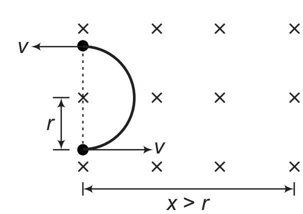
(i) After time t, deviation will be

$$\boxed{\theta = \omega t = \left(\frac{Bq}{m}\right) t} \qquad \left[\text{as } \omega = \frac{Bq}{m}\right]$$

(ii) In terms of the length of the magnetic field (i.e. when the particle leaves the magnetic field) the deviation will be

$$\boxed{\theta = \sin^{-1}\left(\frac{x}{r}\right)}$$

But, since, $\sin\theta \not> 1$, this relation can be used only when $x < r$. For $x \geq r$, the deviation will be $180°$ as shown in figure.



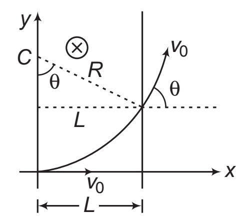
Example 1 *The region between $x = 0$ and $x = L$ is filled with uniform steady magnetic field $-B_0\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field.* **[JEE 1999]**

Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2.1 L.

Solution (a) $\theta = 30°$



$$\sin\theta = \frac{L}{R}$$

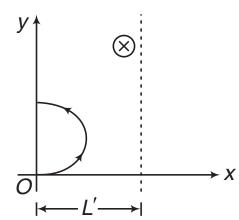
Here, $$R = \frac{mv_0}{B_0 q}$$

$\therefore$ $$\sin 30° = \frac{L}{\dfrac{mv_0}{B_0 q}}$$

or $$\frac{1}{2} = \frac{B_0 qL}{mv_0}$$

$\therefore$ $$L = \frac{mv_0}{2B_0 q}$$

(b) In part (a)



$$\sin 30° = \frac{L}{R} \quad \text{or} \quad \frac{1}{2} = \frac{L}{R}$$

or $$L = R/2$$

Now, when $L' = 2.1\ L$

or $$\frac{2.1}{2} R \Rightarrow L' > R$$

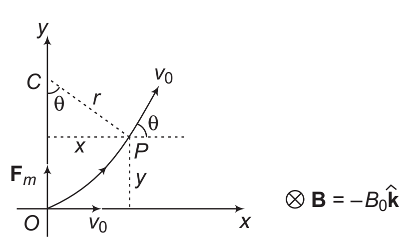
Therefore, deviation of the particle is $\theta = 180°$ as shown in figure.

$\therefore$ $$\mathbf{v}_f = -v_0\hat{\mathbf{i}}$$

and $$t_{AB} = T/2 = \frac{\pi m}{B_0 q}$$

Type 2. *To find coordinates and velocity of particle at any time t in circular path*

Example 2 *A particle of specific charge* α *enters a uniform magnetic field* $\mathbf{B} = -B_0\hat{\mathbf{k}}$ *with velocity* $\mathbf{v} = v_0\hat{\mathbf{i}}$ *from the origin. Find the time dependence of velocity and position of the particle.*



$OC = CP$ = radius of circle

HOW TO PROCEED *In such type of problems first of all see the angle between* **v** *and* **B**. *Because only this angle decides the path of the particle. Here, the angle is 90°. Therefore, the path is a circle. If it is a circle, see the plane of the circle (perpendicular to the magnetic field). Here, the plane is xy. Then, see the sense of the rotation.*

Here, it will be anti-clockwise as shown in figure, because at origin the magnetic force is along positive y-direction (which can be seen from Fleming's left hand rule). Find the deviation and radius of the particle.

$$\theta = \omega t = B_0 \alpha t \quad and \quad r = \frac{v_0}{B_0 \alpha}$$

Now, according to the figure, find $\mathbf{v}(t)$ *and* $\mathbf{r}(t)$.

Solution Velocity of the particle at any time t is

$$\mathbf{v}(t) = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} = v_0 \cos\theta \hat{\mathbf{i}} + v_0 \sin\theta \hat{\mathbf{j}}$$

or $$\mathbf{v}(t) = v_0 \cos(B_0 \alpha t)\hat{\mathbf{i}} + v_0 \sin(B_0 \alpha t)\hat{\mathbf{j}}$$ **Ans.**

Position of particle at time t is

$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = r\sin\theta\hat{\mathbf{i}} + (r - r\cos\theta)\hat{\mathbf{j}}$$

Substituting the values of r and θ, we have

$$\mathbf{r}(t) = \frac{v_0}{B_0\alpha}[\sin(B_0\alpha t)\hat{\mathbf{i}} + \{1 - \cos(B_0\alpha t)\}\hat{\mathbf{j}}]$$ **Ans.**

Type 3. *To find coordinates and velocity of particle at any time t in helical path*

Example 3 *A particle of specific charge* α *is projected from origin with velocity* $\mathbf{v} = v_0\hat{\mathbf{i}} - v_0\hat{\mathbf{k}}$ *in a uniform magnetic field* $\mathbf{B} = -B_0\hat{\mathbf{k}}$. *Find time dependence of velocity and position of the particle.*

HOW TO PROCEED *Here, the angle between* **v** *and* **B** *is*

$$\theta = \cos^{-1}\frac{\mathbf{v}\cdot\mathbf{B}}{|\mathbf{v}||\mathbf{B}|} = \cos^{-1}\frac{B_0 v_0}{\sqrt{2}v_0 \cdot B_0} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

or $$\theta = 45°$$

Hence, the path is a helix. The axis of the helix is along z-axis (parallel to **B***) and plane of the circle of helix is xy (perpendicular to* **B***). So, in xy-plane, the velocity components and x and y-coordinates are same as that of the above problem. The only change is along z-axis. Velocity component in this direction will remain unchanged while the z-coordinate of particle at time t would be* $v_z t$.

Solution Velocity of particle at time t is

$$\mathbf{v}(t) = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$= v_0\cos(B_0\alpha t)\hat{\mathbf{i}} + v_0\sin(B_0\alpha t)\hat{\mathbf{j}} - v_0\hat{\mathbf{k}}$$ **Ans.**

v_x and v_y can be found in the similar manner as done in Example 2.

The position of the particle at time t would be

$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

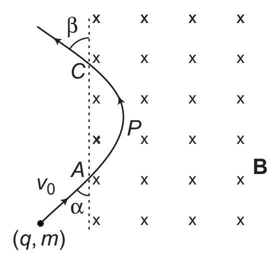
Here, $$z = v_z t = -v_0 t$$

and x and y are same as in Example 2.

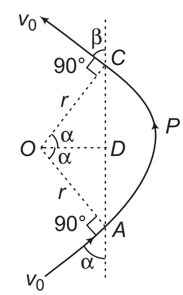
Hence, $$\mathbf{r}(t) = \frac{v_0}{B_0\alpha}[\sin(B_0\alpha t)\hat{\mathbf{i}} + \{1 - \cos(B_0\alpha t)\}\hat{\mathbf{j}}] - v_0 t\hat{\mathbf{k}}$$ **Ans.**

Type 4. *To find the time spent in magnetic field, deviation etc. if a charged particle enters from outside in uniform magnetic field (which extends upto large distance from point of entering)*

Example 4 *A charged particle (q, m) enters a uniform magnetic field $\mathbf{B}$ at angle α as shown in figure with speed v_0. Find*



(a) the angle β at which it leaves the magnetic field.
(b) time spent by the particle in magnetic field and
(c) the distance AC.



Solution (a) Here, velocity of the particle is in the plane of paper while the magnetic field is perpendicular to the paper inwards,. i.e. angle between $\mathbf{v}$ and $\mathbf{B}$ is 90°. So, the path is a circle. The radius of the circle is $r = \dfrac{mv_0}{Bq}$

O is the centre of the circle. In ΔAOC,

$$\angle OCD = \angle OAD$$

or $$90° - \beta = 90° - \alpha$$

$\therefore$ $$\beta = \alpha$$ **Ans.**

(b) $\angle COD = \angle DOA = \alpha$ (as $\angle OCD = \angle OAD = 90° - \alpha$)

$\therefore$ $$\angle AOC = 2\alpha$$

or $$\text{length } APC = r(2\alpha) = \frac{2mv_0}{Bq}.\alpha$$

$\therefore$ $$t_{APC} = \frac{APC}{v_0} = \frac{2m\alpha}{Bq}$$ **Ans.**

Alternate method

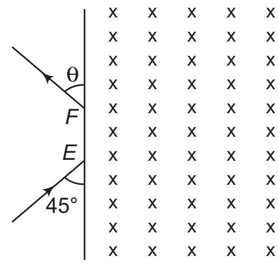
$$t_{APC} = \left(\frac{T}{2\pi}\right)(2\alpha) = \left(\frac{\alpha}{\pi}\right)\cdot T$$

$$= \left(\frac{\alpha}{\pi}\right)\left(\frac{2\pi m}{Bq}\right) = \frac{2\alpha m}{Bq}$$ **Ans.**

(c) Distance, $AC = 2(AD) = 2(r\sin\alpha)$

$$= \frac{2mv_0}{Bq}\sin\alpha$$ **Ans.**

Example 5 *A particle of mass* $m = 1.6 \times 10^{-27}$ *kg and charge* $q = 1.6 \times 10^{-19}$ *C enters a region of uniform magnetic field of strength* 1 *T along the direction shown in figure. The speed of the particle is* 10^7 *m/s.* **(JEE 1984)**

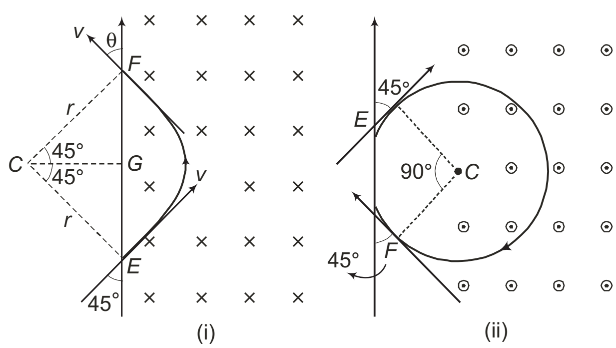


(a) *The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.*

(b) *If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.*

Solution Inside a magnetic field, speed of charged particle does not change. Further, velocity is perpendicular to magnetic field in both the cases hence path of the particle in the magnetic field will be circular. Centre of circle can be obtained by drawing perpendiculars to velocity (or tangent to the circular path) at E and F. Radius and angular speed of circular path would be

$$r = \frac{mv}{Bq} \quad \text{and} \quad \omega = \frac{Bq}{m}$$



(a) Refer figure (i)

$$\angle CFG = 90° - \theta \text{ and } \angle CEG = 90° - 45° = 45°$$

Since, $\quad CF = CE$

$\therefore \quad \angle CFG = \angle CEG$

or $\quad 90° - \theta = 45° \quad \text{or} \quad \theta = 45°$

Further, $\quad FG = GE = r \cos 45°$

$\therefore \quad EF = 2FG = 2r \cos 45° = \dfrac{2mv \cos 45°}{Bq}$

$$= \frac{2\,(1.6 \times 10^{-27})\,(10^7)\left(\frac{1}{\sqrt{2}}\right)}{(1)\,(1.6 \times 10^{-19})} = 0.14 \text{ m}$$

Note *That in this case particle completes 1/4th of circle in the magnetic field because the angle rotated is 90°.*

(b) **Refer figure (ii)** In this case, particle will complete $\frac{3}{4}$th of circle in the magnetic field.

Hence, the time spent in the magnetic field :

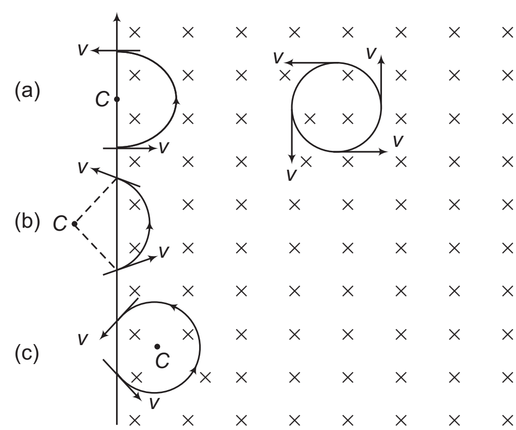
$$t = \frac{3}{4} \text{ (time period of circular motion)}$$

$$= \frac{3}{4}\left(\frac{2\pi m}{Bq}\right) = \frac{3\pi m}{2Bq}$$

$$= \frac{(3\pi)(1.6 \times 10^{-27})}{(2)(1)(1.6 \times 10^{-19})}$$

$$= 4.712 \times 10^{-8} \text{ s}$$ **Ans.**

Note *From the above examples, we can see that particle never completes circular path if it enters from outside in uniform magnetic field at right angles (as in Examples 1, 4 and 5). Circle is completed if magnetic field extends all around (Example-2). Following figures explain these points more clearly. In all figures, particle is positively charged.*



In figure (a) *Centre of circular path is lying on the boundary line of magnetic field. Deviation of the particle is 180° and time spent in magnetic field* $t = \frac{T}{2}$.

In figure (b) *Centre of circular path lies outside the magnetic field. Deviation of the particle is less than 180° and time spent in magnetic field* $t < \frac{T}{2}$.

In figure (c) *Centre of circular path lies inside the magnetic field. Deviation of the particle is more than 180° and time spent in magnetic field* $t > \frac{T}{2}$.

Type 5. *Based on the concept of helical path*

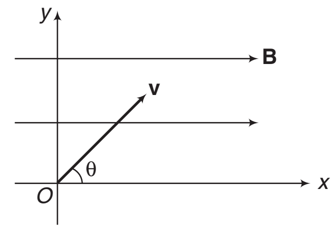
Concept

Following points are worthnoting in case of a helical path.

(i) The plane of the circle of the helix is perpendicular to the magnetic field.

(ii) The axis of the helix is parallel to magnetic field.

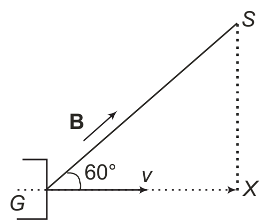
(iii) The particle while moving in helical path in magnetic field touches the line passing through the starting point parallel to the magnetic field after every pitch.



For example, a charged particle is projected from origin in a magnetic field (along x-direction) at angle θ from the x-axis as shown. As the velocity vector $\mathbf{v}$ makes an angle θ with $\mathbf{B}$, its path is a helix. The plane of the circle of the helix is yz (perpendicular to magnetic field) and axis of the helix is parallel to x-axis. The particle while moving in helical path touches the x-axis after every pitch, i.e. it will touch the x-axis at a distance

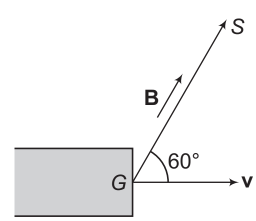
$$\boxed{x = np} \qquad \text{where, } n = 0, 1, 2 \ldots$$

Example 6 *An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where GS = 0.1 m, and the line GS makes an angle of 60° with the x-axis as shown in figure. A uniform magnetic field* $\mathbf{B}$ *parallel to GS exists in the region outside the electron gun.*



Find the minimum value of B needed to make the electrons hit S. **(JEE 1993)**

Solution Kinetic energy of electron,



$$K = \frac{1}{2}mv^2 = 2 \text{ keV}$$

$\therefore$ Speed of electron, $v = \sqrt{\dfrac{2K}{m}}$

$$v = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-16}}{9.1 \times 10^{-31}}} \text{ m/s}$$

$$= 2.65 \times 10^7 \text{ m/s}$$

Since, the velocity ($\mathbf{v}$) of the electron makes an angle of $\theta = 60°$ with the magnetic field $\mathbf{B}$, the path will be a helix. So, the particle will hit S if

$$GS = np$$

Here, $n = 1, 2, 3 \ldots\ldots\ldots\ldots$

$$p = \text{pitch of helix} = \frac{2\pi m}{qB} v\cos\theta$$

But for B to be minimum, $n = 1$

Hence,
$$GS = p = \frac{2\pi m}{qB} v\cos\theta$$

$$B = B_{\min} = \frac{2\pi m v\cos\theta}{q(GS)}$$

Substituting the values, we have

$$B_{min} = \frac{(2\pi)(9.1\times10^{-31})(2.65\times10^{7})\left(\frac{1}{2}\right)}{(1.6\times10^{-19})\,(0.1)}$$

or $$B_{min} = 4.73\times10^{-3}\ \text{T}$$ **Ans.**

Type 6. *Based on calculation of magnetic field due to current carrying wires*

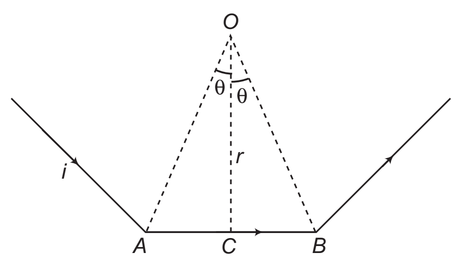
Example 7 *A wire shaped to a regular hexagon of side 2 cm carries a current of 2 A. Find the magnetic field at the centre of the hexagon.*

Solution $\because\ \theta = 30°$

$$\frac{BC}{OC} = \tan\theta \qquad (BC = 1\ \text{cm})$$

$$\therefore\quad \frac{1}{r} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore\quad r = \sqrt{3}\ \text{cm}$$



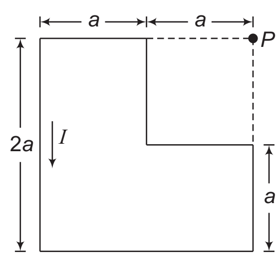
Net magnetic field at O is 6 times the magnetic field due to one side.

$$\therefore\quad B = 6\left[\frac{\mu_0}{4\pi}\frac{i}{r}(\sin\theta + \sin\theta)\right]$$

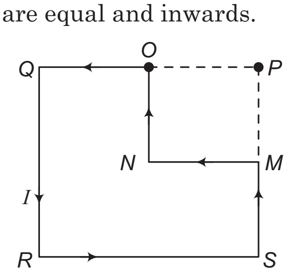
$$= \frac{6\,(10^{-7})\,(2)}{\sqrt{3}\times10^{-2}}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$= 6.9\times10^{-5}\ \text{T}$$ **Ans.**

Example 8 *Find the magnetic field* **B** *at the point P in figure.*



Solution Magnetic field at P due to SM and OQ is zero. Due to QR and RS are equal and outwards. Due to MN and NO are equal and inwards.



Due to QR and RS,

$$B_1 = 2\left[\frac{\mu_0}{4\pi}\frac{I}{2a}(\sin 0° + \sin 45°)\right]$$

$$= \frac{\mu_0 I}{4\sqrt{2}\pi a} \quad \text{[outwards]}$$

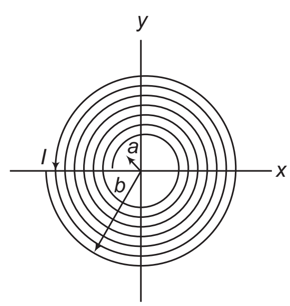
Due to MN and NO,

$$B_2 = 2\left[\frac{\mu_0}{4\pi}\frac{I}{a}(\sin 0° + \sin 45°)\right]$$

$$= \frac{\mu_0 I}{2\sqrt{2}\,\pi a} \quad \text{[inwards]}$$

$$\therefore \quad B_{\text{net}} = B_2 - B_1 = \frac{\mu_0 I}{4\sqrt{2}\,\pi a} \quad \text{[inwards]}$$

Example 9 *A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the xy-plane and a steady current I flows through the wire. The z-component of the magnetic field at the centre of the spiral is* **(JEE 2011)**



(a) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$

(b) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b+a}{b-a}\right)$

(c) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b}{a}\right)$

(d) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b+a}{b-a}\right)$

Solution (a) If we take a small strip of dr at distance r from centre, then number of turns in this strip would be

$$dN = \left(\frac{N}{b-a}\right)dr$$

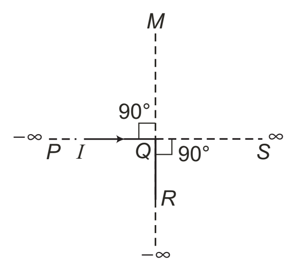
Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0 (dN) I}{2r} = \frac{\mu_0 NI}{(b-a)}\frac{dr}{2r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

$\therefore$ Correct answer is (a).

Example 10 *An infinitely long conductor PQR is bent to form a right angle as shown in figure. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q, so that current is I/2 in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by* **(JEE 2000)**



(a) 1/2 (b) 1 (c) 2/3 (d) 2

Solution H_1 = Magnetic field at M due to PQ + Magnetic field at M due to QR

But magnetic field at M due to $QR = 0$

$\therefore$ Magnetic field at M due to PQ (or due to current I in PQ)

$$= H_1$$

Now, H_2 = Magnetic field at M due to PQ (current I)

+ magnetic field at M due to QS (current $I/2$) + magnetic field at M due to QR

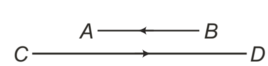
$$= H_1 + \frac{H_1}{2} + 0 = \frac{3}{2} H_1$$

$$\frac{H_1}{H_2} = \frac{2}{3}$$

Note *Magnetic field at any point lying on the current carrying straight conductor is zero.*

Type 7. *Based on the magnetic force on current carrying wire*

Example 11 *A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.* **(JEE 1994)**



Solution Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by

$$F_m = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{x} \quad \text{(upwards)} \quad \ldots\text{(i)}$$

Weight per unit length of wire AB is

$$F_g = mg \quad \text{(downwards)}$$

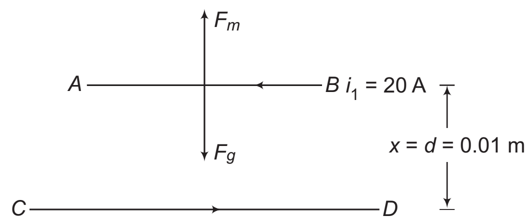
Here, m = mass per unit length of wire AB

At $x = d$, wire is in equilibrium, i.e.

$$F_m = F_g$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d^2} = \frac{mg}{d} \quad \ldots\text{(ii)}$$



When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Change in magnetic force will become the net restoring force, Let AB is displaced by dx downwards.

Differentiating Eq. (i) w.r.t. x, we get

$$dF_m = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}\,.\,dx \quad \ldots\text{(iii)}$$

i.e. restoring force, $F = dF_m \propto -dx$

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right)\cdot dx \quad [\because x = d]$$

$\therefore$ Acceleration of wire, $a = -\left(\frac{g}{d}\right).\,dx$

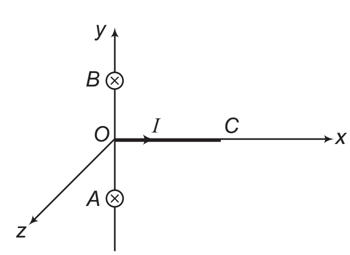
Hence, period of oscillation

$$T = 2\pi\sqrt{\frac{|\text{displacement}|}{|\text{acceleration}|}} = 2\pi\sqrt{\left|\frac{dx}{a}\right|}$$

or $$T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}}$$

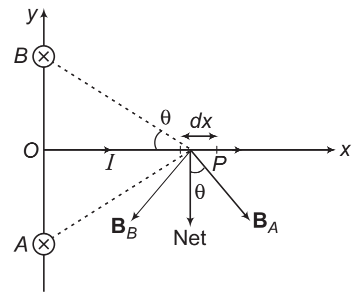
or $$T = 0.2 \text{ s}$$ **Ans.**

Example 12 *A straight segment OC (of length L) of a circuit carrying a current I is placed along the x-axis. Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed at $y = -a$ and $y = +a$ respectively, as shown in the figure. If the wires A and B each carry a current I into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed?*



(JEE 1992)

Solution (a) Let us assume a segment of wire OC at a point P, a distance x from the centre of length dx as shown in figure.

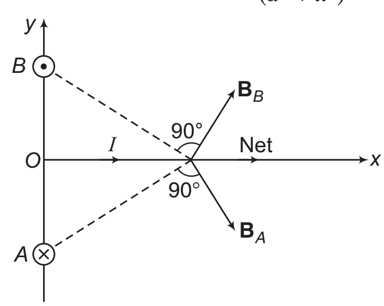


Magnetic field at P due to current in wires A and B will be in the directions perpendicular to AP and BP respectively as shown.

$$|\mathbf{B}| = \frac{\mu_0}{2\pi}\frac{I}{AP}$$

Therefore, net magnetic force at P will be along negative y-axis as shown below

$$B_{\text{net}} = 2|\mathbf{B}|\cos\theta$$
$$= 2\left(\frac{\mu_0}{2\pi}\right)\frac{I}{AP}\left(\frac{x}{AP}\right)$$
$$B_{\text{net}} = \left(\frac{\mu_0}{\pi}\right)\frac{Ix}{(AP)^2}$$
$$B_{\text{net}} = \frac{\mu_0}{\pi}\cdot\frac{Ix}{(a^2 + x^2)}$$



Therefore, force on this element will be

$$dF = I\left\{\frac{\mu_0}{\pi}\frac{Ix}{a^2 + x^2}\right\}dx \qquad \text{[in negative } z\text{-direction]}$$

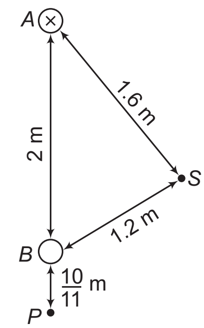
∴ Total force on the wire will be

$$F = \int_{x=0}^{x=L} dF = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{x\,dx}{x^2 + a^2}$$
$$= \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{L^2 + a^2}{a^2}\right) \qquad \text{[in negative } z\text{-axis]}$$

Hence,
$$\mathbf{F} = -\frac{\mu_0 I^2}{2\pi}\ln\left(\frac{L^2 + a^2}{a^2}\right)\hat{\mathbf{k}}$$

(b) When direction of current in B is reversed net magnetic field is along the current. Hence, force is zero.

Example 13 *Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 A, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of 10 / 11 m from the wire B, is zero.*



Find **(JEE 1997)**

(a) the magnitude and direction of the current in B.

(b) the magnitude of the magnetic field of induction at the point S.

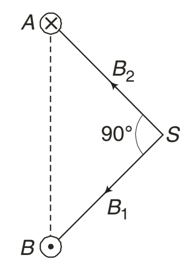
(c) the force per unit length on the wire B.

Solution (a) Direction of current at B should be perpendicular to paper outwards. Let current in this wire be i_B. Then,

$$\frac{\mu_0}{2\pi}\frac{i_A}{\left(2+\frac{10}{11}\right)}=\frac{\mu_0}{2\pi}\frac{i_B}{(10/11)}$$

or $$\frac{i_B}{i_A}=\frac{10}{32}$$

or $$i_B=\frac{10}{32}\times i_A=\frac{10}{32}\times 9.6=3\text{ A}$$



(b) Since, $AS^2+BS^2=AB^2$

$\therefore$ $$\angle ASB=90°$$

At S : B_1 = Magnetic field due to i_A

$$=\frac{\mu_0}{2\pi}\frac{i_A}{1.6}=\frac{(2\times 10^{-7})(9.6)}{1.6}$$

$$=12\times 10^{-7}\text{ T}$$

$$B_2=\text{Magnetic field due to } i_B$$

$$=\frac{\mu_0}{2\pi}\frac{i_B}{1.2}$$

$$=\frac{(2\times 10^{-7})(3)}{1.2}$$

$$=5\times 10^{-7}\text{ T}$$

Since, B_1 and B_2 are mutually perpendicular. Net magnetic field at S would be

$$B=\sqrt{B_1^2+B_2^2}$$

$$=\sqrt{(12\times 10^{-7})^2+(5\times 10^{-7})^2}$$

$$=13\times 10^{-7}\text{ T}$$

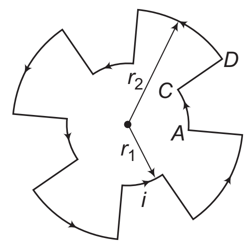
(c) Force per unit length on wire B :

$$\frac{F}{l}=\frac{\mu_0}{2\pi}\frac{i_A i_B}{r} \qquad [\because\ r=AB=2\text{ m}]$$

$$=\frac{(2\times 10^{-7})(9.6\times 3)}{2}$$

$$=2.88\times 10^{-6}\text{ N/m}$$

Example 14 *A current of* 10 *A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii* $r_1 = 0.08$ *m and* $r_2 = 0.12$ *m. Each subtends the same angle at the centre.* **(JEE 2001)**



(a) *Find the magnetic field produced by this circuit at the centre.*

(b) *An infinitely long straight wire carrying a current of* 10 *A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre?*

Solution (a) Given, $i = 10$ A, $r_1 = 0.08$ m and $r_2 = 0.12$ m. Straight portions, i.e. *CD* etc, will produce zero magnetic field at the centre. Rest eight arcs will produce the magnetic field at the centre in the same direction, i.e. perpendicular to the paper outwards or vertically upwards and its magnitude is

$$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$$

$$= \frac{1}{2}\left\{\frac{\mu_0 i}{2 r_1}\right\} + \frac{1}{2}\left\{\frac{\mu_0 i}{2 r_2}\right\}$$

$$= \left(\frac{\mu_0}{4\pi}\right)(\pi i)\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

Substituting the values, we have

$$B = \frac{(10^{-7})(3.14)(10)(0.08 + 0.12)}{(0.08 \times 0.12)}$$

$$B = 6.54 \times 10^{-5} \text{ T}$$

(vertically upward or outward normal to the paper)

(b) Force on *AC*

Force on circular portions of the circuit, i.e. *AC* etc, due to the wire at the centre will be zero because magnetic field due to the central wire at these arcs will be tangential ($\theta = 180°$).

Force on *CD*

Current in central wire is also $i = 10$ A. Magnetic field at distance x due to central wire

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$$

$\therefore$ Magnetic force on element dx due to this magnetic field

$$dF = (i)\left(\frac{\mu_0}{2\pi} \cdot \frac{i}{x}\right) \cdot dx = \left(\frac{\mu_0}{2\pi}\right) i^2 \frac{dx}{x} \qquad [F = ilB \sin 90°]$$

Therefore, net force on CD is

$$F = \int_{x=r_1}^{x=r_2} dF = \frac{\mu_0 i^2}{2\pi}\int_{0.08}^{0.12}\frac{dx}{x} = \frac{\mu_0}{2\pi} i^2 \ln\left(\frac{3}{2}\right)$$

Substituting the values, $F = (2\times10^{-7})\,(10)^2 \ln(1.5)$

or $\qquad F = 8.1\times10^{-6}$ N (inwards)

Force on wire at the centre

Net magnetic field at the centre due to the circuit is in vertical direction and current in the wire in centre is also in vertical direction. Therefore, net force on the wire at the centre will be zero. ($\theta = 180°$). Hence,

(i) Force acting on the wire at the centre is zero.
(ii) Force on arc $AC = 0$.
(iii) Force on segment CD is 8.1×10^{-6} N (inwards).

Type 8. *Based on the magnetic force on a charged particle in electric and (or) magnetic field*

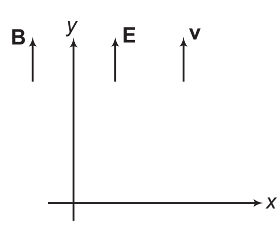
Example 15 *Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields* $\mathbf{E} = E_0\hat{\mathbf{j}}$ *and* $\mathbf{B} = B_0\hat{\mathbf{j}}$. *At time $t = 0$, this charge has velocity $\mathbf{v}$ in the xy-plane making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?* **(JEE 2012)**

(a) If $\theta = 0°$, the charge moves in a circular path in the xz-plane.
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time along the y-axis.
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

Solution Magnetic field will rotate the particle in a circular path (in xz-plane or perpendicular to $\mathbf{B}$). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical or the options (a) and (b) both are wrong.

(c) $v_\perp$ and $\mathbf{B}$ will rotate the particle in a circular path in xz- plane (or perpendicular to $\mathbf{B}$). Further, $v_\parallel$ and $\mathbf{E}$ will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch along the y-axis (or along $\mathbf{B}$ and $\mathbf{E}$). Therefore, option (c) is correct.

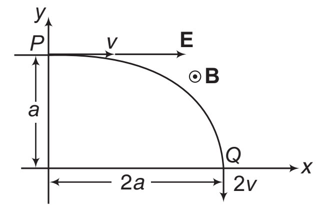
(d)



Magnetic force is zero, as θ between $\mathbf{B}$ and $\mathbf{v}$ is zero. But, electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction).

Therefore, option (d) is also correct.

Example 16 *A particle of charge +q and mass m moving under the influence of a uniform electric field* $E\hat{\mathbf{i}}$ *and uniform magnetic field* $B\hat{\mathbf{k}}$ *follows a trajectory from P to Q as shown in figure. The velocities at P and Q are* $v\hat{\mathbf{i}}$ *and* $-2\hat{\mathbf{j}}$. *Which of the following statement(s) is/are correct ?* **(JEE 1991)**



(a) $E = \frac{3}{4}\left[\frac{mv^2}{qa}\right]$

(b) Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^3}{a}\right]$

(c) Rate of work done by the electric field at P is zero

(d) Rate of work done by both the fields at Q is zero

Solution Magnetic force does not do work. From work-energy theorem :

$$W_{F_e} = \Delta \text{KE}$$

or
$$(qE)(2a) = \frac{1}{2}m[4v^2 - v^2]$$

or
$$E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

$\therefore$ Option (a) is correct.

At P, rate of work done by electric field

$$= \mathbf{F}_e \cdot \mathbf{v} = (qE)(v)\cos 0°$$

$$= q\left(\frac{3}{4}\frac{mv^2}{qa}\right)v = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Therefore, option (b) is also correct.

Rate of work done at Q :

of electric field $= \mathbf{F}_e \cdot \mathbf{v} = (qE)(2v)\cos 90° = 0$ and of magnetic field is always zero.

Therefore, option (d) is also correct.

Note that $\mathbf{F}_e = qE\hat{\mathbf{i}}$

Example 17 *A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields, respectively. Then, this region of space may have* **(JEE 1985)**

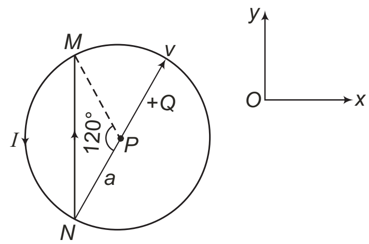
(a) $E = 0, B = 0$
(b) $E = 0, B \neq 0$
(c) $E \neq 0, B = 0$
(d) $E \neq 0, B \neq 0$

Solution If both E and B are zero, then $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, velocity may remain constant. Therefore, option (a) is correct.

If $E = 0$, $B \neq 0$ but velocity is parallel or antiparallel to magnetic field, then also $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, option (b) is also correct.

If $E \neq 0$, $B \neq 0$ but $\mathbf{F}_e + \mathbf{F}_m = 0$, then again velocity may remain constant or option (d) is also correct.

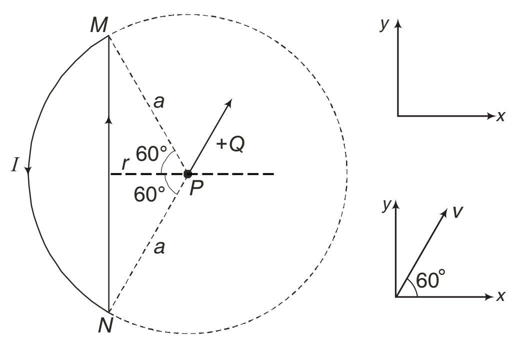
Example 18 *A wire loop carrying a current I is placed in the xy-plane as shown in figure.* **(JEE 1991)**



(a) *If a particle with charge +Q and mass m is placed at the centre P and given a velocity* $\mathbf{v}$ *along NP (see figure), find its instantaneous acceleration.*

(b) *If an external uniform magnetic induction field* $\mathbf{B} = B\hat{\mathbf{i}}$ *is applied, find the force and the torque acting on the loop due to this field.*

Solution (a) Magnetic field at P due to arc of circle,



Subtending an angle of 120° at centre would be

$$B_1 = \frac{1}{3}\text{(field due to circle)} = \frac{1}{3}\left(\frac{\mu_0 I}{2a}\right)$$

$$= \frac{\mu_0 I}{6a} \qquad \text{[outwards]}$$

$$= \frac{0.16\mu_0 I}{a} \qquad \text{[outwards]}$$

or $$\mathbf{B}_1 = \frac{0.16\mu_0 I}{a}\hat{\mathbf{k}}$$

Magnetic field due to straight wire NM at P,

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 60^\circ + \sin 60^\circ)$$

Here, $$r = a\cos 60^\circ$$

$\therefore$ $$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a\cos 60^\circ}(2\sin 60^\circ)$$

or $$B_2 = \frac{\mu_0}{2\pi}\frac{I}{a}\tan 60^\circ$$

$$= \frac{0.27\,\mu_0 I}{a} \qquad \text{(inwards)}$$

or $$\mathbf{B}_2 = -\frac{0.27\,\mu_0 I}{a}\,\hat{\mathbf{k}}$$

$\therefore$ $$\mathbf{B}_{net} = \mathbf{B}_1 + \mathbf{B}_2 = -\frac{0.11\,\mu_0\, I}{a}\,\hat{\mathbf{k}}$$

Now, velocity of particle can be written as

$$\mathbf{v} = v\cos 60°\,\hat{\mathbf{i}} + v\sin 60°\,\hat{\mathbf{j}}$$

$$= \frac{v}{2}\,\hat{\mathbf{i}} + \frac{\sqrt{3}v}{2}\,\hat{\mathbf{j}}$$

Magnetic force,

$$\mathbf{F}_m = Q\,(\mathbf{v} \times \mathbf{B})$$

$$= \frac{0.11\,\mu_0 IQv}{2a}\,\hat{\mathbf{j}} - \frac{0.11\sqrt{3}\,\mu_0 IQv}{2a}\,\hat{\mathbf{i}}$$

$\therefore$ Instantaneous acceleration,

$$\mathbf{a} = \frac{\mathbf{F}_m}{m} = \frac{0.11\,\mu_0 IQv}{2am}\,(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$$

(b) In uniform magnetic field, force on a current loop is zero. Further, magnetic dipole moment of the loop will be

$$\mathbf{M} = (IA)\,\hat{\mathbf{k}}$$

Here, A is the area of the loop.

$$A = \frac{1}{3}(\pi a^2) - \frac{1}{2}[2 \times a\sin 60°]\,[a\cos 60°]$$

$$= \frac{\pi a^2}{3} - \frac{a^2}{2}\sin 120°$$

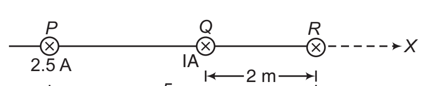
$$= 0.61\,a^2$$

$\therefore$ $$\mathbf{M} = (0.61\,Ia^2)\,\hat{\mathbf{k}}$$

Given, $$\mathbf{B} = B\hat{\mathbf{i}}$$

$\therefore$ $$\tau = \mathbf{M} \times \mathbf{B} = (0.61\,Ia^2B)\,\hat{\mathbf{j}}$$

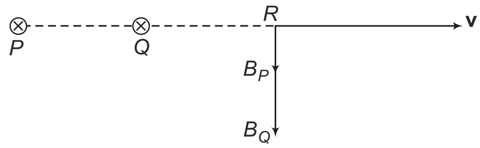
Example 19 *Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R (see figure).* **(JEE 1990)**



(a) An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

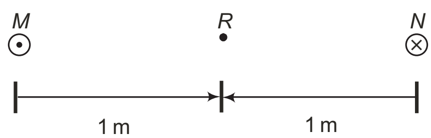
Solution (a) Magnetic field at R due to both the wires P and Q will be downwards as shown in figure.



Therefore, net field at R will be sum of these two.

$$B = B_P + B_Q$$
$$= \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2} = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right)$$
$$= \frac{\mu_0}{4\pi}(I+1) = 10^{-7}(I+1)$$

Net force on the electron will be



$$F_m = Bqv \sin 90°$$

or $$(3.2 \times 10^{-20}) = (10^{-7})(I+1)(1.6 \times 10^{-19})(4 \times 10^5)$$

or $$I + 1 = 5$$

$\therefore$ $$I = 4\text{ A}$$

(b) Net field at R due to wires P and Q is

$$B = 10^{-7}(I+1)\text{ T}$$
$$= 5 \times 10^{-7}\text{ T}$$

Magnetic field due to third wire carrying a current of 2.5 A should be 5×10^{-7} T in upward direction, so that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5 \times 10^{-7}$$

or $$\frac{(2 \times 10^{-7})(2.5)}{r} = 5 \times 10^{-7}\text{ m}$$

or $$r = 1\text{ m}$$

So, the third wire can be put at M or N as shown in figure.

If it is placed at M, then current in it should be outwards and if placed at N, then current be inwards.

Type 9. *Path of charged particle in both electric and magnetic fields*

Concept

Here, normally two cases are popular. In the first case, $\mathbf{E} \uparrow\uparrow \mathbf{B}$ and particle velocity is perpendicular to both of these fields. In the second case, $\mathbf{E} \perp \mathbf{B}$ and the particle is released from rest. Let us now consider both the cases separately.

Example 20 *When* $\mathbf{E} \uparrow\uparrow \mathbf{B}$ *and particle velocity is perpendicular to both of these fields.*

Solution Consider a particle of charge q and mass m released from the origin with velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ into a region of uniform electric and magnetic fields parallel to y-axis, i.e. $\mathbf{E} = E_0\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$. The electric field accelerates the particle in y-direction, i.e. y-component of velocity goes on increasing with acceleration,

$$a_y = \frac{F_y}{m} = \frac{F_e}{m} = \frac{qE_0}{m} \qquad \text{...(i)}$$

The magnetic field rotates the particle in a circle in xz-plane (perpendicular to magnetic field). The resultant path of the particle is a **helix with increasing pitch**. The axis of the plane is parallel to y-axis. Velocity of the particle at time t would be

$$\mathbf{v}(t) = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

Here, $$v_y = a_y t = \frac{qE_0}{m}t$$

and $$v_x^2 + v_z^2 = \text{constant} = v_0^2$$

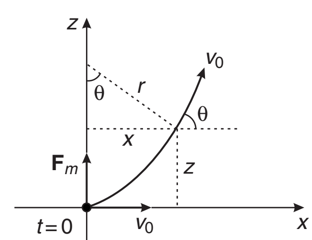
$$\theta = \omega t = \frac{Bq}{m}t$$

$$v_x = v_0\cos\theta = v_0\cos\left(\frac{Bqt}{m}\right)$$

and $$v_z = v_0\sin\theta = v_0\sin\left(\frac{Bqt}{m}\right)$$

$\therefore$ $$\boxed{\mathbf{v}(t) = v_0\cos\left(\frac{Bqt}{m}\right)\hat{\mathbf{i}} + \left(\frac{qE_0}{m}t\right)\hat{\mathbf{j}} + v_0\sin\left(\frac{Bqt}{m}\right)\hat{\mathbf{k}}}$$

Similarly, position vector of particle at time t can be given by



$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Here, $$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2$$

$$x = r\sin\theta = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)$$

and $$z = r(1-\cos\theta) = \left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]$$

$\therefore$ $$\boxed{\mathbf{r}(t) = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)\hat{\mathbf{i}} + \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\hat{\mathbf{j}} + \left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]\hat{\mathbf{k}}}$$

Note *(i) While moving in helical path the particle touches the y-axis after every T or after,*

$$t = nT, \qquad \text{where } n = 0, 1, 2\ldots$$

Here, $$T = \frac{2\pi m}{Bq}$$

(ii) At $t = 0$, velocity is along positive x-axis and magnetic field is along y-axis. Therefore, magnetic force is along positive z-axis and the particle rotates in xz-plane as shown in figure.

Example 21 *When* $\mathbf{E} \perp \mathbf{B}$ *and the particle is released at rest from origin.*

Solution Consider a particle of charge q and mass m emitted at origin with zero initial velocity into a region of uniform electric and magnetic fields. The field $\mathbf{E}$ is acting along x-axis and field $\mathbf{B}$ along y-axis, i.e.

$$\mathbf{E} = E_0\hat{\mathbf{i}}$$

and $$\mathbf{B} = B_0\hat{\mathbf{j}}$$

Electric field will provide the particle an acceleration (and therefore a velocity component) in x-direction and the magnetic field will rotate the particle in xz-plane (perpendicular to $\mathbf{B}$). Hence, at any instant of time its velocity (and hence, position) will have only x and z components. Let at time t its velocity be

$$\mathbf{v} = v_x\hat{\mathbf{i}} + v_z\hat{\mathbf{k}}$$

Net force on it at this instant is

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = q\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B})$$

$$= q\,[E_0\hat{\mathbf{i}} + (v_x\hat{\mathbf{i}} + v_z\hat{\mathbf{k}}) \times (B_0\hat{\mathbf{j}})]$$

or $$\mathbf{F} = q\,(E_0 - v_zB_0)\,\hat{\mathbf{i}} + qv_xB_0\hat{\mathbf{k}}$$

$\therefore$ $$\mathbf{a} = \frac{\mathbf{F}}{m} = a_x\hat{\mathbf{i}} + a_z\hat{\mathbf{k}}$$

where, $$a_x = \frac{q}{m}\,(E_0 - v_zB_0) \qquad \text{...(i)}$$

and $$a_z = \frac{q}{m}\,v_xB_0 \qquad \text{...(ii)}$$

Differentiating Eq. (i) w.r.t. time, we have

$$\frac{d^2v_x}{dt^2} = -\frac{qB_0}{m}\left(\frac{dv_z}{dt}\right)$$

But, $$\frac{dv_z}{dt} = a_z = \frac{qB_0}{m}\,v_x$$

$\therefore$ $$\frac{d^2v_x}{dt^2} = -\left(\frac{qB_0}{m}\right)^2 v_x \qquad \text{...(iii)}$$

Comparing this equation with the differential equation of SHM $\left(\frac{d^2y}{dt^2} = -\,\omega^2y\right)$, we get

$$\omega = \frac{qB_0}{m}$$

and the general solution of Eq. (iii) is

$$v_x = A\sin\,(\omega t + \phi) \qquad \text{...(iv)}$$

At time $t = 0$, $v_x = 0$, hence, $\phi = 0$

Again, $$\frac{dv_x}{dt} = A\omega\cos\omega t \qquad (\text{as } \phi = 0)$$

From Eq. (i), $a_x = \frac{qE_0}{m}$ at $t = 0$, as $v_z = 0$ at $t = 0$

$\therefore$ $$A\omega = \frac{qE_0}{m} \quad \text{or} \quad A = \frac{qE_0}{\omega m}$$

Substituting $\omega = \frac{qB_0}{m}$, we get $$A = \frac{E_0}{B_0}$$

Therefore, Eq. (iv) becomes

$$\boxed{v_x = \frac{E_0}{B_0}\sin\omega t}$$

where,

$$\boxed{\omega = \frac{qB_0}{m}}$$

Now substituting value of v_x in Eq. (ii), we get

$$\frac{dv_z}{dt} = \frac{qE_0}{m}\sin\omega t$$

$\therefore$

$$\int_0^{v_z} dv_z = \frac{qE_0}{m}\int_0^t \sin\omega t\, dt$$

or

$$v_z = \frac{qE_0}{\omega m}(1-\cos\omega t)$$

Substituting $\omega = \dfrac{qB_0}{m}$, we get

$$\boxed{v_z = \frac{E_0}{B_0}(1-\cos\omega t)}$$

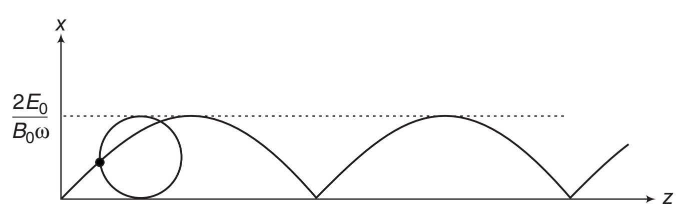
On integrating equations for v_x and v_z and knowing that at $t=0$, $x=0$ and $z=0$, we get

$$\boxed{x = \frac{E_0}{B_0\omega}(1-\cos\omega t)}$$

and

$$\boxed{z = \frac{E_0}{B_0\omega}(\omega t - \sin\omega t)}$$

These equations are the equations for a **cycloid** which is defined as the path generated by the point on the circumference of a wheel rolling on a ground.



In the present case, the radius of the rolling wheel is $\dfrac{E_0}{B_0\omega}$, the maximum displacement along x-direction is $\dfrac{2E_0}{B_0\omega}$. The x-displacement becomes zero at $t = 0, 2\pi/\omega, 4\pi/\omega$, etc.

Note *Path of a charged particle in uniform electric and magnetic field will remain unchanged if*

$$\mathbf{F}_{net} = 0$$

or

$$\mathbf{F}_e + \mathbf{F}_m = 0$$

or

$$q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) = 0$$

or

$$\mathbf{E} = -(\mathbf{v}\times\mathbf{E})$$

$$= (\mathbf{E}\times\mathbf{v})$$

Miscellaneous Examples

Example 22 *A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its dees is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator?*

$$(e = 1.60 \times 10^{-19} C,\ m_p = 1.67 \times 10^{-27} kg,\ 1\ MeV = 1.6 \times 10^{-13} J)$$

Solution **Magnetic field** Cyclotron's oscillator frequency should be same as the proton's revolution frequency (in circular path)

$$\therefore \qquad f = \frac{Bq}{2\pi m}$$

or

$$B = \frac{2\pi m f}{q}$$

Substituting the values in SI units, we have

$$B = \frac{(2)(22/7)(1.67 \times 10^{-27})(10 \times 10^6)}{1.6 \times 10^{-19}}$$

$$= 0.67 \text{ T}$$ **Ans.**

Kinetic energy Let final velocity of proton just after leaving the cyclotron is v. Then, radius of dee should be equal to

$$R = \frac{mv}{Bq} \quad \text{or} \quad v = \frac{BqR}{m}$$

$\therefore$ Kinetic energy of proton,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{BqR}{m}\right)^2 = \frac{B^2q^2R^2}{2m}$$

Substituting the values in SI units, we have

$$K = \frac{(0.67)^2(1.6 \times 10^{-19})^2(0.60)^2}{2 \times 1.67 \times 10^{-27}}$$

$$= 1.2 \times 10^{-12} \text{ J}$$

$$= \frac{1.2 \times 10^{-12}}{(1.6 \times 10^{-19})(10^6)} \text{ MeV}$$

$$= 7.5 \text{ MeV}$$ **Ans.**

Example 23 *A charged particle carrying charge $q = 1\,\mu C$ moves in uniform magnetic field with velocity $v_1 = 10^6$ m/s at angle 45° with x-axis in the xy-plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ m/s along the z-axis, it experiences a force F_2 in y-direction. Find*

(a) the magnitude and direction of the magnetic field

(b) the magnitude of the force F_2.

Solution F_2 is in y-direction when velocity is along z-axis. Therefore, magnetic field should be along x-axis. So let,

$$\mathbf{B} = B_0\hat{\mathbf{i}}$$

(a) Given, $$\mathbf{v}_1 = \frac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \frac{10^6}{\sqrt{2}}\hat{\mathbf{j}}$$

and $$\mathbf{F}_1 = -5\sqrt{2}\times 10^{-3}\,\hat{\mathbf{k}}$$

From the equation, $$\mathbf{F} = q\,(\mathbf{v}\times\mathbf{B})$$

We have $$(-5\sqrt{2}\times 10^{-3})\,\hat{\mathbf{k}} = (10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \frac{10^6}{\sqrt{2}}\hat{\mathbf{j}}\right)\times(B_0\hat{\mathbf{i}})\right]$$

$$= -\frac{B_0}{\sqrt{2}}\hat{\mathbf{k}}$$

$\therefore$ $$\frac{B_0}{\sqrt{2}} = 5\sqrt{2}\times 10^{-3}$$

or $$B_0 = 10^{-2}\ \text{T}$$

Therefore, the magnetic field is

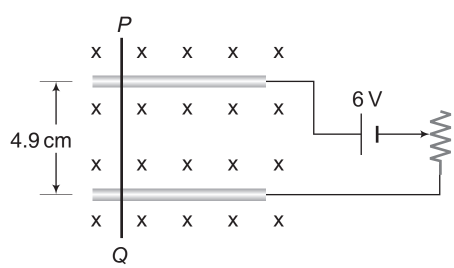
$$\mathbf{B} = (10^{-2}\,\hat{\mathbf{i}})\ \text{T}$$ **Ans.**

(b) $$F_2 = B_0 q v_2 \sin 90°$$

As the angle between **B** and **v** in this case is 90°.

$\therefore$ $$F_2 = (10^{-2})(10^{-6})(10^6)$$
$$= 10^{-2}\ \text{N}$$ **Ans.**

Example 24 *A wire PQ of mass 10 g is at rest on two parallel metal rails. The separation between the rails is 4.9 cm. A magnetic field of 0.80 T is applied perpendicular to the plane of the rails, directed downwards. The resistance of the circuit is slowly decreased. When the resistance decreases to below 20 Ω, the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.*



Solution Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e.

$$\mu\, mg = ilB\sin\theta \qquad (\theta = 90°)$$

Here, $$i = \frac{E}{R} = \frac{6}{20} = 0.3\ \text{A}$$

$\therefore$ $$\mu = \frac{ilB}{mg} = \frac{(0.3)(4.9\times 10^{-2})(0.8)}{(10\times 10^{-3})(9.8)}$$

$$= 0.12$$ **Ans.**

Example 25 *What is the value of B that can be set up at the equator to permit a proton of speed* 10^7 *m/s to circulate around the earth?*

$$[R = 6.4 \times 10^6 \, m, \; m_p = 1.67 \times 10^{-27} \; kg].$$

Solution From the relation

$$r = \frac{mv}{Bq},$$

We have

$$B = \frac{mv}{qr}$$

Substituting the values, we have

$$B = \frac{(1.67 \times 10^{-27})(10^7)}{(1.6 \times 10^{-19})(6.4 \times 10^6)}$$

$$= 1.6 \times 10^{-8} \text{ T}$$ **Ans.**

Example 26 *Deuteron in a cyclotron describes a circle of radius* 32.0 *cm. Just before emerging from the D's. The frequency of the applied alternating voltage is* 10 *MHz. Find*

(a) the magnetic flux density (i.e. the magnetic field).

(b) the energy and speed of the deuteron upon emergence.

Solution (a) Frequency of the applied emf = Cyclotron frequency

or $$f = \frac{Bq}{2\pi m}$$

∴ $$B = \frac{2\pi mf}{q}$$

$$= \frac{(2)(3.14)(2 \times 1.67 \times 10^{-27})(10 \times 10^6)}{1.6 \times 10^{-19}}$$

$$= 1.30 \text{ T}$$ **Ans.**

(b) The speed of deuteron on the emergence from the cyclotron,

$$v = \frac{2\pi R}{T} = 2\pi f R$$

$$= (2)(3.14)(10 \times 10^6)(32 \times 10^{-2})$$

$$= 2.01 \times 10^7 \text{ m/s}$$

∴ Energy of deuteron $$= \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times (2 \times 1.67 \times 10^{-27})(2.01 \times 10^7)^2 \text{ J}$$

$$= 4.22 \text{ MeV}$$ **Ans.**

Note *1 MeV = 1.6 × 10^{-13} J*

Example 27 *In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius* 5×10^{-11} *m at a frequency of* 6.8×10^{15} *Hz.*

(a) What value of magnetic field is set up at the centre of the orbit?

(b) What is the equivalent magnetic dipole moment?

Solution (a) An electron moving around the nucleus is equivalent to a current,

$$i = qf$$

Magnetic field at the centre,

$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R}$$

Substituting the values, we have

$$B = \frac{(4\pi \times 10^{-7})\,(1.6 \times 10^{-19})\,(6.8 \times 10^{15})}{2 \times 5.1 \times 10^{-11}}$$

$$= 13.4 \text{ T}$$ **Ans.**

(b) The current carrying circular loop is equivalent to a magnetic dipole with magnetic dipole moment,

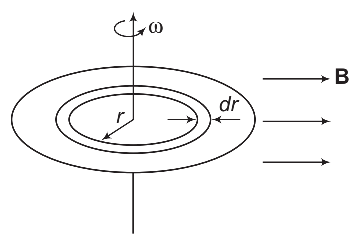
$$M = NiA = (Nqf\,\pi R^2)$$

Substituting the values, we have

$$M = (1)\,(1.6 \times 10^{-19})\,(6.8 \times 10^{15})\,(3.14)\,(5.1 \times 10^{-11})^2$$
$$= 8.9 \times 10^{-24} \text{ A-m}^2$$ **Ans.**

Example 28 *A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the centre with angular velocity ω. Find the torque on the disc if it is placed in a uniform magnetic field B directed perpendicular to the rotation axis.*

Solution Consider an annular ring of radius r and of thickness dr on this disc. Charge within this ring,



$$dq = (\sigma)\,(2\pi r dr)$$

As ring rotates with angular velocity ω, the equivalent current is

$$i = (dq)\,(\text{frequency})$$
$$= (\sigma)\,(2\pi r dr)\left(\frac{\omega}{2\pi}\right) \quad \text{or} \quad i = \sigma\omega r dr$$

Magnetic moment of this annular ring,

$$M = iA = (\sigma\omega r dr)\,(\pi r^2)$$ (along the axis of rotation)

Torque on this ring,

$$d\tau = MB \sin 90° = (\sigma\omega\pi r^3 B)\,dr$$

∴ Total torque on the disc is $$\tau = \int_0^R d\tau = (\sigma\omega\pi B)\int_0^R r^3\,dr$$

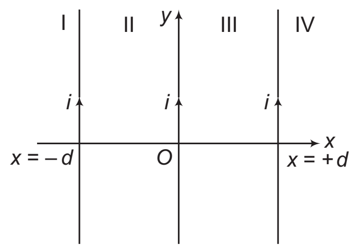
$$= \frac{\sigma\omega\pi B R^4}{4}$$ **Ans.**

Example 29 *Three infinitely long thin wires, each carrying current i in the same direction, are in the xy-plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.*

(i) *Find the locus of the points for which the magnetic field B is zero.*

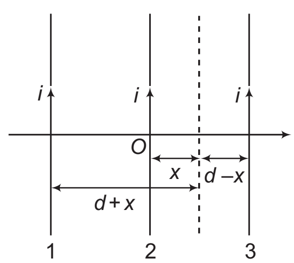
(ii) *If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear mass density of the wires is λ, find the frequency of oscillation.*

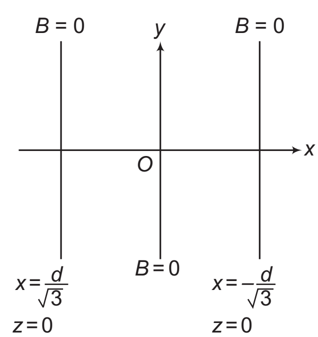
Solution (i) Magnetic field will be zero on the y-axis, i.e. $x = 0 = z$.



Magnetic field cannot be zero in region I and region IV because in region I magnetic field will be along positive z-direction due to all the three wires, while in region IV magnetic field will be along negative z-axis due to all the three wires. It can be zero only in region II and III.

Let magnetic field is zero on line $z = 0$ and $x = x$ (shown as dotted). The magnetic field on this line due to wires 1 and 2 will be along negative z-axis and due to wire 3 along positive z-axis. Thus,





$$B_1 + B_2 = B_3$$

or $$\frac{\mu_0}{2\pi}\frac{i}{(d+x)} + \frac{\mu_0 i}{2\pi x} = \frac{\mu_0}{2\pi}\frac{i}{(d-x)}$$

or $$\frac{1}{d+x} + \frac{1}{x} = \frac{1}{d-x}$$

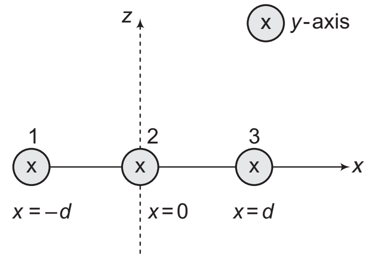
This equation gives $$x = \pm\frac{d}{\sqrt{3}}$$

Hence, there will be two lines $$x = \frac{d}{\sqrt{3}}$$

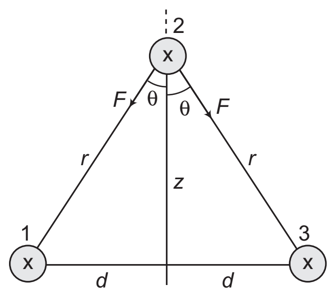
and $$x = -\frac{d}{\sqrt{3}} \qquad (z = 0)$$

where, magnetic field is zero. **Ans.**

(ii) In this part, we change our coordinate axes system, just for better understanding.



There are three wires 1, 2 and 3 as shown in figure. If we displace the wire 2 towards the z-axis, then force of attraction per unit length between wires (1 and 2) and (2 and 3) will be given by



$$F = \frac{\mu_0}{2\pi}\frac{i^2}{r}$$

The components of F along x-axis will be cancelled out. Net resultant force will be towards negative z-axis (or mean position) and will be given by

$$F_{\text{net}} = 2F\cos\theta = 2\left\{\frac{\mu_0}{2\pi}\frac{i^2}{r}\right\}\frac{z}{r}$$

$$F_{\text{net}} = \frac{\mu_0}{\pi}\frac{i^2}{(z^2+d^2)}\cdot z \qquad (r^2 = z^2 + d^2)$$

If $z << d$, then

$$z^2 + d^2 \approx d^2$$

and $$F_{\text{net}} = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right)\cdot z$$

Negative sign implies that F_{net} is restoring in nature.

Therefore, $$F_{\text{net}} \propto -z$$

i.e. the wire will oscillate simple harmonically.

Let a be the acceleration of wire in this position and λ the mass per unit length of wire, then

$$F_{\text{net}} = \lambda a = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right)z$$

or $$a = -\left(\frac{\mu_0 i^2}{\pi\lambda d^2}\right)z$$

$\therefore$ Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|}$$

$$= \frac{1}{2\pi}\sqrt{\left|\frac{a}{z}\right|} = \frac{1}{2\pi}\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

or $$f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$ **Ans.**

Example 30 *Uniform electric and magnetic fields with strength E and B are directed along the y-axis. A particle with specific charge q/m leaves the origin in the direction of x-axis with an initial velocity v_0. Find*

(a) the y-coordinate of the particle when it crosses the y-axis for nth time.

(b) the angle α between the particle's velocity vector and the y-axis at that moment.

Solution (a) As discussed in Type-9 path of the particle is a helix of increasing pitch. The axis of the helix is parallel to y-axis (parallel to **E**) and plane of circle of the helix is xz (perpendicular to **B**). The particle will cross the y-axis after time,

$$t = nT = n\left(\frac{2\pi m}{Bq}\right) = \frac{2\pi mn}{Bq}$$

The y-coordinate of particle at this instant is

$$y = \frac{1}{2}a_y t^2$$

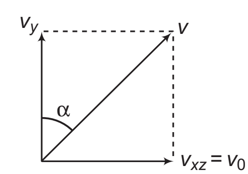
where, $$a_y = \frac{F_y}{m} = \frac{qE}{m}$$

$\therefore$ $$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{Bq}\right)^2$$

$$= \frac{2n^2 mE\pi^2}{qB^2}$$ **Ans.**

(b) At this moment y-component of its velocity is

$$v_y = a_y t = \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{Bq}\right) = 2\pi n\left(\frac{E}{B}\right)$$



The angle α between particle's velocity vector and the y-axis at this moment is

$$\alpha = \tan^{-1}\left(\frac{v_{xz}}{v_y}\right)$$

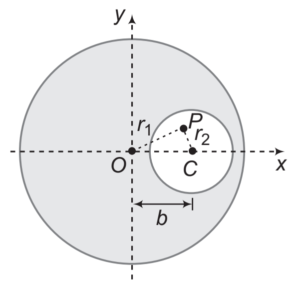
Here, $$v_{xz} = \sqrt{v_x^2 + v_z^2} = v_0$$

or $$\alpha = \tan^{-1}\left(\frac{Bv_0}{2\pi nE}\right)$$ **Ans.**

Example 31 *A current is passing through a cylindrical conductor with a hole (or cavity) inside it. Show that the magnetic field inside the hole is uniform and find its magnitude and direction.*

Solution Let us find the magnetic field at point P inside the cavity at a distance r_1 from O and r_2 from C.

J = current per unit area
R = radius of cylinder
a = radius of cavity
i_1 = whole current from cylinder = $J(\pi R^2)$
i_2 = current from hole = $J(\pi a^2)$

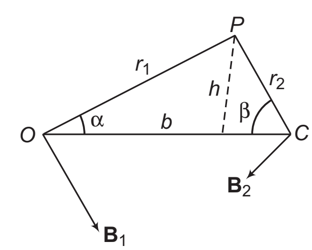


At point P magnetic field due to i_1 is B_1 (perpendicular to OP) and is $\mathbf{B}_2$ due to i_2 (perpendicular to CP) in the directions shown. Although $\mathbf{B}_1$ and $\mathbf{B}_2$ are actually at P, but for better understanding they are drawn at O and C respectively. Let B_x be the x-component of resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ and B_y its y-component. Then,

$$B_x = B_1 \sin\alpha - B_2 \sin\beta$$
$$= \left(\frac{\mu_0}{2\pi}\frac{i_1}{R^2} r_1\right)\sin\alpha - \left(\frac{\mu_0}{2\pi}\frac{i_2}{a^2}\cdot r_2\right)\sin\beta$$
$$= \left(\frac{\mu_0}{2\pi}\frac{J\pi R^2}{R^2}\cdot r_1 \sin\alpha\right) - \left(\frac{\mu_0}{2\pi}\frac{J.\pi a^2}{a^2} r_2 \sin\beta\right)$$
$$= \frac{\mu_0 J}{2}(r_1 \sin\alpha - r_2 \sin\beta) = 0$$

Because in ΔOPC $\quad \dfrac{r_1}{\sin\beta} = \dfrac{r_2}{\sin\alpha} = h \quad$ or $\quad r_1 \sin\alpha - r_2 \sin\beta = 0$

Now,
$$B_y = -(B_1 \cos\alpha + B_2 \cos\beta)$$
$$= -\frac{\mu_0 J}{2}(r_1 \cos\alpha + r_2 \cos\beta)$$



From ΔOPC, we can see that

$$r_1 \cos\alpha + r_2 \cos\beta = b \quad \text{or} \quad B_y = -\frac{\mu_0 J b}{2} = \text{constant}$$

Thus, we can see that net magnetic field at point P is along negative y-direction and constant in magnitude. **Proved**

Note *(i) That ∠OPC is not necessarily 90°. At some point it may be 90°.*

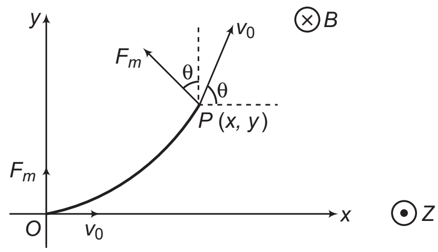
(ii) At point C magnetic field due to i_2 is zero (i. e. $B_2 = 0$) while that due to i_1 is $\frac{\mu_0}{2\pi}\frac{i_1}{R^2} b$ in negative y-direction. Substituting $i_1 = J(\pi R^2)$, we get

$$B = B_2 = \frac{\mu_0 J b}{2} \qquad \textit{(along negative y-direction)}$$

This agrees with the result derived above.

Example 32 *A particle of charge q and mass m is projected from the origin with velocity* $\mathbf{v} = v_0 \hat{\mathbf{i}}$ *in a non-uniform magnetic field* $\mathbf{B} = -B_0 x \hat{\mathbf{k}}$. *Here,* v_0 *and* B_0 *are positive constants of proper dimensions. Find the maximum positive x-coordinate of the particle during its motion.*

Solution Magnetic field is along negative z-direction. So in the coordinate axes shown in figure, it is perpendicular to paper inwards. (⊗) Magnetic force on the particle at origin is along positive y-direction. So, it will rotate in xy-plane as shown. The path is not a perfect circle as the magnetic field is non-uniform. Speed of the particle in magnetic field remains constant. Magnetic force is always perpendicular to velocity. Let at point $P(x, y)$, its velocity vector makes an angle θ with positive x-axis. Then, magnetic force $\mathbf{F}_m$ will be at angle θ with positive y-direction. So,



$$a_y = \left(\frac{F_m}{m}\right)\cos\theta$$

∴ $$\frac{dv_y}{dt} = \frac{(B_0 x)\,(q v_0 \cos\theta)}{m} \qquad [F_m = B q v_0 \sin 90°]$$

∴ $$\left(\frac{dv_y}{dx}\right)\cdot\left(\frac{dx}{dt}\right) = \left(\frac{B_0 q x}{m}\right)(v_0 \cos\theta)$$

Here, $$\frac{dx}{dt} = v_x = v_0 \cos\theta$$

∴ $$\frac{dv_y}{dx} = \left(\frac{B_0 q}{m}\right) x$$

∴ $$\int_0^{v_0} dv_y = \left(\frac{B_0 q}{m}\right)\int_0^{x_{\max}} x\,dx$$

∴ $$v_0 = \left(\frac{B_0 q}{m}\right)\left(\frac{x_{\max}^2}{2}\right)$$

∴ $$x_{\max} = \sqrt{\frac{2 m v_0}{B_0 q}}$$ **Ans.**

Note *At maximum x-displacement velocity is along positive y-direction.*

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(*c*) *If **Assertion** is true, but the **Reason** is false.*

(*d*) *If **Assertion** is false but the **Reason** is true.*

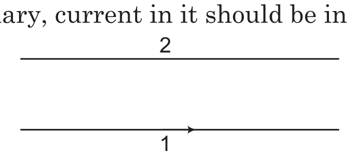
1. Assertion : Path of a charged particle in uniform magnetic field cannot be a parabola.

Reason : For parabolic path acceleration should be constant.

2. Assertion : A beam of protons is moving towards east in vertically upward magnetic field. Then, this beam will deflect towards south.

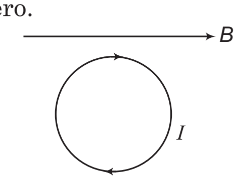
Reason : A constant magnetic force will act on the proton beam.

3. Assertion : Current in wire-1 is in the direction as shown in figure. The bottom wire is fixed. To keep the upper wire stationary, current in it should be in opposite direction.



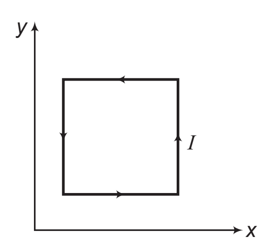
Reason : Under the above condition, equilibrium of upper wire is stable.

4. Assertion : A current carrying loop is placed in uniform magnetic field as shown in figure. Torque in the loop in this case is zero.



Reason : Magnetic moment vector of the loop is perpendicular to paper inwards.

5. Assertion : Force on current carrying loop shown in figure in magnetic field, $\mathbf{B} = (B_0 x)\,\hat{\mathbf{k}}$ is along positive x-axis. Here, B_0 is a positive constant.



Reason : A torque will also act on the loop.

6. Assertion : An electron and a proton are accelerated by same potential difference and then enter in uniform transverse magnetic field. The radii of the two will be different.

Reason : Charges on them are different.

7. Assertion : A charged particle moves along positive y-axis with constant velocity in uniform electric and magnetic fields. If magnetic field is acting along positive x-axis, then electric field should act along positive z-axis.

Reason : To keep the charged particle undeviated the relation $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ must hold good.

8. Assertion : Power of a magnetic force on a charged particle is always zero.

Reason : Power of electric force on charged particle cannot be zero.

9. Assertion : If a charged particle enters from outside at right angles in uniform magnetic field. The maximum time spent in magnetic field may be $\frac{\pi m}{Bq}$.

Reason : It can complete only semi-circle in the magnetic field.

10. Assertion : A charged particle enters in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$ with velocity $\mathbf{v} = v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}$, then minimum speed of charged particle may be v_0.

Reason : A variable acceleration will act on the charged particle.

11. Assertion : A charged particle is moving in a circle with constant speed in uniform magnetic field. If we increase the speed of particle to twice, its acceleration will become four times.

Reason : In circular path of radius R with constant speed v, acceleration is given by $\frac{v^2}{R}$.

Objective Questions

1. The universal property among all substances is

(a) diamagnetism (b) paramagnetism
(c) ferromagnetism (d) non-magnetism

2. A charged particle moves in a circular path in a uniform magnetic field. If its speed is reduced, then its time period will

(a) increase (b) decrease
(c) remain same (d) None of these

3. A straight wire of diameter 0.5 mm carrying a current 2 A is replaced by another wire of diameter 1 mm carrying the same current. The strength of magnetic field at a distance 2 m away from the centre is

(a) half of the previous value (b) twice of the previous value
(c) unchanged (d) quarter of its previous value

4. The path of a charged particle moving in a uniform steady magnetic field cannot be a

(a) straight line (b) circle
(c) parabola (d) None of these

5. The SI unit of magnetic permeability is

(a) $\mathrm{Wb\,m^{-2}A^{-1}}$ (b) $\mathrm{Wb\,m^{-1}A}$
(c) $\mathrm{Wb\,m^{-1}A^{-1}}$ (d) $\mathrm{Wb\,m\,A^{-1}}$

6. Identify the correct statement about the magnetic field lines.

(a) These start from the N-pole and terminate on the S-pole
(b) These lines always form closed loops
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

7. Identify the correct statement related to the direction of magnetic moment of a planar loop.
(a) It is always perpendicular to the plane of the loop
(b) It depends on the direction of current
(c) It is obtained by right hand screw rule
(d) All of the above

8. A non-planar closed loop of arbitrary shape carrying a current I is placed in uniform magnetic field. The force acting on the loop
(a) is zero only for one orientation of loop in magnetic field
(b) is zero for two symmetrically located positions of loop in magnetic field
(c) is zero for all orientations
(d) is never zero

9. The magnetic dipole moment of current loop is independent of
(a) number of turns
(b) area of loop
(c) current in the loop
(d) magnetic field in which it is lying

10. The acceleration of an electron at a certain moment in a magnetic field $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ is $\mathbf{a} = x\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. The value of x is
(a) 0.5 (b) 1
(c) 2.5 (d) 1.5

11. Match the following and select the correct alternatives given below
(p) unit of magnetic induction B (q) dimensions of B
(r) unit of permeability (μ_0) (s) dimensions of μ_0
(t) dimensions of magnetic moment (u) $[MLT^{-2}A^{-2}]$
(v) $[ML^0T^{-2}A^{-1}]$
(x) Newton/amp-metre
(y) Newton/amp^2
(z) $[M^0L^2T^0A]$
(a) p-y, q-v, r-x, s-z, t-u (b) p-x, q-r, r-y, s-z, t-v
(c) p-x, q-v,. r-y, s-u, t-z (d) p-y, q-z, r-x, s-u, t-v

12. A closed loop carrying a current I lies in the xz-plane. The loop will experience a force if it is placed in a region occupied by uniform magnetic field along
(a) x-axis (b) y-axis
(c) z-axis (d) None of these

13. A stream of protons and α-particles of equal momenta enter a uniform magnetic field perpendicularly. The radii of their orbits are in the ratio
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

14. A loop of magnetic moment **M** is placed in the orientation of unstable equilibrium position in a uniform magnetic field **B**. The external work done in rotating it through an angle θ is
(a) $-MB(1-\cos\theta)$ (b) $-MB\cos\theta$
(c) $MB\cos\theta$ (d) $MB(1-\cos\theta)$

15. A current of 50 A is passed through a straight wire of length 6 cm, then the magnetic induction at a point 5 cm from the either end of the wire is (1 gauss = 10^{-4} T)

(a) 2.5 gauss
(b) 1.25 gauss
(c) 1.5 gauss
(d) 3.0 gauss

16. The magnetic field due to a current carrying circular loop of radius 3 m at a point on the axis at a distance of 4 m from the centre is 54 μT. What will be its value at the centre of the loop?

(a) 250 μT
(b) 150 μT
(c) 125 μT
(d) 75 μT

17. A conductor *ab* of arbitrary shape carries current I flowing from b to a. The length vector **ab** is oriented from a to b. The force $\mathbf{F}$ experienced by this conductor in a uniform magnetic field $\mathbf{B}$ is

(a) $\mathbf{F} = -I(\mathbf{ab} \times \mathbf{B})$
(b) $\mathbf{F} = I(\mathbf{B} \times \mathbf{ab})$
(c) $\mathbf{F} = I(\mathbf{ba} \times \mathbf{B})$
(d) All of these

18. When an electron is accelerated through a potential difference V, it experiences a force F through a uniform transverse magnetic field. If the potential difference is increased to 2 V, the force experienced by the electron in the same magnetic field is

(a) $2F$
(b) $2\sqrt{2}F$
(c) $\sqrt{2}F$
(d) $4F$

19. Two long straight wires, each carrying a current I in opposite directions are separated by a distance R. The magnetic induction at a point mid-way between the wires is

(a) zero
(b) $\frac{\mu_0 I}{\pi R}$
(c) $\frac{2\mu_0 I}{\pi R}$
(d) $\frac{\mu_0 I}{4\pi R}$

20. The magnetic field at a distance x on the axis of a circular coil of radius R is $\frac{1}{8}$th of that at the centre. The value of x is

(a) $\frac{R}{\sqrt{3}}$
(b) $\frac{2R}{\sqrt{3}}$
(c) $R\sqrt{3}$
(d) $R\sqrt{2}$

21. Electric field and magnetic field in a region of space is given by $\mathbf{E} = E_0\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$. A particle of specific charge α is released from origin with velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$. Then, path of particle

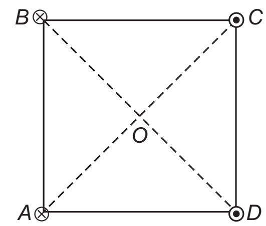
(a) is a circle
(b) is a helix with uniform pitch
(c) is a helix with non-uniform pitch
(d) is cycloid

Note *E_0, B_0 and v_0 are constant values.*

22. An electron having kinetic energy K is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction. If kinetic energy is doubled and magnetic induction tripled, the radius will become

(a) $\frac{2R}{3}$
(b) $\frac{\sqrt{2}}{3}R$
(c) $\sqrt{\frac{2}{3}}R$
(d) $\frac{2}{\sqrt{3}}R$

23. Four long straight wires are located at the corners of a square $ABCD$. All the wires carry equal currents. Current in the wires A and B are inwards and in C and D are outwards. The magnetic field at the centre O is along

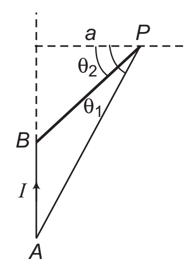


(a) AD
(b) CB
(c) AB
(d) CD

24. A charged particle of mass m and charge q is accelerated through a potential difference of V volts. It enters a region of uniform magnetic field B which is directed perpendicular to the direction of motion of the particle. The particle will move on a circular path of radius

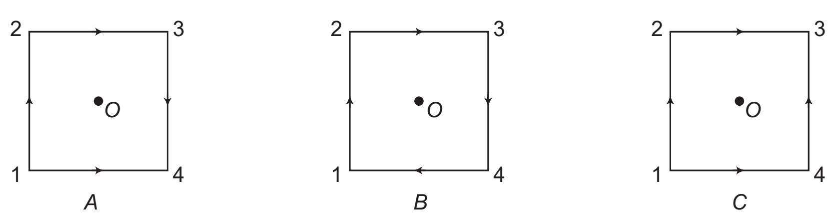
(a) $\sqrt{\dfrac{Vm}{2qB^2}}$
(b) $\dfrac{2\,Vm}{qB^2}$
(c) $\sqrt{\dfrac{2\,Vm}{q}}\left(\dfrac{1}{B}\right)$
(d) $\sqrt{\dfrac{Vm}{q}}\left(\dfrac{1}{B}\right)$

25. The straight wire AB carries a current I. The ends of the wire subtend angles θ_1 and θ_2 at the point P as shown in figure. The magnetic field at the point P is



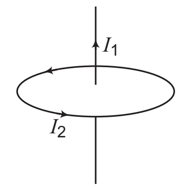
(a) $\dfrac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$
(b) $\dfrac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$
(c) $\dfrac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$
(d) $\dfrac{\mu_0 I}{4\pi a}(\cos\theta_1 + \cos\theta_2)$

26. The figure shows three identical current carrying square loops A, B and C. Identify the correct statement related to magnetic field $\mathbf{B}$ at the centre O of the square loop. Current in each wire is I.



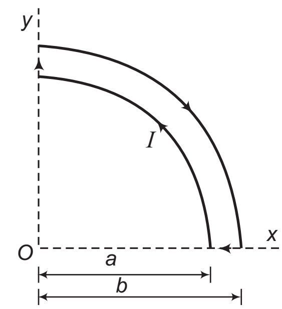
(a) $\mathbf{B}$ is zero in all cases
(b) $\mathbf{B}$ is zero only in case of C
(c) $\mathbf{B}$ is non-zero in all cases
(d) $\mathbf{B}$ is non-zero only in case of B

27. The figure shows a long straight wire carrying a current I_1 along the axis of a circular ring carrying a current I_2. Identify the correct statement.



(a) Straight wire attracts the ring
(b) Straight wire attracts a small element of the ring
(c) Straight wire does not attract any small element of the ring
(d) None of the above

28. The figure shows a wire frame in *xy*-plane carrying a current I. The magnetic field at the point O is



(a) $\frac{\mu_0 I}{8}\left[\frac{1}{a}-\frac{1}{b}\right]\hat{\mathbf{k}}$
(b) $\frac{\mu_0 I}{8}\left[\frac{1}{b}-\frac{1}{a}\right]\hat{\mathbf{k}}$
(c) $\frac{\mu_0 I}{4}\left[\frac{1}{a}-\frac{1}{b}\right]\hat{\mathbf{k}}$
(d) $\frac{\mu_0 I}{4}\left[\frac{1}{b}-\frac{1}{a}\right]\hat{\mathbf{k}}$

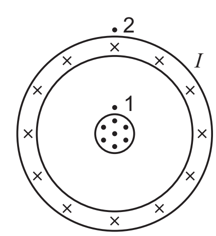
29. An electron moving in a circular orbit of radius R with frequency f. The magnetic field at the centre of the orbit is

(a) $\frac{\mu_0 ef}{2\pi R}$
(b) $\frac{\mu_0 ef}{2R}$
(c) $\frac{\mu ef^2}{2R}$
(d) zero

30. A square loop of side a carries a current I. The magnetic field at the centre of the loop is

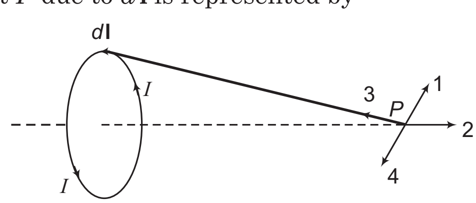
(a) $\frac{2\mu_0 I\sqrt{2}}{\pi a}$
(b) $\frac{\mu_0 I\sqrt{2}}{\pi a}$
(c) $\frac{4\mu_0 I\sqrt{2}}{\pi a}$
(d) $\frac{\mu_0 I}{\pi a}$

31. The figure shows the cross-section of two long coaxial tubes carrying equal currents I in opposite directions. If B_1 and B_2 are magnetic fields at points 1 and 2 as shown in figure, then



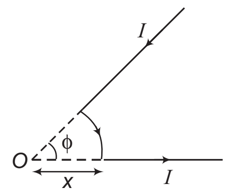
(a) $B_1 \neq 0; B_2 = 0$
(b) $B_1 = 0; B_2 = 0$
(c) $B_1 \neq 0; B_2 \neq 0$
(d) $B_1 = 0, B_2 \neq 0$

32. The figure shows a point P on the axis of a circular loop carrying current I. The correct direction of magnetic field vector at P due to $d\mathbf{l}$ is represented by



(a) 1
(b) 2
(c) 3
(d) 4

33. In figure, the curved part represents arc of a circle of radius x. If it carries a current I, then the magnetic field at the point O is

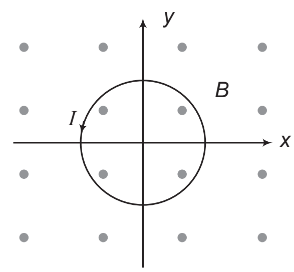


(a) $\frac{\mu_0 I\phi}{2\pi x}$ (b) $\frac{\mu_0 I\phi}{4\pi x}$ (c) $\frac{\mu_0 I\phi}{2x}$ (d) $\frac{\mu_0 I\phi}{4x}$

34. A cylindrical long wire of radius R carries a current I uniformly distributed over the cross-sectional area of the wire. The magnetic field at a distance x from the surface inside the wire is

(a) $\frac{\mu_0 I}{2\pi(R-x)}$ (b) $\frac{\mu_0 I}{2\pi x}$ (c) $\frac{\mu_0 I}{2\pi(R+x)}$ (d) None of these

35. A circular loop carrying a current I is placed in the xy-plane as shown in figure. A uniform magnetic field $\mathbf{B}$ is oriented along the positive z-axis. The loop tends to



(a) expand (b) contract (c) rotate about x-axis (d) rotate about y-axis

Subjective Questions

Note *You can take approximations in the answers.*

1. An electron has velocity $\mathbf{v} = (2.0 \times 10^6 \text{ m/s})\hat{\mathbf{i}} + (3.0 \times 10^6 \text{ m/s})\hat{\mathbf{j}}$. Magnetic field present in the region is $\mathbf{B} = (0.030 \text{ T})\hat{\mathbf{i}} - (0.15 \text{ T})\hat{\mathbf{j}}$.

(a) Find the force on electron.

(b) Repeat your calculation for a proton having the same velocity.

2. An electron moves through a uniform magnetic field given by $\mathbf{B} = B_x\hat{\mathbf{i}} + (3B_x)\hat{\mathbf{j}}$. At a particular instant, the electron has the velocity $\mathbf{v} = (2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19} \text{ N})\hat{\mathbf{k}}$. Find B_x.

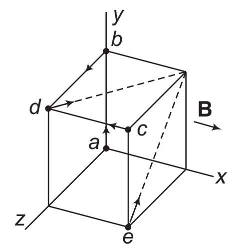
3. A particle with charge $7.80\,\mu\text{C}$ is moving with velocity $\mathbf{v} = -(3.80 \times 10^3 \text{ m/s})\hat{\mathbf{j}}$. The magnetic force on the particle is measured to be $\mathbf{F} = +(7.60 \times 10^{-3} \text{ N})\hat{\mathbf{i}} - (5.20 \times 10^{-3} \text{ N})\hat{\mathbf{k}}$.

(a) Calculate the components of the magnetic field you can find from this information.

(b) Are the components of the magnetic field that are not determined by the measurement of the force? Explain.

(c) Calculate the scalar product $\mathbf{B} \cdot \mathbf{F}$. What is the angle between $\mathbf{B}$ and $\mathbf{F}$?

4. Each of the lettered points at the corners of the cube in figure represents a positive charge q moving with a velocity of magnitude v in the direction indicated. The region in the figure is in a uniform magnetic field **B**, parallel to the x-axis and directed toward the right. Find the magnitude and direction of the force on each charge.



5. An electron in the beam of a TV picture tube is accelerated by a potential difference of 2.00 kV. Then, it passes through region of transverse magnetic field, where it moves in a circular arc with radius 0.180 m. What is the magnitude of the field?

6. A deuteron (the nucleus of an isotope of hydrogen) has a mass of 3.34×10^{-27} kg and a charge of $+e$. The deuteron travels in a circular path with a radius of 6.96 mm in a magnetic field with magnitude 2.50 T.

(a) Find the speed of the deuteron.

(b) Find the time required for it to make half of a revolution.

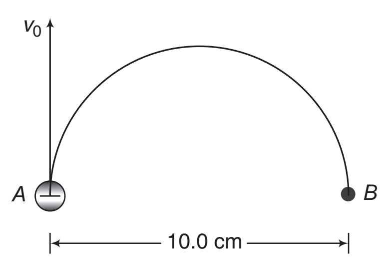
(c) Through what potential difference would the deuteron have to be accelerated to acquire this speed?

7. A neutral particle is at rest in a uniform magnetic field **B**. At time $t = 0$, it decays into two charged particles, each of mass m.

(a) If the charge of one of the particles is $+q$, what is the charge of the other?

(b) The two particles move off in separate paths, both of them lie in the plane perpendicular to **B**. At a later time, the particles collide. Express the time from decay until collision in terms of m, B and q.

8. An electron at point A in figure has a speed $v_0 = 1.41 \times 10^6$ m/s. Find



(a) the magnitude and direction of the magnetic field that will cause the electron to follow the semicircular path from A to B,

(b) the time required for the electron to move from A to B.

9. A proton of charge e and mass m enters a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{i}}$ with an initial velocity $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$. Find an expression in unit vector notation for its velocity at time t.

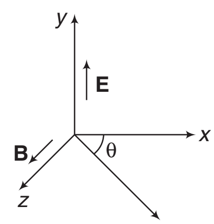
10. A proton moves at a constant velocity of 50 m/s along the x-axis, in uniform electric and magnetic fields. The magnetic field is $\mathbf{B} = (2.0\text{ mT})\hat{\mathbf{j}}$. What is the electric field?

11. A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by

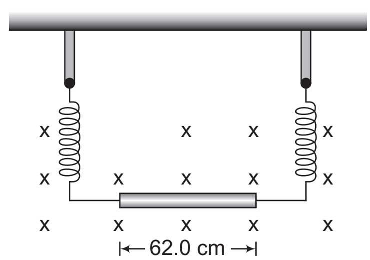
$$\mathbf{B} = -B_0\hat{\mathbf{j}} \quad \text{and} \quad \mathbf{E} = E_0\hat{\mathbf{k}}$$

Find the y-component of the velocity and the speed of the particle as a function of its z-coordinate.

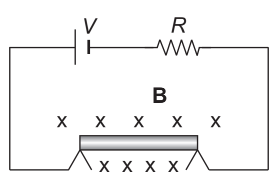
12. Protons move rectilinearly in the region of space where there are uniform mutually perpendicular electric and magnetic fields E and B. The trajectory of protons lies in the plane xz as shown in the figure and forms an angle θ with x-axis. Find the pitch of the helical trajectory along which the protons will move after the electric field is switched off.



13. A wire of 62.0 cm length and 13.0 g mass is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T in figure. What are the magnitude and direction of the current required to remove the tension in the supporting leads? Take $g = 10\ \text{m/s}^2$.



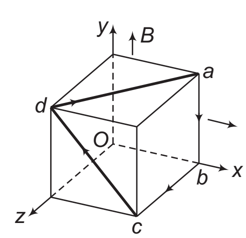
14. A thin, 50.0 cm long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a 0.450 T magnetic field as shown in figure. A battery and a resistance $R = 25.0\ \Omega$ in series are connected to the supports.



(a) What is the largest voltage the battery can have without breaking the circuit at the supports?

(b) The battery voltage has this maximum value calculated. Decreasing the resistance to $2.0\ \Omega$, find the initial acceleration of the bar.

15. In figure, the cube is 40.0 cm on each edge. Four straight segments of wire ab, bc, cd and da form a closed loop that carries a current $I = 5.00$ A, in the direction shown. A uniform magnetic field of magnitude $B = 0.020$ T is in the positive y-direction. Determine the magnitude and direction of the magnetic force on each segment.



16. Find the ratio of magnetic dipole moment and magnetic field at the centre of a disc. Radius of disc is R and it is rotating at constant angular speed ω about its axis. The disc is insulating and uniformly charged.

17. A magnetic dipole with a dipole moment of magnitude 0.020 J/T is released from rest in a uniform magnetic field of magnitude 52 mT. The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientations where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ.

(a) What is the initial angle between the dipole moment and the magnetic field?

(b) What is the angle when the dipole is next (momentarily) at rest?

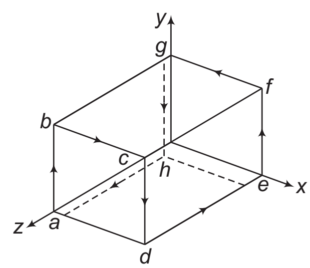
18. In the Bohr model of the hydrogen atom, in the lowest energy state the electron revolves round the proton at a speed of 2.2×10^6 m/s in a circular orbit of radius 5.3×10^{-11} m.

(a) What is the orbital period of the electron?

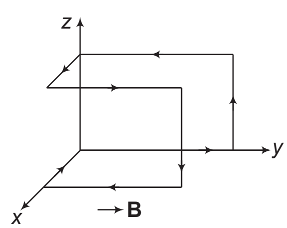
(b) If the orbiting electron is considered to be a current loop, what is the current I?

(c) What is the magnetic moment of the atom due to the motion of the electron?

19. A conductor carries a constant current I along the closed path *abcdefgha* involving 8 of the 12 edges each of length l. Find the magnetic dipole moment of the closed path.

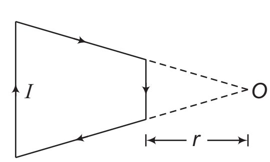


20. Given figure shows a coil bent with all edges of length 1 m and carrying a current of 1 A. There exists in space a uniform magnetic field of 2 T in positive y-direction. Find the torque on the loop.

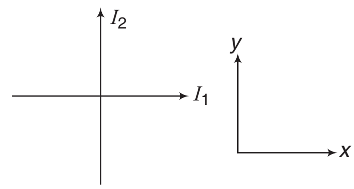


21. A very long wire carrying a current $I = 5.0$ A is bent at right angles. Find the magnetic induction at a point lying on a perpendicular normal to the plane of the wire drawn through the point of bending at a distance $l = 35$ cm from it.

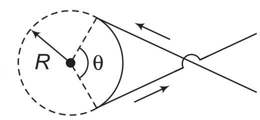
22. A current $I = \sqrt{2}$ A flows in a circuit having the shape of isosceles trapezium. The ratio of the bases of the trapezium is 2. Find the magnetic induction B at symmetric point O in the plane of the trapezium. The length of the smaller base of the trapezium is 100 mm and the distance $r = 50$ mm.



23. Two long mutually perpendicular conductors carrying currents I_1 and I_2 lie in one plane. Find the locus of points at which the magnetic induction is zero.



24. A wire carrying current i has the configuration as shown in figure. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc of central angle θ, along the circumference of the circle, with all sections lying in the same plane. What must θ be for B to be zero at the centre of the circle?

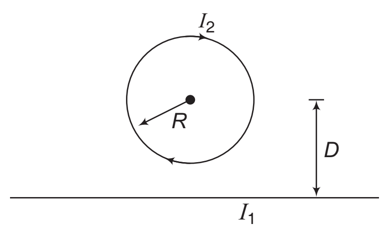


25. Two long parallel transmission lines 40.0 cm apart carry 25.0 A and 75.0 A currents. Find all locations where the net magnetic field of the two wires is zero if these currents are in

(a) the same direction (b) the opposite direction

26. A closely wound coil has a radius of 6.00 cm and carries a current of 2.50 A. How many turns must it have if, at a point on the coil axis 6.00 cm from the centre of the coil, the magnetic field is 6.39×10^{-4} T?

27. A circular loop of radius R carries current I_2 in a clockwise direction as shown in figure. The centre of the loop is a distance D above a long, straight wire. What are the magnitude and direction of the current I_1 in the wire if the magnetic field at the centre of loop is zero?

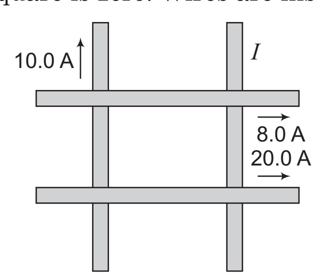


28. A closely wound, circular coil with radius 2.40 cm has 800 turns.

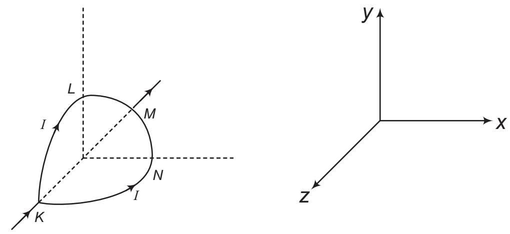
(a) What must the current in the coil be if the magnetic field at the centre of the coil is 0.0580 T?

(b) At what distance x from the centre of the coil, on the axis of the coil, is the magnetic field half its value at the centre?

29. Four very long, current carrying wires in the same plane intersect to form a square 40.0 cm on each side as shown in figure. Find the magnitude and direction of the current I so that the magnetic field at the centre of square is zero. Wires are insulated from each other.



30. A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semicircles (KNM) lies in the xz-plane and the other one (KLM) in the yz-plane with their centres at origin. Current I is flowing through each of the semicircles as shown in figure.



(a) A particle of charge q is released at the origin with a velocity $\mathbf{v} = -v_0\hat{\mathbf{i}}$. Find the instantaneous force $\mathbf{F}$ on the particle. Assume that space is gravity free.

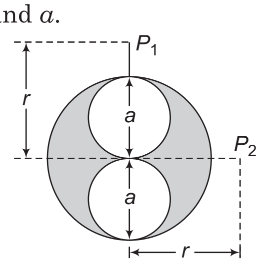
(b) If an external uniform magnetic field $B_0\hat{\mathbf{j}}$ is applied, determine the force $\mathbf{F}_1$ and $\mathbf{F}_2$ on the semicircles KLM and KNM due to the field and the net force $\mathbf{F}$ on the loop.

31. A regular polygon of n sides is formed by bending a wire of total length $2\pi r$ which carries a current i.

(a) Find the magnetic field B at the centre of the polygon.

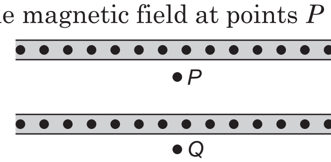
(b) By letting $n \to \infty$, deduce the expression for the magnetic field at the centre of a circular coil.

32. A long cylindrical conductor of radius a has two cylindrical cavities of diameter a through its entire length as shown in cross-section in figure. A current I is directed out of the page and is uniform throughout the cross-section of the conductor. Find the magnitude and direction of the magnetic field in terms of μ_0, I, r and a.

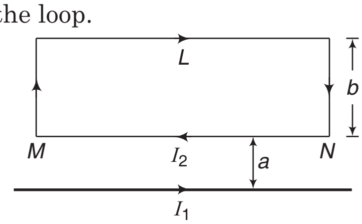


(a) at point P_1 and (b) at point P_2

33. Two infinite plates shown in cross-section in figure carry λ amperes of current out of the page per unit width of plate. Find the magnetic field at points P and Q.



34. For the situation shown in figure, find the force experienced by side MN of the rectangular loop. Also, find the torque on the loop.

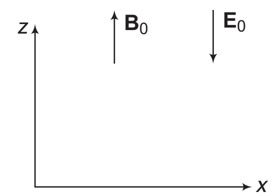


35. In a region of space, a uniform magnetic field B is along positive x-axis. Electrons are emitted from the origin with speed v at different angles. Show that the paraxial electrons are refocused on the x-axis at a distance $\frac{2\pi mv}{Be}$. Here, m is the mass of electron and e the charge on it.

36. A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation of the particle as it comes out of the magnetic field if the width of the region is

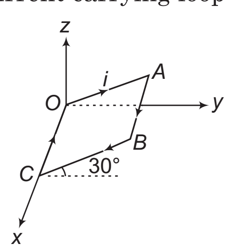
(a) $\frac{2mv}{Bq}$ (b) $\frac{mv}{Bq}$ (c) $\frac{mv}{2Bq}$

37. In a certain region, uniform electric field $\mathbf{E} = -E_0\hat{\mathbf{k}}$ and magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$ are present. At time $t = 0$, a particle of mass m and charge q is given a velocity $\mathbf{v} = v_0\hat{\mathbf{j}} + v_0\hat{\mathbf{k}}$. Find the minimum speed of the particle and the time when it happens so.



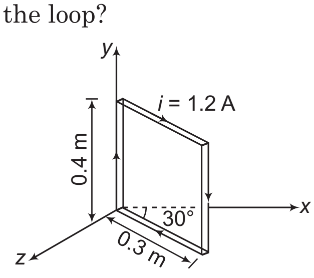
38. A particle of mass m and charge q is lying at the origin in a uniform magnetic field B directed along x-axis. At time $t = 0$, it is given a velocity v_0 at an angle θ with the y-axis in the xy-plane. Find the coordinates of the particle after one revolution.

39. Find the magnetic moment of the current carrying loop $OABCO$ shown in figure.

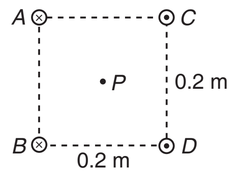


Given that, $i = 4.0$ A, $OA = 20$ cm and $AB = 10$ cm.

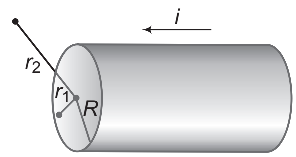
40. A rectangular loop consists of $N = 100$ closed wrapped turns and has dimensions (0.4 m × 0.3 m). The loop is hinged along the y-axis and its plane makes an angle $\theta = 30°$ with the x-axis. What is the magnitude of the torque exerted on the loop by a uniform magnetic field $B = 0.8$ T directed along the x-axis when current is $i = 1.2$ A in the direction shown. What is the expected direction of rotation of the loop?



41. Four long, parallel conductors carry equal currents of 5.0 A. The direction of the currents is into the page at points A and B and out of the page at C and D. Calculate the magnitude and direction of the magnetic field at point P, located at the centre of the square.



42. A long cylindrical conductor of radius R carries a current i as shown in figure. The current density J is a function of radius according to, $J = br$, where b is a constant. Find an expression for the magnetic field B

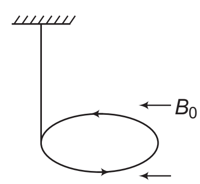


(a) at a distance $r_1 < R$ and
(b) at a distance $r_2 > R$, measured from the axis.

LEVEL 2

Single Correct Option

1. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown. A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is



(a) $\dfrac{mg}{\pi RB_0}$ (b) $\dfrac{mg}{RB_0}$ (c) $\dfrac{mg}{3\pi RB_0}$ (d) $\dfrac{mg}{\pi R^2B_0}$

2. A wire of mass 100 g is carrying a current of 2 A towards increasing x in the form of $y = x^2(-2m \le x \le +2m)$. This wire is placed in a magnetic field $\mathbf{B} = -0.02\,\hat{\mathbf{k}}$ tesla. The acceleration of the wire (in m/s^2) is

(a) $-1.6\,\hat{\mathbf{j}}$ (b) $-3.2\,\hat{\mathbf{j}}$ (c) $1.6\,\hat{\mathbf{j}}$ (d) zero

3. A conductor of length l is placed perpendicular to a horizontal uniform magnetic field B. Suddenly, a certain amount of charge is passed through it, when it is found to jump to a height h. The amount of charge that passes through the conductor is

(a) $\dfrac{m\sqrt{gh}}{Bl}$ (b) $\dfrac{m\sqrt{gh}}{2Bl}$

(c) $\dfrac{m\sqrt{2gh}}{Bl}$ (d) None of these

4. A solid conducting sphere of radius R and total charge q rotates about its diametric axis with constant angular speed ω. The magnetic moment of the sphere is

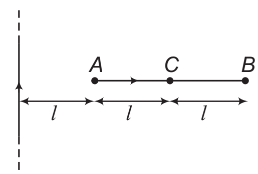
(a) $\frac{1}{3}qR^2\omega$ (b) $\frac{2}{3}qR^2\omega$ (c) $\frac{1}{5}qR^2\omega$ (d) $\frac{2}{5}qR^2\omega$

5. A charged particle moving along positive x-direction with a velocity v enters a region where there is a uniform magnetic field $\mathbf{B} = -B\hat{\mathbf{k}}$, from $x = 0$ to $x = d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is

(a) $\frac{Bd}{v\cos\theta}$ (b) $\frac{v\tan\theta}{Bd}$

(c) $\frac{B\sin\theta}{vd}$ (d) $\frac{v\sin\theta}{Bd}$

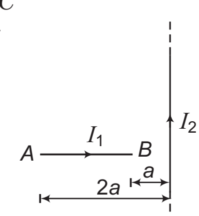
6. A current carrying rod AB is placed perpendicular to an infinitely long current carrying wire as shown in figure. The point at which the conductor should be hinged so that it will not rotate ($AC = CB$)



(a) A (b) somewhere between B and C

(c) C (d) somewhere between A and C

7. The segment AB of wire carrying current I_1 is placed perpendicular to a long straight wire carrying current I_2 as shown in figure. The magnitude of force experienced by the straight wire AB is



(a) $\frac{\mu_0 I_1 I_2}{2\pi}\ln 3$ (b) $\frac{\mu_0 I_1 I_2}{2\pi}\ln 2$

(c) $\frac{2\mu_0 I_1 I_2}{2\pi}$ (d) $\frac{\mu_0 I_1 I_2}{2\pi}$

8. A straight long conductor carries current along the positive x-axis. Identify the correct statement related to the four points $A(a, a, 0)$, $B(a, 0, a)$, $C(a, -a, 0)$ and $D(a, 0, -a)$.

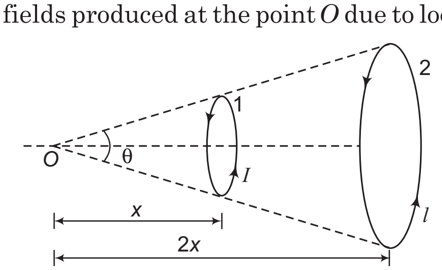
(a) The magnitude of magnetic field at all points is same

(b) Fields at A and B are mutually perpendicular

(c) Fields at A and C are antiparallel

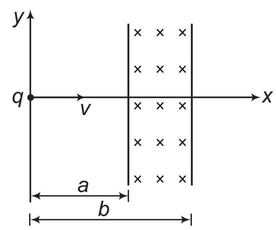
(d) All of the above

9. The figure shows two coaxial circular loops 1 and 2, which forms same solid angle θ at point O. If B_1 and B_2 are the magnetic fields produced at the point O due to loop 1 and 2 respectively, then



(a) $\frac{B_1}{B_2} = 1$ (b) $\frac{B_1}{B_2} = 2$ (c) $\frac{B_1}{B_2} = 8$ (d) $\frac{B_1}{B_2} = 4$

10. In the figure shown, a charge q moving with a velocity v along the x-axis enter into a region of uniform magnetic field. The minimum value of v so that the charge q is able to enter the region $x > b$



(a) $\frac{qBb}{m}$ (b) $\frac{qBa}{m}$

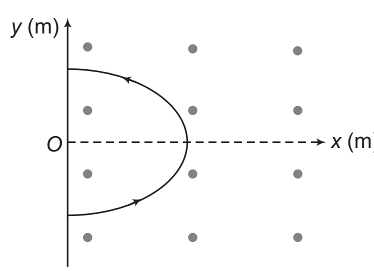
(c) $\frac{qB(b-a)}{m}$ (d) $\frac{qB(b+a)}{2m}$

11. An insulating rod of length l carries a charge q uniformly distributed on it. The rod is pivoted at one of its ends and is rotated at a frequency f about a fixed perpendicular axis. The magnetic moment of the rod is

(a) $\frac{\pi q f l^2}{12}$ (b) $\frac{\pi q f l^2}{2}$

(c) $\frac{\pi q f l^2}{6}$ (d) $\frac{\pi q f l^2}{3}$

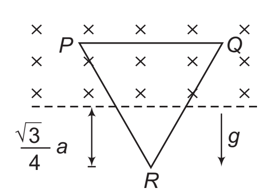
12. A wire carrying a current of 3 A is bent in the form of a parabola $y^2 = 4 - x$ as shown in figure, where x and y are in metre. The wire is placed in a uniform magnetic field $\mathbf{B} = 5\hat{\mathbf{k}}$ tesla. The force acting on the wire is



(a) $60\,\hat{\mathbf{i}}$ N (b) $-60\hat{\mathbf{i}}$ N

(c) $30\,\hat{\mathbf{i}}$ N (d) $-30\,\hat{\mathbf{i}}$ N

13. An equilateral triangle frame PQR of mass M and side a is kept under the influence of magnetic force due to inward perpendicular magnetic field B and gravitational field as shown in the figure. The magnitude and direction of current in the frame so that the frame remains at rest, is



(a) $I = \frac{2Mg}{aB}$; anti-clockwise (b) $I = \frac{2Mg}{aB}$; clockwise

(c) $I = \frac{Mg}{aB}$; anti-clockwise (d) $I = \frac{Mg}{aB}$; clockwise

14. A tightly wound long solenoid has n turns per unit length, radius r and carries a current i. A particle having charge q and mass m is projected from a point on the axis in the direction perpendicular to the axis. The maximum speed for which particle does not strike the solenoid will be

(a) $\dfrac{\mu_0 qrni}{2m}$ (b) $\dfrac{\mu_0 qrni}{m}$

(c) $\dfrac{2\mu_0 qrni}{3m}$ (d) None of these

15. If the acceleration and velocity of a charged particle moving in a constant magnetic region is given by $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{k}}$, $\mathbf{v} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{k}}$. [$a_1$, a_2, b_1 and b_2 are constants], then choose the **wrong** statement.

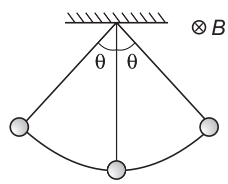
(a) Magnetic field may be along y-axis

(b) $a_1b_1 + a_2b_2 = 0$

(c) Magnetic field is along x-axis

(d) Kinetic energy of particle is always constant

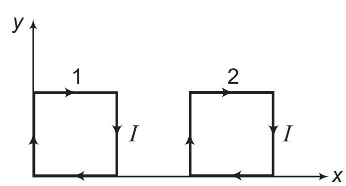
16. A simple pendulum with a charged bob is oscillating as shown in the figure. Time period of oscillation is T and angular amplitude is θ. If a uniform magnetic field perpendicular to the plane of oscillation is switched on, then



(a) T will decrease but θ will remain constant (b) T will remain constant but θ will decrease

(c) Both T and θ will remain the same (d) Both T and θ will decrease

17. Magnetic field in a region is given by $\mathbf{B} = B_0 x\hat{\mathbf{k}}$. Two loops each of side a is placed in this magnetic region in the xy-plane with one of its sides on x-axis. If F_1 is the force on loop 1 and F_2 be the force on loop 2, then



(a) $F_1 = F_2 = 0$ (b) $F_1 > F_2$

(c) $F_2 > F_1$ (d) $F_1 = F_2 \neq 0$

18. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c, respectively. The inner wire carries a current I and outer shell carries an equal and opposite current. The magnetic field at a distance x from the axis where $b < x < c$ is

(a) $\dfrac{\mu_0 I(c^2 - b^2)}{2\pi x(c^2 - a^2)}$ (b) $\dfrac{\mu_0 I(c^2 - x^2)}{2\pi x(c^2 - a^2)}$

(c) $\dfrac{\mu_0 I(c^2 - x^2)}{2\pi x(c^2 - b^2)}$ (d) zero

19. A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity of 1.28×10^6 m/s along positive direction of x-axis enters a region in which a uniform electric field **E** and a uniform magnetic field **B** are present such that

$$E_z = -102.4 \text{ kV/m} \quad \text{and} \quad B_y = 8 \times 10^{-2} \text{ Wb/m}^2$$

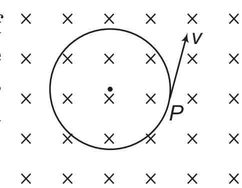
The particle enters this region at origin at time $t = 0$. Then,

(a) net force acts on the particle along the +ve z-direction
(b) net force acts on the particle along –ve z-direction
(c) net force acting on particle is zero
(d) net force acts in xz-plane

20. A wire lying along y-axis from $y = 0$ to $y = 1$ m carries a current of 2 mA in the negative y-direction. The wire lies in a non-uniform magnetic field given by $\mathbf{B} = (0.3 \text{ T/m})\, y\hat{\mathbf{i}} + (0.4 \text{ Tm})\, y\hat{\mathbf{j}}$. The magnetic force on the entire wire is

(a) $-3 \times 10^{-4}\,\hat{\mathbf{j}}$ N
(b) $6 \times 10^{-3}\,\hat{\mathbf{k}}$ N
(c) $-3 \times 10^{-4}\,\hat{\mathbf{k}}$ N
(d) $3 \times 10^{-4}\,\hat{\mathbf{k}}$ N

21. A particle having a charge of 20 μC and mass 20 μg moves along a circle of radius 5 cm under the action of a magnetic field $B = 0.1$ tesla. When the particle is at P, uniform transverse electric field is switched on and it is found that the particle continues along the tangent with a uniform velocity. Find the electric field



(a) 2 V/m
(b) 0.5 V/m
(c) 5 V/m
(d) 1.5 V/m

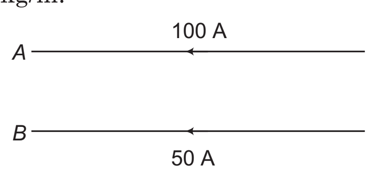
22. Two circular coils A and B of radius $\frac{5}{\sqrt{2}}$ cm and 5 cm carry currents 5 A and $5\sqrt{2}$ A, respectively. The plane of B is perpendicular to plane of A and their centres coincide. Magnetic field at the centre is

(a) 0
(b) $4\pi\sqrt{2} \times 10^{-5}$ T
(c) $4\pi \times 10^{-5}$ T
(d) $2\pi\sqrt{2} \times 10^{-5}$ T

23. A charged particle with specific charge s moves undeflected through a region of space containing mutually perpendicular and uniform electric and magnetic fields E and B. When the electric field is switched off, the particle will move in a circular path of radius

(a) $\frac{E}{Bs}$
(b) $\frac{Es}{B}$
(c) $\frac{Es}{B^2}$
(d) $\frac{E}{B^2 s}$

24. Two long parallel conductors are carrying currents in the same direction as shown in the figure. The upper conductor (A) carrying a current of 100 A is held firmly in position. The lower conductor (B) carries a current of 50 A and free to move up and down. The linear mass density of the lower conductor is 0.01 kg/m.

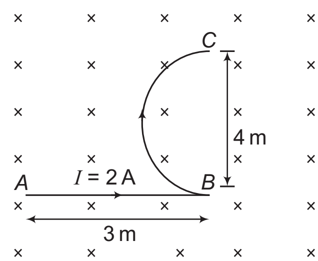


(a) Conductor B will be in equilibrium if the distance between the conductors is 0.1 m
(b) Equilibrium of conductor B is unstable
(c) Both (a) and (b) are wrong
(d) Both (a) and (b) are correct

25. Equal currents are flowing in three infinitely long wires along positive x, y and z-directions. The magnetic field at a point $(0, 0, -a)$ would be (i = current in each wire)

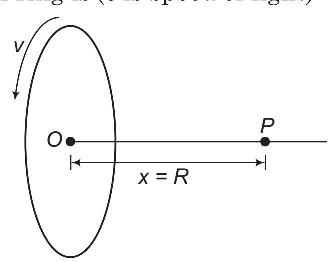
(a) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$
(b) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$
(c) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(d) $\frac{\mu_0 i}{2\pi a}(-\hat{\mathbf{i}} - \hat{\mathbf{j}})$

26. In the figure, the force on the wire ABC in the given uniform magnetic field will be (B = 2 tesla)



(a) $4(3 + 2\pi)\,N$
(b) $20\,N$
(c) $30\,N$
(d) $40\,N$

27. A uniformly charged ring of radius R is rotated about its axis with constant linear speed v of each of its particles. The ratio of electric field to magnetic field at a point P on the axis of the ring distant $x = R$ from centre of ring is (c is speed of light)



(a) $\frac{c^2}{v}$
(b) $\frac{v^2}{c}$
(c) $\frac{v}{c}$
(d) $\frac{c}{v}$

More than One Correct Options

1. Two circular coils of radii 5 cm and 10 cm carry currents of 2 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as their centres coincide. Magnitude of magnetic field at the common centre of coils is

(a) $8\pi \times 10^{-4}$ T if currents in the coils are in same sense
(b) $4\pi \times 10^{-4}$ T if currents in the coils are in opposite sense
(c) zero if currents in the coils are in opposite sense
(d) $8\pi \times 10^{-4}$ T if currents in the coils are in opposite sense

2. A charged particle enters into a gravity free space occupied by an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ and it comes out without any change in velocity. Then, the possible cases may be

(a) $\mathbf{E} = 0$ and $\mathbf{B} \neq 0$
(b) $\mathbf{E} \neq 0$ and $\mathbf{B} = 0$
(c) $\mathbf{E} \neq 0$ and $\mathbf{B} \neq 0$
(d) $\mathbf{E} = 0$, $\mathbf{B} = 0$

3. A charged particle of unit mass and unit charge moves with velocity $\mathbf{v} = (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$ m/s in a magnetic field of $\mathbf{B} = 2\hat{\mathbf{k}}$ T. Choose the correct alternative (s).

(a) The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$
(b) The path of the particle may be $x^2 + y^2 = 25$
(c) The path of the particle may be $y^2 + z^2 = 25$
(d) The time period of the particle will be 3.14 s

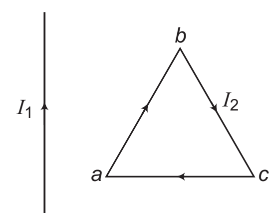
4. When a current carrying coil is placed in a uniform magnetic field with its magnetic moment anti-parallel to the field, then

(a) torque on it is maximum
(b) torque on it is zero
(c) potential energy is maximum
(d) dipole is in unstable equilibrium

5. If a long cylindrical conductor carries a steady current parallel to its length, then

(a) the electric field along the axis is zero
(b) the magnetic field along the axis is zero
(c) the magnetic field outside the conductor is zero
(d) the electric field outside the conductor is zero

6. An infinitely long straight wire is carrying a current I_1. Adjacent to it there is another equilateral triangular wire having current I_2. Choose the **wrong** options.



(a) Net force on loop is leftwards
(b) Net force on loop is rightwards
(c) Net force on loop is upwards
(d) Net force on loop is downwards

7. A charged particle is moving along positive y-axis in uniform electric and magnetic fields

$$\mathbf{E} = E_0\hat{\mathbf{k}}$$

and

$$\mathbf{B} = B_0\hat{\mathbf{i}}$$

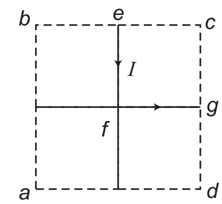
Here, E_0 and B_0 are positive constants. Choose the correct options.

(a) particle may be deflected towards positive z-axis
(b) particle may be deflected towards negative z-axis
(c) particle may pass undeflected
(d) kinetic energy of particle may remain constant

8. A charged particle revolves in circular path in uniform magnetic field after accelerating by a potential difference of V volts. Choose the correct options if V is doubled.

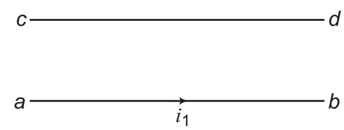
(a) kinetic energy of particle will become two times
(b) radius in circular path will become two times
(c) radius in circular path will become $\sqrt{2}$ times
(d) angular velocity will remain unchanged

9. *abcd* is a square. There is a current I in wire *efg* as shown. Choose the correct options.



(a) Net magnetic field at a is inwards
(b) Net magnetic field at b is zero
(c) Net magnetic field at c is outwards
(d) Net magnetic field at d is inwards

10. There are two wires *ab* and *cd* in a vertical plane as shown in figure. Direction of current in wire *ab* is rightwards. Choose the correct options.



(a) If wire *ab* is fixed, then wire *cd* can be kept in equilibrium by the current in *cd* in leftward direction
(b) Equilibrium of wire *cd* will be stable equilibrium
(c) If wire *cd* is fixed, then wire *ab* can be kept in equilibrium by flowing current in *cd* in rightward direction
(d) Equilibrium of wire *ab* will be stable equilibrium

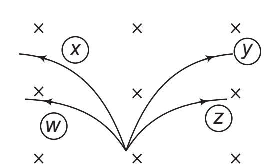
Match the Columns

1. An electron is moving towards positive x-direction. Match the following two columns for deflection of electron just after the fields are switched on. (E_0 and B_0 are positive constants)

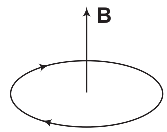
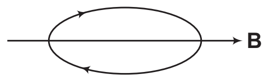
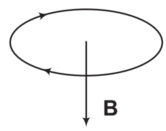
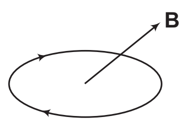
Column I	Column II
(a) If magnetic field $\mathbf{B} = B_0\hat{\mathbf{j}}$ is switched on	(p) Negative y-axis
(b) If magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$ is switched on	(q) Positive y-axis
(c) If magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$ and electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$ is switched on	(r) Negative z-axis
(d) If electric field $\mathbf{E} = E_0\hat{\mathbf{k}}$ is switched on	(s) Positive z-axis

2. Four charged particles, $(-q, m)$, $(-3q, 4m)$, $(+q, m)$ and $(+2q, m)$ enter in uniform magnetic field (in inward direction) with same kinetic energy as shown in figure. Inside the magnetic field their paths are shown. Match the following two columns.

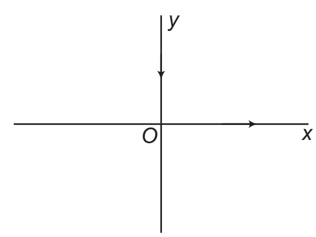
Column I	Column II
(a) Particle $(-q, m)$	(p) w
(b) Particle $(-3q, 4m)$	(q) x
(c) Particle $(+q, m)$	(r) y
(d) Particle $(+2q, m)$	(s) z



3. In Column I, a current carrying loop and a uniform magnetic field are shown. Match this with Column II.

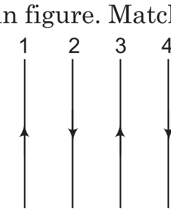
Column I	Column II
(a) 	(p) Force = 0
(b) 	(q) Maximum torque
(c) 	(r) Minimum potential energy
(d) 	(s) Positive potential energy

4. Equal currents are flowing in two infinitely long wires lying along x and y-axes in the directions shown in figure. Match the following two columns.



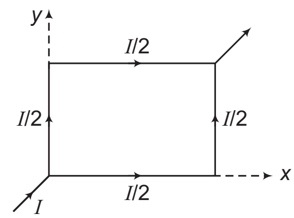
Column I	Column II
(a) Magnetic field at (a, a)	(p) along positive y-axis
(b) Magnetic field at $(-a, -a)$	(q) along positive z-axis
(c) Magnetic field at $(a, -a)$	(r) along negative z-axis
(d) Magnetic field at $(-a, a)$	(s) zero

5. Equal currents are flowing in four infinitely long wires. Distance between two wires is same and directions of currents are shown in figure. Match the following two columns.



Column I	Column II
(a) Force on wire-1	(p) inwards
(b) Force on wire-2	(q) leftwards
(c) Force on wire-3	(r) rightwards
(d) Force on wire-4	(s) zero

6. A square loop of uniform conducting wire is as shown in figure. A current I (in ampere) enters the loop from one end and exits the loop from opposite end as shown in figure.

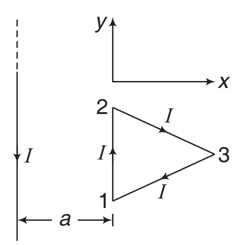


The length of one side of square loop is l metre. The wire has uniform cross-section area and uniform linear mass density.

Column I	Column II
(a) $\mathbf{B} = B_0\hat{\mathbf{i}}$ in tesla	(p) magnitude of net force on loop is $\sqrt{2}B_0Il$ newton
(b) $\mathbf{B} = B_0\hat{\mathbf{j}}$ in tesla	(q) magnitude of net force on loop is zero
(c) $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ in tesla	(r) magnitude of force on loop is 2 B_0Il
(d) $\mathbf{B} = B_0\hat{\mathbf{k}}$ in tesla	(s) magnitude of net force on loop is B_0Il newton

Subjective Questions

1. An equilateral triangular frame with side a carrying a current I is placed at a distance a from an infinitely long straight wire carrying a current I as shown in the figure. One side of the frame is parallel to the wire. The whole system lies in the xy-plane. Find the magnetic force $\mathbf{F}$ acting on the frame.



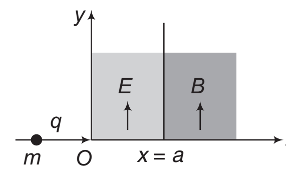
2. Find an expression for the magnetic dipole moment and magnetic field induction at the centre of a Bohr's hypothetical hydrogen atom in the nth orbit of the electron in terms of universal constants.

3. A square loop of side 6 cm carries a current of 30 A. Calculate the magnitude of magnetic field $\mathbf{B}$ at a point P lying on the axis of the loop and a distance $\sqrt{7}$ cm from centre of the loop.

4. A positively charged particle of charge 1 C and mass 40 g, is revolving along a circle of radius 40 cm with velocity 5 m/s in a uniform magnetic field with centre at origin O in xy-plane. At $t = 0$, the particle was at (0, 0.4 m, 0) and velocity was directed along positive x-direction. Another particle having charge 1 C and mass 10 g moving uniformly parallel to z-direction with velocity $\frac{40}{\pi}$ m/s collides with revolving particle at $t = 0$ and gets stuck with it. Neglecting gravitational force and colombians force, calculate x, y and z-coordinates of the combined particle at $t = \frac{\pi}{40}$ sec.

5. A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120 \frac{\text{kV}}{\text{m}}$ and $B = 50$ mT. Then, the beam strikes a grounded target. Find the force imparted by the beam on the target if the beam current is equal to $I = 0.80$ mA.

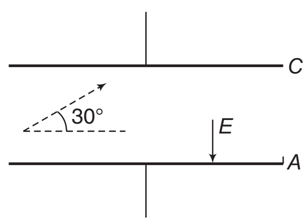
6. A positively charged particle having charge q is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bounded by the lines $x = 0$ and $x = a$. Beyond the line $x = a$ (i.e. in the region $x \geq a$) there exists a magnetic field of strength B, directed along the positive y-axis. Find



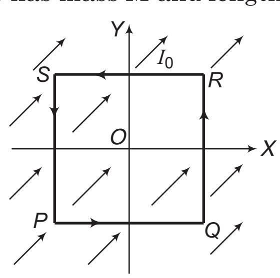
(a) at which point does the particle meet the line $x = a$

(b) the pitch of the helix formed after the particle enters the region $x \geq a$. Mass of the particle is m.

7. A charged particle having charge 10^{-6} C and mass of 10^{-10} kg is fired from the middle of the plate making an angle 30° with plane of the plate. Length of the plate is 0.17 m and it is separated by 0.1 m. Electric field $E = 10^{-3}$ N/C is present between the plates. Just outside the plates magnetic field is present. Find the velocity of projection of charged particle and magnitude of the magnetic field perpendicular to the plane of the figure, if it has to graze the plate at C and A parallel to the surface of the plate. (Neglect gravity)



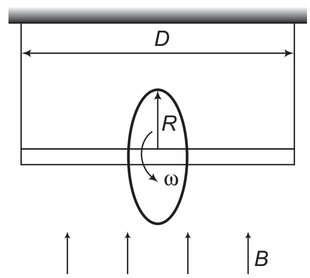
8. A uniform constant magnetic field B is directed at an angle of 45° to the x-axis in xy-plane. $PQRS$ is a rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure with its sides parallel to x and y-axis. Each side of the frame has mass M and length L.



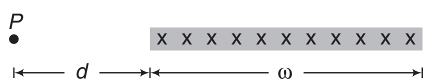
(a) What is the magnitude of torque τ acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which the rotation occurs (Δt is so short that any variation in the torque during this interval may be neglected). Given : The moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3} ML^2$.

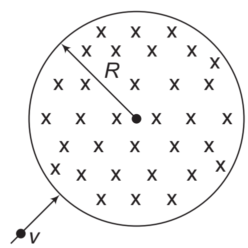
9. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now, a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum value of ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.



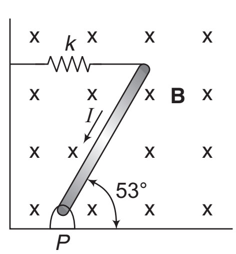
10. Figure shows a cross-section of a long ribbon of width ω that is carrying a uniformly distributed total current i into the page. Calculate the magnitude and direction of the magnetic field **B** at a point P in the plane of the ribbon at a distance d from its edge.



11. A particle of mass m having a charge q enters into a circular region of radius R with velocity v directed towards the centre. The strength of magnetic field is B. Find the deviation in the path of the particle.



12. A thin, uniform rod with negligible mass and length 0.2 m is attached to the floor by a frictionless hinge at point P. A horizontal spring with force constant $k = 4.80$ N/m connects the other end of the rod with a vertical wall. The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the figure. There is current $I = 6.50$ A in the rod, in the direction shown.

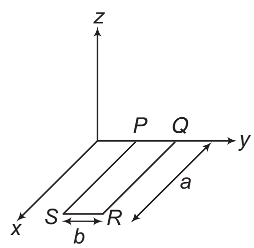


(a) Calculate the torque due to the magnetic force on the rod, for an axis at P. Is it correct to take the total magnetic force to act at the centre of gravity of the rod when calculating the torque?

(b) When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?

(c) How much energy is stored in the spring when the rod is in equilibrium?

13. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0$ exists in the region. The loop is held in the xy-plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.



(a) What is the direction of the current I in PQ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

Answers

Introductory Exercise 26.1

1. $[LT^{-1}]$ **2.** $(\mathbf{F}, \mathbf{v}), (\mathbf{F}, \mathbf{B})$ **3.** No **4.** $(-0.16\,\hat{\mathbf{i}} - 0.32\,\hat{\mathbf{j}} - 0.64\,\hat{\mathbf{k}})$ N

5. Positive **6.** 9.47×10^6 m/s **7.** 2.56×10^{-14} N

Introductory Exercise 26.2

1. D, B **2.** False **3.** False **4.** Yes, No **5.** Along positive z-direction

6. (a) electron (b) electron **7.** 0.0167 cm, 0.7 cm

Introductory Exercise 26.3

1. $\sqrt{2} B_0 il$ **2.** No

3. (a) $(0.023 \text{ N})\,\hat{\mathbf{k}}$ (b) $(0.02 \text{ N})\,\hat{\mathbf{j}}$ (c) zero (d) $(-0.0098 \text{ N})\,\hat{\mathbf{j}}$ (e) $(-0.013 \text{ N})\,\hat{\mathbf{j}} + (-0.026 \text{ N})\,\hat{\mathbf{k}}$

4. 32 N upwards

Introductory Exercise 26.4

1. $\frac{qR^2\omega}{4}$ **2.** (a) $\tau = (-9.6\,\hat{\mathbf{i}} - 7.2\,\hat{\mathbf{j}} + 8.0\,\hat{\mathbf{k}}) \times 10^{-4}$ N-m (b) $U = -(6.0 \times 10^{-4})$ J

4. -2.42 J

Introductory Exercise 26.5

1. (a) 28.3 μT into the page (b) 24.7 μT into the page **2.** $\frac{\mu_0 i}{4\pi x}$ into the page

3. 58.0 μT into the page **4.** 26.2 μT into the page **5.** $\frac{\mu_0 i}{12}\left(\frac{1}{a} - \frac{1}{b}\right)$ out of the page

Introductory Exercise 26.6

1. 200 μT toward the top of the page, 133 μT toward the bottom of the page

2. (a) zero (b) -5.0×10^{-6} T-m (c) 2.5×10^{-6} T-m (d) 5.0×10^{-6} T-m

3. (b) the magnetic field at any point inside the pipe is zero

Introductory Exercise 26.7

1. 90 T **2.** 1.3×10^{-7} A

Exercises

LEVEL 1

Assertion and Reason

1. (c) **2.** (c) **3.** (b) **4.** (d) **5.** (c) **6.** (b) **7.** (a) **8.** (c) **9.** (a) **10.** (d)
11. (d)

Objective Questions

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (c) **6.** (b) **7.** (d) **8.** (c) **9.** (d) **10.** (a)
11. (c) **12.** (d) **13.** (c) **14.** (a) **15.** (c) **16.** (a) **17.** (d) **18.** (c) **19.** (c) **20.** (c)

21. (c) **22.** (b) **23.** (d) **24.** (c) **25.** (a) **26.** (b) **27.** (c) **28.** (a) **29.** (b) **30.** (a)
31. (a) **32.** (a) **33.** (b) **34.** (d) **35.** (a)

Subjective Questions

1. (a) $(6.24 \times 10^{-14}\ \text{N})\hat{\mathbf{k}}$ (b) $-(6.24 \times 10^{-14}\ \text{N})\hat{\mathbf{k}}$

2. $B_x = -2.0$ T

3. (a) $B_x = (-0.175)$T, $B_z = (-0.256)$T (b) Yes, B_y (c) zero, 90°

4. (a) $-qvB\,\hat{\mathbf{k}}$ (b) $+qvB\,\hat{\mathbf{j}}$ (c) zero (d) $\dfrac{-qvB}{\sqrt{2}}\hat{\mathbf{j}}$ (e) $\left(-\dfrac{qvB}{\sqrt{2}}\right)(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

5. 8.38×10^{-4} T

6. (a) 8.35×10^5 m/s (b) 2.62×10^{-8} s (c) 7.26 kV

7. (a) –q (b) $\dfrac{\pi m}{Bq}$

8. (a) 1.6×10^{-4} T into the page (b) 1.11×10^{-7} s

9. $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y \cos\omega t\,\hat{\mathbf{j}} - v_y \sin\omega t\,\hat{\mathbf{k}}$, Here $\omega = \dfrac{Be}{m}$

10. $\mathbf{E} = (-0.1\ \text{V/m})\,\hat{\mathbf{k}}$

11. $v_y = 0$, $v = \sqrt{\dfrac{2qE_0z}{m}}$

12. $\dfrac{2\pi mE\tan\theta}{qB^2}$

13. 0.47 A from left to right

14. (a) 817.5 V (b) 112.8 m/s^2

15. $\mathbf{F}_{ab} = 0$, $\mathbf{F}_{bc} = (-0.04\ \text{N})\,\hat{\mathbf{i}}$, $\mathbf{F}_{cd} = (-0.04\ \text{N})\,\hat{\mathbf{k}}$, $\mathbf{F}_{da} = (0.04\hat{\mathbf{i}} + 0.04\hat{\mathbf{k}})$ N

16. $\dfrac{\pi R^3}{2\mu_0}$

17. (a) 76.7° (b) 76.7°

18. (a) 1.5×10^{-16} s (b) 1.1 mA (c) 9.3×10^{-24} A-m^2

19. $\mathbf{M} = 2Il^2\hat{\mathbf{j}}$

20. zero

21. 2.0 μT

22. 2×10^{-6} T

23. $y = \left(\dfrac{I_1}{I_2}\right)x$

24. 2 rad

25. (a) Between the wires, 30.0 cm from wire carrying 75.0 A
(b) 20.0 cm from wire carrying 25.0 A and 60.0 cm from wire carrying 75.0 A

26. 69

27. $I_1 = \left(\dfrac{\pi D}{R}\right)I_2$, towards right

28. (a) 2.77 A (b) 0.0184 m

29. 2.0 A toward bottom of page

30. (a) $\dfrac{-\mu_0 qv_0 I}{4R}\,\hat{\mathbf{k}}$ (b) $\mathbf{F}_1 = \mathbf{F}_2 = 2B_0IR\hat{\mathbf{i}}$, $\mathbf{F} = 4B_0IR\hat{\mathbf{i}}$

31. (a) $\dfrac{\mu_0 in^2\sin\left(\dfrac{\pi}{n}\right)\tan\left(\dfrac{\pi}{n}\right)}{2\pi^2 r}$ (b) $\dfrac{\mu_0 i}{2r}$

32. (a) $\dfrac{\mu_0 I}{\pi r}\left(\dfrac{2r^2 - a^2}{4r^2 - a^2}\right)$ to the left (b) $\dfrac{\mu_0 I}{\pi r}\left(\dfrac{2r^2 + a^2}{4r^2 + a^2}\right)$ towards the top of the page

33. $B_P = 0$, $B_Q = \mu_0\lambda$

34. $\dfrac{\mu_0 I_1 I_2 L}{2\pi a}$, zero

36. (a) π (b) π (c) $\dfrac{\pi}{6}$

37 v_0, $\dfrac{mv_0}{qE_0}$

38. $\left(\dfrac{2\pi m v_0 \sin\theta}{Bq},\ 0,\ 0\right)$

39. $(0.04\,\hat{\mathbf{j}} - 0.07\,\hat{\mathbf{k}})$ A-m^2

40. 9.98 N·m, clockwise as seen looking down from above.

41. 20.0 μT toward the bottom of the square

42. (a) $\dfrac{\mu_0 b\, r_1^2}{3}$ (b) $\dfrac{\mu_0 b\, R^3}{3r_2}$

LEVEL 2

Objective Questions

1.(a) **2.**(c) **3.**(c) **4.**(c) **5.**(d) **6.**(d) **7.**(b) **8.**(d) **9.**(b) **10.**(c)
11.(d) **12.**(a) **13.**(b) **14.**(a) **15.**(c) **16.**(c) **17.**(d) **18.**(c) **19.**(c) **20.**(d)
21.(b) **22.**(c) **23.**(d) **24.**(d) **25.**(a) **26.**(b) **27.**(a)

More than One Correct Options

1.(a,c) **2.**(a,c,d) **3.**(a,b,d) **4.**(b,c,d) **5.**(b,d) **6.**(a,b,c,d) **7.**(a,b,c,d) **8.**(a,c,d) **9.**(a,c,d) **10.**(a,b,c)

Match the Columns

1.	(a) → r	(b) → q	(c) → p	(d) → r
2.	(a) → r	(b) → s	(c) → q	(d) → p
3.	(a) → p,s	(b) → p,q	(c) → p,r	(d) → p,s
4.	(a) → q	(b) → r	(c) → s	(d) → s
5.	(a) → q	(b) → r	(c) → q	(d) → r
6.	(a) → s	(b) → s	(c) → q	(d) → p

Subjective Questions

1. $\mathbf{F} = \dfrac{\mu_0 I^2}{\pi}\left[\dfrac{1}{2} - \dfrac{1}{\sqrt{3}} \ln\left(\dfrac{2+\sqrt{3}}{2}\right)\right](\hat{\mathbf{i}})$

2. $M = \dfrac{neh}{4\pi\, m}, B = \dfrac{\mu_0 \pi m^2 e^7}{8\,\varepsilon_0^3 h^5 n^5}$

3. 2.7×10^{-4} T

4. (0.2 m, 0.2 m, 0.2 m)

5. 2×10^{-5} N

6. (a) $y = \dfrac{Ea^2}{4V}$ (b) $p = \dfrac{\pi Ea}{B}\sqrt{\dfrac{2m}{qV}}$

7. 2.0 m/s, 3.46 mT

8. (a) $I_0 L^2 B$ (b) $\dfrac{3}{4}\dfrac{I_0 B}{M}(\Delta t)^2$

9. $\omega_{max} = \dfrac{DT_0}{BQR^2}$

10. $B = \dfrac{\mu_0}{2\pi}\dfrac{i}{\omega}\ln\left(\dfrac{d+\omega}{d}\right)$ (upwards)

11. $2\tan^{-1}\left(\dfrac{BqR}{mv}\right)$

12. (a) 0.0442 N-m, clockwise, yes (b) stretched (c) 7.8×10^{-3} J

13. (a) P to Q (b) $IbB_0(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$ (c) $\dfrac{mg}{6bB_0}$

27

Electromagnetic Induction

Chapter Contents

27.1 Introduction

Almost every modern device has electric circuits at its heart. We learned in the chapter of current electricity that an electromagnetic force (emf) is required for a current to flow in a circuit. But for most of the electric devices used in industry the source of emf is not a battery but an electrical generating station. In these stations other forms of energy are converted into electric energy. For example, in a hydroelectric plant gravitational potential energy is converted into electric energy. Similarly, in a nuclear plant nuclear energy is converted into electric energy.

But how this conversion is done? Or what is the physics behind this? The branch of physics, known as **electromagnetic induction** gives the answer to all these queries. If the **magnetic flux (ϕ_B)** through a circuit changes, an emf and a current are induced in the circuit. Electromagnetic induction was discovered in 1830. The central principle of electromagnetic induction is **Faraday's law**. This law relates induced emf to change in magnetic flux in any loop, including a closed circuit. We will also discuss Lenz's law, which helps us to predict the directions of induced emf and current.

27.2 Magnetic Field Lines and Magnetic Flux

Let us first discuss the concept of magnetic field lines and magnetic flux. We can represent any magnetic field by **magnetic field lines**. Unlike the electric lines of force, it is wrong to call them magnetic lines of force, because they do not point in the direction of the force on a charge. The force on a moving charged particle is always perpendicular to the magnetic field (or magnetic field lines) at the particle's position.

The idea of magnetic field lines is same as for the electric field lines as discussed in the chapter of electrostatics. The magnetic field at any point is tangential to the field line at that point. Where the field lines are close, the magnitude of field is large, where the field lines are far apart, the field magnitude is small. Also, because the direction of **B** at each point is unique, field lines never intersect. Unlike the electric field lines, magnetic lines form a closed loop.

Magnetic Flux

The flux associated with a magnetic field is defined in a similar manner to that used to define electric flux. Consider an element of area dS on an arbitrary shaped surface as shown in figure. If the magnetic field at this element is **B**, the magnetic flux through the element is

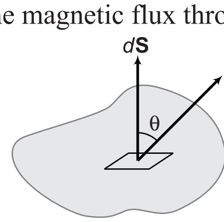


Fig. 27.1

$$\boxed{d\phi_B = \mathbf{B}\cdot d\mathbf{S} = BdS\cos\theta}$$

Here, $d\mathbf{S}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dS and θ is the angle between **B** and $d\mathbf{S}$ at that element. In general, $d\phi_B$ varies from element to element. The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$\therefore$
$$\boxed{\phi_B = \int BdS\cos\theta = \int \mathbf{B}\cdot d\mathbf{S}}$$

Note down the following points regarding the magnetic flux :

(i) Magnetic flux is a scalar quantity (dot product of two vector quantities is a scalar quantity)

(ii) The SI unit of magnetic flux is tesla-metre^2 (1 T-m^2). This unit is called **weber** (1Wb).

$$\boxed{1\ \text{Wb} = 1\ \text{T-m}^2 = 1\ \text{N-m/A}}$$

Thus, unit of magnetic field is also weber/m^2 (1Wb/m^2).

or $$\boxed{1\ \text{T} = 1\ \text{Wb/m}^2}$$

(iii) In the special case in which **B** is uniform over a plane surface with total area S,

$$\phi_B = BS\cos\theta$$

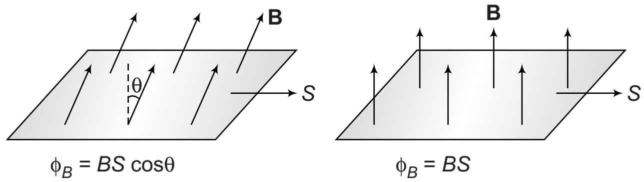


Fig. 27.2

If **B** is perpendicular to the surface, then $\cos\theta = 1$ and

$$\phi_B = BS$$

Gauss's Law for Magnetism

In Gauss's law, the total electric flux through a closed surface is proportional to the total electric charge enclosed by the surface. For example, if a closed surface encloses an electric dipole, the total electric flux is zero because the total charge is zero.

By analogy, if there were such as thing as a single magnetic charge (magnetic monopole), the total magnetic flux through a closed surface would be proportional to the total magnetic charge enclosed. But as no magnetic monopole has ever been observed, we conclude that **the total magnetic flux through a closed surface is zero**.

$$\boxed{\oint \mathbf{B}\cdot d\mathbf{S} = 0}$$

Unlike electric field lines that begin and end on electric charges, magnetic field lines never have end points. Such a point would otherwise indicate the existence of a monopole. For a closed surface, the vector area element $d\mathbf{S}$ always points out of the surface. However, for an open surface we choose one of the possible sides of the surface to be the positive and use that choice consistently.

27.3 Faraday's Law

This law states that, "the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop."

$$\boxed{e = -\frac{d\phi_B}{dt}}$$

If a circuit is a coil consisting of N loops all of the same area and if ϕ_B is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by the expression,

$$\boxed{e = -N\frac{d\phi_B}{dt}}$$

The negative sign in the above equations is of important physical significance, which we will discuss in article. 27.4.

Note down the following points regarding the Faraday's law:

(i) As we have seen, induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by

$$\phi = BS\cos\theta$$

This, flux can be changed in several ways:

(a) The magnitude of **B** can change with time. In the problems if magnetic field is given a function of time, it implies that the magnetic field is changing. Thus,

$$B = B(t)$$

(b) The current producing the magnetic field can change with time. For this, the current can be given as a function of time. Hence,

$$i = i(t)$$

(c) The area of the loop inside the magnetic field can change with time.This can be done by pulling a loop inside (or outside) a magnetic field.

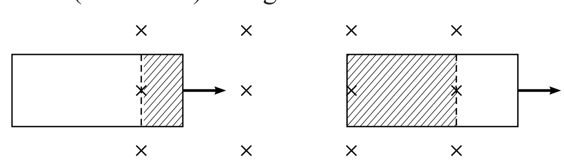


Fig. 27.3

(d) The angle θ between **B** and the normal to the loop (or **s**) can change with time.

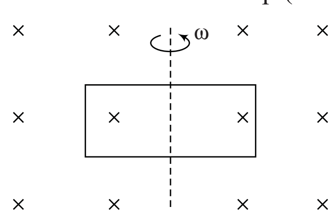


Fig. 27.4

This can be done by rotating a loop in a magnetic field.

(e) Any combination of the above can occur.

(ii) When the magnetic flux passing through a loop is changed, an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, means flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = idt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt we can write the average values as,

$$\boxed{e = -\frac{\Delta\phi_B}{\Delta t}, \quad i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right) \text{ and } \Delta q = \frac{1}{R}(-\Delta\phi_B)}$$

From these equations, we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken in it. This can be explained by the following example.

Example 27.1 *A square loop ACDE of area $20\ cm^2$ and resistance $5\ \Omega$ is rotated in a magnetic field $\mathbf{B} = 2T$ through $180°$, (a) in $0.01\ s$ and (b) in $0.02\ s$*
Find the magnitudes of average values of e, i and Δq in both the cases.

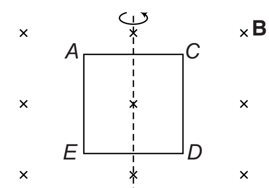


Fig. 27.5

Solution Let us take the area vector **S** perpendicular to plane of loop inwards. So initially, $\mathbf{S} \uparrow\uparrow \mathbf{B}$ and when it is rotated by 180°, $\mathbf{S} \uparrow\downarrow \mathbf{B}$. Hence, initial flux passing through the loop,

$$\phi_i = BS\cos 0° = (2)(20 \times 10^{-4})(1) = 4.0 \times 10^{-3}\ \text{Wb}$$

Flux passing through the loop when it is rotated by 180°,

$$\phi_f = BS\cos 180° = (2)(20 \times 10^{-4})(-1) = -4.0 \times 10^{-3}\ \text{Wb}$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8.0 \times 10^{-3}\ \text{Wb}$$

(a) Given, $\Delta t = 0.01\text{s}, \quad R = 5\ \Omega$

$$\therefore \qquad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8.0 \times 10^{-3}}{0.01} = 0.8\ \text{V}$$

$$i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16\ \text{A}$$

and $$\Delta q = i\Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}\ \text{C}$$

(b) $\Delta t = 0.02\text{s}$

$$\therefore \qquad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8.0 \times 10^{-3}}{0.02} = 0.4\ \text{V}$$

$$i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08\ \text{A}$$

and $$\Delta q = i\Delta t = (0.08)(0.02) = 1.6 \times 10^{-3}\ \text{C}$$

Note *Time interval Δt in part (b) is two times the time interval in part (a), so e and i are half while Δq is same.*

Example 27.2 *A coil consists of 200 turns of wire having a total resistance of 2.0 Ω. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.80 s, what is the magnitude of induced emf and current in the coil while the field is changing?*

Solution From the Faraday's law,

$$\text{Induced emf, } |e| = \frac{N\Delta\phi}{\Delta t} = (NS)\frac{\Delta B}{\Delta t}$$

$$= \frac{(200)(18\times 10^{-2})^2\,(0.5-0)}{0.8}$$

$$= 4.05 \text{ V} \qquad \textbf{Ans.}$$

$$\text{Induced current, } i = \frac{|e|}{R} = \frac{4.05}{2} \approx 2.0 \text{ A} \qquad \textbf{Ans.}$$

Example 27.3 *The magnetic flux passing through a metal ring varies with time t as : $\phi_B = 3(at^3 - bt^2)\,T\text{-}m^2$ with $a = 2.00\ s^{-3}$ and $b = 6.00\ s^{-2}$. The resistance of the ring is 3.0 Ω. Determine the maximum current induced in the ring during the interval from t = 0 to t = 2.0 s.*

Solution Given, $\phi_B = 3(at^3 - bt^2)$

$$\therefore \qquad |e| = \left|\frac{d\phi_B}{dt}\right| = 9at^2 - 6bt$$

$$\therefore \text{ Induced current, } \qquad i = \frac{|e|}{R} = \frac{9at^2 - 6bt}{3} = 3at^2 - 2bt$$

For current to be maximum,

$$\frac{di}{dt} = 0$$

$$\therefore \qquad 6at - 2b = 0$$

$$\text{or} \qquad t = \frac{b}{3a}$$

i.e. at $t = \dfrac{b}{3a}$, current is maximum. This maximum current is

$$i_{\max} = 3a\left(\frac{b}{3a}\right)^2 - 2b\left(\frac{b}{3a}\right)$$

$$= \frac{b^2}{3a} - \frac{2b^2}{3a} = \frac{b^2}{3a}$$

Substituting the given values of a and b, we have

$$i_{\max} = \frac{(6)^2}{3(2)} = 6.0 \text{ A} \qquad \textbf{Ans.}$$

27.4 Lenz's Law

The negative sign in Faraday's equations of electromagnetic induction describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with the help of Lenz's law. This law states that:

"The direction of any magnetic induction effect is such as to oppose the cause of the effect."

For different types of problems, Lenz's law has been further subdivided into following concepts.

1. **Attraction and repulsion concept** If magnetic flux is changed by bringing a magnet and a loop (or solenoid etc.) closer to each other then direction of induced current is so produced, that the magnetic field produced by it always repels the two. Similarly, if they are moved away from each other then they are attracted towards each other. Following two examples will illustrate this.

Example 27.4 *A bar magnet is freely falling along the axis of a circular loop as shown in figure. State whether its acceleration a is equal to, greater than or less than the acceleration due to gravity g.*

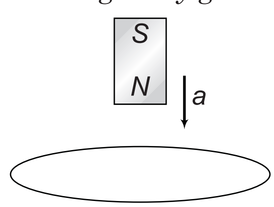


Fig. 27.6

Solution $a < g$. Because according to Lenz's law, whatever may be the direction of induced current, it will oppose the cause.

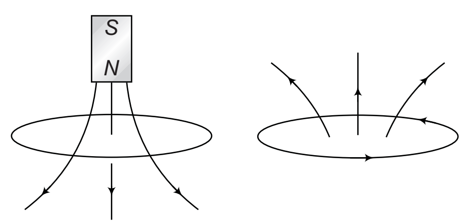


Fig. 27.7

Here, the cause is, the free fall of magnet and so the induced current will oppose it and the acceleration of magnet will be less than the acceleration due to gravity g. This can also be explained in a different manner. When the magnet falls downwards with its north pole downwards.

The magnetic field lines passing through the coil in the downward direction increase. Since, the induced current opposes this, the upper side of the coil will become north pole, so that field lines of coil's magnetic field are upwards. Now, like poles repel each other. Hence, $a < g$.

Example 27.5 *A bar magnet is brought near a solenoid as shown in figure. Will the solenoid attract or repel the magnet?*

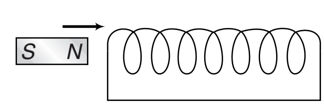


Fig. 27.8

Solution When the magnet is brought near the solenoid, according to Lenz's law, both repel each other. On the other hand, if the magnet is moved away from the solenoid, it attracts the magnet. When the magnet is brought near the solenoid, the nearer side becomes the same pole and when it is moved away it becomes the opposite pole as shown in figure.

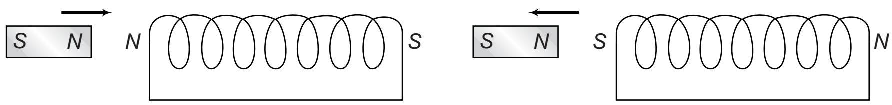


Fig. 27.9

It can also be explained by increasing or decreasing field lines as discussed in example 27.4.

2. **Cross or dot magnetic field increasing or decreasing concept**

If cross magnetic field passing through a loop increases then induced current will produce dot magnetic field. Similarly, if dot magnetic field passing through a loop decreases then dot magnetic field is produced by the induced current. Let us take some examples in support of it.

Example 27.6 *A circular loop is placed in magnetic field $B = 2t$. Find the direction of induced current produced in the loop.*

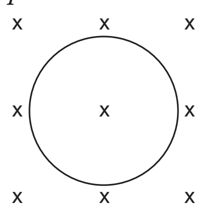


Fig. 27.10

Solution $B = 2t$, means $\otimes$ magnetic field (or we can also say cross magnetic flux) passing through the loop is increasing. So, induced current will produce dot magnetic field. To produce $\odot$ magnetic field, induced current from our side should be anti-clockwise.

Example 27.7 *A rectangular loop is placed to the left of large current carrying straight wire as shown in figure. Current varies with time as $I = 2t$. Find direction of induced current I_{in} in the square loop.*

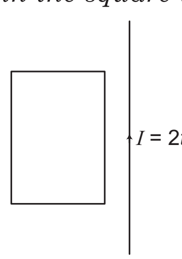


Fig. 27.11

Solution Current I will produce $\odot$ magnetic field passing through the loop. Current I is increasing, so dot magnetic field will also increase. Therefore, induced current should produce cross magnetic field. For producing cross magnetic field in the loop, induced current from our side should be clockwise.

3. **Situations where flux passing through the loop is always zero or change in flux is zero.**

Example 27.8 *A current carrying straight wire passes inside a triangular loop as shown in Fig. 27.12. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop if current in the wire is increased.*

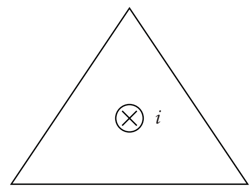


Fig. 27.12

Solution Magnetic field lines round the current carrying wire are as shown in Fig.27.13. Since, the lines are tangential to the loop ($\theta = 90°$) the flux passing through the loop is always zero, whether the current is increased or decreased. Hence, change in flux is also zero. Therefore, induced current in the loop will be zero.

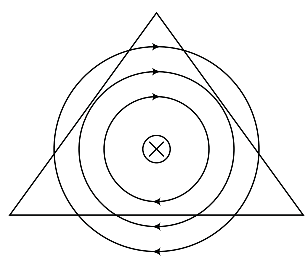


Fig. 27.13

Example 27.9 *A rectangular loop is placed adjacent to a current carrying straight wire as shown in figure. If the loop is rotated about an axis passing through one of its sides, find the direction of induced current in the loop.*

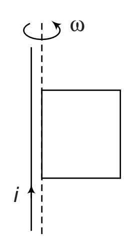


Fig. 27.14

Solution Magnetic field lines around the straight wire are circular. So, same magnetic lines will pass through loop under all conditions.

$$\Delta\phi = 0$$

$$\Rightarrow \qquad \text{emf} = 0$$

$$\Rightarrow \qquad i = 0$$

4. Attraction or repulsion between two loops facing each other if current in one loop is changed

Example 27.10 *Two loops are facing each other as shown in Fig. 27.15. State whether the loops will attract each other or repel each other if current I_1 is increased.*

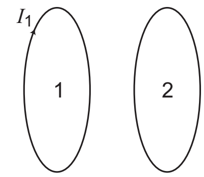


Fig. 27.15

Solution If current I_1 is increased then induced current in loop-2 (say I_2) will be in opposite direction. Now, two wires having currents in opposite directions repel each other. So, the loops will repel each other.

INTRODUCTORY EXERCISE 27.1

1. Figure shows a conducting loop placed near a long straight wire carrying a current i as shown. If the current increases continuously, find the direction of the induced current in the loop.

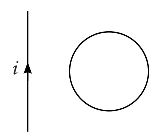


Fig. 27.16

2. A metallic loop is placed in a non-uniform steady magnetic field. Will an emf be induced in the loop?
3. Write the dimensions of $\frac{d\phi_B}{dt}$.
4. A triangular loop is placed in a dot $\odot$ magnetic field as shown in figure. Find the direction of induced current in the loop if magnetic field is increasing.

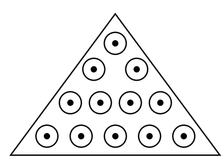


Fig. 27.17

5. Two circular loops lie side by side in the same plane. One is connected to a source that supplies an increasing current, the other is a simple closed ring. Is the induced current in the ring is in the same direction as that in the loop connected to the source or opposite? What if the current in the first loop is decreasing?
6. A wire in the form of a circular loop of radius 10 cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s, find the average induced emf in the loop.
7. A closed coil consists of 500 turns has area $4\ \text{cm}^2$ and a resistance of $50\ \Omega$. The coil is kept with its plane perpendicular to a uniform magnetic field of $0.2\ \text{Wb/m}^2$. Calculate the amount of charge flowing through the coil if it is rotated through 180°.
8. The magnetic field in a certain region is given by $\mathbf{B} = (4.0\ \hat{\mathbf{i}} - 1.8\ \hat{\mathbf{k}}) \times 10^{-3}$ T. How much flux passes through a $5.0\ \text{cm}^2$ area loop in this region if the loop lies flat on the xy-plane?

27.5 Motional Electromotive Force

Till now, we have considered the cases in which an emf is induced in a stationary circuit placed in a magnetic field, when the field changes with time. In this section, we describe what is called **motional emf**, which is the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length l shown in figure is moving through a uniform magnetic field directed into the page. For simplicity we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. The electrons in the conductor experience a force

$$\mathbf{F}_m = -e(\mathbf{v} \times \mathbf{B})$$

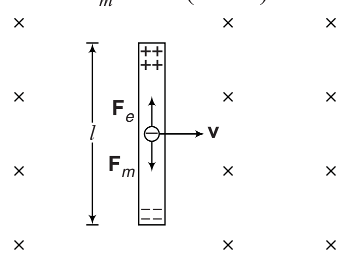


Fig. 27.18

Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced inside the conductor. The charges accumulate at both ends untill the downward magnetic force evB is balanced by the upward electric force eE. At this point, electrons stop moving. The condition for equilibrium requires that,

$$eE = evB \quad \text{or} \quad E = vB$$

The electric field produced in the conductor (once the electrons stop moving and E is constant) is related to the potential difference across the ends of the conductor according to the relationship,

$$\Delta V = El = Blv$$

$$\boxed{\Delta V = Blv}$$

where the upper end is at a higher electric potential than the lower end. Thus,

"a potential difference is maintained between the ends of a straight conductor as long as the conductor continues to move through the uniform magnetic field."

Now, suppose the moving rod slides along a stationary U-shaped conductor forming a complete circuit. We call this a **motional electromagnetic force** denoted by e, we can write

$$\boxed{e = Bvl}$$

If R is the resistance of the circuit, then current in the circuit is

$$\boxed{i = \frac{e}{R} = \frac{Bvl}{R}}$$

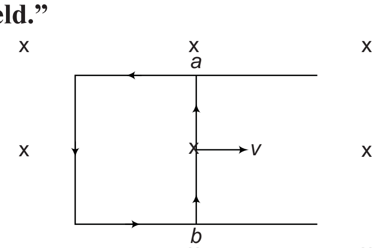


Fig. 27.19

$V_a > V_b$. Therefore, direction of current in the loop is anti-clockwise as shown in figure.

Extra Points to Remember

- The direction of motional emf or current can be given by right hand rule.

  Stretch your right hand.

  The stretched fingers point in the direction of magnetic field. Thumb is along the velocity of conductor. The upper side of the palm is at higher potential and lower side on lower potential. If the circuit is closed, the induced current within the conductor is along perpendicular to palm upwards.

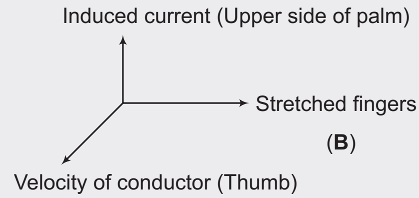


Fig. 27.20

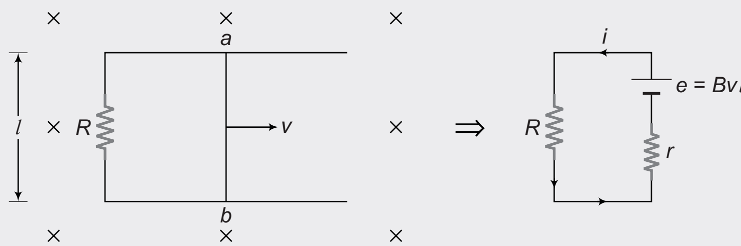
- 

Fig. 27.21

  In the Fig. 27.20, we can replace the moving rod *ab* by a battery of emf *Bvl* with the positive terminal at *a* and the negative terminal at *b*. The resistance *r* of the rod *ab* may be treated as the internal resistance of the battery. Hence, the current in the circuit is

$$i = \frac{e}{R + r} \quad \text{or} \quad \boxed{i = \frac{Bvl}{R + r}}$$

- **Induction and energy transfers** In the Fig. 27.21, if you move the conductor *ab* with a constant velocity *v*, the current in the circuit is

$$i = \frac{Bvl}{R} \qquad (r = 0)$$

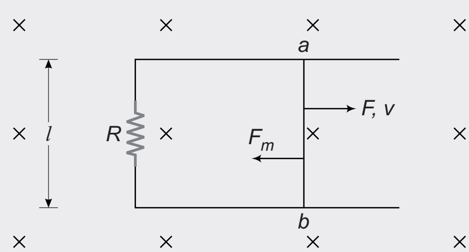


Fig. 27.22

A magnetic force $F_m = ilB = \frac{B^2 l^2 v}{R}$ acts on the conductor in opposite direction of velocity. So, to move the conductor with a constant velocity *v* an equal and opposite force *F* has to be applied in the conductor. Thus,

$$F = F_m = \frac{B^2 l^2 v}{R}$$

The rate at which work is done by the applied force is

$$P_{\text{applied}} = Fv = \frac{B^2 l^2 v^2}{R}$$

and the rate at which energy is dissipated in the circuit is

$$P_{\text{dissipated}} = i^2R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2l^2v^2}{R}$$

This is just equal to the rate at which work is done by the applied force.

- Motional emf is not a different kind of induced emf, it is exactly the induced emf described by Faraday's law, in the case in which there is a conductor moving in a magnetic field. Equation, $e = -\frac{d\phi_B}{dt}$ is best applied to problems in which there is a changing flux through a closed loop while $e = Bvl$ is applied to problems in which a conductor moves through a magnetic field. Note that, if a conductor is moving in a magnetic field but circuit is not closed, then only PD will be asked between two points of the conductor. If the circuit is closed, then current will be asked in the circuit. Now, let us see how these two are similar.

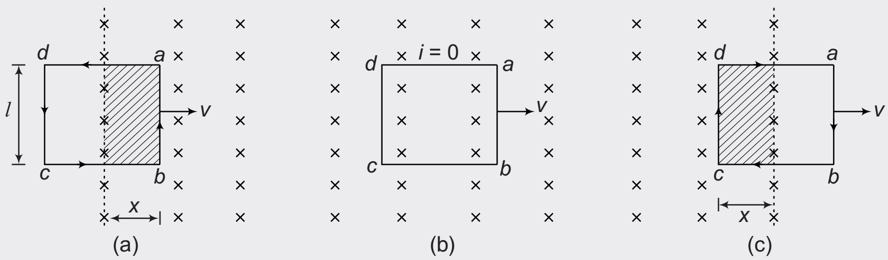


Fig. 27.23

Refer figure (a) A loop *abcd* enters a uniform magnetic field *B* at constant speed *v*.

Using Faraday's equation,

$$|e| = \left|-\frac{d\phi_B}{dt}\right| = \frac{d(BS)}{dt} = \frac{d(Blx)}{dt} = Bl\frac{dx}{dt} = Blv$$

For the direction of current, we can use Lenz's law. As the loop enters the field, ⊗ magnetic field passing through the loop increases, hence, induced current should produce ⊙ magnetic field or current in the loop is anti-clockwise. From the theory of motional emf, $e = Bvl$ and using right hand rule also, current in the circuit is anti-clockwise. Thus, we see that $e = -\frac{d\phi_B}{dt}$ and $e = Bvl$ give the same result. In the similar manner, we can show that current in the loop in figure (b) is zero and in figure (c) it is clockwise.

- We can generalize the concept of motional emf for a conductor with any shape moving in any magnetic field uniform or not. For an element $d\mathbf{l}$ of conductor the contribution de to the emf is the magnitude dl multiplied by the component of $\mathbf{v} \times \mathbf{B}$ parallel to $d\mathbf{l}$, that is

$$de = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

For any two points *a* and *b* the motional emf in the direction from *b* to *a* is,

$$\boxed{e = \int_b^a (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}}$$

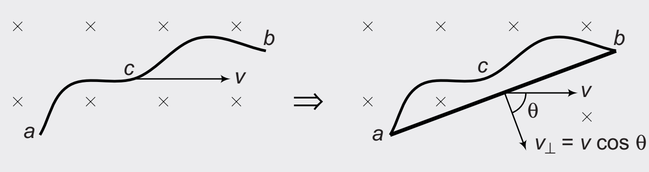


Fig. 27.24

In general, we can say that motional emf in wire *acb* in a uniform magnetic field is the motional emf in an imaginary straight wire *ab*. Thus,

$$e_{acb} = e_{ab} = (\text{length of } ab)\,(v_{\perp})\,(B)$$

Here, $v_{\perp}$ is the component of velocity perpendicular to both **B** and *ab*.

From right hand rule we can see that *b* is at higher potential and *a* at lower potential.

Hence, $$V_{ba} = V_b - V_a = (ab)\,(v\cos\theta)\,(B)$$

- **Motional emf induced in a rotating bar :** A conducting rod of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation as shown in figure. Consider a segment of rod of length dr at a distance r from O. This segment has a velocity,

$$v = r\omega$$

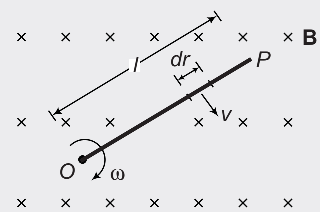


Fig. 27.25

The induced emf in this segment is $de = Bv dr = B(r\omega)\,dr$

Summing the emfs induced across all segments, which are in series, gives the total emf across the rod.

$\therefore$ $$e = \int_0^l de = \int_0^l Br\omega dr = \frac{B\omega l^2}{2}$$

$\therefore$ $$\boxed{e = \frac{B\omega l^2}{2}}$$

From right hand rule we can see that P is at higher potential than O. Thus,

$$V_P - V_O = \frac{B\omega l^2}{2}$$

- Note that in the problems of electromagnetic induction whenever you see a conductor moving in a magnetic field use the motional approach. It is easier than the other approach. But, if the conductor (or loop) is stationary, you have no choice. Use $e = -\dfrac{d\phi_B}{dt}$.

- Now onwards, the following integrations will be used very frequently.

If $$\int_0^x \frac{dx}{a - bx} = \int_0^t c\,dt$$

then, $$x = \frac{a}{b}(1 - e^{-bct})$$

and if $$\int_{x_0}^x \frac{dx}{a - bx} = \int_0^t c\,dt$$

then, $$x = \frac{a}{b} - \left(\frac{a}{b} - x_0\right)e^{-bct}$$

Here a, b and c are positive constants.

Note *In an electrical circuit, a moving or rotating wire may be assumed as a battery of emf Bvl or $\frac{B\omega l^2}{2}$ and then it can be solved with the help of Kirchhoff's laws. The following example will illustrate this concept.*

Example 27.11 *Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the, plane of the rails. Determine the current in the 5.0 Ω resistor.*

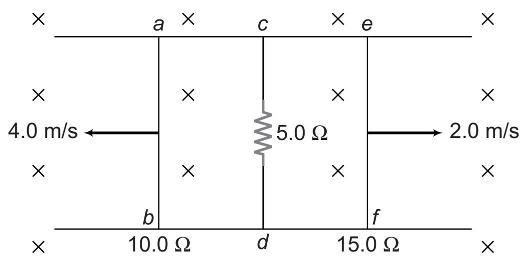


Fig. 27.26

HOW TO PROCEED *Here, two conductors are moving in a uniform magnetic field. So, we will use the motional approach. The rod ab will act as a source of emf,*

$$e_1 = Bvl = (0.01)(4.0)(0.1) = 4 \times 10^{-3}\ V$$

and internal resistance $r_1 = 10.0\ \Omega$
Similarly, rod ef will also act as a source of emf,

$$e_2 = (0.01)(2.0)(0.1) = 2.0 \times 10^{-3}\ V$$

and internal resistance $r_2 = 15.0\ \Omega$.
From right hand rule we can see that, $V_b > V_a$ *and* $V_e > V_f$
Now, either by applying Kirchhoff's laws or applying principle of superposition (discussed in the chapter of current electricity) we can find current through 5.0 Ω resistor. We will here use the superposition principle. You solve it by using Kirchhoff's laws.

Solution In the figures $R = 5.0\,\Omega$, $r_1 = 10\,\Omega$, $r_2 = 15\,\Omega$, $e_1 = 4 \times 10^{-3}$ V and $e_2 = 2 \times 10^{-3}$ V

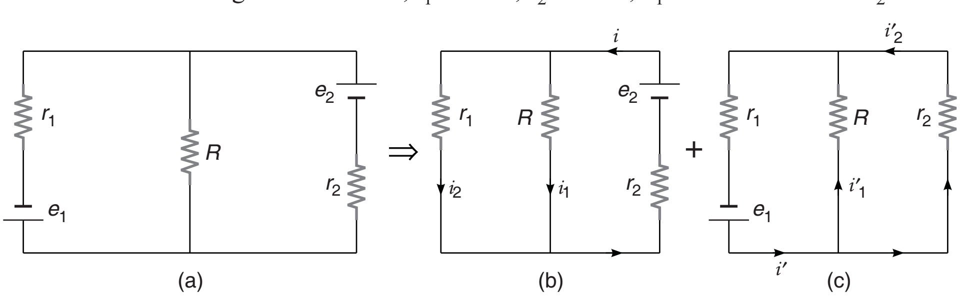


Fig. 27.27

Refer figure (b) Net resistance of the circuit $= r_2 + \dfrac{Rr_1}{R + r_1} = 15 + \dfrac{10 \times 5}{10 + 5} = \dfrac{55}{3}\,\Omega$

$\therefore$ Current, $i = \dfrac{e_2}{\text{Net resistance}} = \dfrac{2 \times 10^{-3}}{55/3} = \dfrac{6}{55} \times 10^{-3}$ A

$\therefore$ Current through R,

$$i_1 = \left(\frac{r_1}{R + r_1}\right) i = \left(\frac{10}{10+5}\right)\left(\frac{6}{55} \times 10^{-3}\right) \text{A}$$

$$= \frac{4}{55} \times 10^{-3}\ \text{A} = \frac{4}{55}\ \text{mA}$$

Refer figure (c) Net resistance of the circuit $= r_1 + \dfrac{Rr_2}{R + r_2}$

$$= 10 + \frac{5 \times 15}{5 + 15} = \frac{55}{4}\,\Omega$$

$\therefore$ Current, $i' = \dfrac{e_1}{\text{Net resistance}}$

$$= \frac{4 \times 10^{-3}}{55/4} = \frac{16}{55} \times 10^{-3}\ \text{A}$$

$\therefore$ Current through R, $i_1' = \left(\dfrac{r_2}{R + r_2}\right) i' = \left(\dfrac{15}{15+5}\right)\left(\dfrac{16}{55}\right) \times 10^{-3}$ A

$$= \frac{12}{55}\ \text{mA}$$

From superposition principle net current through $5.0\,\Omega$ resistor is

$$i_1' - i_1 = \frac{8}{55}\ \text{mA from } d \text{ to } c$$ **Ans.**

Example 27.12 *Figure shows the top view of a rod that can slide without friction. The resistor is* $6.0\,\Omega$ *and a* 2.5 *T magnetic field is directed perpendicularly downward into the paper. Let* $l = 1.20$ *m.*

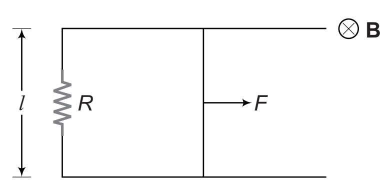


Fig. 27.28

(a) Calculate the force F required to move the rod to the right at a constant speed of 2.0 *m/s.*

(b) At what rate is energy delivered to the resistor?

(c) Show that this rate is equal to the rate of work done by the applied force.

Solution The motional emf in the rod, $e = Bvl$ or $e = (2.5)\,(2.0)\,(1.2)\ \text{V} = 6.0\ \text{V}$

The current in the circuit, $$i = \frac{e}{R} = \frac{6.0}{6.0} = 1.0\ \text{A}$$

(a) The magnitude of force F required will be equal to the magnetic force acting on the rod, which opposes the motion.

$\therefore$ $$F = F_m = ilB \quad \text{or} \quad F = (1.0)\,(1.2)\,(2.5)\ \text{N} = 3\ \text{N}$$ **Ans.**

(b) Rate by which energy is delivered to the resistor is

$$P_1 = i^2R = (1)^2\ (6.0) = 6\ \text{W}$$ **Ans.**

(c) The rate by which work is done by the applied force is

$$P_2 = F \cdot v = (3)\,(2.0) = 6\ \text{W}$$

$$P_1 = P_2$$ **Hence proved.**

INTRODUCTORY EXERCISE 27.2

1. A horizontal wire 0.8 m long is falling at a speed of 5 m/s perpendicular to a uniform magnetic field of 1.1 T, which is directed from east to west. Calculate the magnitude of the induced emf. Is the north or south end of the wire positive?
2. As shown in figure, a metal rod completes the circuit. The circuit area is perpendicular to a magnetic field with $B = 0.15$ T. If the resistance of the total circuit is 3 Ω, how large a force is needed to move the rod as indicated with a constant speed of 2 m/s?

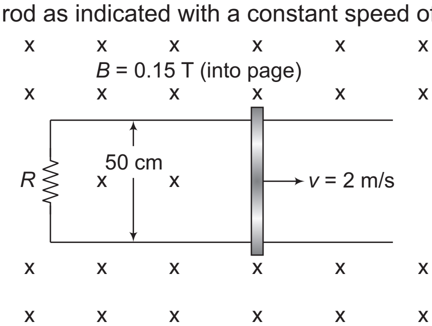


Fig. 27.29

3. A rod of length $3l$ is rotated with an angular velocity ω as shown in figure. The uniform magnetic field B is into the paper. Find
   (a) $V_A - V_C$ (b) $V_A - V_D$

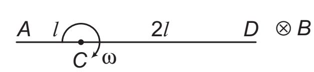


Fig. 27.30

4. As the bar shown in figure moves in a direction perpendicular to the field, is an external force required to keep it moving with constant speed.

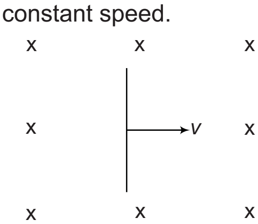


Fig. 27.31

27.6 Self-inductance and Inductors

Consider a single isolated circuit. When a current is present in the circuit, it sets up a magnetic field that causes a magnetic flux through the same circuit. This flux changes as the current in the circuit is changed. According to Faraday's law any change in flux in a circuit produces an induced emf in it. Such an emf is called a **self-induced emf**. The name is so called because the source of this induced emf is the change of current in the same circuit.

According to Lenz's law the self-induced emf always opposes the change in the current that caused the emf and so tends to make it more difficult for variations in current to occur. We will here like to define a term **self-inductance *L*** of a circuit which is of great importance in our proceeding discussions. It can be defined in the following two ways :

First Definition Suppose a circuit includes a coil with N turns of wire. It carries a current i. The total flux ($N\phi_B$) linked with the coil is directly proportional to the current (i) in the coil, i.e.

$$N\phi_B \propto i$$

When the proportionality sign is removed a constant L comes in picture, which depends on the dimensions and number of turns in the coil. This constant is called self-inductance. Thus,

$$N\phi_B = Li \qquad \text{or} \qquad \boxed{L = \frac{N\phi_B}{i}}$$

From here we can define self-inductance (L) of any circuit as, **the total flux per unit current**. The SI unit of self-inductance is henry (1H).

Second Definition If a current i is passed in a circuit and it is changed with a rate di/dt, the induced emf e produced in the circuit is directly proportional to the rate of change of current. Thus,

$$e \propto \frac{di}{dt}$$

When the proportionality constant is removed, the same constant L again comes here.

Hence, $$e = -L\frac{di}{dt}$$

The minus sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit. From the above equation,

$$\boxed{L = \left|\frac{-e}{di/dt}\right|}$$

This equation states that, **the self-inductance of a circuit is the magnitude of self induced emf per unit rate of change of current**.

A circuit or part of a circuit, that is designed to have a particular inductance is called an **inductor**. The usual symbol for an inductor is

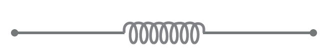


Fig. 27.32

Thus, an inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid etc.

Significance of Self-inductance and Inductor

Like capacitors and resistors, inductors are among the circuit elements of modern electronics. Their purpose is to oppose any variations in the current through the circuit. In a DC circuit, an inductor helps to maintain a steady state current despite fluctuations in the applied emf. In an AC circuit, an inductor tends to suppress variations of the current that are more rapid than desired. An inductor plays a dormant role in a circuit so far as current is constant. It becomes active when current changes in the circuit. Every inductor has some self-inductance which depends on the size, shape and the number of turns etc. For N turns close together, it is always proportional to N^2. It also depends on the magnetic properties of the material enclosed by the circuit. When the current passing through it is changed, an emf of magnitude $L\,di/dt$ is induced across it. Later in this article, we will explore the method of finding the self-inductance of an inductor.

Potential Difference Across an Inductor

We can find the polarities of self-induced emf across an inductor from Lenz's law.

The induced emf is produced whenever there is a change in the current in the inductor. This emf always acts to oppose this change. Figure shows three cases. Assume that the inductor has negligible resistance, so the PD, $V_{ab} = V_a - V_b$ between the inductor terminals a and b is equal in magnitude to the self-induced emf.

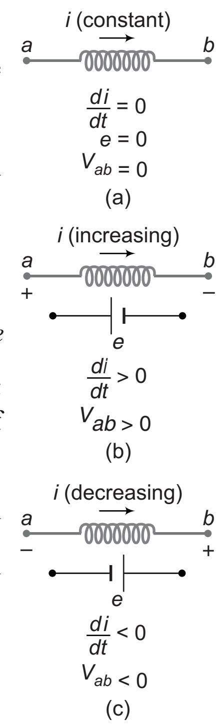


Fig. 27.33

Refer figure (a) The current is constant, and there is no self-induced emf. Hence, $V_{ab} = 0$

Refer figure (b) The current is increasing, so $\dfrac{di}{dt}$ is positive. The induced emf e must oppose the increasing current, so it must be in the sense from b to a, a becomes the higher potential terminal and V_{ab} is positive. The direction of the emf is analogous to a battery with a as its positive terminal.

Refer figure (c) The current is decreasing and $\dfrac{di}{dt}$ is negative. The self-induced emf e opposes this decrease and V_{ab} is negative. This is analogous to a battery with b as its positive terminal.

In each case, we can write the PD, V_{ab} as

$$\boxed{V_{ab} = -e = L\frac{di}{dt}}$$

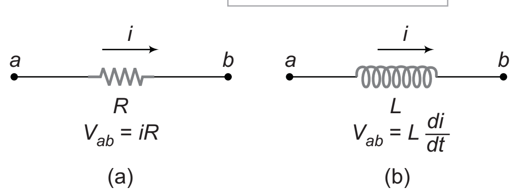


Fig. 27.34

The circuit's behaviour of an inductor is quite different from that of a resistor. While a resistor opposes the current i, an inductor opposes the change (di/dt) in the current.

Kirchhoff's potential law with an inductor In Kirchhoff's potential law when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L\,di/dt$, where di/dt is to be substituted with sign.

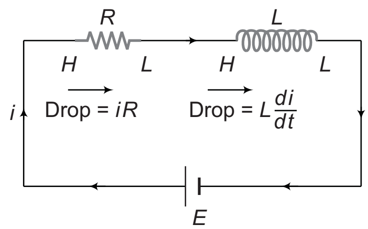


Fig. 27.35

For example in the loop shown in figure, Kirchhoff's second law gives the equation.

$$E - iR - L\frac{di}{dt} = 0$$

Example 27.13 *The inductor shown in the figure has inductance* $0.54\,H$ *and carries a current in the direction shown that is decreasing at a uniform rate* $\frac{di}{dt} = -0.03\ A/s$.

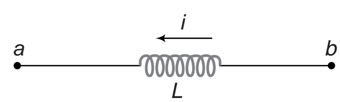


Fig. 27.36

(a) Find the self-induced emf.

(b) Which end of the inductor a or b is at a higher potential?

Solution (a) Self-induced emf,

$$e = -L\frac{di}{dt} = (-0.54)(-0.03)\ \text{V}$$

$$= 1.62 \times 10^{-2}\ \text{V}$$ **Ans.**

(b) $V_{ba} = L\frac{di}{dt} = -1.62 \times 10^{-2}\ \text{V}$

Since, $V_{ba}\ (= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential. **Ans.**

Example 27.14 *In the circuit diagram shown in figure,* $R = 10\ \Omega$, $L = 5H$, $E = 20\ V$, $i = 2\ A$. *This current is decreasing at a rate of* $-1.0\ A/s$. *Find* V_{ab} *at this instant.*

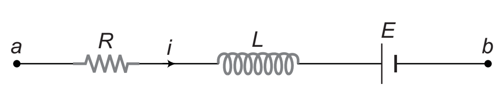


Fig. 27.37

Solution PD across inductor,

$$V_L = L\frac{di}{dt} = (5)(-1.0) = -5\ \text{V}$$

Now, $V_a - iR - V_L - E = V_b$

$\therefore$ $V_{ab} = V_a - V_b = E + iR + V_L$

$= 20 + (2)(10) - 5 = 35\text{ V}$ **Ans.**

Note *As the current is decreasing, the inductor can be replaced by a source of emf* $e = \left|L \cdot \dfrac{di}{dt}\right| = 5\text{ V}$ *in such a manner that this emf supports the decreasing current, or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards b. Thus, the given circuit can be drawn as shown below,*

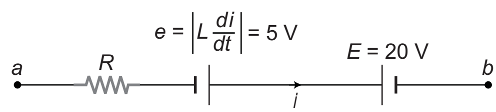


Fig. 27.38

Now, we can find V_{ab}.

Method of Finding Self-inductance of a Circuit

We use the equation, $L = N\phi_B / i$ to calculate the inductance of given circuit.

A good approach for calculating the self-inductance of a circuit consists of the following steps:

(a) Assume that there is a current i flowing through the circuit (we can call the circuit as inductor).

(b) Determine the magnetic field **B** produced by the current.

(c) Obtain the magnetic flux ϕ_B.

(d) With the flux known, the self-inductance can be found from $L = N\phi_B / i$.

To demonstrate this procedure, we now calculate the self-inductance of two inductors.

Inductance of a Solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air.

We can assume that the interior magnetic field due to a current i is uniform and given by equation,

$$B = \mu_0 n i = \mu_0 \left(\frac{N}{l}\right) i$$

where, $n = \dfrac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is, $\phi_B = BS = \mu_0 \dfrac{NS}{l} i$

Here, S is the cross-sectional area of the solenoid. Now,

$$L = \frac{N\phi_B}{i} = \frac{N}{i}\left(\frac{\mu_0 NSi}{l}\right) = \frac{\mu_0 N^2 S}{l}$$

$\therefore$ $$\boxed{L = \frac{\mu_0 N^2 S}{l}}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$\boxed{L \propto N^2}$$

Because $N = nl$, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} S = \mu_0 n^2 Sl = \mu_0 n^2 V \quad \text{or} \quad \boxed{L = \mu_0 n^2 V}$$

Here, $V = Sl$ is the volume of the solenoid.

Inductance of a Rectangular Toroid

A toroid with a rectangular cross-section is shown in figure. The inner and outer radii of the toroid are R_1 and R_2 and h is the height of the toroid. Applying Ampere's law for a toroid, we can show that magnetic field inside a rectangular toroid is given by

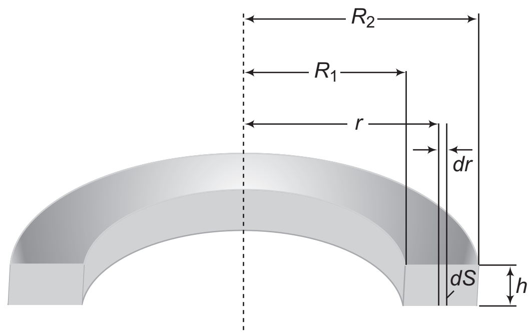


Fig. 27.39

$$B = \frac{\mu_0 Ni}{2\pi r}$$

where, r is the distance from the central axis of the toroid. Because the magnitude of magnetic field changes within the toroid, we must calculate the flux by integrating over the toroid's cross-section. Using the infinitesimal cross-sectional area element $dS = hdr$ shown in the figure, we obtain

$$\phi_B = \int B\, dS = \int_{R_1}^{R_2} \left(\frac{\mu_0 Ni}{2\pi r}\right)(hdr)$$

$$= \frac{\mu_0 Nhi}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

Now,

$$L = \frac{N\phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

or

$$\boxed{L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{R_2}{R_1}\right)}$$

As expected, the self-inductance is a constant determined by only the physical properties of the toroid.

Energy Stored in an Inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

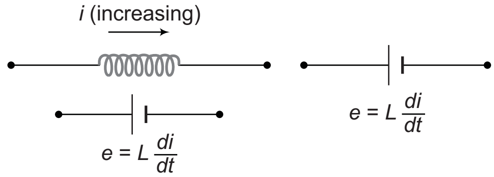


Fig. 27.40

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power.

$$P = \frac{dW}{dt} = -\,ei = -\,Li\frac{di}{dt}$$

From $\qquad dW = -\,dU \quad \text{or} \quad P = \frac{dW}{dt} = -\frac{dU}{dt}$

We have, $\qquad \frac{dU}{dt} = Li\frac{di}{dt} \quad \text{or} \quad dU = Li\,di$

The total energy U supplied while the current increases from zero to a final value i is

$$U = L\int_0^i i\,di = \frac{1}{2}Li^2$$

$$\therefore \qquad \boxed{U = \frac{1}{2}Li^2}$$

This is the expression for the energy stored in the magnetic field of an inductor when a current i flows through it. The source of this energy is the external source of emf that supplies the current.

Note *(i) After the current has reached its final steady state value i, $di/dt = 0$ and no more energy is taken by the inductor.*

(ii) When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch, the current decreases very rapidly, the induced emf is very large and the energy may be dissipated as a spark across the switch.

(iii) If we compare the behaviour of a resistor and an inductor towards the current flow we can observe that energy flows into a resistor whenever a current passes through it. Whether the current is steady (constant) or varying this energy is dissipated in the form of heat. By contrast energy flows into an ideal, zero resistance inductor only when the current in the inductor increases. This energy is not dissipated, it is stored in the inductor and released when the current decreases.

(iv) As we said earlier also, the energy in an inductor is actually stored in the magnetic field within the coil. We can develop relations of magnetic energy density u (energy stored per unit volume) analogous to those we obtained in electrostatics. We will concentrate on one simple case of an ideal long cylindrical solenoid. For a long solenoid its magnetic field can be assumed completely of within the solenoid. The energy U stored in the solenoid when a current i is

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(\mu_0 n^2 V)\,i^2 \quad \text{as} \quad L = \mu_0 n^2 V$$

The energy per unit volume is $u = \frac{U}{V}$

$$\therefore \qquad u = \frac{U}{V} = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\frac{(\mu_0 ni)^2}{\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0}$$

as $\qquad B = \mu_0 ni$

Thus,

$$\boxed{u = \frac{1}{2}\frac{B^2}{\mu_0}}$$

This expression is similar to $u = \frac{1}{2}\varepsilon_0 E^2$ *used in electrostatics. Although, we have derived it for one special situation, it turns out to be correct for any magnetic field configuration.*

Example 27.15 *(a) Calculate the inductance of an air core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm*^{2}.

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution (a) The inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 S}{l}$$

Substituting the values, we have

$$L = \frac{(4\pi \times 10^{-7})(300)^2 (4.00 \times 10^{-4})}{(25.0 \times 10^{-2})} \text{ H}$$

$$= 1.81 \times 10^{-4} \text{ H} \qquad \textbf{Ans.}$$

(b) $e = -L\frac{di}{dt}$

Here, $\qquad \frac{di}{dt} = -50.0 \text{ A/s}$

$$\therefore \qquad e = -(1.81 \times 10^{-4})(-50.0)$$

$$= 9.05 \times 10^{-3} \text{ V}$$

or $\qquad e = 9.05 \text{ mV} \qquad$ **Ans.**

Example 27.16 *What inductance would be needed to store 1.0 kWh of energy in a coil carrying a 200 A current.* $(1\ kWh = 3.6 \times 10^6\ J)$

Solution We have, $i = 200$ A

and $\qquad U = 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

$$\therefore \qquad L = \frac{2U}{i^2} \qquad \left(U = \frac{1}{2}Li^2\right)$$

$$= \frac{2(3.6 \times 10^6)}{(200)^2} = 180 \text{ H} \qquad \textbf{Ans.}$$

Example 27.17 *(a) What is the magnetic flux through one turn of a solenoid of self-inductance 8.0×10^{-5} H when a current of 3.0 A flows through it? Assume that the solenoid has 1000 turns and is wound from wire of diameter 1.0 mm. (b) What is the cross-sectional area of the solenoid?*

Solution Given, $L = 8.0 \times 10^{-5}$ H, $i = 3.0$ A and $N = 1000$ turns

(a) From the relation, $L = \dfrac{N\phi}{i}$

The flux linked with one turn,

$$\phi = \frac{Li}{N} = \frac{(8.0 \times 10^{-5})(3.0)}{1000}$$

$$= 2.4 \times 10^{-7} \text{ Wb}$$

(b) This $\phi = BS = (\mu_0 ni)(S)$

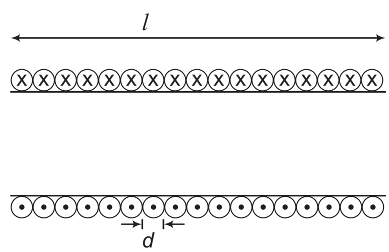


Fig. 27.41

Here, n = number of turns per unit length

$$= \frac{N}{l} = \frac{N}{Nd} = \frac{1}{d}$$

∴ $$\phi = \frac{\mu_0 iS}{d}$$

or $$S = \frac{\phi d}{\mu_0 i} = \frac{(2.4 \times 10^{-7})\ (1.0 \times 10^{-3})}{(4\pi \times 10^{-7})\ (3.0)}$$

$$= 6.37 \times 10^{-5} \text{ m}^2$$ **Ans.**

Example 27.18 *A 10 H inductor carries a current of 20 A. How much ice at $0°$C could be melted by the energy stored in the magnetic field of the inductor? Latent heat of ice is 22.6×10^3 J/kg.*

Solution Energy stored is $\dfrac{1}{2}Li^2$.

This energy is completely used in melting the ice.

Hence, $$\frac{1}{2}Li^2 = mL_f$$

Here, L_f = latent heat of fusion

Hence, mass of ice melted, $$m = \frac{Li^2}{2L_f}$$

Substituting the values, we have

$$m = \frac{(10)(20)^2}{2(2.26 \times 10^3)}$$

$$= 0.88 \text{ kg}$$ **Ans.**

INTRODUCTORY EXERCISE 27.3

1. The current through an inductor of 1H is given by $i = 3t \sin t$. Find the voltage across the inductor.
2. In the figure shown $i = 10e^{-4t}$ A. Find V_L and V_{ab}.

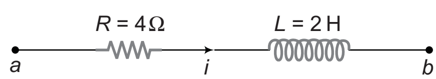


Fig. 27.42

3. The current (in Ampere) in an inductor is given by $I = 5 + 16t$, where t is in seconds. The self-induced emf in it is 10 mV. Find
   (a) the self-inductance, and
   (b) the energy stored in the inductor and the power supplied to it at $t = 1$s
4. (a) Calculate the self-inductance of a solenoid that is tightly wound with wire of diameter 0.10 cm, has a cross-sectional area 0.90 cm^2 and is 40 cm long.
   (b) If the current through the solenoid decreases uniformly from 10 A to 0 A in 0.10 s, what is the emf induced between the ends of the solenoid?

27.7 Mutual Inductance

We have already discussed in Chapter 26, the magnetic interaction between two wires carrying steady currents. The current in one wire causes a magnetic field, which exerts a force on the current in the second wire.

An additional interaction arises between two circuits when there is a changing current in one of the circuits.

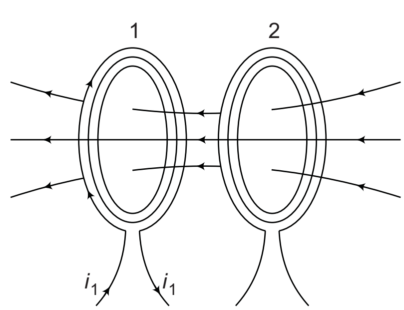


Fig. 27.43

Consider two neighbouring coils of wire as shown in Fig. 27.42. A current flowing in coil 1 produces magnetic field and hence, a magnetic flux through coil 2. If the current in coil 1 changes, the flux through coil 2 changes as well. According to Faraday's law this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as **mutual induction**. Like the self-inductance (L), two circuits have mutual inductance (M). It also have two definitions as under:

First Definition Suppose the circuit 1 has a current i_1 flowing in it. Then, total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in 1. Thus,

$$N_2\,\phi_{B_2} \propto i_1$$

or

$$N_2\phi_{B_2} = Mi_1$$

Here, the proportionality constant M is known as the mutual inductance M of the two circuits.

Thus,

$$\boxed{M = \frac{N_2\phi_{B_2}}{i_1}}$$

From this expression, M can be defined as the **total flux $N_2\phi_{B_2}$ linked with circuit 2 per unit current in circuit 1**.

Second Definition If we change the current in circuit 1 at a rate di_1/dt, an induced emf e_2 is developed in circuit 1, which is proportional to the rate di_1/dt. Thus,

$$e_2 \propto di_1/dt$$

or

$$e_2 = -\,Mdi_1/dt$$

Here, the proportionality constant is again M. Minus sign indicates that e_2 is in such a direction that it opposes any change in the current in circuit 1. From the above equation,

$$\boxed{M = \left|\frac{-e_2}{di_1/dt}\right|}$$

This equation states that, **the mutual inductance of two circuits is the magnitude of induced emf e_2 per unit rate of change of current di_1/dt**.

Note down the following points regarding the mutual inductance:

1. The SI unit of mutual inductance is **henry** (1H).
2. M depends upon closeness of the two circuits, their orientations and sizes and the number of turns etc.
3. **Reciprocity theorem :** $M_{21} = M_{12} = M$

$$e_2 = -\,M(di_1/dt)$$

and

$$e_1 = -\,M(di_2/dt)$$

$$M_{12} = \frac{N_2\phi_{B_2}}{i_1}$$

and

$$M_{21} = \frac{N_1\phi_{B_1}}{i_2}$$

4. A good approach for calculating the mutual inductance of two circuits consists of the following steps:
   (a) Assume any one of the circuits as primary (first) and the other as secondary (second).
   (b) Pass a current i_1 through the primary circuit.
   (c) Determine the magnetic field **B** produced by the current i_1.
   (d) Obtain the magnetic flux ϕ_{B_2}.
   (e) With this flux, the mutual inductance can be found from,
$$M = \frac{N_2\,\phi_{B_2}}{i_1}$$

Mutual Inductance of a Solenoid Surrounded by a Coil

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.

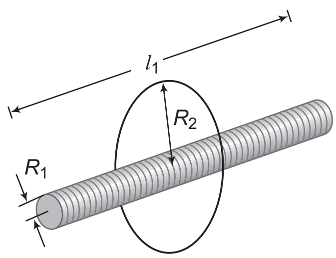


Fig. 27.44

To calculate M between them, let us assume a current i_1 in solenoid.

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0 \left(\frac{N_1}{l_1}\right) i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B_2} through the surrounding coil is, therefore,

$$\phi_{B_2} = B(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\,\pi R_1^2$$

Now,
$$M = \frac{N_2 \phi_{B_2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

∴
$$\boxed{M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}}$$

Note that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior. In principle, we can also calculate M by finding the magnetic flux through the solenoid produced by the current in the surrounding coil. This approach is much more difficult, because ϕ_{B_1} is so complicated. However, since $M_{12} = M_{21}$, we do know the result of this calculation.

Combination of Inductances

In series If several inductances are in series so that there are no interactions through mutual inductance.

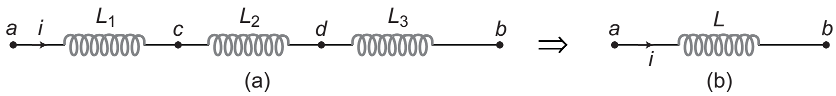


Fig. 27.45

Refer figure (a)

$$V_a - V_c = L_1 \frac{di}{dt}$$

$$V_c - V_d = L_2 \frac{di}{dt}$$

and $$V_d - V_b = L_3 \frac{di}{dt}$$

Adding all these equations, we have

$$V_a - V_b = (L_1 + L_2 + L_3)\frac{di}{dt} \quad \ldots(i)$$

Refer figure (b)

$$V_a - V_b = L\frac{di}{dt} \quad \ldots(ii)$$

Here, L = equivalent inductance.

From Eqs. (i) and (ii), we have

$$\boxed{L = L_1 + L_2 + L_3}$$

In parallel

Refer figure (a)

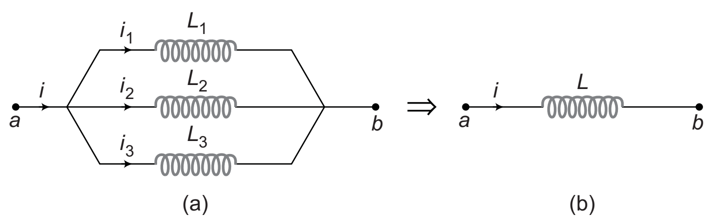


Fig. 27.46

$$i = i_1 + i_2 + i_3$$

or $$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

or $$\frac{di}{dt} = \frac{V_a - V_b}{L_1} + \frac{V_a - V_b}{L_2} + \frac{V_a - V_b}{L_3} \quad \ldots(i)$$

Refer figure (b)

$$\frac{di}{dt} = \frac{V_a - V_b}{L} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Example 27.19 *A straight solenoid has 50 turns per cm in primary and total 200 turns in the secondary. The area of cross-section of the solenoids is 4 cm^2. Calculate the mutual inductance. Primary is tightly kept inside the secondary.*

Solution The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation,

$$B = \mu_0 n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given as

$$N_2 \phi_2 = N_2 (BS) = \mu_0 n_1 N_2 i_1 S$$

$$\therefore \qquad M = \frac{N_2 \phi_2}{i_1} = \mu_0 n_1 N_2 S$$

Substituting the values, we get

$$M = (4\pi \times 10^{-7}) \left(\frac{50}{10^{-2}} \right) (200)\,(4 \times 10^{-4})$$

$$= 5.0 \times 10^{-4}\ \text{H}$$ **Ans.**

Example 27.20 *Two solenoids A and B spaced close to each other and sharing the same cylindrical axis have 400 and 700 turns, respectively. A current of 3.50 A in coil A produced an average flux of 300 μT-m^2 through each turn of A and a flux of 90.0 μT-m^2 through each turn of B.*

(a) Calculate the mutual inductance of the two solenoids.

(b) What is the self-inductance of A?

(c) What emf is induced in B when the current in A increases at the rate of 0.5 A/s?

Solution (a)

$$M = \frac{N_B \phi_B}{i_A}$$

$$= \frac{(700)\,(90 \times 10^{-6})}{3.5}$$

$$= 1.8 \times 10^{-2}\ \text{H}$$ **Ans.**

(b) $L_A = \dfrac{N_A \phi_A}{i_A}$

$$= \frac{(400)\,(300 \times 10^{-6})}{3.5}$$

$$= 3.43 \times 10^{-2}\ \text{H}$$ **Ans.**

(c) $$e_B = M\left(\frac{di_A}{dt}\right)$$
$$= (1.8 \times 10^{-2})(0.5)$$
$$= 9.0 \times 10^{-3}\ \text{V}$$ **Ans.**

INTRODUCTORY EXERCISE 27.4

1. Calculate the mutual inductance between two coils when a current of 4 A changes to 12 A in 0.5 s in primary and induces an emf of 50 mV in the secondary. Also, calculate the induced emf in the secondary if current in the primary changes from 3 A to 9 A is 0.02 s.
2. A coil has 600 turns which produces 5×10^{-3} Wb / turn of flux when 3 A current flows in the wire. This produced 6×10^{-3} Wb/turn in 1000 turns secondary coil. When the switch is opened, the current drops to zero in 0.2 s in primary. Find
   (a) mutual inductance,
   (b) the induced emf in the secondary,
   (c) the self-inductance of the primary coil.
3. Two coils have mutual inductance $M = 3.25 \times 10^{-4}$ H. The current i_1 in the first coil increases at a uniform rate of 830 A/s.
   (a) What is the magnitude of the induced emf in the second coil? Is it constant?
   (b) Suppose that the current described is in the second coil rather than the first. What is the induced emf in the first coil?

27.8 Growth and Decay of Current in an *L-R* Circuit

Growth of Current

Let us consider a circuit consisting of a battery of emf E, a coil of self-inductance L and a resistor R. The resistor R may be a separate circuit element, or it may be the resistance of the inductor windings. By closing switch S_1, we connect R and L in series with constant emf E. Let i be the current at some time t after switch S_1 is closed and di/dt be its rate of increase at that time. Applying Kirchhoff's loop rule starting at the negative terminal and proceeding counterclockwise around the loop

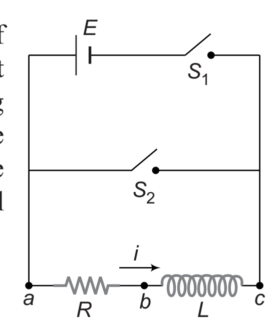


Fig. 27.47

$$E - V_{ab} - V_{bc} = 0 \quad \text{or} \quad E - iR - L\frac{di}{dt} = 0$$

$$\therefore \quad \int_0^i \frac{di}{E - iR} = \int_0^t \frac{dt}{L} \quad \text{or} \quad i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

By letting $E/R = i_0$ and $L/R = \tau_L$, the above expression reduces to

$$\boxed{i = i_0\,(1 - e^{-t/\tau_L})}$$

Here, $i_0 = E/R$ is the current at $t = \infty$. It is also called the steady state current or the maximum current in the circuit.

And $\tau_L = \dfrac{L}{R}$ is called **time constant** of the *L-R* circuit. At a time equal to one time constant the current has risen to $(1 - 1/e)$ or about 63% of its final value i_0.

The *i-t* graph is as shown in figure.

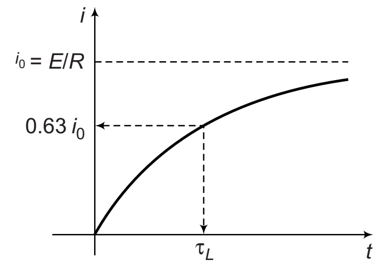


Fig. 27.48

Note that the final current i_0 does not depend on the inductance *L*, it is the same as it would be if the resistance *R* alone were connected to the source with emf *E*.

Let us have an insight into the behaviour of an *L - R* circuit from energy considerations.

The instantaneous rate at which the source delivers energy to the circuit $(P = Ei)$ is equal to the instantaneous rate at which energy is dissipated in the resistor $(= i^2 R)$ plus the rate at which energy is stored in the inductor $\left(= iV_{bc} = Li\dfrac{di}{dt}\right)$ or $\dfrac{d}{dt}\left(\dfrac{1}{2}Li^2\right) = Li\dfrac{di}{dt}$.

Thus, $$\boxed{Ei = i^2 R + Li\frac{di}{dt}}$$

Decay of Current

Now suppose switch S_1 in the circuit shown in figure has been closed for a long time and that the current has reached its steady state value i_0. Resetting our stopwatch to redefine the initial time we close switch S_2 at time $t = 0$ and at the same time we should open the switch S_1 to by pass the battery. The current through *L* and *R* does not instantaneously go to zero but decays exponentially. To apply Kirchhoff's loop rule and to find current in the circuit at time *t*, let us draw the circuit once more.

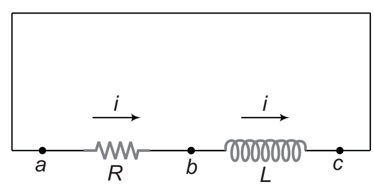


Fig. 27.49

Applying loop rule we have,

$$(V_a - V_b) + (V_b - V_c) = 0 \qquad \text{(as } V_a = V_c\text{)}$$

or $$iR + L\left(\frac{di}{dt}\right) = 0$$

Note *Don't bother about the sign of* $\dfrac{di}{dt}$.

$\therefore$ $$\frac{di}{i} = -\frac{R}{L}dt$$

$\therefore$ $$\int_{i_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

$\therefore$ $$\boxed{i = i_0 e^{-t/\tau_L}}$$

where, $\tau_L = \dfrac{L}{R}$, *is the time for current to decrease to* $1/e$ *or about 37% of its original value. The i-t graph is as shown in Fig. 27.49.`*

The energy that is needed to maintain the current during this decay is provided by energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor = rate at which the stored energy decreases in magnetic field of inductor

or $$i^2R = -\frac{dU}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right)$$

or $$i^2R = Li\left(\frac{-di}{dt}\right)$$

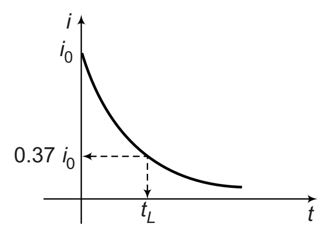


Fig. 27.50

Example 27.21 *A coil of resistance 20 Ω and inductance 0.5 H is switched to DC 200 V supply. Calculate the rate of increase of current*
(a) at the instant of closing the switch and
(b) after one time constant.
(c) Find the steady state current in the circuit.

Solution (a) This is the case of growth of current in an *L-R* circuit. Hence, current at time *t* is given by

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Rate of increase of current,

$$\frac{di}{dt} = \frac{i_0}{\tau_L} e^{-t/\tau_L}$$

At $t = 0$, $$\frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$$

Substituting the value, we have

$$\frac{di}{dt} = \frac{200}{0.5} = 400 \text{ A/s}$$ **Ans.**

(b) At $t = \tau_L$,

$$\frac{di}{dt} = (400)\,e^{-1} = (0.37)\,(400)$$

$$= 148 \text{ A/s}$$ **Ans.**

(c) The steady state current in the circuit,

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10 \text{ A}$$ **Ans.**

Example 27.22 *A 5 H inductor is placed in series with a 10 Ω resistor. An emf of 5 V being suddenly applied to the combination. Using these values prove the principle of conservation of energy, for time equal to the time constant.*

Solution At any instant t, current in L-R circuit is given as

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Here,
$$i_0 = \frac{E}{R} \text{ and } \tau_L = \frac{L}{R}$$

After one time constant ($t = \tau_L$), current in the circuit is

$$i = \frac{E}{R}\left(1 - \frac{1}{e}\right) = \frac{5}{10}\left(1 - \frac{1}{e}\right) = 0.316 \text{ A}$$

The rate at which the energy is delivered by the battery is

$$P_1 = Ei = (5)\,(0.316) = 1.58 \text{ W} \qquad \text{...(i)}$$

At this time rate by which energy is dissipated in the resistor is

$$P_2 = i^2 R = (0.316)^2\ (10) = 0.998 \text{ W} \qquad \text{...(ii)}$$

The rate at which energy is stored in the inductor is

$$P_3 = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(\frac{di}{dt}\right)$$

Here,
$$\frac{di}{dt} = \frac{i_0}{\tau_L}e^{-1} = \frac{E}{eL} \qquad \text{(after one time constant)}$$

Substituting the values, we get

$$P_3 = (L)\,(i)\left(\frac{E}{eL}\right) = \frac{Ei}{e}$$

$$= \frac{5 \times 0.316}{2.718} = 0.582 \text{ W} \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we have

$$P_1 = P_2 + P_3$$

It is the same as required by the principle of conservation of energy.

INTRODUCTORY EXERCISE 27.5

1. Show that $\frac{L}{R}$ has units of time.
2. A coil of inductance 2 H and resistance 10 Ω are in a series circuit with an open key and a cell of constant 100 V with negligible resistance. At time $t = 0$, the key is closed. Find
   (a) the time constant of the circuit.
   (b) the maximum steady current in the circuit.
   (c) the current in the circuit at $t = 1$s.
3. In the simple L-R circuit, can the emf induced across the inductor ever be greater than the emf of the battery used to produce the current?

27.9 Oscillations in *L-C* Circuit

If a charged capacitor C is short-circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealized situation in which energy is not radiated away from the circuit. With these idealizations-zero resistance and no radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor's magnetic field and back. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy. Later, we will see that this analogy goes much further.

Let us now derive an equation for the oscillations in an L-C circuit.

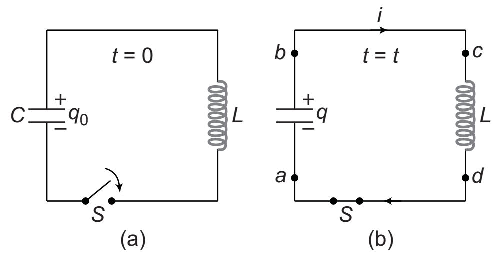


Fig. 27.51

Refer figure (a) A capacitor is charged to a PD, $V_0 = q_0 C$

Here, q_0 is the maximum charge on the capacitor. At time $t = 0$, it is connected to an inductor through a switch S. At time $t = 0$, switch S is closed.

Refer figure (b) When the switch is closed, the capacitor starts discharging. Let at time t charge on the capacitor is $q\ (< q_0)$ and since, it is further decreasing there is a current i in the circuit in the direction shown in figure. Later we will see that, as the charge is oscillating there may be a situation when q will be increasing, but in that case direction of current is also reversed and the equation remains unchanged.

The potential difference across capacitor = potential difference across inductor, or

$$V_b - V_a = V_c - V_d$$

$\therefore$
$$\frac{q}{C} = L\left(\frac{di}{dt}\right) \qquad \ldots\text{(i)}$$

Now, as the charge is decreasing,
$$i = \left(\frac{-dq}{dt}\right)$$

or
$$\frac{di}{dt} = -\frac{d^2q}{dt^2}$$

Substituting in Eq. (i), we get
$$\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$$

or
$$\boxed{\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q} \qquad \ldots\text{(ii)}$$

This is the standard equation of simple harmonic motion $\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$.

Here, $$\boxed{\omega = \frac{1}{\sqrt{LC}}} \quad \text{...(iii)}$$

The general solution of Eq. (ii), is $q = q_0 \cos(\omega t \pm \phi)$

For example in our case $\phi = 0$ as $q = q_0$ at $t = 0$.

Hence, $$q = q_0 \cos \omega t \quad \text{...(iv)}$$

Thus, we can say that charge in the circuit oscillates simple harmonically with angular frequency given by Eq. (iii). Thus,

$$\boxed{\omega = \frac{1}{\sqrt{LC}}, \; f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad T = \frac{1}{f} = 2\pi\sqrt{LC}}$$

The oscillations of the L-C circuit are an electromagnetic analog to the mechanical oscillations of a block-spring system.

Table below shows a comparison of oscillations of a mass-spring system and an L-C circuit.

Table 27.1

S.No.	Mass spring system	Inductor-capacitor circuit
1.	Displacement (x)	Charge (q)
2.	Velocity (v)	Current (i)
3.	Acceleration (a)	Rate of change of current $\left(\frac{di}{dt}\right)$
4.	$\frac{d^2x}{dt^2} = -\omega^2 x$, where $\omega = \sqrt{\frac{k}{m}}$	$\frac{d^2q}{dt^2} = -\omega^2 q$, where $\omega = \frac{1}{\sqrt{LC}}$
5.	$x = A\sin(\omega t \pm \phi)$ or $x = A\cos(\omega t \pm \phi)$	$q = q_0 \sin(\omega t \pm \phi)$ or $q = q_0 \cos(\omega t \pm \phi)$
6.	$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$	$i = \frac{dq}{dt} = \omega\sqrt{q_0^2 - q^2}$
7.	$a = \frac{dv}{dt} = -\omega^2 x$	Rate of change of current $= \frac{di}{dt} = -\omega^2 q$
8.	Kinetic energy $= \frac{1}{2}mv^2$	Magnetic energy $= \frac{1}{2}Li^2$
9.	Potential energy $= \frac{1}{2}kx^2$	Potential energy $= \frac{1}{2}\frac{q^2}{C}$
10.	$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$	$\frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} = \text{constant} = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}Li_{max}^2$
11.	$\|v_{max}\| = A\omega$	$i_{max} = q_0\omega$
12	$\|a_{max}\| = \omega^2 A$	$\left\|\left(\frac{di}{dt}\right)_{max}\right\| = \omega^2 q_0$
13.	$\frac{1}{k}$	C
14.	m	L

A graphical description of the energy transfer between the inductor and the capacitor in an L-C circuit is shown in the figure. The right side of the figure shows the analogous energy transfer in the oscillating block-spring system.

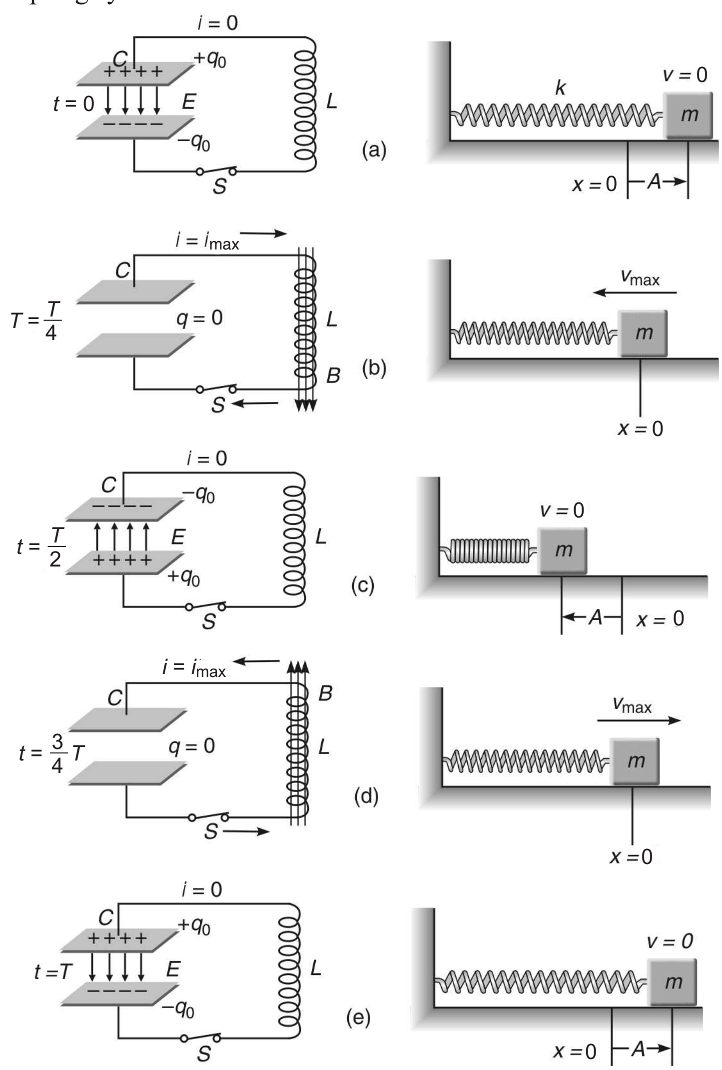


Fig. 27.52

Note *In L-C oscillations, q, i and* $\frac{di}{dt}$ *all oscillate simple harmonically with same angular frequency* ω. *But the phase difference between q and i or between i and* $\frac{di}{dt}$ *is* $\frac{\pi}{2}$, *while that between i and* $\frac{di}{dt}$ *is* π. *Their amplitudes are* q_0, $q_0\omega$ *and* $\omega^2 q_0$ *respectively. So, now suppose*

$$q = q_0 \cos \omega t, \text{ then}$$

$$i = \frac{dq}{dt} = -\, q_0\omega \sin \omega t \text{ and}$$

$$\frac{di}{dt} = -\, q_0\omega^2 \cos \omega t$$

Similarly, potential energy across capacitor (U_C) and across inductor (U_L) also oscillate with double the frequency 2ω but not simple harmonically. The different graphs are as shown in Fig. 27.52.

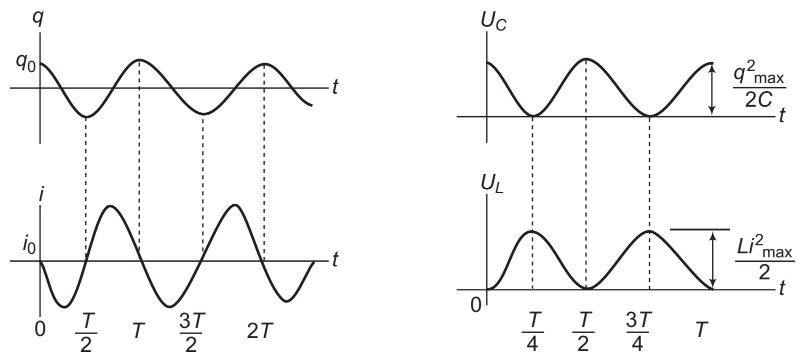


Fig. 27.53

Example 27.23 *A capacitor of capacitance $25\ \mu F$ is charged to $300\ V$. It is then connected across a $10\ mH$ inductor. The resistance in the circuit is negligible.*

(a) Find the frequency of oscillation of the circuit.

(b) Find the potential difference across capacitor and magnitude of circuit current $1.2\ ms$ after the inductor and capacitor are connected.

(c) Find the magnetic energy and electric energy at $t = 0$ and $t = 1.2\ ms$.

Solution (a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Substituting the given values, we have

$$f = \frac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}} = 318.3\ \text{Hz}$$ **Ans.**

(b) Charge across the capacitor at time t will be

$$q = q_0 \cos\omega t \quad \text{and} \quad i = -q_0\omega\sin\omega t$$

Here, $q_0 = CV_0 = (25\times10^{-6})(300) = 7.5\times10^{-3}\ \text{C}$

Now, charge in the capacitor after $t = 1.2\times10^{-3}$ s is

$$q = (7.5\times10^{-3})\cos(2\pi\times318.3)(1.2\times10^{-3})\ \text{C}$$

$$= -5.53\times10^{-3}\ \text{C}$$

$\therefore$ PD across capacitor, $V = \dfrac{|q|}{C} = \dfrac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2$ volt **Ans.**

The magnitude of current in the circuit at $t = 1.2\times10^{-3}$ s is

$$|i| = q_0\omega\sin\omega t$$

$$= (7.5\times10^{-3})(2\pi)(318.3)\sin(2\pi\times318.3)(1.2\times10^{-3})\ \text{A}$$

$$= 10.13\ \text{A}$$ **Ans.**

(c) **At $t = 0$** Current in the circuit is zero.

Hence, $U_L = 0$

Charge in the capacitor is maximum.

Hence, $$U_C = \frac{1}{2}\frac{q_0^2}{C}$$

or $$U_C = \frac{1}{2} \times \frac{(7.5 \times 10^{-3})^2}{(25 \times 10^{-6})}$$

$$= 1.125 \text{ J}$$ **Ans.**

$\therefore$ Total energy, $E = U_L + U_C = 1.125$ J

At $t = 1.2$ ms

$$U_L = \frac{1}{2} Li^2$$

$$= \frac{1}{2}(10 \times 10^{-3})(10.13)^2$$

$$= 0.513 \text{ J}$$

$\therefore$ $$U_C = E - U_L = 1.125 - 0.513$$

$$= 0.612 \text{ J}$$ **Ans.**

Otherwise U_C can be calculated as

$$U_C = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{1}{2} \times \frac{(5.53 \times 10^{-3})^2}{(25 \times 10^{-6})}$$

$$= 0.612 \text{ J}$$

INTRODUCTORY EXERCISE 27.6

1. Show that $\sqrt{LC}$ has units of time.
2. While comparing the *L-C* oscillations with the oscillations of spring-block system, with whom the magnetic energy can be compared and why?
3. In an *L-C* circuit, $L = 0.75$ H and $C = 18\ \mu$F,
   (a) At the instant when the current in the inductor is changing at a rate of 3.40 A/s, what is the charge on the capacitor?
   (b) When the charge on the capacitor is 4.2×10^{-4} C, what is the induced emf in the inductor?
4. An *L-C* circuit consists of a 20.0 mH inductor and a $0.5\ \mu$F capacitor. If the maximum instantaneous current is 0.1 A, what is the greatest potential difference across the capacitor?

27.10 Induced Electric Field

When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on charges in the conductor as described in Art. 27.5. But an induced emf also occurs when there is a changing flux through a stationary conductor. What is it that pushes the charges around the circuit in this type of situation?

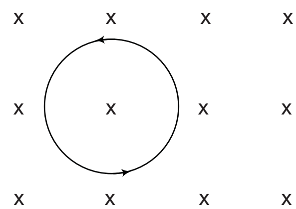


Fig. 27.54

As an example, let's consider the situation shown in figure. A conducting circular loop is placed in a magnetic field which is directed perpendicular to the paper inwards. When the magnetic field changes with time (suppose it increases with time) the magnetic flux ϕ_B also changes and according to Faraday's law the induced emf $e = -\frac{d\phi_B}{dt}$ is produced in the loop. If the total resistance of the loop is R, the induced current in the loop is given by

$$i = \frac{e}{R}$$

But what force makes the charges move around the loop? It can't be the magnetic force, because the charges are not moving in the magnetic field.

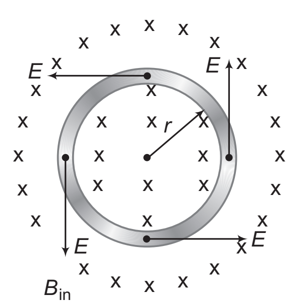


Fig. 27.55

Actually, there is an **induced electric field** in the conductor **caused by the changing magnetic flux.** *This electric field has the following important properties:*

1. It is non-conservative in nature. The line integral of **E** around a closed path is not zero. This line integral is given by

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}} \qquad \text{...(i)}$$

   Note that this equation is valid only if the path around which we integrate is stationary.

2. Because of symmetry, the electric field **E** has the same magnitude at every point on the circle and is tangent to it at each point. The directions of **E** at several points on the loop are shown in figure.
3. Being a non-conservative field, the concept of potential has no meaning for such a field.
4. This field is different from the electrostatic field produced by stationary charges (which is conservative in nature).
5. The relation $\mathbf{F} = q\mathbf{E}$ is still valid for this field.
6. This field can vary with time.

So, **a changing magnetic field acts as a source of electric field of a sort that we cannot produce with any static charge distribution.** This may seen strange but its the way nature behaves.

Note *1. For symmetrical situations (as shown in figure) Eq. (i), in simplified form can be written as*

$$\boxed{El = \left|\frac{d\phi_B}{dt}\right| = S\left|\frac{dB}{dt}\right|}$$

Here, l is the length of closed loop in which electric field is to be calculated and S is the area in which magnetic field is changing.

2. Direction of electric field is the same as the direction of induced current.

Example 27.24 *The magnetic field at all points within the cylindrical region whose cross-section is indicated in the accompanying figure start increasing at a constant rate α T/s. Find the magnitude of electric field as a function of r, the distance from the geometric centre of the region.*

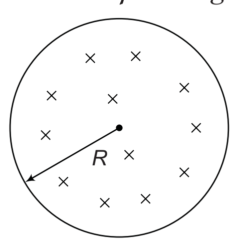


Fig. 27.56

Solution **For $r \le R$**

Using $El = S\left|\frac{dB}{dt}\right|$ or $E\,(2\pi r) = (\pi r^2)\,\alpha$

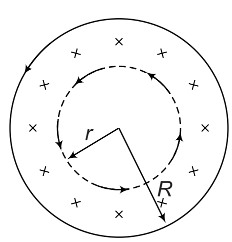


Fig. 27.57

$$\therefore \qquad E = \frac{r\alpha}{2}$$

$\therefore \quad E \propto r$, i.e. E-r graph is a straight line passing through origin.

At $\quad r = R,\ E = \dfrac{R\alpha}{2}$

For $r \geq R$

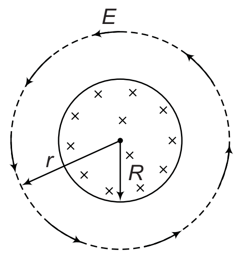


Fig. 27.58

Using $\quad El = S \left|\dfrac{dB}{dt}\right|$,

$$\therefore \qquad E\,(2\pi r) = (\pi R^2)\,(\alpha)$$

$$\therefore \qquad E = \frac{\alpha R^2}{2r}$$

$\therefore \qquad E \propto \dfrac{1}{r}$, i.e. E-r graph is a rectangular hyperbola.

The E-r graph is as shown in figure.

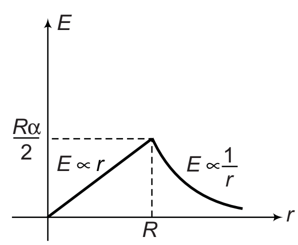


Fig. 27.59

The direction of electric field is shown in above figure.

Example 27.25 *A long thin solenoid has* 900 *turns/metre and radius* 2.50 *cm. The current in the solenoid is increasing at a uniform rate of* 60 *A/s. What is the magnitude of the induced electric field at a point?*

(a) 0.5 *cm from the axis of the solenoid.*

(b) 1.0 *cm from the axis of the solenoid.*

Solution $\quad \because \quad B = \mu_0 ni$

$$\therefore \qquad \frac{dB}{dt} = \mu_0 n \frac{di}{dt}$$

$$= (4\pi \times 10^{-7})\,(900)\,(60)$$

$$= 0.068 \text{ T/s}$$

Using the result of electric field derived in above problem (as both points lie inside the solenoid).

$$E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

(a) $E = \left(\frac{0.5 \times 10^{-2}}{2}\right)(0.068) = 1.7 \times 10^{-4}$ V/m

(b) $E = \frac{(1.0 \times 10^{-2})}{2}(0.068) = 3.4 \times 10^{-4}$ V/m

INTRODUCTORY EXERCISE 27.7

1. A long solenoid of cross-sectional area $5.0\ \text{cm}^2$ is wound with 25 turns of wire per centimetre. It is placed in the middle of a closely wrapped coil of 10 turns and radius 25 cm as shown.

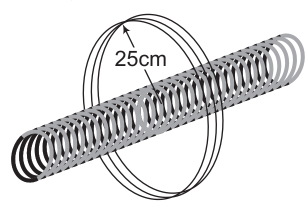


Fig. 27.60

(a) What is the emf induced in the coil when the current through the solenoid is decreasing at a rate −0.20 A/s?

(b) What is the electric field induced in the coil?

2. For the situation described in figure, the magnetic field changes with time according to

$$B = (2.00t^3 - 4.00t^2 + 0.8)\ \text{T} \quad \text{and} \quad r_2 = 2R = 5.0\ \text{cm}$$

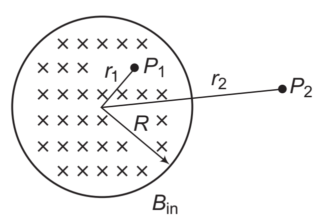


Fig. 27.61

(a) Calculate the force on an electron located at P_2 at $t = 2.00$ s

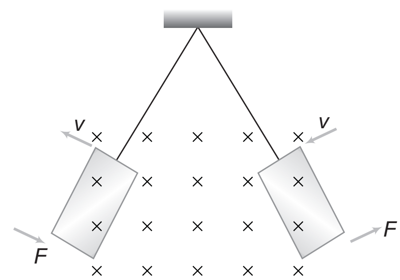
(b) What are the magnitude and direction of the electric field at P_1 when $t = 3.00$ s and $r_1 = 0.02$ m.

Hint : For the direction, see whether the field is increasing or decreasing at given times.

Final Touch Points

1. **Eddy currents** When a changing magnetic flux is applied to a piece of conducting material, circulating currents called eddy currents are induced in the material. These eddy currents often have large magnitudes and heat up the conductor.

   When a metal plate is allowed to swing through a strong magnetic field, then in entering or leaving the field the eddy currents are set up in the plate which opposes the motion as shown in figure. The kinetic energy dissipates in the form of heat. The slowing down of the plate is called the **electromagnetic damping.**

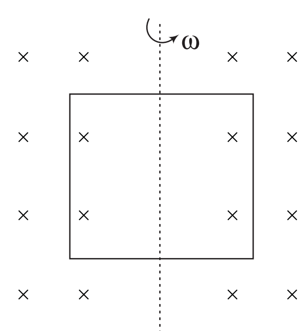


   The electromagnetic damping is used to damp the oscillations of a galvanometer coil or chemical balance and in braking electric trains. Otherwise, the eddy currents are often undesirable. To reduce the eddy currents some slots are cut into moving metallic parts of machinery. These slots intercept the conducting paths and decreases the magnitudes of the induced currents.

2. **Back EMF of Motors** An electric motor converts electrical energy into mechanical energy and is based on the fact that a current carrying coil in a uniform magnetic field experiences a torque. As the coil rotates in the magnetic field, the flux linked with the rotating coil will change and hence, an emf called back emf is produced in the coil.

   When the motor is first turned on, the coil is at rest and so there is no back emf. The 'start up' current can be quite large. To reduce 'start up' current a resistance called **'starter'** is put in series with the motor for a short period when the motor is started. As the rotation rate increases the back emf increases and hence, the current reduces.

3. **Electric Generator or Dynamo** A dynamo converts mechanical energy (rotational kinetic energy) into electrical energy. It consists of a coil rotating in a magnetic field. Due to rotation of the coil magnetic flux linked with it changes, so an emf is induced in the coil.



   Suppose at time $t = 0$, plane of coil is perpendicular to the magnetic field.

The flux linked with it at any time t will be given by

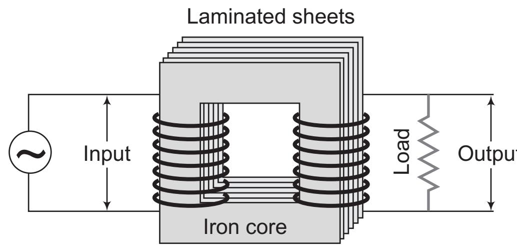
$$\phi = NBA \cos \omega t \qquad (N = \text{number of turns in the coil})$$

$\therefore$
$$e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t$$

or
$$e = e_0 \sin \omega t$$

where,
$$e_0 = NBA\,\omega$$

4. **Transformer** It is a device which is either used to increase or decrease the voltage in AC circuits through mutual induction. A transformer consists of two coils wound on the same core.



The coil connected to input is called primary while the other connected to output is called secondary coil. An alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.

As magnetic lines of force are closed curves, the flux per turn of primary must be equal to flux per turn of the secondary. Therefore,

$$\frac{\phi_P}{N_P} = \frac{\phi_S}{N_S}$$

or
$$\frac{1}{N_P} \cdot \frac{d\phi_P}{dt} = \frac{1}{N_S} \cdot \frac{d\phi_S}{dt} \qquad \left(\text{as } e \propto \frac{d\phi}{dt}\right)$$

$\therefore$
$$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

In an ideal transformer, there is no loss of power. Hence,

$$ei = \text{constant}$$

$\therefore$
$$\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$$

Regarding a transformer, the following are few important points.

(i) In **step-up** transformer, $N_S > N_P$. It increases voltage and reduces current

(ii) In **step-down** transformer, $N_P > N_S$. It increases current and reduces voltage

(iii) It works only on AC

(iv) A transformer cannot increase (or decrease) voltage and current simultaneously. As, $ei = \text{constant}$

(v) Some power is always lost due to eddy currents, hysteresis, etc.

Solved Examples

TYPED PROBLEMS

Type 1. *Based on Faraday's and Lenz's law*

Concept

Problems of induced emf or induced current can be solved by the following two methods.

Method 1 Magnitudes are given by

$$|e| = \left| N \frac{d\phi_B}{dt} \right| \quad \text{and} \quad |i| = \frac{|e|}{R}$$

Direction is given by Lenz's law.

Method 2 Magnitudes are given by

$$|e| = |Bvl| \quad \text{or} \quad |e| = \left| \frac{B\omega l^2}{2} \right| \quad \text{and} \quad |i| = \frac{|e|}{R}$$

Direction is given by right hand rule.

Note *In the first method, we have to first find the magnetic flux passing through the loop and then differentiate it with respect to time. Second method is simple but it can be applied if and only if some conductor is either in translational or rotational motion.*

Example 1 *Current in a long current carrying wire is*

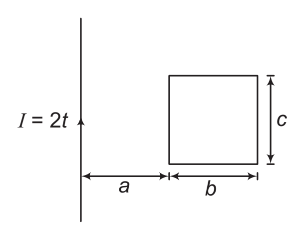
$$I = 2t$$

A conducting loop is placed to the right of this wire. Find

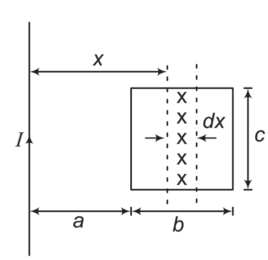
(a) magnetic flux ϕ_B passing through the loop.

(b) induced emf $|e|$ produced in the loop.

(c) if total resistance of the loop is R, then find induced current I_{in} in the loop.



Solution Here, no conductor is in motion. So, we can apply only method-1. Further, magnetic field of straight wire is non-uniform. Therefore, magnetic flux can be obtained by integration.



(a) At a distance x from the straight wire, magnetic field is

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} \qquad [\text{in } \otimes \text{ direction}]$$

Let us take a small strip of width dx.

$\therefore$ Area of this strip is

$$dS = c\,(dx)$$

Now, $d\mathbf{S}$ can also be assumed inwards. Or, angle between $\mathbf{B}$ and $d\mathbf{S}$ may be assumed to be 0°. Therefore, small magnetic flux passing through the loop is

$$d\phi_B = BdS \cos 0° = \frac{\mu_0}{2\pi}\frac{I}{x}\,c\,dx$$

Total magnetic flux is

$$\phi_B = \int_{x=a}^{x=a+b} d\phi_B = \int_a^{a+b}\left(\frac{\mu_0 Ic}{2\pi}\right)\frac{dx}{x} = \frac{\mu_0 Ic}{2\pi}\ln\left(\frac{a+b}{a}\right)$$

Substituting the values of I, we get

$$\phi_B = \frac{\mu_0 ct}{\pi}\ln\left(\frac{a+b}{a}\right)$$ **Ans.**

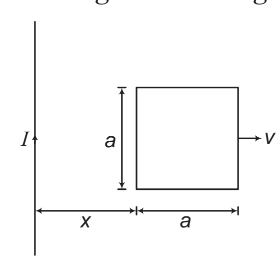
(b)
$$|e| = \left|\frac{d\phi_B}{dt}\right| = \frac{d}{dt}\left[\frac{\mu_0 ct}{\pi}\ln\left(\frac{a+b}{a}\right)\right] = \frac{\mu_0 c}{\pi}\ln\left(\frac{a+b}{a}\right)$$ **Ans.**

(c) Induced current,

$$I_{\text{in}} = \frac{|e|}{R} = \frac{\mu_0 c}{\pi R}\ln\left(\frac{a+b}{a}\right)$$

Note *The main current $I\ (=2t)$ is increasing with time. Hence, $\otimes$ magnetic field passing through the loop will also increase. So, induced current I_{in} will produce $\odot$ magnetic field. Or, induced current is anti-clockwise.*

Example 2 *A constant current I flows through a long straight wire as shown in figure. A square loop starts moving towards right with a constant speed v.*



(a) Find induced emf produced in the loop as a function of x.

(b) If total resistance of the loop is R, then find induced current in the loop.

Note *In this problem, loop is in motion therefore both methods can be applied.*

Solution **Method 1**

(a) Using the result of magnetic flux obtained in Example-1, we have

$$\phi_B = \frac{\mu_0 Ic}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

Here, $a = x, b = c = a$

Substituting the values, we get

$$\phi_B = \frac{\mu_0 Ia}{2\pi} \ln\left(\frac{x+a}{x}\right)$$

$$= \frac{\mu_0 Ia}{2\pi} \ln\left(1 + \frac{a}{x}\right)$$

Now, $$|e| = \left|\frac{d\phi_B}{dt}\right| = \frac{\mu_0 Ia}{2\pi}\left(\frac{x}{x+a}\right)\left(\frac{a}{x^2}\right)\frac{dx}{dt}$$

Putting $\dfrac{dx}{dt} = v$, we have

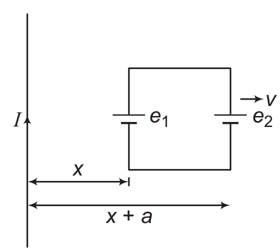
$$|e| = \frac{\mu_0 Ia^2}{2\pi x(x+a)} v$$ **Ans.**

(b) Induced current,

$$I_{\text{in}} = \frac{|e|}{R} = \frac{\mu_0 Ia^2 v}{2\pi Rx(x+a)}$$ **Ans.**

Note *Near the wire (towards right) value of ⊗ magnetic field is high. So, the loop is moving from higher magnetic field to lower magnetic field. or, ⊗ magnetic field passing through the loop is decreasing. Hence, induced current will produce ⊗ magnetic field or it should be clockwise..*

Method 2



$$e_1 = B_1 vl = \frac{\mu_0}{2\pi}\frac{I}{x} va$$

$$e_2 = B_2 vl = \frac{\mu_0}{2\pi}\frac{I}{x+a} va$$

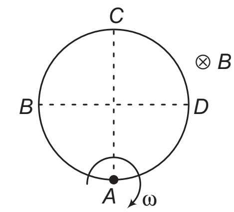
$$e_1 > e_2$$

$\therefore$ $$e_{\text{net}} = e_1 - e_2 = \frac{\mu_0 Iva}{2\pi}\left(\frac{1}{x} - \frac{1}{x+a}\right)$$

$$= \frac{\mu_0 Iva^2}{2\pi x(x+a)}$$

This is the same result as was obtained in Method 1.

Example 3 *A conducting circular ring is rotated with angular velocity ω about point A as shown in figure. Radius of ring is a. Find*

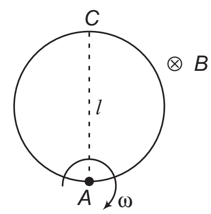


(a) potential difference between points A and C

(b) potential difference between the points A and D.

Solution Here, the loop is rotating. So, we can applying $e = \dfrac{B\omega l^2}{2}$

(a)

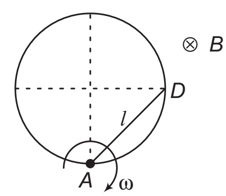


Using right hand rule, we can see that

$$V_C > V_A$$

$$\therefore \qquad V_C - V_A = \frac{B\omega l^2}{2} = \frac{B\omega (2a)^2}{2} = 2B\omega a^2$$ **Ans.**

(b)



Using right hand rule, we can see that

$$V_D > V_A$$

$$\therefore \qquad V_D - V_A = \frac{B\omega l^2}{2} = \frac{B\omega (\sqrt{2}\, a)^2}{2} = B\omega a^2$$ **Ans.**

Type 2. *Based on potential difference across an inductor*

Example 4 *Two different coils have self-inductances $L_1 = 8\ mH$ and $L_2 = 2\ mH$. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then,* **(JEE 1994)**

(a) $\dfrac{i_1}{i_2} = \dfrac{1}{4}$

(b) $\dfrac{i_1}{i_2} = 4$

(c) $\dfrac{W_1}{W_2} = \dfrac{1}{4}$

(d) $\dfrac{V_1}{V_2} = 4$

Solution **Potential difference across an inductor :**

$V \propto L$, if rate of change of current is constant $\left(V = -L\dfrac{di}{dt}\right)$

$$\therefore \qquad \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$$

$$\text{or} \qquad \frac{V_1}{V_2} = 4$$

Power given to the two coils is same, i.e.

$$V_1 i_1 = V_2 i_2$$

$$\text{or} \qquad \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

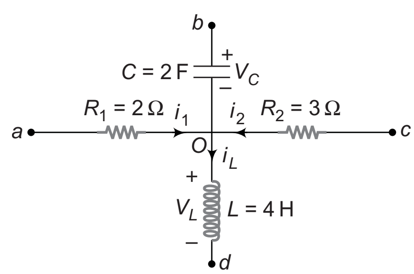
Energy stored, $W = \dfrac{1}{2}Li^2$

$$\therefore \qquad \frac{W_2}{W_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{i_2}{i_1}\right)^2 = \left(\frac{1}{4}\right)(4)^2$$

$$\text{or} \qquad \frac{W_1}{W_2} = \frac{1}{4}$$

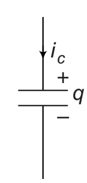
$\therefore$ The correct options are (a), (c) and (d).

Example 5 *In the figure shown,* $i_1 = 10\,e^{-2t}$ *A,* $i_2 = 4$ *A and* $V_C = 3e^{-2t}$ *V. Determine* (JEE 1992)



(a) i_L *and* V_L *(b)* V_{ac}, V_{ab} *and* V_{cd}.

Solution (a) Charge stored in the capacitor at time t,



$$q = CV_C$$
$$= (2)(3e^{-2t})$$
$$= 6e^{-2t}\ \text{C}$$

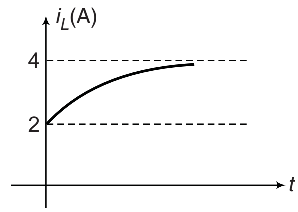
$$\therefore \qquad i_c = \frac{dq}{dt} = -\,12e^{-2t}\ \text{A}$$

(Direction of current is from b to O)

Applying junction rule at O,

$$i_L = i_1 + i_2 + i_c = 10e^{-2t} + 4 - 12e^{-2t}$$
$$= (4 - 2e^{-2t})\ \text{A}$$
$$= [2 + 2\,(1 - e^{-2t})]\ \text{A} \qquad \textbf{Ans.}$$

i_L *versus* time graph is as shown in figure.

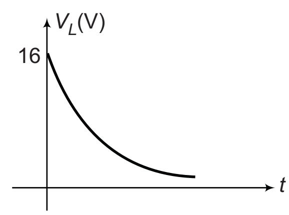


i_L increases from 2 A to 4 A exponentially.

$$V_L = V_{Od} = L\frac{di_L}{dt} = (4)\frac{d}{dt}(4 - 2e^{-2t})$$

$$= 16e^{-2t} \text{ V}$$ **Ans.**

V_L *versus* time graph is as shown in figure.



V_L decreases exponentially from 16 V to 0.

(b) $\boldsymbol{V_{ac} = V_a - V_c}$

$$V_a - i_1R_1 + i_2R_2 = V_c$$

$\therefore$ $$V_a - V_c = V_{ac} = i_1R_1 - i_2R_2$$

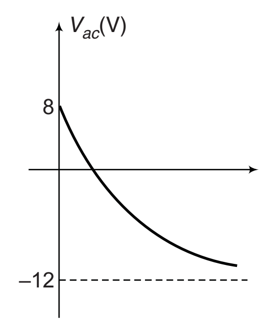
Substituting the values, we have

$$V_{ac} = (10e^{-2t})(2) - (4)(3)$$

$$V_{ac} = (20e^{-2t} - 12) \text{ V}$$

At $t = 0$, $V_{ac} = 8$ V and at $t = \infty$, $V_{ac} = -12$V

Therefore, V_{ac} decreases exponentially from 8 V to –12 V.



$\boldsymbol{V_{ab} = V_a - V_b}$

$$V_a - i_1R_1 + V_C = V_b$$

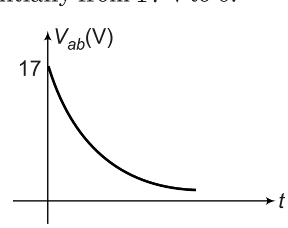
$\therefore$ $$V_a - V_b = V_{ab} = i_1R_1 - V_C$$

Substituting the values, we have

$$V_{ab} = (10e^{-2t})(2) - 3e^{-2t}$$

or $$V_{ab} = 17e^{-2t} \text{ V}$$ **Ans.**

Thus, V_{ab} decreases exponentially from 17 V to 0.



V_{ab} *versus* t graph is shown in figure.

$V_{cd} = V_c - V_d$

$$V_c - i_2R_2 - V_L = V_d$$

$$\therefore \qquad V_c - V_d = V_{cd} = i_2R_2 + V_L$$

Substituting the values, we have

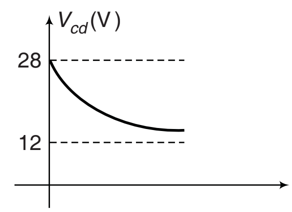
$$V_{cd} = (4)(3) + 16e^{-2t}$$

or $$V_{cd} = (12 + 16e^{-2t})\ \text{V} \qquad \textbf{Ans.}$$

At $t = 0$, $V_{cd} = 28$ V and at $t = \infty$, $V_{cd} = 12$ V

i.e. V_{cd} decreases exponentially from 28 V to 12 V.

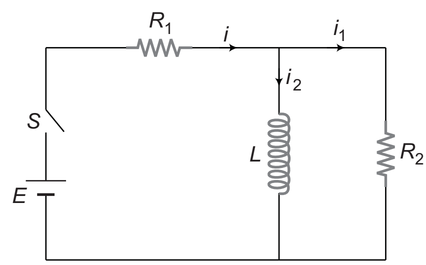
V_{cd} *versus* t graph is shown in figure.



Type 3. *Based on L - R circuit*

Concept

At time $t = 0$, when there is zero current in the circuit, an inductor offers infinite resistance and at $t = \infty$, when steady state is reached an ideal inductor (of zero resistance) offers zero resistance.



Thus, in the circuit shown, if switch S is closed at time $t = 0$, then

$$i_2 = 0$$

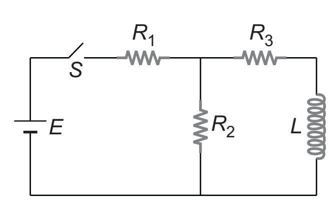
and $$i = i_1 = \frac{E}{R_1 + R_2} \quad \text{at} \quad t = 0$$

as initially the inductor offers infinite resistance and at $t = \infty$,

$$i_1 = 0, \quad \text{while} \quad i = i_2 = \frac{E}{R_1}$$

as in steady state the inductor offers zero resistance.

Example 6 *For the circuit shown in figure, $E = 50$ V, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$ and $L = 2.0$ mH. Find the current through R_1 and R_2.*



(a) Immediately after switch S is closed.

(b) A long time after S is closed.

(c) Immediately after S is reopened.

(d) A long time after S is reopened.

Solution (a) Resistance offered by inductor immediately after switch is closed will be infinite. Therefore, current through R_3 will be zero and

$$\text{current through } R_1 = \text{current through } R_2 = \frac{E}{R_1 + R_2}$$

$$= \frac{50}{10+20} = \frac{5}{3}\text{ A} \qquad \textbf{Ans.}$$

(b) After long time of closing the switch, resistance offered by inductor will be zero.

In that case R_2 and R_3 are in parallel, and the resultant of these two is then in series with R_1. Hence,

$$R_{\text{net}} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$= 10 + \frac{(20)(30)}{20+30} = 22\ \Omega$$

Current through the battery (or through R_1)

$$= \frac{E}{R_{\text{net}}} = \frac{50}{22}\text{ A} \qquad \textbf{Ans.}$$

This current will distribute in R_2 and R_3 in inverse ratio of resistance. Hence,

$$\text{Current through } R_2 = \left(\frac{50}{22}\right)\left(\frac{R_3}{R_2 + R_3}\right)$$

$$= \left(\frac{50}{22}\right)\left(\frac{30}{30+20}\right) = \frac{15}{11}\text{ A} \qquad \textbf{Ans.}$$

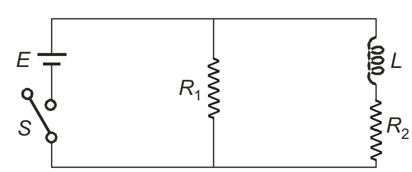
(c) Immediately after switch is reopened, the current through R_1 will become zero.

But current through R_2 will be equal to the steady state current through R_3, which is equal to,

$$\left(\frac{50}{22} - \frac{15}{11}\right)\text{A} = 0.91\text{ A} \qquad \textbf{Ans.}$$

(d) A long after S is reopened, current through all resistors will be zero.

Example 7 *An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\,\Omega$ and $R_2 = 2\,\Omega$ are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$.*



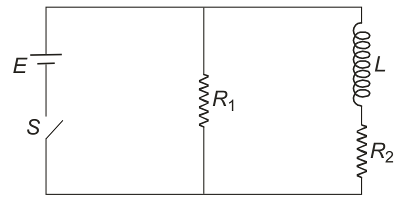
What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time?

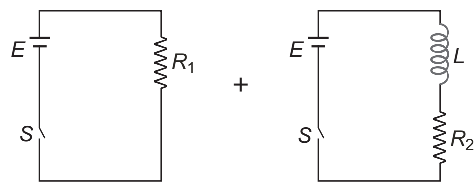
(JEE 2001)

Solution (a) Given, $R_1 = R_2 = 2\ \Omega$, $E = 12$ V and $L = 400$ mH $= 0.4$ H.

Two parts of the circuit are in parallel with the applied battery.

So, the given circuit can be broken as :





(a) (b)

Now refer Fig. (b)

This is a simple L-R circuit, whose time constant

$$\tau_L = L/R_2 = \frac{0.4}{2} = 0.2 \text{ s}$$

and steady state current

$$i_0 = \frac{E}{R_2} = \frac{12}{2} = 6 \text{ A}$$

Therefore, if switch S is closed at time $t = 0$, then current in the circuit at any time t will be given by

$$i(t) = i_0 (1 - e^{-t/\tau_L})$$
$$i(t) = 6(1 - e^{-t/0.2})$$
$$= 6(1 - e^{-5t}) = i \qquad \text{(say)}$$

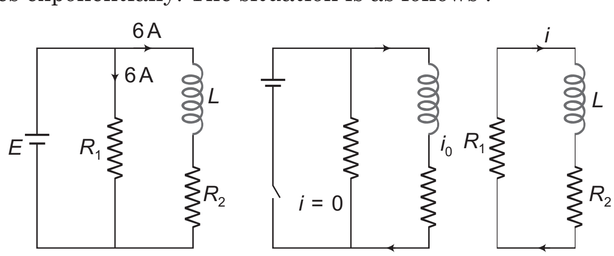
Therefore, potential drop across L at any time t is

$$V = \left| L\frac{di}{dt} \right| = L(30e^{-5t}) = (0.4)(30)\, e^{-5t} \quad \text{or} \quad V = 12\, e^{-5t} \text{ volt}$$

(b) The steady state current in L or R_2 is

$$i_0 = 6 \text{ A}$$

Now, as soon as the switch is opened, current in R_1 is reduced to zero immediately. But in L and R_2 it decreases exponentially. The situation is as follows :



Steady state condition
(c) (d) (e)

Refer figure (e)

Time constant of this circuit would be

$$\tau_L' = \frac{L}{R_1 + R_2} = \frac{0.4}{(2+2)} = 0.1 \text{ s}$$

$\therefore$ Current through R_1 at any time t is

$$i = i_0 e^{-t/\tau_{L'}} = 6\, e^{-t/0.1}$$

or $$i = 6\, e^{-10t} \text{ A}$$

Direction of current in R_1 is as shown in figure or clockwise.

Example 8 *A solenoid has an inductance of 10 H and a resistance of 2 Ω. It is connected to a 10 V battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value?* **(JEE 1996)**

Solution $U = \frac{1}{2} L i^2$, i.e. $U \propto i^2$

U will reach $\frac{1}{4}$th of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$$i = i_0 (1 - e^{-t/\tau_L})$$

Here, i_0 = Maximum value of current

τ_L = Time constant = L/R

$$\tau_L = \frac{10 \text{ H}}{2\ \Omega} = 5 \text{ s}$$

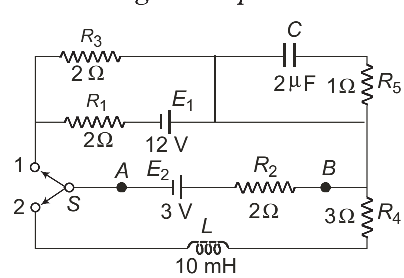
Therefore, $$i = i_0/2 = i_0 (1 - e^{-t/5})$$

or $$\frac{1}{2} = 1 - e^{-t/5} \quad \text{or} \quad e^{-t/5} = \frac{1}{2}$$

or $$-t/5 = \ln\left(\frac{1}{2}\right) \quad \text{or} \quad t/5 = \ln(2) = 0.693$$

$\therefore$ $$t = (5)(0.693) \text{ or } t = 3.465 \text{ s}$$

Example 9 *A circuit containing a two position switch S is shown in figure.*

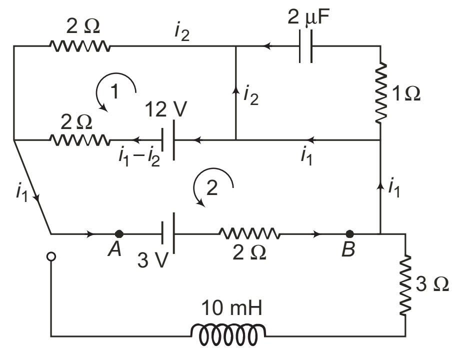


(a) *The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.*

(b) *If now the switch S is put in position 2 at $t = 0$. Find*

(i) steady current in R_4 and (ii) the time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time.

Solution (a) In steady state, no current will flow through capacitor.



Applying Kirchhoff's second law in loop 1,

$$-2i_2 + 2(i_1 - i_2) + 12 = 0$$

$\therefore$ $$2i_1 - 4i_2 = -12$$

or $$i_1 - 2i_2 = -6 \qquad \text{...(i)}$$

Applying Kirchhoff's second law in loop 2,

$$-12 - 2(i_1 - i_2) + 3 - 2i_1 = 0$$

$\therefore$ $$4i_1 - 2i_2 = -9 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$i_2 = 2.5 \text{ A} \quad \text{and} \quad i_1 = -1 \text{ A}$$

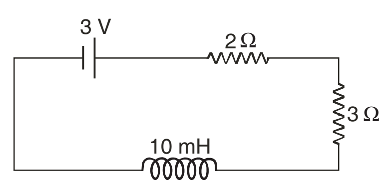
Now, $$V_A + 3 - 2i_1 = V_B \quad \text{or} \quad V_A - V_B = 2i_1 - 3$$

$$= 2(-1) - 3 = -5 \text{ V}$$

$$P_{R_1} = (i_1 - i_2)^2 R_1 = (-1 - 2.5)^2 (2)$$

$$= 24.5 \text{ W}$$

(b) **In position 2** Circuit is as below



Steady current in R_4,

$$i_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

Time when current in R_4 is half the steady value,

$$i = i_0(1 - e^{-t/\tau_L})$$

$$i = i_0/2 \text{ at } t = t_{1/2}, \text{ where}$$

$$t_{1/2} = \tau_L (\ln 2) = \frac{L}{R} \ln(2) = \frac{(10 \times 10^{-3})}{5} \ln(2)$$

$$= 1.386 \times 10^{-3} \text{ s}$$

$$U = \frac{1}{2} Li^2 = \frac{1}{2}(10 \times 10^{-3})(0.3)^2$$

$$= 4.5 \times 10^{-4} \text{ J}$$

Type 4. *Based on L-C oscillations*

Example 10 *In an L-C circuit, $L = 3.3$ H and $C = 840$ pF. At $t = 0$, charge on the capacitor is $105\ \mu C$ and maximum. Compute the following quantities at $t = 2.0$ ms:*

(a) The energy stored in the capacitor.

(b) The total energy in the circuit,

(c) The energy stored in the inductor.

Solution Given, $L = 3.3$ H, $C = 840 \times 10^{-12}$ F and $q_0 = 105 \times 10^{-6}$ C

The angular frequency of L-C oscillations is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3 \times 840 \times 10^{-12}}}$$

$$= 1.9 \times 10^4 \text{ rad/s}$$

Charge stored in the capacitor at time t would be

$$q = q_0 \cos \omega t$$

(a) At $t = 2 \times 10^{-3}$ s,

$$q = (105 \times 10^{-6}) \cos [1.9 \times 10^4][2 \times 10^{-3}]$$

$$= 100.3 \times 10^{-6} \text{ C}$$

$\therefore$ Energy stored in the capacitor,

$$U_C = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{(100.3 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}}$$

$$= 6.0 \text{ J}$$ **Ans.**

(b) Total energy in the circuit,

$$U = \frac{1}{2}\frac{q_0^2}{C} = \frac{(105 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}}$$

$$= 6.56 \text{ J}$$ **Ans.**

(c) Energy stored in inductor in the given time

= total energy in circuit – energy stored in capacitor

$= (6.56 - 6.0)$ J

$= 0.56$ J **Ans.**

Example 11 *An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance $5.0\ \mu$F and the resulting L-C circuit is set oscillating at its natural frequency. Let Q denotes the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is $200\ \mu C$.* **(JEE 1998)**

(a) When $Q = 100\ \mu C$, what is the value of $|dI/dt|$?

(b) When $Q = 200\ \mu C$, what is the value of I?

(c) Find the maximum value of I.

(d) When I is equal to one-half of its maximum value, what is the value of $|Q|$?

Solution This is a problem of L-C oscillations.

Charge stored in the capacitor oscillates simple harmonically as

$$Q = Q_0 \sin\ (\omega t \pm \phi)$$

Here, Q_0= maximum value of $Q = 200\,\mu\text{C} = 2 \times 10^{-4}$ C

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2 \times 10^{-3})(5.0 \times 10^{-6})}} = 10^4\text{s}^{-1}$$

Let at $t = 0$, $Q = Q_0$, then

$$Q(t) = Q_0 \cos \omega t \qquad \text{...(i)}$$

$$I(t) = \frac{dQ}{dt} = -Q_0\omega \sin \omega t \text{ and} \qquad \text{...(ii)}$$

$$\frac{dI(t)}{dt} = -Q_0\omega^2 \cos \omega t \qquad \text{...(iii)}$$

(a) $Q = 100\,\mu\text{C}$

or $$\frac{Q_0}{2} \text{ at } \cos \omega t = \frac{1}{2}$$

or $$\omega t = \frac{\pi}{3}$$

At $\cos \omega t = \frac{1}{2}$, from Eq. (iii) :

$$\left|\frac{dI}{dt}\right| = (2.0 \times 10^{-4}\text{C})(10^4\text{s}^{-1})^2 \left(\frac{1}{2}\right)$$

$$\left|\frac{dI}{dt}\right| = 10^4 \text{ A/s}$$

(b) $Q = 200\,\mu\text{C}$ or Q_0 when $\cos \omega t = 1$, i. e. $\omega t = 0, 2\pi$ ...

At this time $$I(t) = -Q_0\omega \ \sin \omega t$$

or $$I(t) = 0 \qquad (\sin 0° = \sin 2\pi = 0)$$

(c) $I(t) = -Q_0\omega \sin \omega t$

$\therefore$ Maximum value of I is $Q_0\,\omega$

$$I_{\text{max}} = Q_0\,\omega = (2.0 \times 10^{-4})(10^4)$$

$$I_{\text{max}} = 2.0 \text{ A}$$

(d) From energy conservation,

$$\frac{1}{2} LI_{\text{max}}^2 = \frac{1}{2} LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

or $$Q = \sqrt{LC(I_{\text{max}}^2 - I^2)}$$

$$I = \frac{I_{\text{max}}}{2} = 1.0 \text{ A}$$

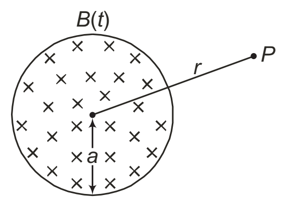
$\therefore$ $$Q = \sqrt{(2.0 \times 10^{-3})(5.0 \times 10^{-6})(2^2 - 1^2)}$$

$$Q = \sqrt{3} \times 10^{-4} \text{ C}$$

or $$Q = 1.732 \times 10^{-4} \text{ C}$$

Type 5. *Based on induced electric field*

Example 12 *A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region* **(JEE 2000)**



(a) is zero *(b) decreases as 1/r* *(c) increases as r* *(d) decreases as* $1/r^2$

Solution $\int \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$ or $E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$

For $r \geq a$,

$\therefore$ $$E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

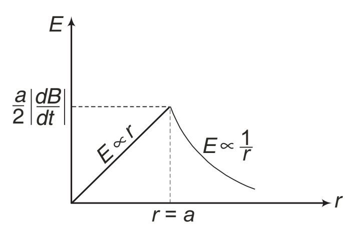
$\therefore$ Induced electric field $\propto 1/r$

For $r \leq a$,

$$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right| \quad \text{or} \quad E = \frac{r}{2}\left|\frac{dB}{dt}\right| \quad \text{or} \quad E \propto r$$

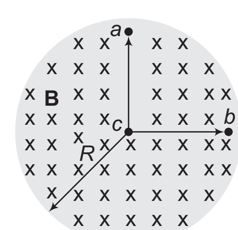
At $r = a$, $E = \frac{a}{2}\left|\frac{dB}{dt}\right|$

Therefore, variation of E with r (distance from centre) will be as follows



$\therefore$ The correct option is (b).

Example 13 *The magnetic field* **B** *at all points within a circular region of radius R is uniform in space and directed into the plane of the page in figure. If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b and c? (Point a is a distance r above the centre of the region, point b is a distance r to the right of the centre, and point c is at the centre of the region.)*



Solution Inside the circular region at distance r,

$$El = \frac{d\phi}{dt} = S\left(\frac{dB}{dt}\right)$$

$$\therefore \qquad E\,(2\pi r) = (\pi r^2) \cdot \frac{dB}{dt}$$

$$\therefore \qquad E = \frac{r}{2}\frac{dB}{dt}$$

$$F = qE = \frac{qr}{2}\frac{dB}{dt}$$

At points a and b, distance from centre is r.

$$\therefore \qquad F = \frac{qr}{2}\frac{dB}{dt}$$

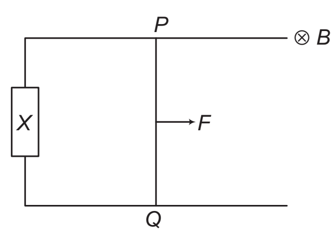
At point C, distance $r = 0$

$$\therefore \qquad F = 0$$

$\otimes$ magnetic field is increasing. Hence, induced current in an imaginary loop passing through a and b should produce $\odot$ magnetic field. Hence, induced current through an imaginary circular loop passing through a and b should be anti-clockwise. Force on positive charge is in the direction of induced current. Hence, force at a is towards left and force at b is upwards.

Type 6. *Based on motion of a wire in uniform magnetic field with other element like resistance, capacitor or an inductor*

Concept



A constant force F is applied on wire PQ of length l and mass m. There is an electrical element X in the box as shown in figure. There are the following three different cases :

Case 1 If X is a resistance, then velocity of the wire increases exponentially.

Case 2 If X is a capacitor, then wire moves with a constant acceleration a $(< F/m)$.

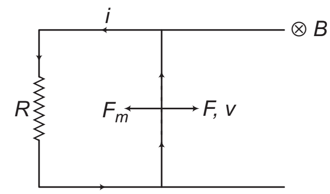
Case 3 If X is an inductor and instead of constant force F an initial velocity v_0 is given to the wire then the wire starts simple harmonic motion with v_0 as the maximum velocity $(= \omega A)$ at mean position.

Example 14 *In the above case if X is a resistance R, then find velocity of wire as a function of time t.*

Solution At time t suppose velocity of wire is v, then due to motional emf a current i flows in the closed circuit in anti-clockwise direction.

$$i = \frac{e}{R} = \frac{Bvl}{R}$$

Due to this current magnetic force will act on the wire in the direction shown in figure,



$$F_m = ilB = \left(\frac{B^2l^2}{R}\right)v$$

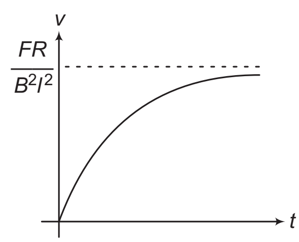
$$F_{\text{net}} = F - F_m = F - \left(\frac{B^2\,l^2}{R}\right)v = m\frac{dv}{dt}$$

$\therefore$
$$\int_0^v \frac{dv}{F - \left(\frac{B^2\,l^2}{R}\right)v} = \frac{1}{m}\int_0^t dt$$

Solving this equation, we get

$$v = \frac{FR}{B^2l^2}\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$$

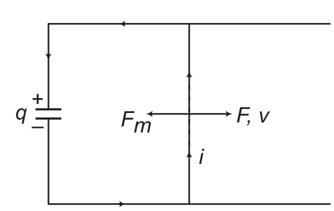
Thus, velocity of the wire increases exponentially. v- t graph is as shown below.



Example 15 *If X is a capacitor C, then find the constant acceleration a of the wire.*

Solution At time t *suppose velocity of wire is v. Then, due to motional emf* $e = Bvl$ *capacitor gets charged .*

$$q = CV = C\,(Bvl)$$



This charge is increasing as v will be increasing.

Hence, there will be a current in the circuit as shown in figure.

$$i = \frac{dq}{dt} = (BlC)\frac{dv}{dt}$$

or $$i = (BlC)\,a \qquad \left(\text{as } \frac{dv}{dt} = a\right)$$

Due to this current a magnetic force F_m will act in the direction shown in figure,

$$F_m = ilB = (B^2l^2C)\,a$$

Now,

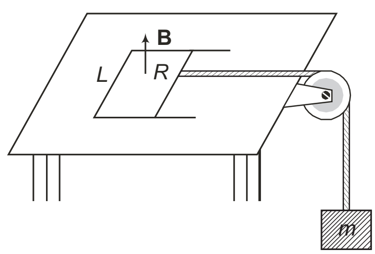
$$F_{\text{net}} = F - F_m$$

or $$ma = F - (B^2l^2C)\,a$$

∴ $$a = \frac{F}{m + B^2l^2C}$$ **Ans.**

Now, we can see that this acceleration is constant but less than F/m.

Example 16 *A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate* **(JEE 1997)**



(a) *the terminal velocity achieved by the rod and*

(b) *the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.*

Solution (a) Let v be the velocity of the wire (as well as block) at any instant of time t.

Motional emf, $e = BvL$

Motional current, $i = \dfrac{e}{r} = \dfrac{BvL}{R}$

and magnetic force on the wire

$$F_m = iLB = \frac{vB^2L^2}{R}$$

Net force on the system at this moment will be

$$F_{\text{net}} = mg - F_m = mg - \frac{vB^2L^2}{R}$$

or $$ma = mg - \frac{vB^2L^2}{R}$$

$$a = g - \frac{vB^2L^2}{mR} \qquad \text{...(i)}$$

Velocity will acquire its terminal value, i.e. $v = v_T$ when F_{net} or acceleration a of the particle becomes zero.

Thus, $$0 = g - \frac{v_T B^2 L^2}{mR}$$

or $$v_T = \frac{mgR}{B^2 L^2}$$

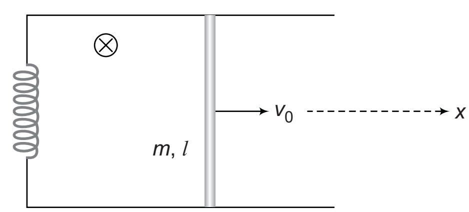
(b) When $$v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$$

Then from Eq. (i), acceleration of the block,

$$a = g - \left(\frac{mgR}{2B^2L^2}\right)\left(\frac{B^2L^2}{mR}\right) = g - \frac{g}{2}$$

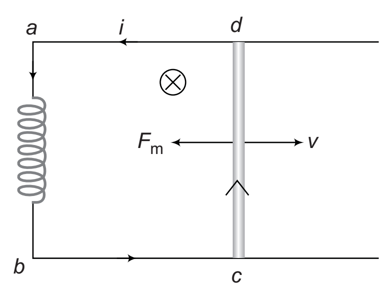
or $$a = \frac{g}{2}$$

Example 17 *A loop is formed by two parallel conductors connected by a solenoid with inductance L and a conducting rod of mass m which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane in a uniform vertical magnetic field B. The distance between the conductors is l.*



At the moment $t = 0$, the rod is imparted an initial velocity v_0 directed to the right. Find the law of its motion x (t) if the electric resistance of the loop is negligible.

Solution Let at any instant of time, velocity of the rod is v towards right. The current in the circuit is i. In the figure,



$$V_a - V_b = V_d - V_c$$

or $$L\frac{di}{dt} = Bvl = Bl\frac{dx}{dt} \qquad \left(\text{as } v = \frac{dx}{dt}\right)$$

i.e. $$Ldi = Bldx$$

Integrating on both sides, we get

$$Li = Blx$$

or $$i = \frac{Bl}{L}x \qquad \text{...(i)}$$

Magnetic force on the rod at this instant is

$$F_m = ilB = \frac{B^2l^2}{L}x \qquad \text{...(ii)}$$

Since, this force is in opposite direction of **v**, so from Newton's second law we can write

$$m\left(\frac{d^2x}{dt^2}\right) = -\frac{B^2l^2}{L}x$$

or $$\left(\frac{d^2x}{dt^2}\right) = -\frac{B^2l^2}{mL}x$$

Comparing this with equation of SHM,

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

We have, $$\omega = \frac{Bl}{\sqrt{mL}}$$

Therefore, the rod will oscillate simple harmonically with angular frequency $\omega = \frac{Bl}{\sqrt{mL}}$. At time $t = 0$, rod was at $x = 0$ and it was moving towards positive x-axis. Hence, x-t equation of the rod is

$$x = A\sin\omega t \qquad \text{...(iii)}$$

To find A, we use the fact that at $t = 0$, v or $\frac{dx}{dt}$ has a value v_0. Hence,

$$\frac{dx}{dt} = v = A\omega\cos\omega t$$

or $$A\omega = v_0 \qquad \text{(at } t = 0\text{)}$$

or $$A = \frac{v_0}{\omega}$$

Substituting in Eq. (iii), we have

$$x = \frac{v_0}{\omega}\sin\omega t, \text{ where } \omega = \frac{Bl}{\sqrt{mL}} \qquad \textbf{Ans.}$$

Alternate method of finding *A*

At $x = A$, $v = 0$, i.e. whole of its kinetic energy is converted into magnetic energy. Thus,

$$\frac{1}{2}Li^2 = \frac{1}{2}mv_0^2$$

Substituting value of i from Eq. (i), with $x = A$, we have

$$L\left(\frac{Bl}{L}A\right)^2 = mv_0^2$$

or $$A = \frac{\sqrt{mL}}{Bl}v_0 = \frac{v_0}{\omega}$$

as $$\omega = \frac{Bl}{\sqrt{mL}} \qquad \textbf{Ans.}$$

Miscellaneous Examples

Example 18 *A sensitive electronic device of resistance 175 Ω is to be connected to a source of emf by a switch. The device is designed to operate with a current of 36 mA, but to avoid damage to the device, the current can rise to no more than 4.9 mA in the first 58 μs after the switch is closed. To protect the device it is connected in series with an inductor.*

(a) What emf must the source have?

(b) What inductance is required?

(c) What is the time constant?

Solution (a) Given, $R = 175\ \Omega$ and peak value current

$$i_0 = 36 \times 10^{-3}\ \text{A}$$

Applied voltage, $V = i_0 R = (175)(36 \times 10^{-3})$ volt $= 6.3$ V **Ans.**

(b) From the relation,

$$i = i_0 (1 - e^{-t/\tau_L})$$

We have, $(4.9) = (36)[1 - e^{-t/\tau_L}]$

or $e^{-t/\tau_L} = 1 - \dfrac{4.9}{36} = 0.864$

∴ $\dfrac{t}{\tau_L} = -\ln(0.864) = 0.146$

or $\dfrac{t}{L/R} = 0.146$

∴ $\dfrac{Rt}{L} = 0.146$

or $L = \dfrac{Rt}{0.146} = \dfrac{(175)(58 \times 10^{-6})}{0.146}$

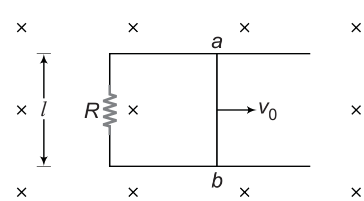
$= 7.0 \times 10^{-2}$ H **Ans.**

(c) Time constant of the circuit,

$$\tau_L = \frac{L}{R} = \frac{7.0 \times 10^{-2}}{175}$$

$= 4.0 \times 10^{-4}$ **Ans.**

Example 19 *A conducting rod shown in figure of mass m and length l moves on two frictionless horizontal parallel rails in the presence of a uniform magnetic field directed into the page. The rod is given an initial velocity v_0 to the right and is released at $t = 0$. Find as a function of time,*



(a) the velocity of the rod

(b) the induced current and

(c) the magnitude of the induced emf.

HOW TO PROCEED *The initial velocity will produce an induced emf and hence, an induced current in the circuit. The current carrying wire will now experience a magnetic force* ($\mathbf{F}_m$) *in opposite direction of its velocity. The force will retard the motion of the conductor. Thus,*

Initial velocity → motional emf → induced current → magnetic force → retardation.

Solution (a) Let v be the velocity of the rod at time t.

Current in the circuit at this moment is

$$i = \frac{Bvl}{R} \qquad \ldots\text{(i)}$$

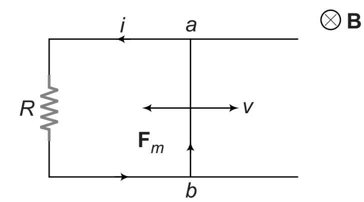
From right hand rule, we can see that this current is in counterclockwise direction.

The magnetic force is,

$$F_m = -\, ilB = -\frac{B^2l^2}{R}\, v$$

Here, negative sign denotes that the force is to the left and retards the motion. This is the only horizontal force acting on the bar, and hence, Newton's second law applied to motion in horizontal direction gives

$$m\frac{dv}{dt} = F_m = -\frac{B^2l^2}{R}\, v$$



$$\therefore \qquad \frac{dv}{v} = -\left(\frac{B^2l^2}{mR}\right) dt$$

Integrating this equation using the initial condition that, $v = v_0$ at $t = 0$, we find that

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

Solving this equation, we find that

$$v = v_0 e^{-t/\tau} \qquad \ldots\text{(ii)} \quad \textbf{Ans.}$$

where, $$\tau = \frac{mR}{B^2l^2}$$

This expression indicates that the velocity of the rod decreases exponentially with time under the action of the magnetic retarding force.

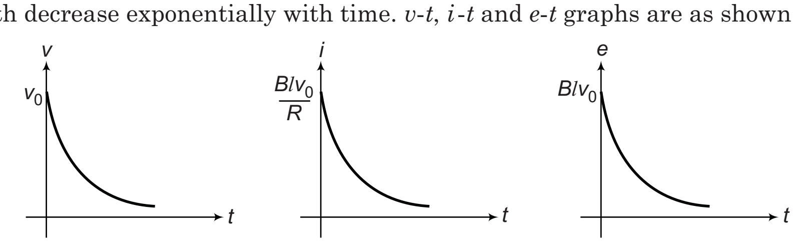
(b) $i = \dfrac{Bvl}{R}$

Substituting the value of v from Eq. (ii), we get

$$i = \frac{Blv_0}{R}\, e^{-t/\tau} \qquad \textbf{Ans.}$$

(c) $$e = iR = Blv_0 e^{-t/\tau} \qquad \textbf{Ans.}$$

i and e both decrease exponentially with time. v-t, i-t and e-t graphs are as shown in figure



Alternate solution This problem can also be solved by energy conservation principle. Let at some instant velocity of the rod is v. As no external force is present. Energy is dissipated in the resistor at the cost of kinetic energy of the rod. Hence,

$$\left(-\frac{dK}{dt}\right) = \text{power dissipated in the resistor}$$

or
$$-\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{e^2}{R}$$

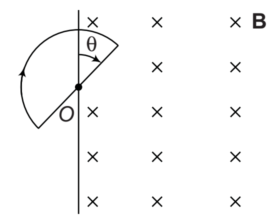
or
$$-mv\left(\frac{dv}{dt}\right) = \frac{B^2l^2v^2}{R} \qquad (\text{as } e = Bvl)$$

$\therefore$
$$\frac{dv}{v} = -\frac{B^2l^2}{mR}dt$$

$\therefore$
$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

or
$$v = v_0e^{-t/\tau}, \quad \text{where } \tau = \frac{mR}{B^2l^2}$$

Example 20 *A wire loop enclosing a semicircle of radius R is located on the boundary of a uniform magnetic field B. At the moment $t = 0$, the loop is set into rotation with a constant angular acceleration α about an axis O coinciding with a line of vector* **B** *on the boundary. Find the emf induced in the loop as a function of time. Draw the approximate plot of this function. The arrow in the figure shows the emf direction taken to be positive.*



Solution
$$\theta = \frac{1}{2}\alpha t^2$$

$\therefore$
$$t = \sqrt{\frac{2\theta}{\alpha}} = \text{time taken to rotate an angle } \theta$$

where, $\theta = 0$ to $\pi, 2\pi$ to 3π, 4π to 5π etc.

$\otimes$ magnetic field passing through the loop is increasing. Hence, current in the loop is anti-clockwise or induced emf is negative. And for, $\theta = \pi$ to 2π, 3π to 4π, 5π to 6π etc.

$\otimes$ magnetic field passing through the loop is decreasing. Hence, current in the loop is clockwise or emf is positive.

So,

$$t_1 = \text{time taken to rotate an angle } \pi = \sqrt{\frac{2\pi}{\alpha}}$$

$$t_2 = \text{time taken to rotate an angle } 2\pi = \sqrt{\frac{4\pi}{\alpha}}$$

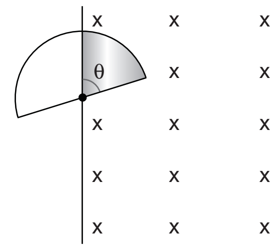
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$t_n = \text{time taken to rotate an angle } n\pi = \sqrt{\frac{2n\pi}{\alpha}}$$

Now, from 0 to t_1 emf is negative
t_1 to t_2 emf is positive
t_2 to t_3 emf is again negative

and so on.

Now, at time t, angle rotated is $$\theta = \frac{1}{2}\alpha t^2$$



Area inside the field is
$$S = (\pi R^2)\left(\frac{\theta}{2\pi}\right) = \frac{1}{2}R^2\theta$$

or $$S = \frac{1}{4}R^2\alpha t^2$$

So, flux passing through the loop, $$\phi = BS = \frac{1}{4}BR^2\alpha t^2$$

$$e = \left|\frac{d\phi}{dt}\right| = \frac{1}{2}BR^2\alpha t$$

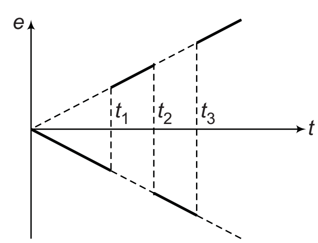
$$e \propto t$$

i.e. e-t graph is a straight line passing through origin. e-t equation with sign can be written as

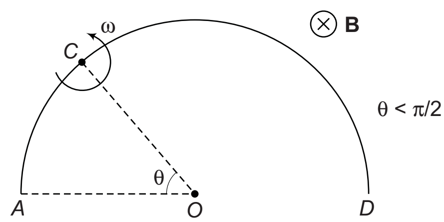
$$e = (-1)^n\left(\frac{1}{2}BR^2\alpha t\right)$$ **Ans.**

Here, $n = 1, 2, 3 \ldots$ is the number of half revolutions that the loop performs at the given moment t.

The e-t graph is as shown in figure.



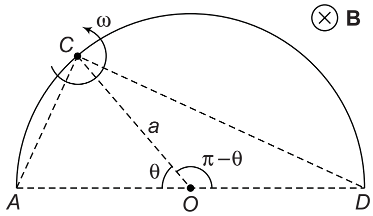
Example 21 *A uniform wire of resistance per unit length λ is bent into a semicircle of radius a. The wire rotates with angular velocity ω in a vertical plane about a horizontal axis passing through C. A uniform magnetic field B exists in space in a direction perpendicular to paper inwards.*



(a) *Calculate potential difference between points A and D. Which point is at higher potential?*

(b) *If points A and D are connected by a conducting wire of zero resistance, find the potential difference between A and C.*

Solution (a) Length of straight wire AC is $l_1 = 2a\sin\left(\frac{\theta}{2}\right)$



Therefore, the motional emf (or potential difference) between points C and A is

$$V_{CA} = V_C - V_A = \frac{1}{2}B\omega l_1^2 = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right) \quad \text{...(i)}$$

From right hand rule, we can see that $V_C > V_A$

Similarly, length of straight wire CD is

$$l_2 = 2a\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = 2a\cos\left(\frac{\theta}{2}\right)$$

Therefore, the PD between points C and D is

$$V_{CD} = V_C - V_D = \frac{1}{2}B\omega l_2^2 = 2a^2B\omega\cos^2\left(\frac{\theta}{2}\right) \quad \text{...(ii)}$$

with $V_C > V_D$

Eq. (ii) − Eq.(i) gives,

$$V_A - V_D = 2a^2B\omega\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)$$

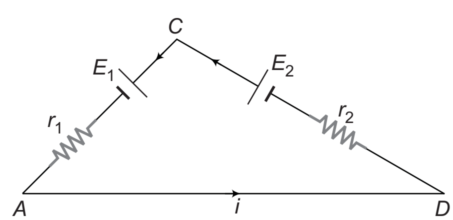
$$= 2a^2B\omega\cos\theta \quad \textbf{Ans.}$$

A is at higher potential.

(b) When A and D are connected from a wire current starts flowing in the circuit as shown in figure :

Resistance between A and C is r_1 = (length of arc AC) $\lambda = a\theta\lambda$

and between C and D is r_2 = (length of arc CD) $\lambda = (\pi - \theta)\,a\lambda$



In the figure, $E_1 = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right)$ and $E_2 = 2a^2B\omega\cos^2\left(\frac{\theta}{2}\right)$

with $E_2 > E_1$

∴ Current in the circuit is

$$i = \frac{E_2 - E_1}{r_1 + r_2} = \frac{2a^2B\omega\cos\theta}{\pi a\lambda} = \frac{2aB\omega\cos\theta}{\pi\lambda}$$

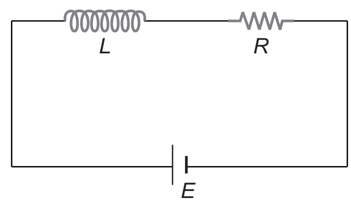
and potential difference between points C and A is

$$V'_{CA} = E_1 + ir_1 = 2a^2 B\omega \sin^2\left(\frac{\theta}{2}\right) + \left(\frac{2aB\omega\cos\theta}{\pi\lambda}\right)(a\theta\lambda)$$

$$= 2a^2 B\omega\left(\sin^2\frac{\theta}{2} + \frac{\theta}{\pi}\cos\theta\right) \qquad \textbf{Ans.}$$

Note *$V_{CA} = E_1$ when no current flows through the circuit and $V'_{CA} = E_1 + ir_1$ when a current i flows in the circuit.*

Example 22 *A battery of emf E and of negligible internal resistance is connected in an L-R circuit as shown in figure. The inductor has a piece of soft iron inside it. When steady state is reached the piece of soft iron is abruptly pulled out suddenly so that the inductance of the inductor decreases to nL with $n < 1$ with battery remaining connected. Calculate*



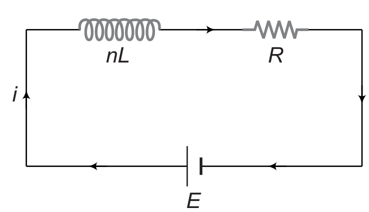
(a) current as a function of time assuming $t = 0$ at the instant when piece is pulled.
(b) the work done to pull out the piece.
(c) thermal power generated in the circuit as a function of time.
(d) power supplied by the battery as a function of time.

HOW TO PROCEED *When the inductance of an inductor is abruptly changed, the flux passing through it remains constant.*

$$\phi = \text{constant}$$

$$\therefore \qquad Li = \text{constant} \qquad \left(L = \frac{\phi}{i}\right)$$

Solution (a) At time $t = 0$, steady state current in the circuit is $i_0 = E/R$. Suddenly, L reduces to nL ($n < 1$), so current in the circuit at time $t = 0$ will increase to $\frac{i_0}{n} = \frac{E}{nR}$. Let i be the current at time t.



Applying Kirchhoff's loop rule, we have

$$E - nL\left(\frac{di}{dt}\right) - iR = 0$$

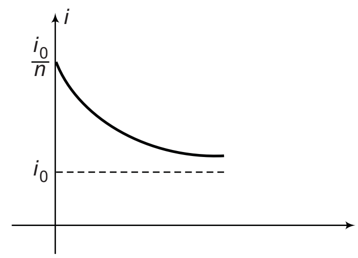
$$\therefore \qquad \frac{di}{E - iR} = \frac{1}{nL}\,dt$$

$$\therefore \qquad \int_{i_0/n}^{i} \frac{di}{E - iR} = \frac{1}{nL}\int_0^t dt$$

Solving this equation, we get $$i = i_0 - \left(i_0 - \frac{i_0}{n}\right) e^{-t/\tau_L}$$ **Ans.**

Here, $$i_0 = \frac{E}{R}$$

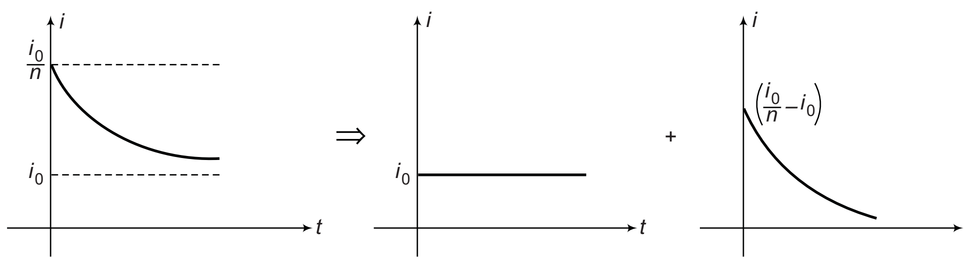
and $$\tau_L = \frac{nL}{R}$$



From the i-t equation, we get $i = \dfrac{i_0}{n}$ at $t = 0$ and $i = i_0$ at $t = \infty$

The i-t graph is as shown in figure.

Note *At $t = 0$, current in the circuit is $\frac{i_0}{n}$. Current in the circuit in steady state will be again i_0. So, it will decrease exponentially from $\frac{i_0}{n}$ to i_0. From the i-t graph, the equation can be formed without doing any calculation.*



$\therefore$ $$i = i_0 + \left(\frac{i_0}{n} - i_0\right) e^{-t/\tau_L}$$

(b) Work done to pull out the piece,

$$W = U_f - U_i = \frac{1}{2} L_f i_f^2 - \frac{1}{2} L_i i_i^2$$

$$= \frac{1}{2}(nL)\left(\frac{E}{nR}\right)^2 - \frac{1}{2}(L)\left(\frac{E}{R}\right)^2$$

$$= \frac{1}{2} L \left(\frac{E}{R}\right)^2 \left(\frac{1}{n} - 1\right)$$

$$= \frac{1}{2} L \left(\frac{E}{R}\right)^2 \left(\frac{1-n}{n}\right)$$ **Ans.**

(c) Thermal power generated in the circuit as a function of time is

$$P_1 = i^2 R$$ **Ans.**

Here, i is the current calculated in part (a).

(d) Power supplied by the battery as a function of time is

$$P_2 = Ei$$ **Ans.**

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

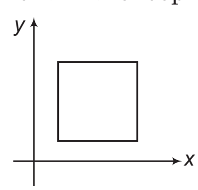
(*a*) *If both* ***Assertion*** *and* ***Reason*** *are true and the* ***Reason*** *is correct explanation of the* ***Assertion.***

(*b*) *If both* ***Assertion*** *and* ***Reason*** *are true; but* ***Reason*** *is not the correct explanation of* ***Assertion.***

(*c*) *If* ***Assertion*** *is true, but the* ***Reason*** *is false.*

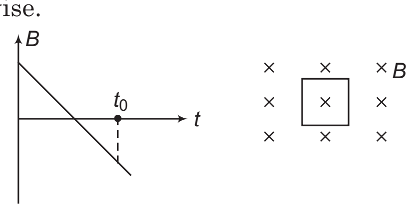
(*d*) *If* ***Assertion*** *is false, but the* ***Reason*** *is true.*

1. Assertion : A square loop is placed in x-y plane as shown in figure. Magnetic field in the region is $\mathbf{B} = -B_0 x\,\hat{\mathbf{k}}$. The induced current in the loop is anti-clockwise.



Reason : If inward magnetic field from such a loop increases, then current should be anti-clockwise.

2. Assertion : Magnetic field B (shown inwards) varies with time t as shown. At time t_0 induced current in the loop is clockwise.

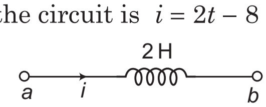


Reason : If rate of change of magnetic flux from a coil is constant, charge should flow in the coil at a constant rate.

3. Assertion : Electric field produced by a variable magnetic field can't exert a force on a charged particle.

Reason : This electric field is non-conservative in nature.

4. Assertion : Current flowing in the circuit is $i = 2t - 8$



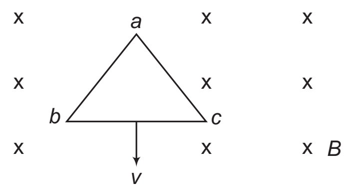
At $t = 1$ s, $V_a - V_b = +4$ V

Reason : $V_a - V_b$ is + 4 V all the time.

5. Assertion : Angular frequency of L-C oscillations is 2 rad/s and maximum current in the circuit is 1 A. Then, maximum rate of change of current should be 2 A/s.

Reason : $\left(\frac{dI}{dt}\right)_{\max} = (I_{\max})\omega$.

6. Assertion : A conducting equilateral loop abc is moved translationally with constant speed v in uniform inward magnetic field B as shown. Then : $V_a - V_b = V_b - V_c$.



Reason : Point a is at higher potential than point b.

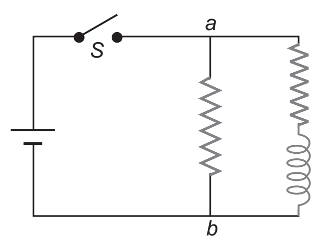
7. Assertion : Motional induced emf $e = Bvl$ can be derived from the relation $e = -\frac{d\phi}{dt}$.

Reason : Lenz's law is a consequence of law of conservation of energy.

8. Assertion : If some ferromagnetic substance is filled inside a solenoid, its coefficient of self induction L will increase.

Reason : By increasing the current in a coil, its coefficient of self induction L can be increased.

9. Assertion : In the circuit shown in figure, current in wire ab will become zero as soon as switch is opened.



Reason : A resistance does not oppose increase or decrease of current through it.

10. Assertion : In parallel, current distributes in inverse ratio of inductance

$$i \propto \frac{1}{L}$$

Reason : In electrical circuits, an inductor can be treated as a resistor.

Objective Questions

1. The dimensions of self inductance are

(a) $[MLT^{-2}A^{-2}]$ (b) $[ML^2T^{-1}A^{-2}]$
(c) $[ML^2T^{-2}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$

2. When the number of turns in the two circular coils closely wound are doubled (in both), their mutual inductance becomes

(a) four times (b) two times
(c) remains same (d) sixteen times

3. Two coils carrying current in opposite direction are placed co-axially with centres at some finite separation. If they are brought close to each other then, current flowing in them should

(a) decrease
(b) increase
(c) remain same
(d) become zero

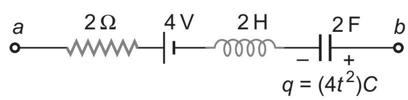
4. A current carrying ring is placed in a horizontal plane. A charged particle is dropped along the axis of the ring to fall under the influence of gravity

(a) the current in the ring may increase
(b) the current in the ring may decrease
(c) the velocity of the particle will increase till it reaches the centre of the ring
(d) the acceleration of the particle will decrease continuously till it reaches the centre of the ring

5. Identify the **incorrect** statement. Induced electric field

(a) is produced by varying magnetic field
(b) is non-conservative in nature
(c) cannot exist in a region not occupied by magnetic field
(d) None of the above

6. In the figure shown, V_{ab} at $t = 1$ s is



(a) 30 V
(b) – 30 V
(c) 20 V
(d) – 20 V

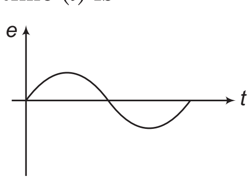
7. Two coils have a mutual inductance of 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rad/s. The maximum value of emf (in volt) in the second coil is

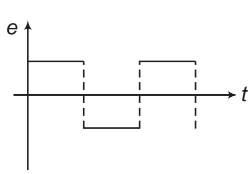
(a) 2π
(b) 5π
(c) π
(d) 4π

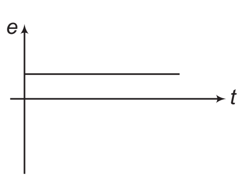
8. An inductance of 2 H carries a current of 2 A. To prevent sparking when the circuit is broken a capacitor of 4 μF is connected across the inductance. The voltage rating of the capacitor is of the order of

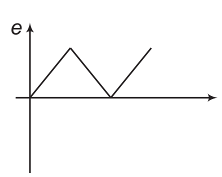
(a) 10^3 V
(b) 10 V
(c) 10^5 V
(d) 10^6 V

9. A conducting rod is rotated about one end in a plane perpendicular to a uniform magnetic field with constant angular velocity. The correct graph between the induced emf (e) across the rod and time (t) is

(a)


(b)


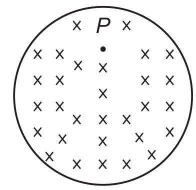
(c)


(d)


10. A magnet is taken towards a conducting ring in such a way that a constant current of 10 mA is induced in it. The total resistance of the ring is 0.5 Ω. In 5 s, the magnetic flux through the ring changes by

(a) 0.25 mWb (b) 25 mWb
(c) 50 mWb (d) 15 mWb

11. A uniform but increasing with time magnetic field exists in a cylindrical region. The direction of force on an electron at P is

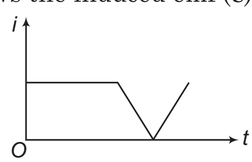


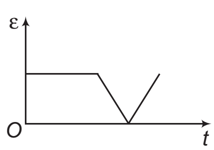
(a) towards right
(b) towards left
(c) into the plane of paper
(d) out of the plane of paper

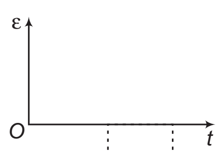
12. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\phi = at(\tau - t)$. Find the amount of heat generated in the loop during that time

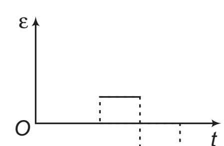
(a) $\dfrac{a\tau^2}{2R}$ (b) $\dfrac{a^2\tau^3}{3R}$
(c) $\dfrac{2a^2\tau^3}{3R}$ (d) $\dfrac{a\tau}{3R}$

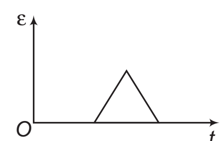
13. The current i in an induction coil varies with time t according to the graph shown in the figure. Which of the following graphs shows the induced emf (ε) in the coil with time?



(a) 

(b) 

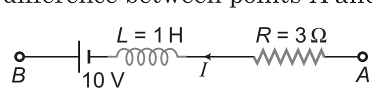
(c) 

(d) 

14. The network shown in the figure is a part of complete circuit. What is the potential difference $V_B - V_A$ when the current I is 5 A and is decreasing at a rate of 10^3 A/s?

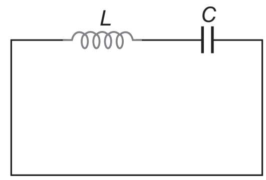
(a) 5 V (b) 10 V
(c) 15 V (d) 20 V

15. In the given branch AB of a circuit a current, $I = (10t + 5)$ A is flowing, where t is time in second. At $t = 0$, the potential difference between points A and $B (V_A - V_B)$ is



(a) 15 V (b) – 5 V
(c) – 15 V (d) 5 V

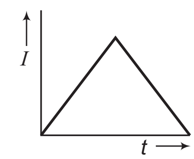
16. In an LC circuit, the capacitor has maximum charge q_0. The value of $\left(\dfrac{dI}{dt}\right)_{\max}$ is



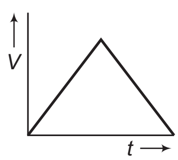
(a) $\dfrac{q_0}{LC}$ (b) $\dfrac{q_0}{\sqrt{LC}}$

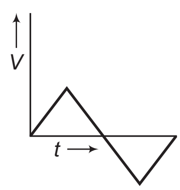
(c) $\dfrac{q_0}{LC}-1$ (d) $\dfrac{q_0}{LC}+1$

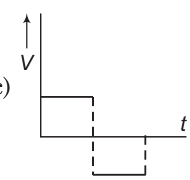
17. An alternating current I in an inductance coil varies with time t according to the graph as shown :

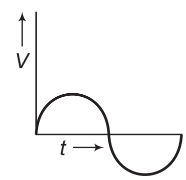


Which one of the following graphs gives the variation of voltage with time?

(a) 

(b) 

(c) 

(d) 

18. A loop of area 1 m^2 is placed in a magnetic field $B = 2$T, such that plane of the loop is parallel to the magnetic field. If the loop is rotated by 180°, the amount of net charge passing through any point of loop, if its resistance is 10 Ω, is

(a) 0.4 C (b) 0.2 C

(c) 0.8 C (d) 0 C

19. A rectangular loop of sides a and b is placed in xy-plane. A uniform but time varying magnetic field of strength $\mathbf{B} = 20\,t\,\hat{\mathbf{i}} + 10\,t^2\hat{\mathbf{j}} + 50\hat{\mathbf{k}}$ is present in the region. The magnitude of induced emf in the loop at time t is

(a) $20 + 20\,t$ (b) 20

(c) $20\,t$ (d) zero

20. The armature of a DC motor has 20 Ω resistance. It draws a current of 1.5 A when run by 200 V DC supply. The value of back emf induced in it will be

(a) 150 V (b) 170 V

(c) 180 V (d) 190 V

21. In a transformer, the output current and voltage are respectively 4 A and 20 V. If the ratio of number of turns in the primary to secondary is 2 : 1, what is the input current and voltage?

(a) 2 A and 40 V (b) 8 A and 10 V

(c) 4 A and 10 V (d) 8 A and 40 V

22. When a loop moves towards a stationary magnet with speed v, the induced emf in the loop is E. If the magnet also moves away from the loop with the same speed, then the emf induced in the loop is

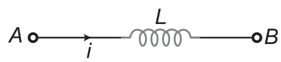
(a) E (b) $2E$

(c) $\dfrac{E}{2}$ (d) zero

23. A short magnet is allowed to fall from rest along the axis of a horizontal conducting ring. The distance fallen by the magnet in one second may be

(a) 5 m (b) 6 m
(c) 4 m (d) None of these

24. In figure, if the current i decreases at a rate α, then $V_A - V_B$ is



(a) zero (b) $-\alpha L$
(c) αL (d) No relation exists

25. A coil has an inductance of 50 mH and a resistance of $0.3\ \Omega$. If a 12 V emf is applied across the coil, the energy stored in the magnetic field after the current has built up to its steady state value is

(a) 40 J (b) 40 mJ
(c) 20 J (d) 20 mJ

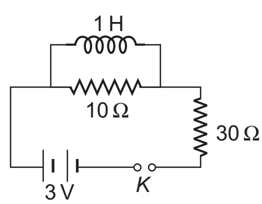
26. A constant voltage is applied to a series R-L circuit by closing the switch. The voltage across inductor ($L = 2$ H) is 20 V at $t = 0$ and drops to 5 V at 20 ms. The value of R in Ω is

(a) $100 \ln 2\Omega$ (b) $100\ (1 - \ln 2)\ \Omega$
(c) $100 \ln 4\Omega$ (d) $100(1 - \ln 4)$

27. A coil of area $10\ \text{cm}^2$ and 10 turns is in magnetic field directed perpendicular to the plane and changing at a rate of 10^8 gauss/s. The resistance of coil is $20\ \Omega$. The current in the coil will be

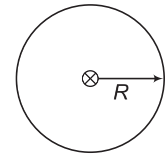
(a) 0.5 A (b) 5×10^{-3} A (c) 0.05 A (d) 5 A

28. In figure, final value of current in $10\ \Omega$ resistor, when plug of key K is inserted is



(a) $\dfrac{3}{10}$ A (b) $\dfrac{3}{20}$ A (c) $\dfrac{3}{11}$ A (d) zero

29. A circuit consists of a circular loop of radius R kept in the plane of paper and an infinitely long current carrying wire kept perpendicular to the plane of paper and passing through the centre of loop. The mutual inductance of wire and loop will be

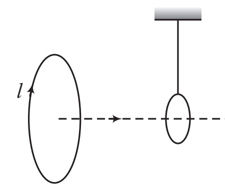


(a) $\dfrac{\mu_0 \pi R}{2}$ (b) 0
(c) $\mu_0 \pi R^2$ (d) $\dfrac{\mu_0 R^2}{2}$

30. A flat circular coil of n turns, area A and resistance R is placed in a uniform magnetic field B. The plane of coil is initially perpendicular to B. When the coil is rotated through an angle of 180° about one of its diameter, a charge Q_1 flows through the coil. When the same coil after being brought to its initial position, is rotated through an angle of 360° about the same axis a charge Q_2 flows through it. Then, Q_2/Q_1 is

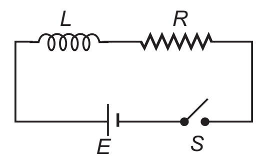
(a) 1 (b) 2 (c) 1/2 (d) 0

31. A small circular loop is suspended from an insulating thread. Another coaxial circular loop carrying a current I and having radius much larger than the first loop starts moving towards the smaller loop. The smaller loop will



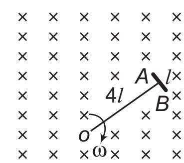
(a) be attracted towards the bigger loop
(b) be repelled by the bigger loop
(c) experience no force
(d) All of the above

32. In the circuit shown in figure, $L = 10\,\text{H}, R = 5\,\Omega, E = 15$ V. The switch S is closed at $t = 0$. At $t = 2$ s, the current in the circuit is



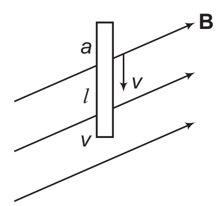
(a) $3\left(1-\frac{1}{e}\right)$ A
(b) $3\left(1-\frac{1}{e^2}\right)$ A
(c) $3\left(\frac{1}{e}\right)$ A
(d) $3\left(\frac{1}{e^2}\right)$ A

33. In the figure shown, a T-shaped conductor moves with constant angular velocity ω in a plane perpendicular to uniform magnetic field **B**. The potential difference $V_A - V_B$ is



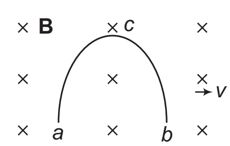
(a) zero
(b) $\frac{1}{2}B\omega l^2$
(c) $2B\omega l^2$
(d) $B\omega l^2$

34. A conducting rod of length l falls vertically under gravity in a region of uniform magnetic field **B**. The field vectors are inclined at an angle θ with the horizontal as shown in figure. If the instantaneous velocity of the rod is v, the induced emf in the rod ab is



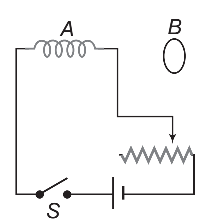
(a) Blv
(b) $Blv \cos\theta$
(c) $Blv \sin\theta$
(d) zero

35. A semi-circular conducting ring acb of radius R moves with constant speed v in a plane perpendicular to uniform magnetic field B as shown in figure. Identify the correct statement.



(a) $V_a - V_c = BRv$
(b) $V_b - V_c = BRv$
(c) $V_a - V_b = 0$
(d) None of these

36. The ring B is coaxial with a solenoid A as shown in figure. As the switch S is closed at $t = 0$, the ring B

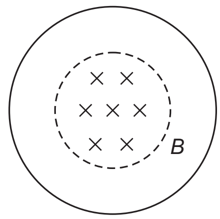


(a) is attracted towards A
(b) is repelled by A
(c) is initially repelled and then attracted
(d) is initially attracted and then repelled

37. If the instantaneous magnetic flux and induced emf produced in a coil is ϕ and E respectively, then according to Faraday's law of electromagnetic induction

(a) E must be zero if $\phi = 0$
(b) $E \neq 0$ if $\phi = 0$
(c) $E \neq 0$ but ϕ may or may not be zero
(d) $E = 0$ then ϕ must be zero

38. The figure shows a conducting ring of radius R. A uniform steady magnetic field B lies perpendicular to the plane of the ring in a circular region of radius r ($< R$). If the resistance per unit length of the ring is λ, then the current induced in the ring when its radius gets doubled is



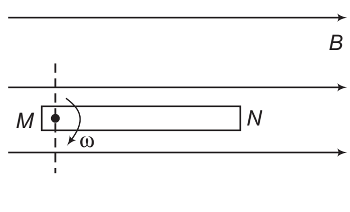
(a) $\dfrac{BR}{\lambda}$

(b) $\dfrac{2BR}{\lambda}$

(c) zero

(d) $\dfrac{Br^2}{4R\lambda}$

39. A metallic rod of length l is hinged at the point M and is rotating about an axis perpendicular to the plane of paper with a constant angular velocity ω. A uniform magnetic field of intensity B is acting in the region (as shown in the figure) parallel to the plane of paper. The potential difference between the points M and N



(a) is always zero

(b) varies between $\frac{1}{2}B\omega l^2$ to 0

(c) is always $\frac{1}{2}B\omega l^2$

(d) is always $B\omega l^2$

Subjective Questions

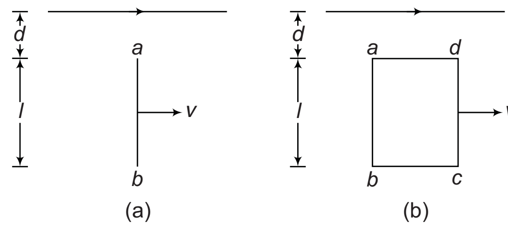
Note *You can take approximations in the answers.*

1. An inductor is connected to a battery through a switch. The emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when the switch is closed. Is this statement true or false?

2. A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30°, with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.4 s, an emf of magnitude 80.0 mV is induced in the coil. What is the total length of the wire?

3. A loop of wire enclosing an area S is placed in a region where the magnetic field is perpendicular to the plane. The magnetic field **B** varies with time according to the expression $B = B_0\, e^{-at}$ where a is some constant. That is, at $t = 0$. The field is B_0 and for $t > 0$, the field decreases exponentially. Find the induced emf in the loop as a function of time.

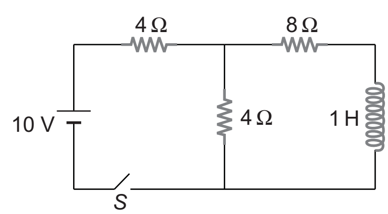
4. The long straight wire in figure (a) carries a constant current i. A metal bar of length l is moving at constant velocity **v** as shown in figure. Point a is a distance d from the wire.



(a) (b)

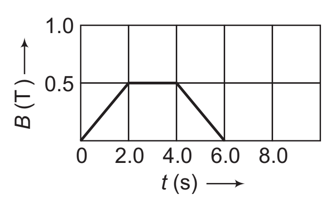
(a) Calculate the emf induced in the bar.
(b) Which point a or b is at higher potential?
(c) If the bar is replaced by a rectangular wire loop of resistance R, what is the magnitude of current induced in the loop?

5. The switch in figure is closed at time $t = 0$. Find the current in the inductor and the current through the switch as functions of time thereafter.



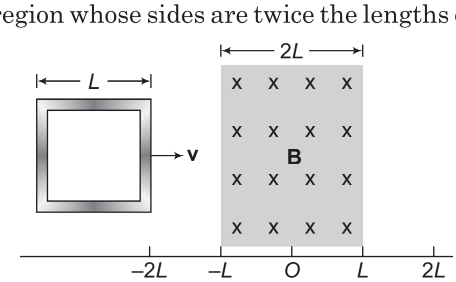
6. A small coil is introduced between the poles of an electromagnet so that its axis coincides with the magnetic field direction. The cross-sectional area of the coil is equal to $S = 3.0\ \text{mm}^2$, the number of turns is $N = 60$. When the coil turns through 180° about its diameter, a galvanometer connected to the coil indicates a charge $q = 4.5\ \mu\text{C}$ flowing through it. Find the magnetic induction magnitude between the poles, provided the total resistance of the electric circuit equals $R = 40\ \Omega$.

7. The magnetic field through a single loop of wire, 12 cm in radius and of 8.5 Ω resistance, changes with time as shown in figure. Calculate the emf in the loop as a function of time. Consider the time intervals



(a) $t = 0$ to $t = 2.0$ s (b) $t = 2.0$ s to $t = 4.0$ s (c) $t = 4.0$ s to $t = 6.0$ s.
The magnetic field is perpendicular to the plane of the loop.

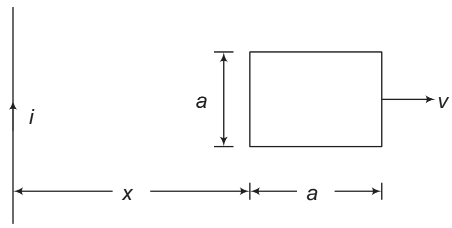
8. A square loop of wire with resistance R is moved at constant speed v across a uniform magnetic field confined to a square region whose sides are twice the lengths of those of the square loop.



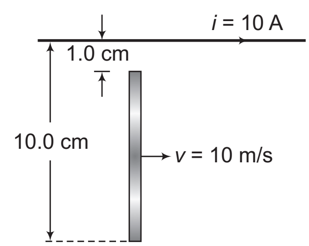
(a) Sketch a graph of the external force F needed to move the loop at constant speed, as a function of the coordinate x, from $x = -2L$ to $x = +2L$. (The coordinate x is measured from the centre of the magnetic field region to the centre of the loop. It is negative when the centre of the loop is to the left of the centre of the magnetic field region. Take positive force to be to the right).

(b) Sketch a graph of the induced current in the loop as a function of x. Take counterclockwise currents to be positive.

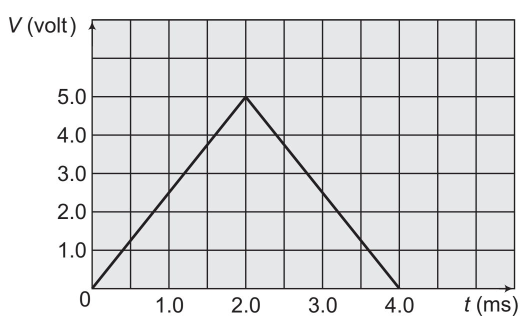
9. A square frame with side a and a long straight wire carrying a current i are located in the same plane as shown in figure. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



10. In figure, a wire perpendicular to a long straight wire is moving parallel to the later with a speed $v = 10$ m/s in the direction of the current flowing in the later. The current is 10 A. What is the magnitude of the potential difference between the ends of the moving wire?



11. The potential difference across a 150 mH inductor as a function of time is shown in figure. Assume that the initial value of the current in the inductor is zero. What is the current when $t = 2.0$ ms? and $t = 4.0$ ms ?



12. At the instant when the current in an inductor is increasing at a rate of 0.0640 A/s, the magnitude of the self-induced emf is 0.0160 V.

(a) What is the inductance of the inductor?

(b) If the inductor is a solenoid with 400 turns, what is the average magnetic flux through each turn when the current is 0.720 A?

13. Two toroidal solenoids are wound around the same pipe so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb.

(a) What is the mutual inductance of the pair of solenoids?

(b) When the current in solenoid 2 is 2.54 A, what is the average flux through each turn of solenoid 1?

14. A coil of inductance 1 H and resistance 10 Ω is connected to a resistanceless battery of emf 50 V at time $t = 0$. Calculate the ratio of the rate at which magnetic energy is stored in the coil to the rate at which energy is supplied by the battery at $t = 0.1$ s.

15. A 3.56 H inductor is placed in series with a 12.8 Ω resistor. An emf of 3.24 V is then suddenly applied across the RL combination.

(a) At 0.278 s after the emf is applied what is the rate at which energy is being delivered by the battery?

(b) At 0.278 s, at what rate is energy appearing as thermal energy in the resistor?

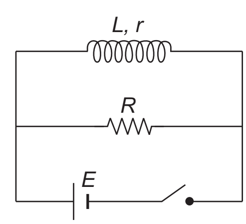
(c) At 0.278 s, at what rate is energy being stored in the magnetic field?

16. A 35.0 V battery with negligible internal resistance, a 50.0 Ω resistor, and a 1.25 mH inductor with negligible resistance are all connected in series with an open switch. The switch is suddenly closed

(a) How long after closing the switch will the current through the inductor reach one-half of its maximum value?

(b) How long after closing the switch will the energy stored in the inductor reach one-half of its maximum value?

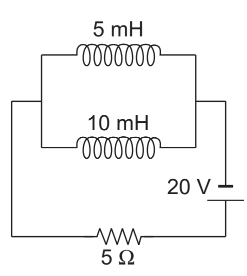
17. A solenoid of inductance L with resistance r is connected in parallel to a resistance R. A battery of emf E and of negligible internal resistance is connected across the parallel combination as shown in the figure. At time $t = 0$, switch S is opened, calculate



(a) current through the solenoid after the switch is opened.

(b) amount of heat generated in the solenoid

18. In the given circuit, find the current through the 5 mH inductor in steady state.



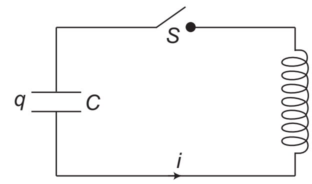
19. In an oscillating L-C circuit in which $C = 4.00\,\mu F$, the maximum potential difference across the capacitor during the oscillations is 1.50 V and the maximum current through the inductor is 50.0 mA.

(a) What is the inductance L?

(b) What is the frequency of the oscillations?

(c) How much time does the charge on the capacitor take to rise from zero to its maximum value?

20. In the L-C circuit shown, $C = 1\,\mu F$. With capacitor charged to 100 V, switch S is suddenly closed at time $t = 0$. The circuit then oscillates at 10^3 Hz.



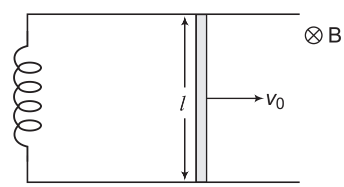
(a) Calculate ω and T
(b) Express q as a function of time
(c) Calculate L
(d) Calculate the average current during the first quarter-cycle.

21. An L-C circuit consists of an inductor with $L = 0.0900$ H and a capacitor of $C = 4 \times 10^{-4}$ F. The initial charge on the capacitor is $5.00\,\mu C$, and the initial current in the inductor is zero.
(a) What is the maximum voltage across the capacitor?
(b) What is the maximum current in the inductor?
(c) What is the maximum energy stored in the inductor?
(d) When the current in the inductor has half its maximum value, what is the charge on the capacitor and what is the energy stored in the inductor?

LEVEL 2

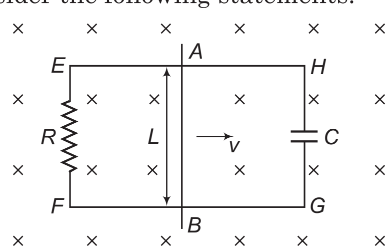
Single Correct Option

1. Two ends of an inductor of inductance L are connected to two parallel conducting wires. A rod of length l and mass m is given velocity v_0 as shown. The whole system is placed in perpendicular magnetic field B. Find the maximum current in the inductor. (Neglect gravity and friction)



(a) $\dfrac{mv_0}{L}$
(b) $\sqrt{\dfrac{m}{L}}v_0$
(c) $\dfrac{mv_0^2}{L}$
(d) None of these

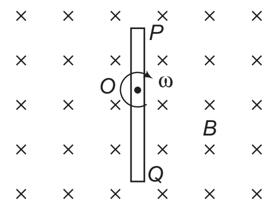
2. A conducting rod is moving with a constant velocity v over the parallel conducting rails which are connected at the ends through a resistor R and capacitor C as shown in the figure. Magnetic field B is into the plane. Consider the following statements.



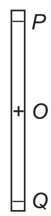
(i) Current in loop $AEFBA$ is anti-clockwise
(ii) Current in loop $AEFBA$ is clockwise
(iii) Current through the capacitor is zero
(iv) Energy stored in the capacitor is $\frac{1}{2}CB^2L^2v^2$

Which of the following options is correct?
(a) Statements (i) and (iii) are correct
(b) Statements (ii) and (iv) are correct
(c) Statements (i), (iii) and (iv) are correct
(d) None of these

3. A rod is rotating with a constant angular velocity ω about point O (its centre) in a magnetic field B as shown. Which of the following figure correctly shows the distribution of charge inside the rod?



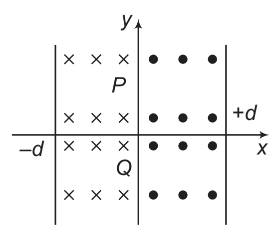
(a) 

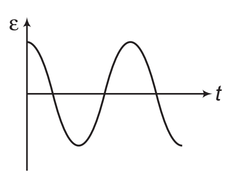
(b) 

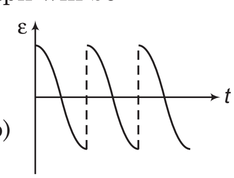
(c) 

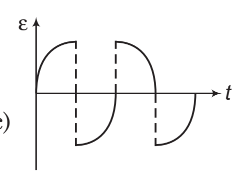
(d) 

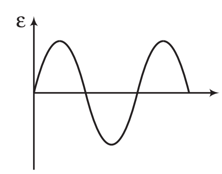
4. A straight conducting rod PQ is executing SHM in xy-plane from $x = -d$ to $x = +d$. Its mean position is $x = 0$ and its length is along y-axis. There exists a uniform magnetic field B from $x = -d$ to $x = 0$ pointing inward normal to the paper and from $x = 0$ to $= +d$ there exists another uniform magnetic field of same magnitude B but pointing outward normal to the plane of the paper. At the instant $t = 0$, the rod is at $x = 0$ and moving to the right. The induced emf (ε) across the rod PQ *vs* time (t) graph will be



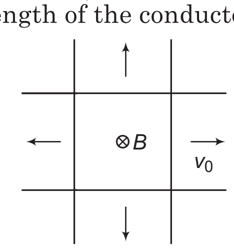
(a) 

(b) 

(c) 

(d) 

5. Two parallel long straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now, conductors start moving outward with a constant velocity v_0 at $t = 0$. Then, induced current in the loop at any time t is (λ is resistance per unit length of the conductors)



(a) $\dfrac{aBv_0}{\lambda\,(a + v_0 t)}$

(b) $\dfrac{aBv_0}{2\lambda}$

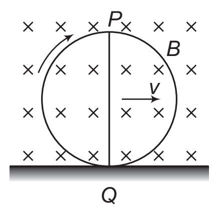
(c) $\dfrac{Bv_0}{\lambda}$

(d) $\dfrac{Bv_0}{2\lambda}$

6. A conducting square loop is placed in a magnetic field B with its plane perpendicular to the field. Now the sides of the loop start shrinking at a constant rate α. The induced emf in the loop at an instant when its side is a, is

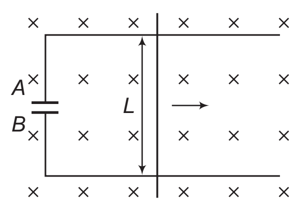
(a) $2a\alpha B$ (b) $a^2\alpha B$ (c) $2a^2\alpha B$ (d) $a\alpha B$

7. A conducting straight wire PQ of length l is fixed along a diameter of a non-conducting ring as shown in the figure. The ring is given a pure rolling motion on a horizontal surface such that its centre of mass has a velocity v. There exists a uniform horizontal magnetic field B in horizontal direction perpendicular to the plane of ring. The magnitude of induced emf in the wire PQ at the position shown in the figure will be



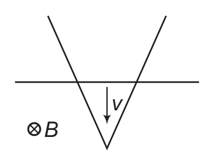
(a) Bvl (b) $2Bvl$ (c) $3Bvl/2$ (d) zero

8. A conducting rod of length $L = 0.1$ m is moving with a uniform speed $v = 0.2$ m/s on conducting rails in a magnetic field $B = 0.5$ T as shown. On one side, the end of the rails is connected to a capacitor of capacitance $C = 20\,\mu$F. Then, the charges on the capacitor's plates are



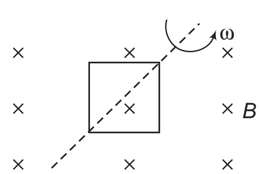
(a) $q_A = 0 = q_B$ (b) $q_A = +20\,\mu C$ and $q_B = -20\,\mu C$
(c) $q_A = +0.2\,\mu C$ and $q_B = -0.2\,\mu C$ (d) $q_A = -0.2$ C and $q_B = -0.2\,\mu C$

9. A wire is bent in the form of a V shape and placed in a horizontal plane. There exists a uniform magnetic field B perpendicular to the plane of the wire. A uniform conducting rod starts sliding over the V shaped wire with a constant speed v as shown in the figure. If the wire has no resistance, the current in rod will



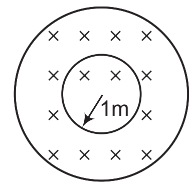
(a) increase with time (b) decrease with time
(c) remain constant (d) always be zero

10. A square loop of side b is rotated in a constant magnetic field B at angular frequency ω as shown in the figure. What is the emf induced in it?



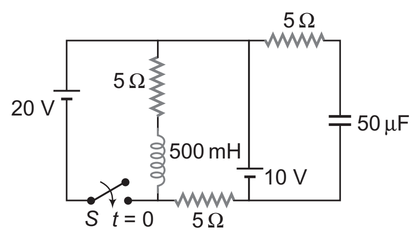
(a) $b^2B\omega \sin \omega t$ (b) $bB\omega \sin^2 \omega t$
(c) $bB^2\omega \cos \omega t$ (d) $b^2B\omega$

11. A uniform but time varying magnetic field exists in a cylindrical region as shown in the figure. The direction of magnetic field is into the plane of the paper and its magnitude is decreasing at a constant rate of 2×10^{-3} T/s. A particle of charge 1 μC is moved slowly along a circle of radius 1m by an external force as shown in figure. The plane of the circle lies in the plane of the paper and it is concentric with the cylindrical region. The work done by the external force in moving this charge along the circle will be



(a) zero
(b) $2\pi \times 10^{-9}$ J
(c) $\pi \times 10^{-9}$ J
(d) $4\pi \times 10^{-6}$ J

12. Switch S is closed at $t = 0$, in the circuit shown. The change in flux in the inductor ($L = 500$ mH) from $t = 0$ to an instant when it reaches steady state is



(a) 2 Wb
(b) 1.5 Wb
(c) 0 Wb
(d) None of these

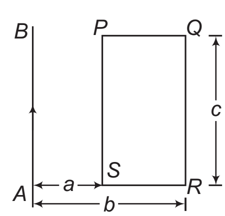
13. An L-R circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

(a) $\frac{R}{L} \ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$
(b) $\frac{L}{R} \ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$
(c) $\frac{L}{R} \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$
(d) $\frac{R}{L} \ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

14. Electric charge q is distributed uniformly over a rod of length l. The rod is placed parallel to a long wire carrying a current i. The separation between the rod and the wire is a. The force needed to move the rod along its length with a uniform velocity v is

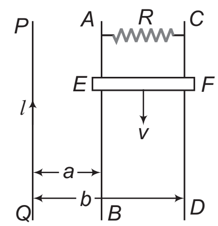
(a) $\frac{\mu_0 iqv}{2\pi a}$
(b) $\frac{\mu_0 iqv}{4\pi a}$
(c) $\frac{\mu_0 iqvl}{2\pi a}$
(d) $\frac{\mu_0 iqvl}{4\pi a}$

15. AB is an infinitely long wire placed in the plane of rectangular coil of dimensions as shown in the figure. Calculate the mutual inductance of wire AB and coil $PQRS$



(a) $\frac{\mu_0 b}{2\pi} \ln \frac{a}{b}$
(b) $\frac{\mu_0 c}{2\pi} \ln \frac{b}{a}$
(c) $\frac{\mu_0 abc}{2\pi(b-a)^2}$
(d) None of these

16. PQ is an infinite current carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity v as shown. The force needed to maintain constant speed of EF is



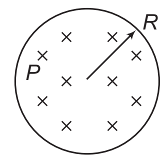
(a) $\frac{1}{vR}\left[\frac{\mu_0 I v}{2\pi}\ln\frac{(b)}{(a)}\right]^2$

(b) $\frac{v}{R}\left[\frac{\mu_0 I v}{2\pi}\ln\frac{(a)}{(b)}\right]^2$

(c) $\frac{v}{R}\left[\frac{\mu_0 I v}{2\pi}\ln\frac{(b)}{(a)}\right]^2$

(d) None of these

17. The figure shows a circular region of radius R occupied by a time varying magnetic field $\mathbf{B}(t)$ such that $\frac{dB}{dt} < 0$. The magnitude of induced electric field at the point P at a distance $r < R$ is



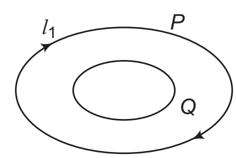
(a) decreasing with r

(b) increasing with r

(c) not varying with r

(d) varying as r^{-2}

18. Two circular loops P and Q are concentric and coplanar as shown in figure. The loop Q is smaller than P. If the current I_1 flowing in loop P is decreasing with time, then the current I_2 in the loop Q



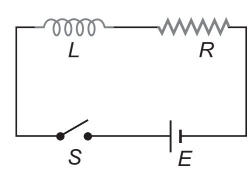
(a) flows in the same direction as that of P

(b) flows in the opposite direction as that of Q

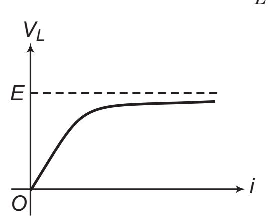
(c) is zero

(d) None of the above

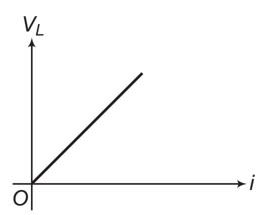
19. In the circuit shown in figure, the switch S is closed at $t = 0$. If V_L is the voltage induced across the inductor and i is the instantaneous current, the correct variation of V_L *versus* i is given by



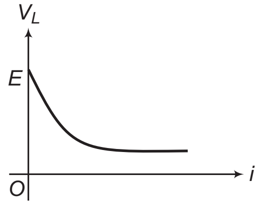
(a)



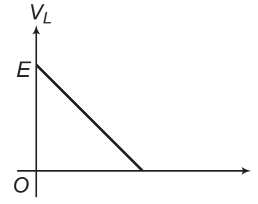
(b)



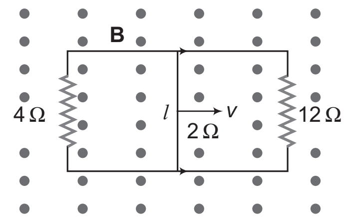
(c)



(d)

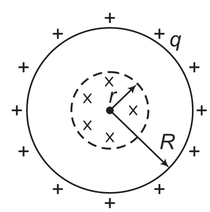


20. In the figure shown, a uniform magnetic field $|\mathbf{B}| = 0.5$ T is perpendicular to the plane of circuit. The sliding rod of length $l = 0.25$ m moves uniformly with constant speed $v = 4\ \text{ms}^{-1}$. If the resistance of the slides is 2 Ω, then the current flowing through the sliding rod is



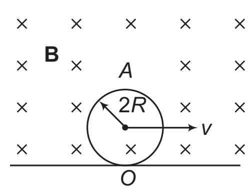
(a) 0.1 A (b) 0.17 A
(c) 0.08 A (d) 0.03 A

21. The figure shows a non-conducting ring of radius R carrying a charge q. In a circular region of radius r, a uniform magnetic field $\mathbf{B}$ perpendicular to the plane of the ring varies at a constant rate $\frac{dB}{dt} = \beta$. The torque acting on the ring is



(a) $\frac{1}{2} qr^2\beta$ (b) $\frac{1}{2} qR^2\beta$ (c) $qr^2\beta$ (d) zero

22. A conducting ring of radius $2R$ rolls on a smooth horizontal conducting surface as shown in figure. A uniform horizontal magnetic field B is perpendicular to the plane of the ring. The potential of A with respect to O is



(a) $2\ BvR$ (b) $\frac{1}{2}\ BvR$
(c) $8\ BvR$ (d) $4\ BvR$

23. A uniformly wound long solenoid of inductance L and resistance R is cut into two parts in the ratio $\eta : 1$, which are then connected in parallel. The combination is then connected to a cell of emf E. The time constant of the circuit is

(a) $\frac{L}{R}$ (b) $\frac{L}{(\eta + 1) R}$
(c) $\left(\frac{\eta}{\eta + 1}\right) \frac{L}{R}$ (d) $\left(\frac{\eta + 1}{\eta}\right) \frac{L}{R}$

24. When a choke coil carrying a steady current is short-circuited, the current in it decreases to $\beta\,(< 1)$ times its initial value in a time T. The time constant of the choke coil is

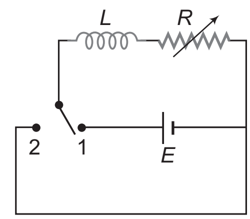
(a) $\dfrac{T}{\beta}$ (b) $\dfrac{T}{\ln\left(\dfrac{1}{\beta}\right)}$ (c) $\dfrac{T}{\ln \beta}$ (d) $T \ln \beta$

25. In the steady state condition, the rate of heat produced in a choke coil is P. The time constant of the choke coil is τ. If now the choke coil is short-circuited, then the total heat dissipated in the coil is

(a) $P\tau$ (b) $\dfrac{1}{2}P\tau$

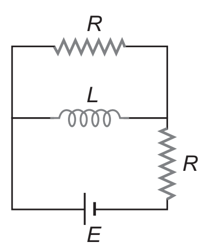
(c) $\dfrac{P\tau}{\ln 2}$ (d) $P\tau \ln 2$

26. In the circuit shown in figure initially the switch is in position 1 for a long time, then suddenly at $t = 0$, the switch is shifted to position 2. It is required that a constant current should flow in the circuit, the value of resistance R in the circuit



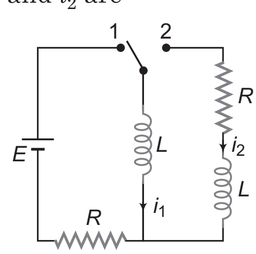
(a) should be decreased at a constant rate
(b) should be increased at a constant rate
(c) should be maintained constant
(d) Not possible

27. The figure shows an L-R circuit, the time constant for the circuit is



(a) $\dfrac{L}{2R}$ (b) $\dfrac{2L}{R}$ (c) $\dfrac{2R}{L}$ (d) $\dfrac{R}{2L}$

28. In figure, the switch is in the position 1 for a long time, then the switch is shifted to position 2 at $t = 0$. At this instant the value of i_1 and i_2 are



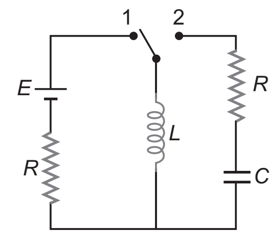
(a) $\dfrac{E}{R}, 0$ (b) $\dfrac{E}{R}, \dfrac{-E}{R}$

(c) $\dfrac{E}{2R}, \dfrac{-E}{2R}$ (d) None of these

29. In a decaying L-R circuit, the time after which energy stored in the inductor reduces to one-fourth of its initial value is

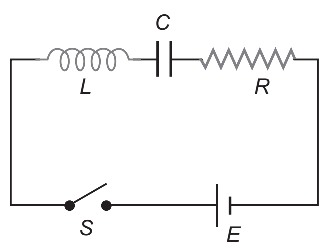
(a) $(\ln 2)\dfrac{L}{R}$
(b) $0.5\dfrac{L}{R}$
(c) $\sqrt{2}\dfrac{L}{R}$
(d) $\left(\dfrac{\sqrt{2}}{\sqrt{2}-1}\right)\dfrac{L}{R}$

30. Initially, the switch is in position 1 for a long time and then shifted to position 2 at $t = 0$ as shown in figure. Just after closing the switch, the magnitude of current through the capacitor is



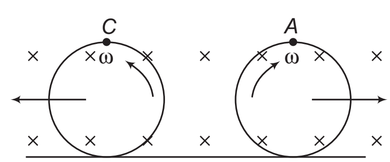
(a) zero
(b) $\dfrac{E}{2R}$
(c) $\dfrac{E}{R}$
(d) None of these

31. When the switch S is closed at $t = 0$, identify the correct statement just after closing the switch as shown in figure



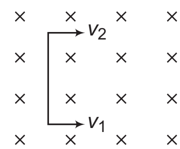
(a) The current in the circuit is maximum
(b) Equal and opposite voltages are dropped across inductor and resistor
(c) The entire voltage is dropped across inductor
(d) All of the above

32. Two metallic rings of radius R are rolling on a metallic rod. A magnetic field of magnitude B is applied in the region. The magnitude of potential difference between points A and C on the two rings (as shown), will be



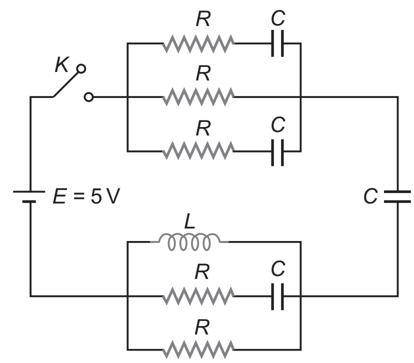
(a) 0
(b) $4\,B\omega R^2$
(c) $8\,B\omega R^2$
(d) $2\,B\omega R^2$

33. In the figure, magnetic field points into the plane of paper and the conducting rod of length l is moving in this field such that the lowest point has a velocity v_1 and the topmost point has the velocity $v_2 (v_2 > v_1)$. The emf induced is given by



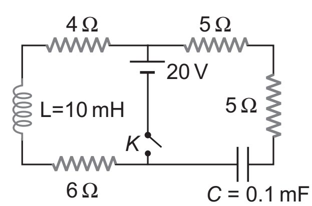
(a) $Bv_1 l$
(b) $Bv_2 l$
(c) $\frac{1}{2} B (v_2 + v_1) l$
(d) $\frac{1}{2} B (v_2 - v_1) l$

34. Find the current passing through battery immediately after key (K) is closed. It is given that initially all the capacitors are uncharged. (Given that $R = 6\,\Omega$ and $C = 4\mu\text{F}$)



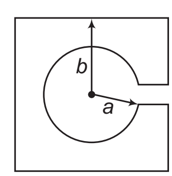
(a) 1 A
(b) 5 A
(c) 3 A
(d) 2 A

35. In the circuit shown, the key (K) is closed at $t = 0$, the current through the key at the instant $t = 10^{-3} \ln 2$, is



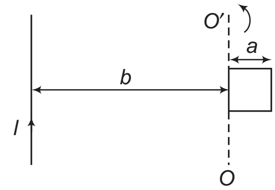
(a) 2 A
(b) 8 A
(c) 4 A
(d) zero

36. A loop shown in the figure is immersed in the varying magnetic field $B = B_0 t$, directed into the page. If the total resistance of the loop is R, then the direction and magnitude of induced current in the inner circle is



(a) clockwise $\frac{B_0(\pi a^2 - b^2)}{R}$
(b) anti-clockwise $\frac{B_0 \pi (a^2 + b^2)}{R}$
(c) clockwise $\frac{B_0(\pi a^2 + 4b^2)}{R}$
(d) clockwise $\frac{B_0(4b^2 - \pi a^2)}{R}$

37. A square loop of side a and a straight long wire are placed in the same plane as shown in figure. The loop has a resistance R and inductance L. The frame is turned through 180° about the axis OO'. What is the electric charge that flows through the loop?



(a) $\dfrac{\mu_0 Ia}{2\pi R}\ln\left(\dfrac{2a+b}{b}\right)$

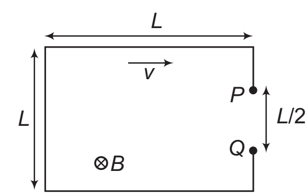
(b) $\dfrac{\mu_0 Ia}{2\pi R}\ln\left(\dfrac{b}{b^2-a^2}\right)$

(c) $\dfrac{\mu_0 Ia}{2\pi R}\ln\left(\dfrac{a+2b}{b}\right)$

(d) None of these

More than One Correct Options

1. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between points P and Q is e. Then,



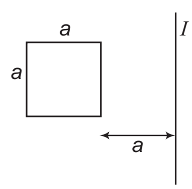
(a) $e = \dfrac{1}{2}BLv$

(b) $e = BLv$

(c) P is positive with respect to Q

(d) Q is positive with respect to P

2. An infinitely long wire is placed near a square loop as shown in figure. Choose the correct options.



(a) The mutual inductance between the two is $\dfrac{\mu_0 a}{2\pi}\ln(2)$

(b) The mutual inductance between the two is $\dfrac{\mu_0 a^2}{2\pi}\ln(2)$

(c) If a constant current is passed in the straight wire in upward direction and loop is brought close to the wire, then induced current in the loop is clockwise

(d) In the above condition, induced current in the loop is anti-clockwise

3. Choose the correct options.

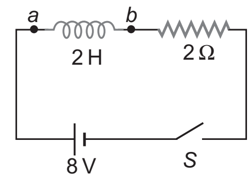
(a) SI unit of magnetic flux is henry-ampere

(b) SI unit of coefficient of self-inductance is J/A

(c) SI unit of coefficient of self-inductance is $\dfrac{\text{volt-second}}{\text{ampere}}$

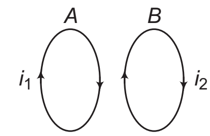
(d) SI unit of magnetic induction is weber

4. In the circuit shown in figure, circuit is closed at time $t = 0$. At time $t = \ln(2)$ second



(a) rate of energy supplied by the battery is 16 J/s
(b) rate of heat dissipated across resistance is 8 J/s
(c) rate of heat dissipated across resistance is 16 J/s
(d) $V_a - V_b = 4$ V

5. Two circular coils are placed adjacent to each other. Their planes are parallel and currents through them i_1 and i_2 are in same direction. Choose the correct options.

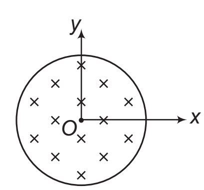


(a) When A is brought near B, current i_2 will decrease
(b) In the above process, current i_2 will increase
(c) When current i_1 is increased, current i_2 will decrease
(d) In the above process, current i_2 will increase

6. A coil of area 2 m^2 and resistance 4 Ω is placed perpendicular to a uniform magnetic field of 4 T. The loop is rotated by 90° in 0.1 second. Choose the correct options.
(a) Average induced emf in the coil is 8 V
(b) Average induced current in the circuit is 20 A
(c) 2 C charge will flow in the coil in above period
(d) Heat produced in the coil in the above period can't be determined from the given data

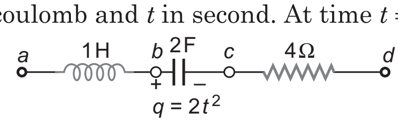
7. In L-C oscillations,
(a) time period of oscillation is $\dfrac{2\pi}{\sqrt{LC}}$
(b) maximum current in circuit is $\dfrac{q_0}{\sqrt{LC}}$
(c) maximum rate of change of current in circuit is $\dfrac{q_0}{LC}$
(d) maximum potential difference across the inductor is $\dfrac{q_0}{2C}$. Here, q_0 is maximum charge on capacitor

8. Magnetic field in a cylindrical region of radius R in inward direction is as shown in figure.



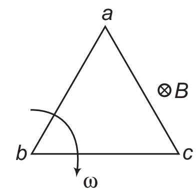
(a) an electron will experience no force kept at $(2R, 0, 0)$ if magnetic field increases with time
(b) in the above situation, electron will experience the force in negative y-axis
(c) If a proton is kept at $\left(0, \dfrac{R}{2}, 0\right)$ and magnetic field is decreasing, then it will experience the force in positive x-direction
(d) if a proton is kept at $(-R, 0, 0)$ and magnetic field is increasing, then it will experience force in negative y-axis

9. In the figure shown, q is in coulomb and t in second. At time $t = 1$ s



(a) $V_a - V_b = 4$ V
(b) $V_b - V_c = 1$ V
(c) $V_c - V_d = 16$ V
(d) $V_a - V_d = 20$ V

10. An equilateral triangular conducting frame is rotated with angular velocity ω in a uniform magnetic field B as shown. Side of triangle is l. Choose the correct options.



(a) $V_a - V_c = 0$

(b) $V_a - V_c = \dfrac{B\omega l^2}{2}$

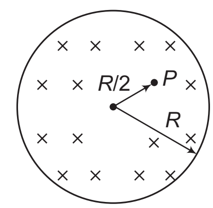
(c) $V_a - V_b = \dfrac{B\omega l^2}{2}$

(d) $V_c - V_b = -\dfrac{B\omega l^2}{2}$

Comprehension Based Questions

Passage I (Q. No. 1 to 3)

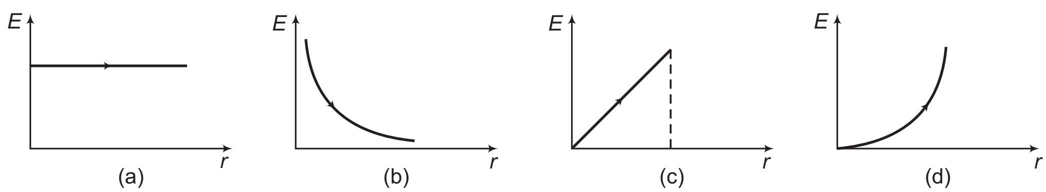
A uniform but time varying magnetic field $B = (2t^3 + 24t)\,T$ is present in a cylindrical region of radius $R = 2.5$ cm as shown in figure.



1. The force on an electron at P at $t = 2.0$ s is

(a) 96×10^{-21} N
(b) 48×10^{-21} N
(c) 24×10^{-21} N
(d) zero

2. The variation of electric field at any instant as a function of distance measured from the centre of cylinder in first problem is



3. In the previous problem, the direction of circular electric lines at $t = 1$ s is

(a) clockwise
(b) anti-clockwise
(c) no current is induced
(d) cannot be predicted

Passage II (Q. No. 4 to 7)

A thin non-conducting ring of mass m, radius a carrying a charge q can rotate freely about its own axis which is vertical. At the initial moment, the ring was at rest in horizontal position and no magnetic field was present. At instant $t = 0$, a uniform magnetic field is switched on which is vertically downward and increases with time according to the law $B = B_0 t$. Neglecting magnetism induced due to rotational motion of ring.

4. The magnitude of induced emf on the closed surface of ring will be

(a) $\pi a^2 B_0$

(b) $2a^2 B_0$

(c) zero

(d) $\dfrac{1}{2}\pi a^2 B_0$

5. The magnitude of an electric field on the circumference of the ring is

(a) aB_0

(b) $2aB_0$

(c) $\dfrac{1}{2}aB_0$

(d) zero

6. Angular acceleration of ring is

(a) $\frac{qB_0}{2m}$ (b) $\frac{qB_0}{4m}$

(c) $\frac{qB_0}{m}$ (d) $\frac{2qB_0}{m}$

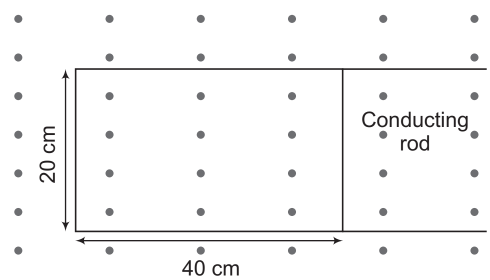
7. Find instantaneous power developed by electric force acting on the ring at $t = 1$ s.

(a) $\frac{2q^2B_0^2a^2}{14m}$ (b) $\frac{q^2B_0^2a^2}{8m}$

(c) $\frac{3q^2B_0^2a^2}{m}$ (d) $\frac{q^2B_0^2a^2}{4m}$

Passage III (Q. No. 8 to 10)

Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance $1\,\Omega/m$. Position of the conducting rod at $t = 0$ is shown. A time dependent magnetic field $B = 2t$ tesla is switched on at $t = 0$.



8. The current in the loop at $t = 0$ due to induced emf is

(a) 0.16 A, clockwise (b) 0.08 A, clockwise

(c) 0.16 A, anti-clockwise (d) zero

9. At $t = 0$, when the magnetic field is switched on, the conducting rod is moved to the left at constant speed 5 cm/s by some external means. At $t = 2$ s, net induced emf has magnitude

(a) 0.12 V (b) 0.08 V

(c) 0.04 V (d) 0.02 V

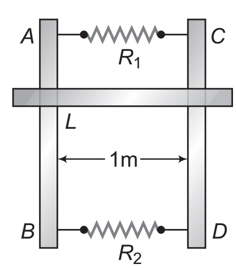
10. The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at the same instant $t = 2$s, is equal to

(a) 0.096 N (b) 0.12 N

(c) 0.08 N (d) 0.064 N

Passage IV (Q. No. 11 to 13)

Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively ($g = 9.8\ m/s^2$)



11. The terminal velocity of the bar L will be

(a) 2 m/s (b) 3 m/s (c) 1 m/s (d) None of these

12. The value of R_1 is

(a) $0.47\,\Omega$ (b) $0.82\,\Omega$
(c) $0.12\,\Omega$ (d) None of these

13. The value of R_2 is

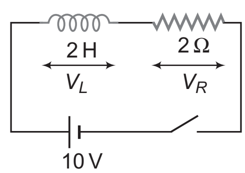
(a) $0.6\,\Omega$ (b) $0.5\,\Omega$
(c) $0.4\,\Omega$ (d) $0.3\,\Omega$

Match the Columns

1. Match the following two columns.

Column I	Column II
(a) Magnetic induction	(p) $[MT^{-2}A^{-1}]$
(b) Coefficient of self-induction	(q) $[L^2T^{-2}]$
(c) LC	(r) $[ML^2T^{-2}A^{-2}]$
(d) Magnetic flux	(s) None of these

2. In the circuit shown in figure, switch is closed at time $t = 0$. Match the following two columns.

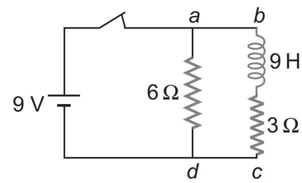


Column I	Column II
(a) V_L at $t = 0$	(p) zero
(b) V_R at $t = 0$	(q) 10 V
(c) V_L at $t = 1$ s	(r) $\dfrac{10}{e}$ V
(d) V_R at $t = 1$ s	(s) $\left(1 - \dfrac{1}{e}\right) 10$ V

3. In an L-C oscillation circuit, $L = 1\,\text{H}$, $C = \dfrac{1}{4}\,\text{F}$ and maximum charge in the capacitor is $4\,\text{C}$. Match the following two columns. Note that in Column II, all values are in SI units.

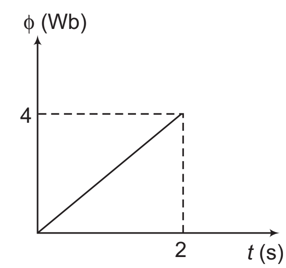
Column I	Column II
(a) Maximum current in the circuit	(p) 16
(b) Maximum rate of change of current in the circuit	(q) 4
(c) Potential difference across inductor when $q = 2C$	(r) 2
(d) Potential difference across capacitor when rate of change of current is half its maximum value	(s) 8

4. In the circuit shown in figure, switch remains closed for long time. It is opened at time $t = 0$. Match the following two columns at $t = (\ln 2)$ second.



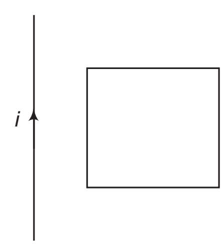
Column I	Column II
(a) Potential differences across inductor	(p) 9 V
(b) Potential difference across 3 Ω resistance	(q) 4.5 V
(c) Potential difference across 6 Ω resistance	(r) 6 V
(d) Potential difference between points b and c	(s) None of these

5. Magnetic flux passing through a coil of resistance 2 Ω is as shown in figure. Match the following two columns. In Column II all physical quantities are in SI units.



Column I	Column II
(a) Induced emf produced	(p) 4
(b) Induced current	(q) 1
(c) Charge flow in 2 s	(r) 8
(d) Heat generation in 2 s	(s) 2

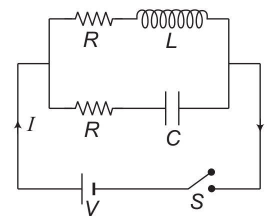
6. A square loop is placed near a long straight current carrying wire as shown. Match the following two columns.



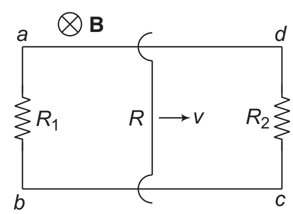
Column I	Column II
(a) If current is increased	(p) induced current in loop is clockwise
(b) If current is decreased	(q) induced current in loop is anti-clockwise
(c) If loop is moved away from the wire	(r) wire will attract the loop
(d) If loop is moved towards the wire	(s) wire will repel the loop

Subjective Questions

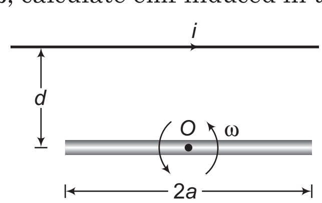
1. In the circuit diagram shown, initially there is no energy in the inductor and the capacitor. The switch is closed at $t = 0$. Find the current I as a function of time if $R = \sqrt{L/C}$.



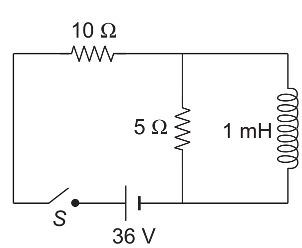
2. A rectangular loop with a sliding connector of length l is located in a uniform magnetic field perpendicular to the loop plane. The magnetic induction is equal to B. The connector has an electric resistance R, the sides ab and cd have resistances R_1 and R_2. Neglecting the self-inductance of the loop, find the current flowing in the connector during its motion with a constant velocity v.



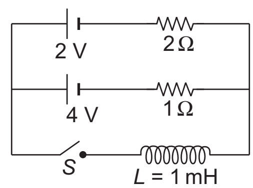
3. A rod of length $2a$ is free to rotate in a vertical plane, about a horizontal axis O passing through its mid-point. A long straight, horizontal wire is in the same plane and is carrying a constant current i as shown in figure. At initial moment of time, the rod is horizontal and starts to rotate with constant angular velocity ω, calculate emf induced in the rod as a function of time.



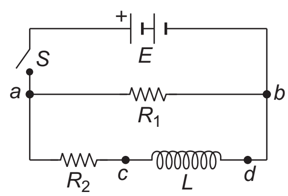
4. In the circuit arrangement shown in figure, the switch S is closed at $t = 0$. Find the current in the inductance as a function of time? Does the current through $10\ \Omega$ resistor vary with time or remains constant.



5. In the circuit shown, switch S is closed at time $t = 0$. Find the current through the inductor as a function of time t.

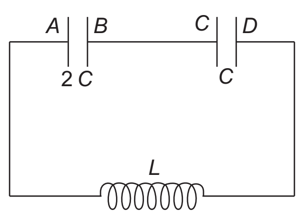


6. In the circuit shown in figure, $E = 120$ V, $R_1 = 30.0\ \Omega$, $R_2 = 50.0\ \Omega$, and $L = 0.200$ H. Switch S is closed at $t = 0$. Just after the switch is closed.



(a) What is the potential difference V_{ab} across the inductor R_1?
(b) Which point, a or b, is at higher potential?
(c) What is the potential difference V_{cd} across the inductor L?
(d) Which point, c or d, is at a higher potential?
The switch is left closed for a long time and then is opened. Just after the switch is opened
(e) What is the potential difference V_{ab} across the resistor R_1?
(f) Which point a or b, is at a higher potential?
(g) What is the potential difference V_{cd} across the inductor L?
(h) Which point, c or d, is at a higher potential?

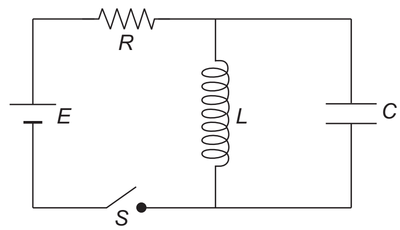
7. Two capacitors of capacitances $2C$ and C are connected in series with an inductor of inductance L. Initially, capacitors have charge such that $V_B - V_A = 4V_0$ and $V_C - V_D = V_0$. Initial current in the circuit is zero. Find



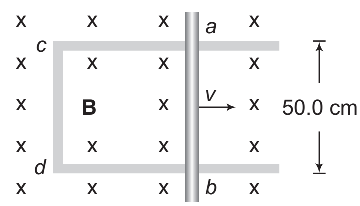
(a) maximum current that will flow in the circuit,
(b) potential difference across each capacitor at that instant,
(c) equation of current flowing towards left in the inductor.

8. A 1.00 mH inductor and a 1.00 μF capacitor are connected in series. The current in the circuit is described by $i = 20\,t$, where t is in second and i is in ampere. The capacitor initially has no charge. Determine
(a) the voltage across the inductor as a function of time,
(b) the voltage across the capacitor as a function of time,
(c) the time when the energy stored in the capacitor first exceeds that in the inductor.

9. In the circuit shown in the figure, $E = 50.0$ V, $R = 250\ \Omega$ and $C = 0.500\ \mu$F. The switch S is closed for a long time, and no voltage is measured across the capacitor. After the switch is opened, the voltage across the capacitor reaches a maximum value of 150 V. What is the inductance L?



10. The conducting rod ab shown in figure makes contact with metal rails ca and db. The apparatus is in a uniform magnetic field 0.800 T, perpendicular to the plane of the figure.



(a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s.

(b) In what direction does the current flow in the rod?

(c) If the resistance of the circuit $abdc$ is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. You can ignore friction.

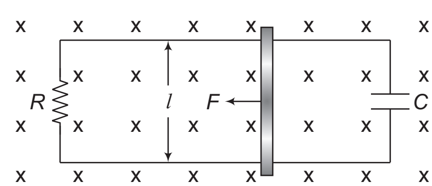
(d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

11. A non-conducting ring of mass m and radius R has a charge Q uniformly distributed over its circumference. The ring is placed on a rough horizontal surface such that plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0 t^2$ tesla is switched on. After 2 s from switching on the magnetic field the ring is just about to rotate about vertical axis through its centre.

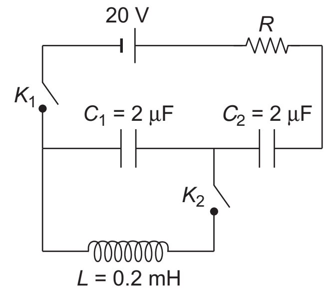
(a) Find friction coefficient μ between the ring and the surface.

(b) If magnetic field is switched off after 4 s, then find the angle rotated by the ring before coming to stop after switching off the magnetic field.

12. Two parallel long smooth conducting rails separated by a distance l are connected by a movable conducting connector of mass m. Terminals of the rails are connected by the resistor R and the capacitor C as shown in figure. A uniform magnetic field B perpendicular to the plane of the rail is switched on. The connector is dragged by a constant force F. Find the speed of the connector as a function of time if the force F is applied at $t = 0$. Also find the terminal velocity of the connector.



13. A circuit containing capacitors C_1 and C_2, shown in the figure is in the steady state with key K_1 closed and K_2 opened. At the instant $t = 0$, K_1 is opened and K_2 is closed.

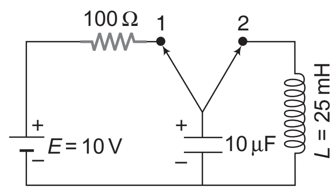


(a) Find the angular frequency of oscillations of L-C circuit.

(b) Determine the first instant t, when energy in the inductor becomes one third of that in the capacitor.

(c) Calculate the charge on the plates of the capacitor at that instant.

14. Initially, the capacitor is charged to a potential of 5 V and then connected to position 1 with the shown polarity for 1 s. After 1 s it is connected across the inductor at position 2.

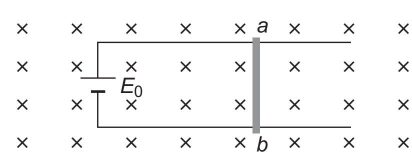


(a) Find the potential across the capacitor after 1 s of its connection to position 1.

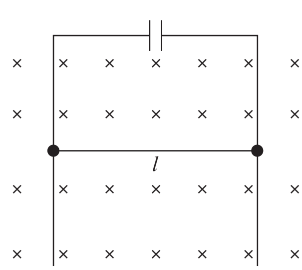
(b) Find the maximum current flowing in the L-C circuit when capacitor is connected across the inductor. Also, find the frequency of LC oscillations.

15. A rod of mass m and resistance R slides on frictionless and resistanceless rails a distance l apart that include a source of *emf* E_0. (see figure). The rod is initially at rest. Find the expression for the

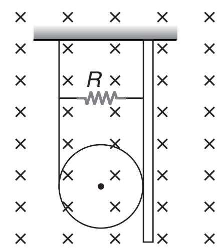
(a) velocity of the rod $v\ (t)$. (b) current in the loop $i(t)$.



16. Two metal bars are fixed vertically and are connected on the top by a capacitor C. A sliding conductor of length l and mass m slides with its ends in contact with the bars. The arrangement is placed in a uniform horizontal magnetic field directed normal to the plane of the figure. The conductor is released from rest. Find the displacement $x(t)$ of the conductor as a function of time t.

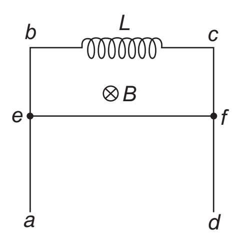


17. A conducting light string is wound on the rim of a metal ring of radius r and mass m. The free end of the string is fixed to the ceiling. A vertical infinite smooth conducting plane is always tangent to the ring as shown in the figure. A uniform magnetic field B is applied perpendicular to the plane of the ring. The ring is always inside the magnetic field. The plane and the string are connected by a resistance R. When the ring is released, find



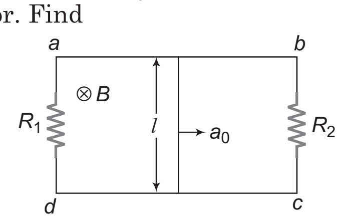
(a) the current in the resistance R as a function of time.
(b) the terminal velocity of the ring.

18. A conducting frame $abcd$ is kept in a vertical plane. A conducting rod ef of mass m and length l can slide smoothly on it remaining always horizontal. The resistance of the loop is negligible and inductance is constant having value L. The rod is left from rest and allowed to fall under gravity and inductor has no initial current. A magnetic field of constant magnitude B is present throughout the loop pointing inwards. Determine



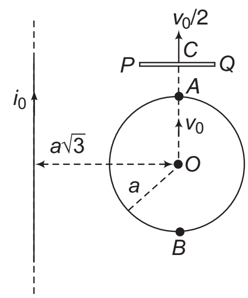
(a) position of the rod as a function of time assuming initial position of the rod to be $x = 0$ and vertically downward as the positive x-axis.
(b) the maximum current in the circuit.
(c) maximum velocity of the rod

19. A rectangular loop with a sliding conductor of length l is located in a uniform magnetic field perpendicular to the plane of loop. The magnetic induction perpendicular to the plane of loop is equal to B. The part ad and bc has electric resistance R_1 and R_2, respectively. The conductor starts moving with constant acceleration a_0 at time $t = 0$. Neglecting the self-inductance of the loop and resistance of conductor. Find



(a) the current through the conductor during its motion.
(b) the polarity of $abcd$ terminal.
(c) external force required to move the conductor with the given acceleration.

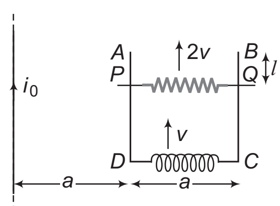
20. A conducting circular loop of radius a and resistance per unit length R is moving with a constant velocity v_0, parallel to an infinite conducting wire carrying current i_0. A conducting rod of length $2a$ is approaching the centre of the loop with a constant velocity $\frac{v_0}{2}$ along the direction of the current. At the instant $t = 0$, the rod comes in contact with the loop at A and starts sliding on the loop with the constant velocity. Neglecting the resistance of the rod and the self-inductance of the circuit, find the following when the rod slides on the loop.



(a) The current through the rod when it is at a distance of $\left(\frac{a}{2}\right)$ from the point A of the loop.

(b) Force required to maintain the velocity of the rod at that instant.

21. U-frame $ABCD$ and a sliding rod PQ of resistance R, start moving with velocities v and $2v$ respectively, parallel to a long wire carrying current i_0. When the distance $AP = l$ at $t = 0$, determine the current through the inductor of inductance L just before connecting rod PQ loses contact with the U-frame.



Answers

Introductory Exercise 27.1

1. Anti-clockwise
2. No
3. $[ML^2A^{-1}T^{-3}]$
4. Clockwise
5. Same direction, opposite direction.
6. 6.74 V
7. 1600 μC
8. 9.0×10^{-7} Wb

Introductory Exercise 27.2

1. 4.4 V, north
2. 0.00375 N
3. (a) $\frac{B\omega l^2}{2}$ (b) $\frac{-3B\omega l^2}{2}$
4. No

Introductory Exercise 27.3

1. $3(t \cos t + \sin t)$
2. $-80e^{-4t}$, $-40e^{-4t}$
3. (a) 0.625 mH (b) 0.13 J, 0.21 J/s
4. (a) 4.5×10^{-5} H (b) 4.5×10^{-3} V

Introductory Exercise 27.4

1. 3.125 mH, 0.9375 V
2. (a) 2 H (b) 30 V (c) 1 H
3. (a) 0.27 V, Yes (b) 0.27 V

Introductory Exercise 27.5

2. (a) 0.2 s (b) 10 A (c) 9.93 A
3. No

Introductory Exercise 27.6

2. With KE as $v \Leftrightarrow i$ and $m \Leftrightarrow L$. Therefore, $\frac{1}{2}mv^2 = \frac{1}{2}Li^2$ **3.** (a) 45.9 μC (b) 23.3 V **4.** 20.0 V

Introductory Exercise 27.7

1. (a) 3.1×10^{-6} V (b) 2.0×10^{-6} V/m
2. (a) 8.0×10^{-21} N (downward and to the right perpendicular to r_2)
(b) 0.36 V/m (upwards and to the left perpendicular to r_1)

Exercises

LEVEL 1

Assertion and Reason

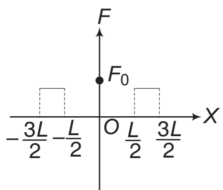
1. (d) **2.** (b) **3.** (d) **4.** (a,b) **5.** (a) **6.** (d) **7.** (b) **8.** (c) **9.** (d) **10.** (c)

Objective Questions

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (b)
11. (a)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (a)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (d)
31. (b)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (b)	**37.** (c)	**38.** (c)	**39.** (a)	

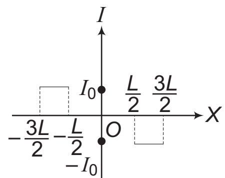
Subjective Questions

1. True
2. 272 m
3. $e = aSB_0 e^{-at}$
4. (a) $\dfrac{\mu_0 iv}{2\pi} \ln\left(1 + \dfrac{l}{d}\right)$ (b) a (c) zero
5. (a) $(0.5)(1 - e^{-10t})\,A$ (b) $1.50\text{ A} - (0.25\text{ A})\,e^{-10t}$
6. 0.5 T
7. (a) 0.011 V (b) zero (c) – 0.011 V
8.



$F_0 = \dfrac{B^2L^2V}{R}$

(a)



$i_0 = \dfrac{BLV}{R}$

(b)

9. $e = \dfrac{\mu_0}{4\pi} \dfrac{2ia^2v}{x(x+a)}$
10. $(2 \times 10^{-5}) \ln(10)$ V
11. 3.33×10^{-2} A, 6.67×10^{-2} A
12. (a) 0.25 H (b) 4.5×10^{-4} Wb
13. (a) 1.96 H (b) 7.12×10^{-3} Wb
14. 0.37
15. (a) 518 mW (b) 328 mW (c) 191 mW
16. (a) 17.3 μs (b) 30.7 μs
17. (a) $\dfrac{E}{r} e^{-\left(\frac{R+r}{L}\right)t}$ (b) $\dfrac{E^2L}{2r(R+r)}$
18. $\dfrac{8}{3}$ A
19. (a) 3.6 mH (b) 1.33 kHz (c) 0.188 ms
20. (a) 6.28×10^3 rad/s, 10^{-3} s (b) $10^{-4} \cos(6.28 \times 10^3 t)$ (c) 0.0253 H (d) 0.4 A
21. (a) 1.25×10^{-2} V (b) 8.33×10^{-4} A (c) 3.125×10^{-8} J (d) 4.33×10^{-6} C, 7.8×10^{-7} J

LEVEL 2

Single Correct Option

1.(b)	**2.**(c)	**3.**(a)	**4.**(b)	**5.**(c)	**6.**(a)	**7.**(a)	**8.**(c)	**9.**(c)	**10.**(a)
11.(b)	**12.**(b)	**13.**(c)	**14.**(a)	**15.**(b)	**16.**(a)	**17.**(b)	**18.**(a)	**19.**(d)	**20.**(a)
21.(a)	**22.**(d)	**23.**(a)	**24.**(b)	**25.**(b)	**26.**(d)	**27.**(b)	**28.**(b)	**29.**(a)	**30.**(c)
31.(c)	**32.**(b)	**33.**(c)	**34.**(a)	**35.**(a)	**36.**(d)	**37.**(d)			

More than One Correct Options

1. (a,c) **2.** (a,c) **3.** (a,c) **4.** (a,b,d) **5.** (a,c) **6.** (b,c,d) **7.** (b,c) **8.** (b,c,d) **9.** (a,b,c) **10.** (a,c)

Comprehension Based Questions

1.(b)	**2.**(c)	**3.**(b)	**4.**(a)	**5.**(c)	**6.**(a)	**7.**(d)	**8.**(a)	**9.**(b)	**10.**(c)
11.(c)	**12.**(a)	**13.**(d)							

Match the Columns

1.	(a) → p	(b) → r	(c) → s	(d) → s
2.	(a) → q	(b) → p	(c) → r	(d) → s
3.	(a) → s	(b) → p	(c) → s	(d) → s
4.	(a) → s	(b) → q	(c) → p	(d) → p
5.	(a) → s	(b) → q	(c) → s	(d) → p
6.	(a) → q, s	(b) → p, r	(c) → p, r	(d) → q, s

Subjective Questions

1. $I = \dfrac{V}{R}$ **2.** $i = \dfrac{Bvl}{R + R'}$ where $R' = \dfrac{R_1 R_2}{R_1 + R_2}$ **3.** $e = \dfrac{\mu_0 i\omega}{2\pi \sin\omega t}\left[\dfrac{d}{\sin\omega t}\ln\left(\dfrac{d - a\sin\omega t}{d + a\sin\omega t}\right) + 2a\right]$

4. $3.6\,(1 - e^{-t/\tau_L})$ A. Here, $\tau_L = 300\,\mu$s. Current through $10\,\Omega$ resistor varies with time.

5. $i = 5(1 - e^{-2000t/3})$ A

6. (a) 120 V (b) a (c) 120 V (d) c (e) -72 V (f) b (g) -192 V (h) d

7. (a) $q_0\omega$ (b) $3V_0 : 3V_0$ (c) $i = q_0 \sin\omega t$. Here, $q_0 = 2CV_0$ and $\omega = \sqrt{\dfrac{3}{2LC}}$

8. (a) 20 mV (b) $10^7 t^2$ V (c) 63.2 μs

9. 0.28 H

10. (a) 3 V (b) b to a (c) 0.8 N towards right (d) both are 6 W

11. (a) $\dfrac{2B_0QR}{mg}$ (b) $\dfrac{B_0Q}{m}$

12. $V = \dfrac{FR}{B^2l^2}(1 - e^{-\alpha t})$ Here, $\alpha = \dfrac{B^2l^2}{mR + RB^2l^2C}$, $v_T = \dfrac{FR}{B^2l^2}$

13. (a) 5×10^4 rad/s (b) 1.05×10^{-5} s (c) $10\sqrt{3}\,\mu$C

14. (a) 8.16 V (b) 5.16 A, 10 Hz

15. (a) $v = \dfrac{E_0}{Bl}\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$ (b) $i = \dfrac{E_0 - Blv}{R}$

16. $x = \dfrac{mgt^2}{2\,(m + CB^2l^2)}$

17. (a) $i = \dfrac{mg}{2Br}\left(1 - e^{\frac{-2B^2r^2}{mR}t}\right)$ (b) $v_T = \dfrac{mgR}{4B^2r^2}$

18. (a) $x = \dfrac{v_0}{\omega}(1 - \cos\omega t)$, $\omega = \dfrac{Bl}{\sqrt{mL}}$ (b) $i_{max} = \dfrac{2mg}{Bl}$ (c) $v_{max} = v_0 = \dfrac{g\sqrt{mL}}{Bl}$

19. (a) $i = \dfrac{Bla_0t}{R_1R_2}(R_1 + R_2)$ (b) Polarity of a, b is positive and polarity of c, d is negative

(c) $F_{ext} = a_0\left[m + \dfrac{B^2l^2t}{R_1R_2}(R_1 + R_2)\right]$

20. (a) $i = \dfrac{9v_0\mu_0 i_0}{16aR\pi^2}\ln(3)$ (b) $\dfrac{9\,\mu_0^2 i_0^2 v_0}{32aR\pi^3}(\ln 3)^2$

21. $i = \left(\dfrac{e}{R}\right)[1 - e^{-lR/Lv}]$, where $e = \dfrac{\mu_0 i_0 v}{2\pi}\ln(2)$

28 Alternating Current

Chapter Contents

28.1 Introduction

A century ago, one of the great technological debates was whether the electrical distribution system should be AC or DC. Thomas Edison favoured direct current (DC), that is, steady current that does not vary with time. George Westinghouse favoured alternating current (AC), with sinusoidally varying voltages and currents. He argued that transformers can be used to step the voltage up or down with AC but not with DC. Low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long distances power transmission to minimize i^2R losses in the cables. Eventually, Westinghouse prevailed, and most present day household and industrial power distribution systems operate with alternating current.

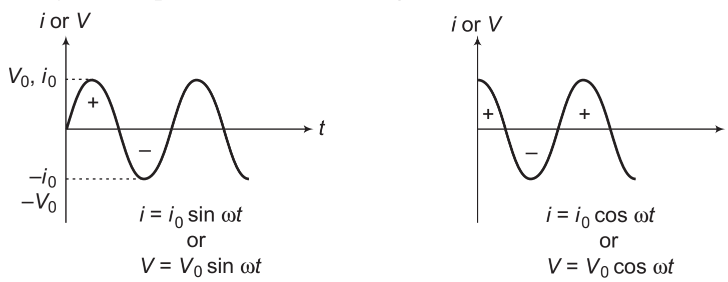


Fig. 28.1

A time varying current or voltage may be periodic and non-periodic. In case of periodic current or voltage, the current or voltage is said to be alternating if its amplitude is constant and alternate half cycle is positive and half negative. If the current or voltage varies periodically as sine or cosine function of time, the current or voltage is said to be sinusoidal and is what we usually mean by it.

28.2 Alternating Currents and Phasors

The basic principle of the AC generator is a direct consequence of Faraday's law of induction. When a conducting loop is rotated in a magnetic field at constant angular frequency ω a sinusoidal voltage (emf) is induced in the loop. This instantaneous voltage is

$$\boxed{V = V_0 \sin \omega t} \qquad \ldots\text{(i)}$$

The usual circuit diagram symbol for an AC source is shown in Fig. 28.2. In Eq. (i), V_0 is the maximum output voltage of the AC generator or the **voltage amplitude** and ω is the **angular frequency** equal to 2π times the frequency f.

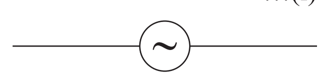


Fig. 28.2

$$\boxed{\omega = 2\pi f}$$

The frequency of AC in India is 50 Hz, i.e.

$$f = 50 \text{ Hz}$$

So, $$\omega = 2\pi f \approx 314 \text{ rad/s}$$

The time of one cycle is known as **time period *T***, the number of cycles per second the frequency f.

$$\boxed{T = \frac{1}{f}} \quad \text{or} \quad \boxed{T = \frac{2\pi}{\omega}}$$

A sinusoidal current might be described as

$$i = i_0 \sin \omega t$$

If an alternating current is passed through an ordinary ammeter or voltmeter, it will record the mean value for the complete cycle, as the quantity to be measured varies with time. The **average** value of current for one cycle is

$$\langle i \rangle_{\text{One cycle}} = (i_{\text{av}})_T = \frac{\int_0^T i\,dt}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} (i_0 \sin \omega t)\,dt}{\int_0^{2\pi/\omega} dt} = 0$$

Thus, $$\boxed{\langle i \rangle_{\text{One cycle}} = 0}$$

Similarly, the average value of the voltage (or emf) for one cycle is zero.

$$\boxed{\langle V \rangle_{\text{One cycle}} = 0}$$

Since, these averages for the whole cycle are zero, the DC instrument will indicate zero deflection. In AC, the average value of current is defined as its average taken over half the cycle. Hence,

$$\langle i \rangle_{\text{Half cycle}} = (i_{\text{av}})_{T/2} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{\pi/\omega} (i_0 \sin \omega t)\,dt}{\int_0^{\pi/\omega} dt} = \frac{2}{\pi} i_0$$

This is sometimes simply written as i_{av}. Hence,

$$i_{\text{av}} = \langle i \rangle_{\text{Half cycle}} = \frac{2}{\pi} i_0 \approx 0.637\, i_0$$

Similarly, $$V_{\text{av}} = \frac{2}{\pi} V_0 \approx 0.637\, V_0$$

A DC meter can be used in an AC circuit if it is connected in the full wave rectifier circuit. The average value of the rectified current is the same as the average current in any half cycle, i.e. $\frac{2}{\pi}$ times the maximum current i_0. A more useful way to describe a quantity is the **root mean square (rms)** value. We square the instantaneous current, take the average (mean) value of i^2 and finally take the square root of that average. This procedure defines the root-mean-square current denoted as i_{rms}. Even when i is negative, i^2 is always positive so i_{rms} is never zero (unless i is zero at every instant). Hence,

$$\langle i^2 \rangle_{\text{One cycle}} = \frac{\int_0^T i^2\,dt}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} (i_0^2 \sin^2 \omega t)\,dt}{\int_0^{2\pi/\omega} dt} = \frac{i_0^2}{2}$$

∴ $$i_{\text{rms}} = \sqrt{\langle i^2 \rangle_{\text{One cycle}}} = \frac{i_0}{\sqrt{2}} \approx 0.707\, i_0$$

Thus, $$\boxed{i_{\text{rms}} = \frac{i_0}{\sqrt{2}} \approx 0.707\, i_0}$$

Similarly, we get

$$\boxed{V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \approx 0.707\, V_0}$$

The square root of the mean square value is called the **virtual value** and is the value given by AC instruments.

Thus, when we speak of our house hold power supply as 220 V AC, this means that the rms voltage is 220 V and its voltage amplitude is

$$V_0 = \sqrt{2}\, V_{\text{rms}} = 311 \text{ V}$$

Form Factor

The ratio, $= \dfrac{\text{rms value}}{\text{average value}} = \dfrac{V_0/\sqrt{2}}{2V_0/\pi} = \dfrac{\pi}{2\sqrt{2}} = 1.11$

is known as form factor.

The different values i_0, i_{av} and i_{rms} are shown in Fig. 28.3.

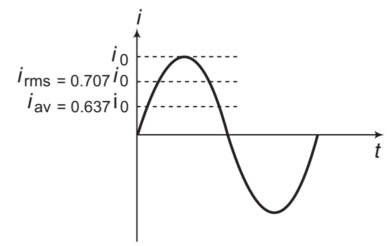


Fig. 28.3

Note (1) *The average value of* $\sin \omega t$, $\cos \omega t$, $\sin 2\omega t$, $\cos 2\omega t$, *etc., is zero because it is positive in half of the time and negative in rest half of the time. Thus,*

$$\langle \sin \omega t \rangle = \langle \cos \omega t \rangle = \langle \sin 2\,\omega t \rangle = \langle \cos 2\,\omega t \rangle = 0$$

If $$i = i_0 \sin \omega t$$

then $$\langle i \rangle = \langle i_0 \sin \omega t \rangle = i_0 \langle \sin \omega t \rangle = 0$$

(2) *The average value of* $\sin^2 \omega t$ *and* $\cos^2 \omega t$ *is* $\frac{1}{2}$.

or $$\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = \frac{1}{2}$$

If $$i^2 = i_0^2 \sin^2 \omega t$$

then $$\langle i^2 \rangle = \langle i_0^2 \sin^2 \omega t \rangle = i_0^2 \langle \sin^2 \omega t \rangle = \frac{i_0^2}{2}$$

(3) *Like SHM, general expressions of current/voltage in an sinusoidal AC are*

$$i = i_0 \sin(\omega t \pm \phi)$$

$$V = V_0 \sin(\omega t \pm \phi)$$

or $$i = i_0 \cos(\omega t \pm \phi)$$

and $$V = V_0 \cos(\omega t \pm \phi)$$

(4) *Average value of current or voltage over a half cycle can be zero also. This depends on the time interval (of course $T/2$) over which average value is to be found. Think why?*

Phasors

If an AC generator is connected to a series circuit containing resistors, inductors and capacitors and we want to know the amplitude and time characteristics of the alternating current. To simplify our analysis of circuits containing two or more elements, we use graphical constructions called **phasor diagrams.** In these constructions, alternating (sinusoidal) quantities, such as current and voltage are rotating vectors called **phasors**.

In these diagrams, the instantaneous value of a quantity that varies sinusoidally with time is represented by the projection onto a vertical axis (if it is a sine function) or onto a horizontal axis (if it is a cosine function) of a vector with a length equal to the amplitude (i_0) of the quantity. The vector rotates counterclockwise with constant angular velocity ω.

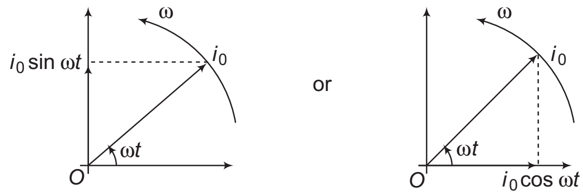


Fig. 28.4

A phasor is not a real physical quantity with a direction in space, such as velocity, momentum or electric field. Rather, it is a geometric entity that helps us to describe and analyze physical quantities that vary sinusoidally with time.

Example 28.1 *Show that average heat produced during a cycle of AC is same as produced by DC with $i = i_{rms}$.*

Solution For an AC, $i = i_0 \sin \omega t$

Therefore, instantaneous value of heat produced in time dt across a resistance R is

$$dH = i^2 R dt = i_0^2 R \sin^2 \omega t \, dt$$

∴ Average value of heat produced during a cycle,

$$H_{av} = \frac{\int_0^T dH}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} (i_0^2 R \sin^2 \omega t)\, dt}{\int_0^{2\pi/\omega} dt}$$

$$= \frac{i_0^2}{2} R\left(\frac{2\pi}{\omega}\right) = i_{rms}^2 RT$$

i.e. AC produces same heating effects as DC of value $i = i_{rms}$.

Example 28.2 *If the current in an AC circuit is represented by the equation,*

$$i = 5 \sin (300t - \pi/4)$$

Here, t is in second and i in ampere. Calculate

(a) peak and rms value of current (b) frequency of AC (c) average current

Solution (a) As in case of AC,

$$i = i_0 \sin (\omega t \pm \phi)$$

∴ The peak value, $i_0 = 5$ A **Ans.**

and $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.535$ A **Ans.**

(b) Angular frequency, $\omega = 300$ rad/s

∴ $f = \frac{\omega}{2\pi} = \frac{300}{2\pi} \approx 47.75$ Hz **Ans.**

(c) $i_{av} = \left(\frac{2}{\pi}\right) i_0 = \left(\frac{2}{\pi}\right)(5) = 3.18$ A **Ans.**

28.3 Current and Potential Relations

In this section, we will derive voltage current relations for individual circuit elements carrying a sinusoidal current. We will consider resistors, inductors and capacitors.

Resistor in an AC Circuit

Consider a resistor with resistance R through which there is a sinusoidal current given by

$$i = i_0 \sin \omega t \qquad \ldots\text{(i)}$$

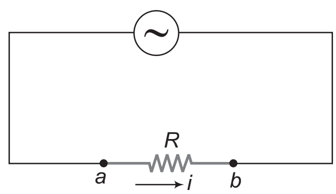


Fig. 28.5

Here, i_0 is the current amplitude (maximum current). From Ohm's law, the instantaneous PD between points a and b is

$$V_R = iR = (i_0 R) \sin \omega t$$

We can write as

$$i_0 R = V_0, \quad \text{the voltage amplitude}$$

$$\therefore \qquad V_R = V_0 \sin \omega t \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we can see that current and voltage are **in phase** if only resistance is in the circuit. Fig. 28.6 shows graphs of i and V_R as functions of time.

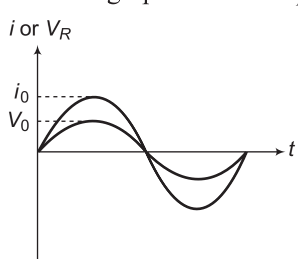


Fig. 28.6

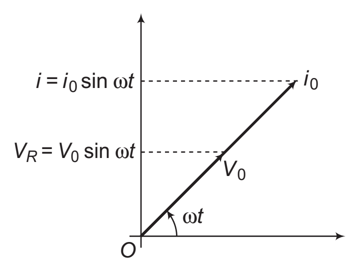


Fig. 28.7

The corresponding phasor diagram is shown in Fig. 28.7.

Because i and V_R are in phase and have the same frequency, the current and voltage phasors rotate together, they are parallel at each instant. Their projection on vertical axis represents the instantaneous current and voltage respectively.

Note *Direction of an alternating current is not shown in a circuit, as it keeps on changing. In the figure, the direction of instantaneous current is only shown.*

Capacitor in an AC Circuit

If a capacitor of capacitance C is connected across the alternating source, the instantaneous charge on the capacitor is

$$q = CV_C = CV_0 \sin \omega t$$

and the instantaneous current i passing through it is given by

$$i = \frac{dq}{dt} = CV_0\omega \cos \omega t$$

$$= \frac{V_0}{1/\omega C} \sin (\omega t + \pi/2)$$

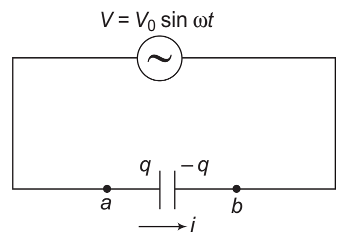


Fig. 25.8

or $$i = i_0 \sin (\omega t + \pi/2)$$

Here, $$\boxed{V_0 = \frac{i_0}{\omega C}}$$

This relation shows that the quantity $\frac{1}{\omega C}$ is the effective AC resistance or the **capacitive reactance** of the capacitor and is represented as X_C. It has unit as ohm. Thus,

$$\boxed{X_C = \frac{1}{\omega C}}$$

It is clear that the **current leads the voltage by 90°** or the potential drop across the capacitor lags the current passing it by 90°.

Fig. 28.9 shows V and i as functions of time t.

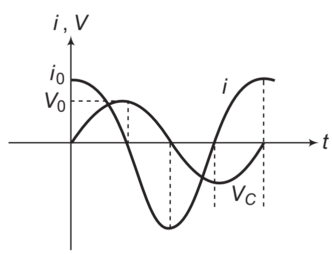


Fig. 28.9

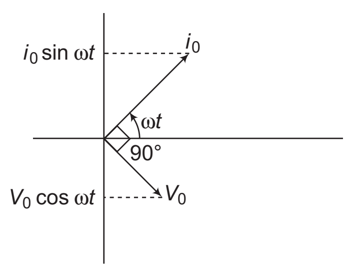


Fig. 28.10

The phasor diagram 28.10 shows that voltage phasor is behind the current phasor by a quarter cycle or 90°.

Inductor in an AC Circuit

Consider a pure inductor of self-inductance L and zero resistance connected to an alternating source. Again we assume that an instantaneous current $i = i_0 \sin \omega t$ flows through the inductor. Although, there is no resistance, there is a potential difference V_L between the inductor terminals a and b because the current varies with time giving rise to a self-induced emf.

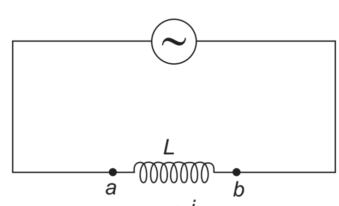


Fig. 28.11

$$V_L = V_{ab} = -\,(\text{induced emf}) = -\left(-L\frac{di}{dt}\right)$$

or $$V_L = L\frac{di}{dt} = Li_0\omega \cos \omega t$$

or $$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(i)}$$

Here, $$\boxed{V_0 = i_0(\omega L)} \qquad \text{...(ii)}$$

or $$i_0 = \frac{V_0}{\omega L}$$

$\therefore$ $$\boxed{i = \frac{V_0}{\omega L}\sin \omega t} \qquad \text{...(iii)}$$

Eq. (iii) shows that effective AC resistance, i.e. **inductive reactance of inductor** is

$$\boxed{X_L = \omega L}$$

and the maximum current,

$$\boxed{i_0 = \frac{V_0}{X_L}}$$

The unit of X_L is also ohm.

From Eqs. (i) and (iii), we see that **the voltage across the inductor leads the current passing through it by 90°**.

Fig. 28.12 shows V_L and i as functions of time.

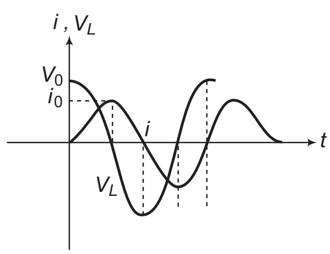


Fig. 28.12

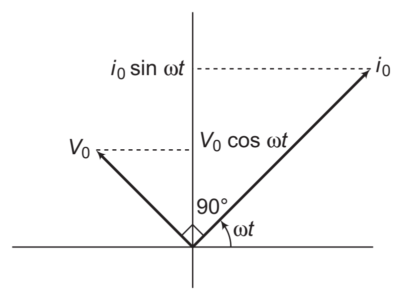


Fig. 28.13

Phasor diagram in Fig. 28.13 shows that V_L leads the current i by 90°.

Extra Points to Remember

- Circuit elements with AC

Circuit elements	Amplitude relation	Circuit quantity	Phase of V
Resistor	$V_0 = i_0 R$	R	in phase with i
Capacitor	$V_0 = i_0 X_C$	$X_C = \frac{1}{\omega C}$	lags i by 90°
Inductor	$V_0 = i_0 X_L$	$X_L = \omega L$	leads i by 90°

- In DC, $\omega = 0$, therefore, $X_L = 0$ and $X_C = \infty$

- The potential of point a with respect to point b is given by $V_L = +L\dfrac{di}{dt}$, the negative of the induced emf. This expression gives the correct sign of V_L in all cases.

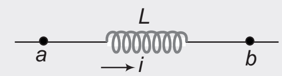


Fig. 28.14

- If an oscillating voltage of a given amplitude V_0 is applied across an inductor, the resulting current will have a smaller amplitude i_0 for larger value of ω. Since, X_L is proportional to frequency, a high frequency voltage applied to the inductor gives only a small current while a lower frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio interference filters to block high frequencies while permitting lower frequencies to pass through. A circuit device that uses an inductor for this purpose is called a **low pass filter**.

- The capacitive reactance of a capacitor is inversely proportional to the capacitance C and the angular frequency ω. The greater the capacitance and the higher the frequency, the smaller is the capacitive reactance X_C. Capacitors tend to pass high frequency current and to block low frequency current, just the opposite of inductors. A device that passes signals of high frequency is called a **high pass filter**.

- Figure shows the graphs of R, X_L and X_C as functions of angular frequency ω.

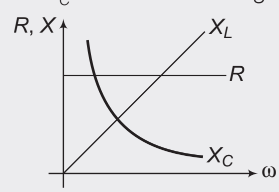


Fig. 28.15

- Remember that we can write, $V_R = iR$, $(V_0)_R = i_0R$, $(V_0)_L = i_0X_L$ and $(V_0)_C = i_0X_C$ but can't write (for instantaneous voltages).

$$V_L = iX_L$$

or

$$V_C = iX_C$$

This is because there is a phase difference between the voltage and current in both an inductor and a capacitor.

Example 28.3 *A 100 Ω resistance is connected in series with a 4 H inductor. The voltage across the resistor is $V_R = (2.0\ V)\sin(10^3\ rad/s)\,t$:*

(a) Find the expression of circuit current
(b) Find the inductive reactance
(c) Derive an expression for the voltage across the inductor.

Solution (a)

$$i = \frac{V_R}{R} = \frac{(2.0\ \text{V})\sin(10^3\ \text{rad/s})\,t}{100}$$

$$= (2.0 \times 10^{-2}\ \text{A})\sin(10^3\ \text{rad/s})\,t \qquad \textbf{Ans.}$$

(b)

$$X_L = \omega L = (10^3\ \text{rad/s})(4\ \text{H})$$

$$= 4.0 \times 10^3\ \Omega \qquad \textbf{Ans.}$$

(c) The amplitude of voltage across inductor,

$$V_0 = i_0X_L = (2.0 \times 10^{-2}\ \text{A})(4.0 \times 10^3\ \Omega)$$

$$= 80\ \text{V} \qquad \textbf{Ans.}$$

In an AC, voltage across the inductor leads the current by 90° or $\pi/2$ rad. Hence,

$$V_L = V_0 \sin(\omega t + \pi/2)$$
$$= (80\text{ V})\sin\left\{(10^3\text{ rad/s})\,t + \frac{\pi}{2}\text{ rad}\right\}$$

Note *That the amplitude of voltage across the resistor* ($= 2.0$ V) *is not same as the amplitude of the voltage across the inductor* ($= 80$ V), *even though the amplitude of the current through both devices is the same.*

28.4 Phasor Algebra

The complex quantities normally employed in AC circuit analysis, can be added and subtracted like coplanar vectors. Such coplanar vectors which represent sinusoidally time varying quantities are known as **phasors**.

In Cartesian form, a phasor **A** can be written as

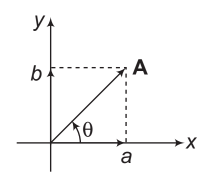


Fig. 28.16

$$\boxed{\mathbf{A} = a + jb}$$

where, a is the x-component and b is the y-component of phasor **A**.

The magnitude of **A** is $\boxed{|\mathbf{A}| = \sqrt{a^2 + b^2}}$

and the angle between the direction of phasor **A** and the positive x-axis is

$$\boxed{\theta = \tan^{-1}\left(\frac{b}{a}\right)}$$

When a given phasor **A**, the direction of which is along the x-axis is multiplied by the operator j, a new phasor $j\mathbf{A}$ is obtained which will be 90° anti-clockwise from **A**, i.e. along y-axis. If the operator j is multiplied now to the phasor $j\mathbf{A}$, a new phasor $j^2\mathbf{A}$ is obtained which is along $-x$-axis and having same magnitude as of **A**. Thus,

$$j^2\mathbf{A} = -\mathbf{A}$$

$\therefore$ $$\boxed{j^2 = -1 \quad \text{or} \quad j = \sqrt{-1}}$$

Now, using the j operator, let us discuss different circuits of an AC.

28.5 Series *L-R* Circuit

As we know, potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor.

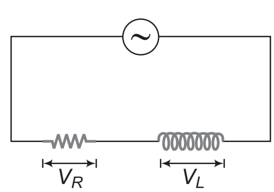


Fig. 28.17

Suppose in phasor diagram current is taken along positive x-direction. Then, V_R is also along positive x-direction and V_L along positive y-direction, so, we can write

$$\begin{aligned} V &= V_R + jV_L = iR + j(iX_L) \\ &= iR + j(i\omega L) \qquad \text{(as } X_L = \omega L) \\ &= i\mathbf{Z} \end{aligned}$$

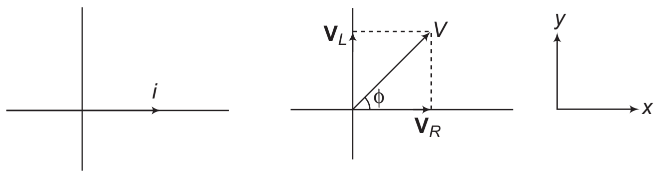


Fig. 28.18

Here, $\mathbf{Z} = R + jX_L = R + j(\omega L)$ is called as **impedance** of the circuit. Impedance plays the same role in AC circuits as the ohmic resistance does in DC circuits. The modulus of impedance is

$$|\mathbf{Z}| = \sqrt{R^2 + (\omega L)^2}$$

The potential difference leads the current by an angle,

$$\phi = \tan^{-1}\frac{|\mathbf{V}_L|}{|\mathbf{V}_R|} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

or
$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

28.6 Series *C-R* Circuit

Potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit.

Suppose in phasor diagram current is taken along positive *x*-direction. Then, V_R is also along positive *x*-direction but V_C is along negative *y*-direction. So, we can write

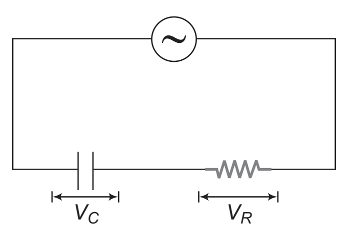


Fig. 28.19

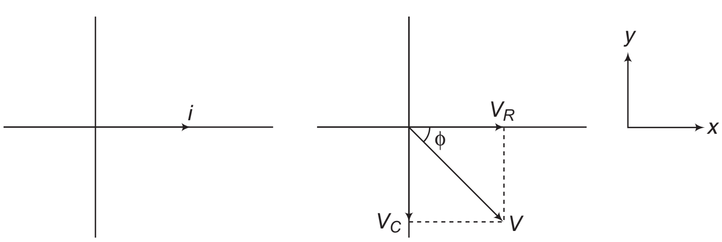


Fig. 28.20

$$\begin{aligned} V &= V_R - jV_C = iR - j(iX_C) \\ &= iR - j\left(\frac{i}{\omega C}\right) = i\boldsymbol{Z} \end{aligned}$$

Here, impedance is $\boldsymbol{Z} = R - j\left(\dfrac{1}{\omega C}\right)$

The modulus of impedance is

$$\boxed{|\mathbf{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right| = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{1/\omega C}{R}\right)$$

or $$\boxed{\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)}$$

28.7 Series *L-C-R* Circuit

Potential difference across an inductor leads the current by 90° in phase while that across a capacitor, it lags in phase by 90°.

Suppose in a phasor diagram current is taken along positive x-direction. Then, V_R is along positive x-direction, V_L along positive y-direction and V_C along negative y-direction.

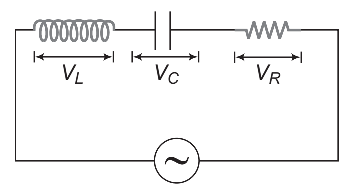


Fig. 28.21

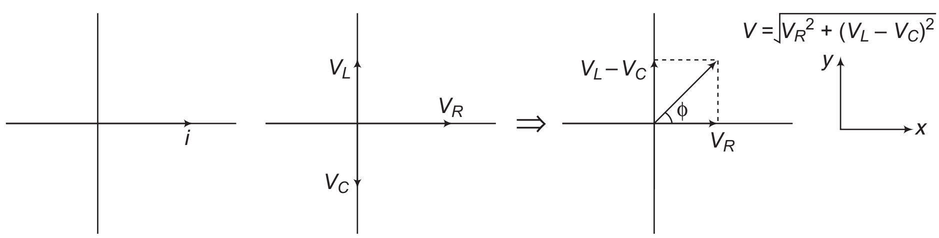


Fig. 28.22

Let us assume that $X_L > X_C$ or $V_L > V_C$

So, we can write $$V = V_R + jV_L - jV_C = iR + j(iX_L) - j(iX_C)$$
$$= iR + j[i(X_L - X_C)] = iZ$$

Here, impedance is $$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The modulus of impedance is $$|\mathbf{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and the potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

or $$\boxed{\phi = \tan^{-1}\left(\frac{\omega L - \dfrac{1}{\omega C}}{R}\right)}$$

Note *Let us take the most general case of a series L-C-R circuit in an AC.*

$$|\mathbf{Z}| = \sqrt{R^2 + (X_L \sim X_C)^2}$$

If $X_L = X_C$ *or* $\omega L = \frac{1}{\omega C}$ *or* $\omega = \frac{1}{\sqrt{LC}}$ *or* $f = \frac{1}{2\pi\sqrt{LC}}$ *the modulus of impedance*

$$|\mathbf{Z}| = R$$

and the current is in phase with voltage, i.e. if $V = V_0 \sin \omega t$, *then*

$$i = i_0 \sin \omega t$$

where,

$$i_0 = \frac{V_0}{|\mathbf{Z}|} = \frac{V_0}{R}$$

Such a condition is known as resonance and frequency known as resonance frequency and is given by

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

The current in such a case is maximum.

If $X_L > X_C$, *then the modulus of the impedance*

$$|\mathbf{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$

and the voltage leads the current by an angle given by

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

i.e. if $V = V_0 \sin \omega t$, *then*

$$i = i_0 \sin(\omega t - \phi)$$

where,

$$i_0 = \frac{V_0}{|\mathbf{Z}|}$$

If $X_C > X_L$, *then the modulus of the impedance is*

$$|\mathbf{Z}| = \sqrt{R^2 + (X_C - X_L)^2}$$

and the current leads the voltage by an angle given by

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

i.e. if $V = V_0 \sin \omega t$, *then*

$$i = i_0 \sin(\omega t + \phi)$$

where,

$$i_0 = \frac{V_0}{|\mathbf{Z}|}$$

Extra Points to Remember

- $i_0 = \frac{V_0}{|\mathbf{Z}|}$, $i_{rms} = \frac{V_{rms}}{|\mathbf{Z}|}$. But in general $i \neq \frac{V}{|\mathbf{Z}|}$.
- In *L-C-R* circuit, whenever voltage across various elements is asked, find rms values unless stated in the question for the peak or instantaneous value.

  The rms values are

  $$V_R = i_{rms}R,\ V_L = i_{rms}X_L$$

  and

  $$V_C = i_{rms}X_C$$

  The peak values can be obtained by multiplying the rms values by $\sqrt{2}$. The instantaneous values across different elements is rarely asked.

- **Voltage magnification in series resonance circuit** At resonance $\left(f = \dfrac{1}{2\pi\sqrt{LC}}\right)$, the PD across the inductor and the capacitor are equal and 180° out of phase and therefore, cancel out. Hence, the applied emf is merely to overcome the resistance opposition only. If an inductance or capacitance of very large reactance (X_L or X_C) is connected with $X_L = X_C$ (at resonance) then PD across them increases to a very high value. The ratio is known as **voltage magnification** and is given by,

$$\frac{\text{PD across inductance (or capacitance)}}{\text{Applied emf}} = \frac{i_{rms}(\omega L)}{i_{rms}(R)} = \frac{\omega L}{R} \quad \text{or} \quad \frac{i_{rms}\left(\frac{1}{\omega C}\right)}{i_{rms}\,(R)} = \frac{1}{\omega CR}$$

This ratio is greater than unity.

- **Response curves of series circuit** The impedance of an L-C-R circuit depends on the frequency. The dependence is shown in figure. The frequency is taken on logarithmic scale because of its wide range.

From the figure, we can see that at resonance,

(i) $X_L = X_C$ or $\omega = \dfrac{1}{\sqrt{LC}}$

(ii) $Z = Z_{min} = R$ and

(iii) i is maximum.

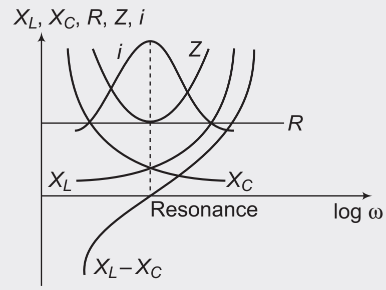


Fig. 28.23

Note *Here, by Z we mean the modulus of Z and i means* i_{rms}.

- **Acceptor circuit** If the frequency of the AC supply can be varied (e.g. in radio or television signal), then in series L-C-R circuit, at a frequency $f = 1/2\pi\sqrt{LC}$ maximum current flows in the circuit and have a maximum PD across its inductance (or capacitance). This is the method by which a radio or television set is tuned at a particular frequency. The circuit is known as **acceptor circuit**.

Example 28.4 *An alternating emf 200 virtual volts at 50 Hz is connected to a circuit of resistance 1 Ω and inductance 0.01 H. What is the phase difference between the current and the emf in the circuit? Also, find the virtual current in the circuit.*

Solution In case of an L-R AC circuit, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $$X_L = \omega L = (2\pi f L)$$

$$= (2\pi)(50)(0.01) = \pi\,\Omega$$

and $$R = 1\,\Omega$$

∴ $$\phi = \tan^{-1}(\pi) \approx 72.3°$$ **Ans.**

Further, $$i_{rms} = \frac{V_{rms}}{|\mathbf{Z}|} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$$

Substituting the values, we have

$$i_{rms} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}} = 60.67\text{ A}$$ **Ans.**

Example 28.5 *A resistance and inductance are connected in series across a voltage,*

$$V = 283 \sin 314t$$

The current is found to be $4 \sin (314t - \pi/4)$. *Find the values of the inductance and resistance.*

Solution In L-R series circuit, current lags the voltage by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $$\phi = \frac{\pi}{4}$$

$\therefore$ $$X_L = R \quad \text{or} \quad \omega L = R \qquad (\omega = 314 \text{ rad/s})$$

$\therefore$ $$314\,L = R \qquad \text{...(i)}$$

Further, $$V_0 = i_0 |\mathbf{Z}|$$

$\therefore$ $$283 = 4\sqrt{R^2 + X_L^2}$$

or $$R^2 + (\omega L)^2 = \left(\frac{283}{4}\right)^2 = 5005.56$$

or $$2R^2 = 5005.56 \qquad (\text{as } \omega L = R)$$

$\therefore$ $$R \approx 50\,\Omega$$ **Ans.**

and from Eq. (i), $$L = 0.16 \text{ H}$$ **Ans.**

Example 28.6 *Find the voltage across the various elements, i.e. resistance, capacitance and inductance which are in series and having values* $1000\ \Omega$, $1\ \mu F$ *and* $2.0\ H$, *respectively. Given emf is*

$$V = 100\sqrt{2} \sin 1000\, t \text{ volt}$$

Solution The rms value of voltage across the source,

$$V_{\text{rms}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

$\therefore$ $$\omega = 1000 \text{ rad/s}$$

$\therefore$ $$i_{\text{rms}} = \frac{V_{\text{rms}}}{|\mathbf{Z}|} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L \sim X_C)^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$= \frac{100}{\sqrt{(1000)^2 + \left(1000 \times 2 - \dfrac{1}{1000 \times 1 \times 10^{-6}}\right)^2}}$$

$$= 0.0707 \text{ A}$$

The current will be same every where in the circuit, therefore,

PD across resistor, $V_R = i_{rms} R = 0.0707 \times 1000 = 70.7$ V

PD across inductor, $V_L = i_{rms} X_L = 0.0707 \times 1000 \times 2 = 141.4$ V and

PD across capacitor, $V_C = i_{rms} X_C = 0.0707 \times \dfrac{1}{1 \times 1000 \times 10^{-6}}$

$= 70.7$ V **Ans.**

Note *The rms voltages do not add directly as `$V_R + V_L + V_C = 282.8$ V which is not the source voltage 100 V. The reason is that these voltages are not in phase and can be added by vector or by phasor algebra.*

$$V = \sqrt{V_R^2 + (V_L \sim V_C)^2}$$

INTRODUCTORY EXERCISE 28.1

1. (a) What is the reactance of a 2.00 H inductor at a frequency of 50.0 Hz?
(b) What is the inductance of an inductor whose reactance is 2.00 Ω at 50.0 Hz?
(c) What is the reactance of a 2.00 μF capacitor at a frequency of 50.0 Hz?
(d) What is the capacitance of a capacitor whose reactance is 2.00 Ω at 50.0 Hz?

2. An electric lamp which runs at 100 V DC and consumes 10 A current is connected to AC mains at 150 V, 50 Hz cycles with a choke coil in series. Calculate the inductance and drop of voltage across the choke. Neglect the resistance of choke.

3. A circuit operating at $\dfrac{360}{2\pi}$ Hz contains a 1 μF capacitor and a 20 Ω resistor. How large an inductor must be added in series to make the phase angle for the circuit zero? Calculate the current in the circuit if the applied voltage is 120 V.

28.8 Power in an AC Circuit

In case of a steady current, the rate of doing work is given by

$$P = Vi$$

In an alternating circuit, current and voltage both vary with time and also they differ in time. So, we cannot use $P = Vi$ for the power generated.

Suppose in an AC, the voltage is leading the current by an angle ϕ. Then, we can write

$$V = V_0 \sin \omega t \quad \text{and} \quad i = i_0 \sin (\omega t - \phi)$$

The instantaneous value of power in that case is

$$P = Vi = V_0 i_0 \sin \omega t \sin (\omega t - \phi)$$

or
$$P = V_0 i_0 \left[\sin^2 \omega t \cos \phi - \frac{1}{2} \sin 2\omega t \sin \phi\right] \quad \text{...(i)}$$

Now, the average rate of doing work (power) in one cycle will be

$$\langle P \rangle_{\text{One cycle}} = \frac{\int_0^{T = 2\pi/\omega} P\,dt}{\int_0^{T = 2\pi/\omega} dt} \quad \text{...(ii)}$$

Substituting the value of P from Eq. (i) in Eq. (ii) and then integrating it with proper limits, we get

$$\langle P \rangle_{\text{One cycle}} = \frac{1}{2} V_0 i_0 \cos\phi = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos\phi$$

or
$$\boxed{\langle P \rangle_{\text{One cycle}} = V_{\text{rms}} i_{\text{rms}} \cos\phi}$$

Here, the term $\cos\phi$ is known as **power factor**.

It is said to be leading if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1} 0.5 = 60°$). The product of V_{rms} and i_{rms} gives the **apparent power**. While the **true power** is obtained by multiplying the apparent power by the power factor $\cos\phi$. Thus,

$$\text{Apparent power} = V_{\text{rms}} \times i_{\text{rms}}$$

and
$$\text{True power} = \text{apparent power} \times \text{power factor}$$

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum $(= V_{\text{rms}} \times i_{\text{rms}})$. For $\phi = 90°$, the power is zero. The current is then stated **wattless**. Such a case will arise when resistance in the circuit is zero. The circuit is purely inductive or capacitive. The case is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum in a cycle is zero.

Extra Points to Remember

Let us consider a choke coil (used in tube lights) of large inductance L and low resistance R. The power factor for such a coil is given by

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \qquad (\text{as } R << \omega L)$$

As $R << \omega L$, $\cos\phi$ is very small. Thus, the power absorbed by the coil $V_{\text{rms}}\, i_{\text{rms}} \cos\phi$ is very small. On account of its large impedance $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. Such a coil is used in AC circuits for the purpose of adjusting current to any required value without waste of energy. The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current instead of a choke coil.

Example 28.7 *A 750 Hz, 20 V source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. Calculate the time in which the resistance (thermal capacity 2 J/°C) will get heated by 10° C.*

Solution The impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left\{(2\pi f L) - \frac{1}{(2\pi f C)}\right\}^2}$$

$$= \sqrt{(100)^2 + \left\{(2 \times 3.14 \times 750 \times 0.1803) - \frac{1}{(2 \times 3.14 \times 750 \times 10^{-5})}\right\}^2}$$

$$= 834\ \Omega$$

In case of an AC,

$$\langle P\rangle = V_{\text{rms}} i_{\text{rms}} \cos\phi = (V_{\text{rms}})\left(\frac{V_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right) = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R$$

$$= \left(\frac{20}{834}\right)^2 \times 100 = 0.0575 \text{ J/s}$$

Now, $\langle P\rangle \times t = S\Delta\theta$

Here, S = thermal capacity

$\therefore$ $$t = \frac{S\Delta\theta}{\langle P\rangle} = \frac{2\times 10}{0.0575} = 348 \text{ s}$$ **Ans.**

Note *In AC, the whole energy or power is consumed by resistance.*

Example 28.8 *In an L-C-R series circuit, $R = 150\ \Omega$, $L = 0.750\ H$ and $C = 0.0180\ \mu F$. The source has voltage amplitude $V = 150\ V$ and a frequency equal to the resonance frequency of the circuit.*

(a) What is the power factor?

(b) What is the average power delivered by the source?

(c) The capacitor is replaced by one with $C = 0.0360\ \mu F$ and the source frequency is adjusted to the new resonance value. Then, what is the average power delivered by the source?

Solution (a) At resonance frequency,

$$X_L = X_C,\ Z = R \text{ and power factor } \cos\phi = \frac{R}{Z} = 1.0$$

(b) $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{(150/\sqrt{2})^2}{150} = 75 \text{ W}$

(c) Again, $P = \dfrac{V_{\text{rms}}^2}{R} = 75 \text{ W}$

INTRODUCTORY EXERCISE 28.2

1. If a 0.03 H inductor, a 10 Ω resistor and a 2 μF capacitor are connected in series. At what frequency will they resonate? What will be the phase angle at resonance?
2. An arc lamp consumes 10 A at 40 V. Calculate the power factor when it is connected with a suitable value of choke coil required to run the arc lamp on AC mains of 200 V (rms) and 50 Hz.

Final Touch Points

1. Frequency of AC in India is 50 Hz.
2. The AC is converted into DC with the help of rectifier while DC is converted into AC with the help of inverter.
3. An AC cannot produce electroplating or electrolysis.
4. The AC is measured by hot wire ammeter.
5. An AC can be stepped up or down with the help of a transformer.
6. An AC can be transmitted over long distances without much power loss.
7. An AC can be regulated by using choke coil without any significant waste of energy.
8. In an AC (sinusoidal), current or voltage can have the following four values
   (i) instantaneous value
   (ii) peak value (i_0 or V_0)
   (iii) rms value (i_{rms} or V_{rms})
   (iv) average value : In full cycle, average value is zero while in half cycle it is non-zero.

Note *That in sinusoidal AC the average value in half cycle can also be zero. It depends on the time interval over which half average value is desired.*

9. In an series L-C-R circuit,
   (i) Capacitive reactance, $X_C = \frac{1}{\omega C}$
   (ii) Inductive reactance, $X_L = \omega L$
   (iii) Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$
   (iv) If $X_C > X_L$, current leads and if $X_L > X_C$, voltage leads by an angle ϕ given by

$$\cos\phi = \frac{R}{Z} \quad \text{and} \quad \tan\phi = \frac{X_C \sim X_L}{R}$$

   (v) Instantaneous power = instantaneous current $\times$ instantaneous voltage
   (vi) Average power $= V_{rms}\, i_{rms} \cos\phi$, where

$$\cos\phi = \frac{R}{Z} = \text{power factor.}$$

Note *Power is also equal to* $P = i_{rms}^2 R$

But this is not equal to $P \neq \frac{V_{rms}^2}{R}$

This is because $V_{rms} = i_{rms} Z$ *and* $\cos\phi = \frac{R}{Z}$. *If we substitute in* $P = V_{rms}\, i_{rms} \cos\phi$, *then we get the first relation but not the second one. This implies that power is consumed only across resistance.*

   (vii) $i_0 = \frac{V_0}{Z}$ or $i_{rms} = \frac{V_{rms}}{Z}$
   (viii) $(V_C)_{rms} = (i_{rms})\, X_C$, $(V_L)_{rms} = (i_{rms})\, X_L$ and $(V_R)_{rms} = (i_{rms})\, R$
   (ix) $V = \sqrt{V_R^2 + (V_C \sim V_L)^2}$

   Here, V is the rms value of applied voltage
   V_R is the rms value of voltage across resistance.
   V_C across capacitor and V_L across inductor etc.

10. $\omega = \omega_r = \dfrac{1}{\sqrt{LC}}$ is called resonance frequency.

11. At $\omega = \omega_r$,

(i) $X_L = X_C$

(ii) Z = minimum value = R

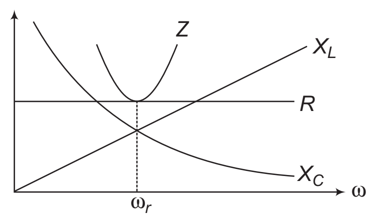
(iii) i_{rms} = maximum value $= \dfrac{V_{\text{rms}}}{Z_{\text{min}}} = \dfrac{V_{\text{rms}}}{R}$

(iv) i_0 = maximum value $= \dfrac{V_0}{Z_{\text{min}}} = \dfrac{V_0}{R}$

(v) Power factor $\cos\phi = 1$

12. In one complete cycle, power is consumed only by resistance. No power is consumed by a capacitor or an inductor.

13.



$$X_C = \frac{1}{\omega C} \quad \Rightarrow \quad X_C \propto \frac{1}{\omega}$$

$$X_L = \omega L \quad \Rightarrow \quad X_L \propto \omega$$

R does not depend on ω. It is a constant.

At $\omega = \omega_r$: $X_C = X_L$ and $Z = Z_{\text{min}} = R$

14. For $\omega > \omega_r$, $X_L > X_C$. Hence, voltage will lead the current or circuit is inductive.

For $\omega < \omega_r$, $X_C > X_L$. Hence, current will lead the voltage function or circuit is capacitive.

At $\omega = \omega_r$, $X_C = X_L$. Hence, current function and voltage function are in same phase.

15.

Conditions	Phase angle	Power factor
$R = 0$	90°	0
$X_C = X_L \neq 0$ $R \neq 0$	0°	1
$X_C = X_L = 0$ $R \neq 0$	0°	1
$\omega = \omega_r$	0°	1

In all other cases, phase difference between current function and voltage function is

$$0° < \phi < 90°$$

If $\quad X_C > X_L, \phi = \tan^{-1}\left(\dfrac{X_C - X_L}{R}\right) \quad$ or $\quad \cos^{-1}\left(\dfrac{R}{Z}\right)$

If $\quad X_L > X_C, \phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) \quad$ or $\quad \cos^{-1}\left(\dfrac{R}{Z}\right)$

Solved Examples

TYPED PROBLEMS

Type 1. *Based on a real inductor*

Concept

An ideal inductor wire has zero resistance ($R = 0$). So, in an AC, only X_L is the impedance. But, a real inductor has some resistance also. In DC, total resistance is only R. In AC, total resistance called impedance is $Z = \sqrt{R^2 + X_L^2}$

Therefore,

$$I_{\text{DC}} = \frac{V_{\text{DC}}}{R}$$

and
$$I_{\text{AC}} = \frac{V_{\text{AC}}}{Z} = \frac{V_{\text{AC}}}{\sqrt{R^2 + X_L^2}}$$

If
$$V_{\text{AC}} = V_{\text{DC}}, \quad \text{then} \quad I_{\text{AC}} < I_{\text{DC}}$$

Example 1 *A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rad/s AC source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also, find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.*

Solution (i) A coil consists of an inductance (L) and a resistance (R).

In DC, only resistance is effective. Hence,

$$R = \frac{V}{i} = \frac{12}{4} = 3\ \Omega$$

In AC,
$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}}$$

$\therefore$
$$L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{\text{rms}}}{i_{\text{rms}}}\right)^2 - R^2\right]$$

$\therefore$
$$L = \frac{1}{\omega}\sqrt{\left(\frac{V_{\text{rms}}}{i_{\text{rms}}}\right)^2 - R^2}$$

Substituting the values, we have

$$L = \frac{1}{50}\sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2}$$

$$= 0.08\ \text{H}$$ **Ans.**

(ii) When capacitor is connected to the circuit, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $R = 3\ \Omega$

$$X_L = \omega L = (50)(0.08) = 4\ \Omega$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{(50)(2500 \times 10^{-6})} = 8\ \Omega$$

$\therefore$ $$Z = \sqrt{(3)^2 + (4-8)^2} = 5\ \Omega$$

Now, $$\langle P \rangle = V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$

$$= V_{\text{rms}} \times \frac{V_{\text{rms}}}{Z} \times \frac{R}{Z}$$

$$= \left(\frac{V_{\text{rms}}}{Z}\right)^2 \times R$$

Substituting the values, we have

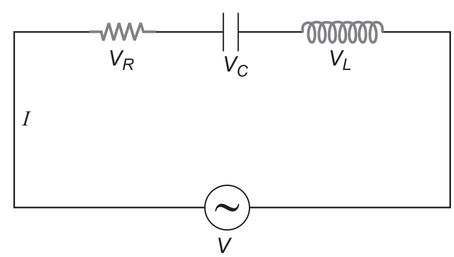
$$\langle P \rangle = \left(\frac{12}{5}\right)^2 \times 3$$

$$= 17.28\ \text{W}$$ **Ans.**

Type 2. *Different time functions in AC*

Concept

In an L-C-R series circuit, there are total five functions of time, V, I, V_R, V_C and V_L. Now, the following points are important in these functions.



(i) V and I have a phase difference of ϕ where

$$0° \le \phi \le 90°$$

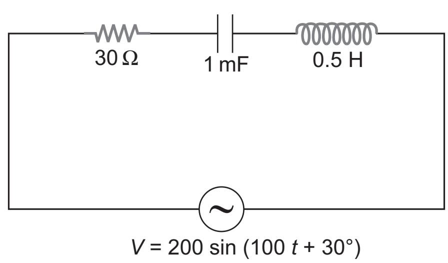
(ii) V_R and I are in same phase

(iii) V_C lags behind I by 90°

(iv) V_L leads I by 90°

(v) The functions, $V = V_L + V_C + V_L$ (all the time)

Example 2



In the diagram shown in figure, V function is given. Find other four functions of time I, V_C, V_R and V_L. Also, find power consumed in the circuit, V is given in volts and ω in rad/s.

Solution Given, $\omega = 100$ rad/s

$$X_L = \omega L = 50\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-3}} = 10\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(30)^2 + (50 - 10)^2}$$

$$= 50\ \Omega$$

Current function

Maximum value of current,

$$I_0 = \frac{V_0}{Z} = \frac{200}{50} = 4\ \text{A}$$

$X_L > X_C$, therefore voltage leads the current by a phase difference ϕ where,

$$\cos\phi = \frac{R}{Z} = \frac{30}{50} = \frac{3}{5}$$

or $$\phi = 53°$$

$\therefore$ $$I = 4 \sin(100\,t + 30° - 53°)$$

or $$I = 4 \sin(100\,t - 23°)$$ **Ans.**

V_R, V_C and V_L functions

Maximum value of $V_R = I_0 R = 4 \times 30 = 120$ volt, V_R and I are in same phase.

Therefore,

$$V_R = 120 \sin(100\,t - 23°)$$ **Ans.**

Maximum value of $V_C = I_0 X_C = 4 \times 10 = 40$ volt

Now, V_C function lags the current function by 90°.

Therefore,

$$V_C = 40 \sin(100\,t - 23° - 90°)$$

or $$V_C = 40 \sin(100\,t - 113°)$$ **Ans.**

Maximum value of $V_L = I_0 X_L = 4 \times 50 = 200$ volt, V_L function leads the current function by 90°. Therefore,

$$V_L = 200 \sin(100\,t - 23° + 90°)$$

or $$V_L = 200 \sin(100\,t + 67°)$$ **Ans.**

Note *We can check at any time that,*

$$V = V_R + V_L + V_C$$

Power Power is consumed in an AC circuit only across a resistance and this power is given by

$$P = V_{\text{rms}}\, I_{\text{rms}} \cos\phi$$

$$= I^2_{\text{rms}} R$$

Let us use the first formula,

$$P = \left(\frac{200}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right)\left(\frac{3}{5}\right)$$

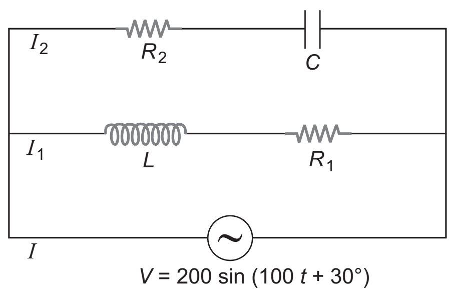
$$= 240\ \text{watt}$$ **Ans.**

Type 3. *Parallel circuits*

Concept

Two or more than two sine or cosine functions of same ω can be added by vector method. Actually, their amplitudes are added by vectors method.

Example 3 *In the circuit shown in figure,*



$$R_1 = 30\ \Omega,\ R_2 = 40\ \Omega,\ L = 0.4\ H \text{ and } C = \frac{1}{3}\ mF.$$

Find seven functions of time $I, I_1, I_2, V_{R_1}, V_L, V_{R_2}$ and V_C. Also, find total power consumed in the circuit. In the given potential function, V is in volts and ω in rad/s.

Solution **Circuit 1 (containing L and R_1)**

$\boldsymbol{I_1}$:
$$X_L = \omega L = 100 \times 0.4 = 40\ \Omega$$
$$R_1 = 30\ \Omega$$
$\therefore$
$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(30)^2 + (40)^2}$$
$$= 50\ \Omega$$

Maximum value of current, $I_1 = \dfrac{V_0}{Z_1} = \dfrac{200}{50} = 4$ A

Since, there is only X_L, so voltage function will lead the current function by an angle ϕ_1, where

$$\cos \phi_1 = \frac{R_1}{Z_1} = \frac{30}{50} = \frac{3}{5}$$

$\therefore$ $\phi_1 = 53°$

$\therefore$ $I_1 = 4 \sin (100\,t + 30° - 53°)$

or $I_1 = 4 \sin (100\,t - 23°)$ **Ans.**

$\boldsymbol{V_{R_1}}$: V_{R_1} function is in phase with I_1 function.

$$\text{Maximum value of } V_{R_1} = (\text{maximum value of } I_1)\,(R_1)$$
$$= (4)\,(30)$$
$$= 120 \text{ volt}$$

$\therefore$ $V_{R_1} = 120 \sin (100\,t - 23°)$ **Ans.**

$\boldsymbol{V_L}$: V_L function is $90°$ ahead of I_1 function.

$$\text{Maximum value of } V_L = (\text{maximum value of } I_1)\,(X_L)$$
$$= (4)(40) = 160 \text{ volt}$$

$\therefore \quad V_L = 160 \sin (100\,t - 23° + 90°)$

or $\quad V_L = 160 \sin (100\,t + 67°)$ **Ans.**

Power In this circuit, power will be consumed only across R_1. This power is given by

$$P_{R_1} = (\text{ rms value of } I_1)^2\, R_1$$
$$= \left(\frac{4}{\sqrt{2}}\right)^2 (30)$$
$$= 240 \text{ watt}$$

Circuit 2 (containing C and R_2)

I_2 : $$X_C = \frac{1}{\omega C} = \frac{1}{100 \times \frac{1}{3} \times 10^{-3}} = 30\ \Omega$$

$$R_2 = 40\ \Omega$$

$\therefore$ $$Z_2 = \sqrt{R_2^2 + X_C^2}$$
$$= \sqrt{(40)^2 + (30)^2}$$
$$= 50\ \Omega$$

Maximum value of $I_2 = \frac{V_0}{Z_2} = \frac{200}{50} = 4\text{ A}$

Since, there is only X_C, so I_2 function will lead the V function by an angle ϕ_2, where

$$\cos \phi_2 = \frac{R_2}{Z_2} = \frac{40}{50} = \frac{4}{5}$$

$\therefore \quad \phi_2 = 37°$

$\therefore \quad I_2 = 4 \sin (100\,t + 30° + 37°) = 4 \sin (100\,t + 67°)$ **Ans.**

V_{R_2} : V_{R_2} function is in phase with I_2 function.

$$\text{Maximum value of } V_{R_2} = (\text{ maximum value of } I_2)\,(R_2)$$
$$= 4 \times 40 = 160 \text{ volt}$$

$\therefore \quad V_{R_2} = 160 \sin (100\,t + 67°)$ **Ans.**

V_C : V_C function lags I_2 function by 90°

$$\text{Maximum value of } V_C = (\text{Maximum value of } I_2)(X_C)$$
$$= 4 \times 30$$
$$= 120 \text{ volt}$$

$\therefore \quad V_C = 120 \sin (100\,t + 67° - 90°)$

or $\quad V_C = 120 \sin (100\,t - 23°)$ **Ans.**

Power In this circuit, power will be consumed only across R_2 and this power is given by

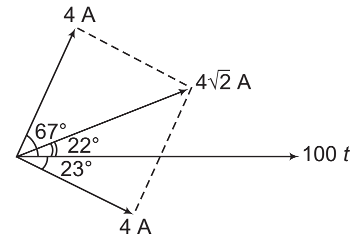
$$P_{R_2} = (\text{rms value of } I_2)^2\, R_2$$
$$= \left(\frac{4}{\sqrt{2}}\right)^2 (40)$$
$$= 320 \text{ W}$$

$\therefore$ Total power consumed in the circuit,

$$P = P_{R_1} + P_{R_2}$$
$$= (240 + 320) \text{ W}$$
$$= 560 \text{ W}$$ **Ans.**

***I* :** $$I = I_1 + I_2$$
$$I = 4\sin(100\,t - 23°) + 4\sin(100\,t + 67°)$$

Now, the amplitudes can be added by vector method.



Resultant of 4 A and 4 A at 90° is $4\sqrt{2}$ A at 45° from both currents or at 22° from 100 t line.

$\therefore$ $$I = 4\sqrt{2}\sin(100\,t + 22°)$$ **Ans.**

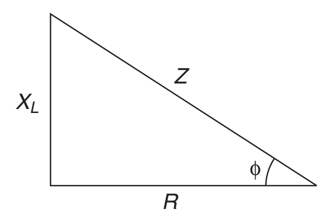
Miscellaneous Examples

Example 4 *An AC circuit consists of a 220 Ω resistance and a 0.7 H choke. Find the power absorbed from 220 V and 50 Hz source connected in this circuit if the resistance and choke are joined*

(a) in series *(b) in parallel.*

Solution (a) In series, the impedance of the circuit is

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$$
$$= \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2}$$
$$= 311\ \Omega$$



$\therefore$ $$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{311} = 0.707\ \text{A}$$

and $$\cos\phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

$\therefore$ The power absorbed in the circuit,

$$P = V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$
$$= (220)\,(0.707)\,(0.707)$$
$$= 110.08\ \text{W}$$ **Ans.**

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power.

$\therefore$ $$P = \frac{V_{\text{rms}}^2}{R} = \frac{(220)^2}{220} = 220\ \text{W}$$ **Ans.**

Example 5 *A sinusoidal voltage of frequency 60 Hz and peak value 150 V is applied to a series L-R circuit, where $R = 20\ \Omega$ and $L = 40$ mH.*

(a) Compute T, ω, X_L, Z and ϕ

(b) Compute the amplitudes of current, V_R and V_L

Solution (a) $$T = \frac{1}{f} = \frac{1}{60}\text{ s}$$ **Ans.**

$$\omega = 2\pi f = (2\pi)(60) = 377\text{ rad/s}$$ **Ans.**

$$X_L = \omega L = (377)(0.040)$$
$$= 15.08\ \Omega$$ **Ans.**

$$Z = \sqrt{X_L^2 + R^2}$$
$$= \sqrt{(15.08)^2 + (20)^2}$$
$$= 25.05\ \Omega$$ **Ans.**

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{15.08}{20}\right) = \tan^{-1}(0.754)$$
$$= 37°$$ **Ans.**

(b) Amplitudes (maximum value) are

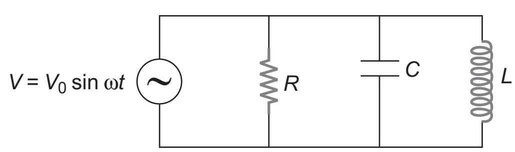
$$i_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6\text{ A}$$ **Ans.**

$$(V_0)_R = i_0 R = (6)(20) = 120\text{ V}$$ **Ans.**

$$(V_0)_L = i_0 X_L$$
$$= (6)(15.08) = 90.5\text{ V}$$ **Ans.**

Note $V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$

Example 6 *For the circuit shown in figure, find the instantaneous current through each element.*



Solution The three current equations are

$$V = i_R R, \quad V = L\frac{di_L}{dt}$$

and $$V = \frac{q}{C} \Rightarrow \frac{dV}{dt} = \frac{1}{C} i_C \qquad \text{...(i)}$$

The steady state solutions of Eq. (i) are

$$i_R = \frac{V_0}{R}\sin\omega t \equiv (i_0)_R \sin\omega t$$

$$i_L = -\frac{V_0}{\omega L}\cos\omega t \equiv -\frac{V_0}{X_L}\cos\omega t \equiv -(i_0)_L\cos\omega t$$

and $$i_C = V_0\omega C\cos\omega t \equiv \frac{V_0}{X_C}\cos\omega t \equiv (i_0)_C\cos\omega t$$

where, the reactances X_L and X_C are as defined.

Example 7 *In the above problem find the total instantaneous current through the source, and find expressions for phase angle of this current and the impedance of the circuit.*

Solution For the total current, we have

$$i = i_R + i_L + i_C$$
$$= V_0\left[\frac{1}{R}\sin\omega t + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)\cos\omega t\right]$$

Using the trigonometric identity,

$$A\sin\theta + B\cos\theta = \sqrt{A^2 + B^2}\sin(\theta + \phi)$$

where, $\phi = \tan^{-1}(B/A)$

We can write, $$i \equiv i_0 \sin(\omega t + \phi)$$

Here, $$i_0 = \frac{V_0}{Z}$$

where, $$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

and $$\tan\phi = \frac{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)}{(1/R)}$$

Example 8 *An L-C-R series circuit with* $100\ \Omega$ *resistance is connected to an AC source of* $200\ V$ *and angular frequency* $300\ rad/s$*. When only the capacitance is removed, the current lags behind the voltage by* $60°$*. When only the inductance is removed, the current leads the voltage by* $60°$*. Calculate the current and the power dissipated in the L-C-R circuit*

Solution When capacitance is removed, then

$$\tan\phi = \frac{X_L}{R}$$

or $$\tan 60° = \frac{X_L}{R}$$

$\therefore$ $$X_L = \sqrt{3}R \quad \text{...(i)}$$

When inductance is removed, then

$$\tan\phi = \frac{X_C}{R}$$

or $$\tan 60° = \frac{X_C}{R}$$

$\therefore$ $$X_C = \sqrt{3}\ R \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we see that $X_C = X_L$

So, the L-C-R circuit is in resonance.

Hence, $Z = R$

$\therefore$ $$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{100} = 2\ \text{A}$$ **Ans.**

$$\langle P \rangle = V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$

At resonance current and voltage are in phase,

or $$\phi = 0°$$

$\therefore$ $$\langle P \rangle = (200)\,(2)\,(1) = 400\ \text{W}$$ **Ans.**

Example 9 *A series L-C-R circuit containing a resistance of* $120\ \Omega$ *has resonance frequency* 4×10^5 *rad/s. At resonance the voltages across resistance and inductance are* $60\ V$ *and* $40\ V$*, respectively. Find the values of L and C. At what angular frequency the current in the circuit lags the voltage by* $\pi/4$*?*

Solution At resonance, $X_L - X_C = 0$

and $$Z = R = 120\ \Omega$$

$\therefore$ $$i_{\text{rms}} = \frac{(V_R)_{\text{rms}}}{R} = \frac{60}{120} = \frac{1}{2}\ \text{A}$$

Also, $$i_{\text{rms}} = \frac{(V_L)_{\text{rms}}}{\omega L}$$

$\therefore$ $$L = \frac{(V_L)_{\text{rms}}}{\omega i_{\text{rms}}} = \frac{40}{(4 \times 10^5)\left(\frac{1}{2}\right)}$$

$$= 2.0 \times 10^{-4}\ \text{H}$$

$$= 0.2\ \text{mH}$$ **Ans.**

The resonance frequency is given by

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad C = \frac{1}{\omega^2 L}$$

Substituting the values, we have

$$C = \frac{1}{(4 \times 10^5)^2\,(2.0 \times 10^{-4})}$$

$$= 3.125 \times 10^{-8}\ \text{F}$$ **Ans.**

Current lags the voltage by 45°, when

$$\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

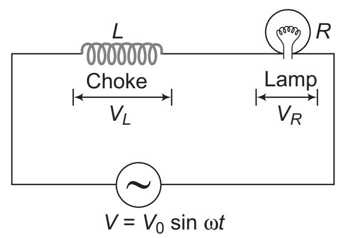
Substituting the values of *L, C, R* and tan 45°, we get

$$\omega = 8 \times 10^5\ \text{rad/s}$$ **Ans.**

Example 10 *A choke coil is needed to operate an arc lamp at* $160\ V$ *(rms) and* $50\ Hz$*. The lamp has an effective resistance of* $5\ \Omega$ *when running at* $10\ A$ *(rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on* $160\ V$ *(DC), what additional resistance is required? Compare the power loses in both cases.*

Solution For lamp,

$$(V_{\text{rms}})_R = (i_{\text{rms}})(R) = 10 \times 5 = 50 \text{ V}$$



In series,

$$(V_{\text{rms}})^2 = (V_{\text{rms}})_R^2 + (V_{\text{rms}})_L^2$$

$\therefore$
$$(V_{\text{rms}})_L = \sqrt{(V_{\text{rms}})^2 - (V_{\text{rms}})_R^2}$$
$$= \sqrt{(160)^2 - (50)^2}$$
$$= 152 \text{ V}$$

As,
$$(V_{\text{rms}})_L = (i_{\text{rms}})X_L = (i_{\text{rms}})(2\pi f L)$$

$\therefore$
$$L = \frac{(V_{\text{rms}})_L}{(2\pi f)(i_{\text{rms}})}$$

Substituting the values, we get

$$L = \frac{152}{(2\pi)(50)(10)}$$
$$= 4.84 \times 10^{-2} \text{ H}$$ **Ans.**

Now, when the lamp is operated at 160 V, DC and instead of choke let an additional resistance R' is put in series with it, then

$$V = i(R + R')$$

or
$$160 = 10(5 + R')$$

$\therefore$
$$R' = 11\ \Omega$$ **Ans.**

In case of AC, as the choke has no resistance, power loss in choke is zero.

In case of DC, the loss in additional resistance R' is

$$P = i^2R' = (10)^2(11)$$
$$= 1100 \text{ W}$$ **Ans.**

Exercises

LEVEL 1

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(*c*) *If **Assertion** is true, but the **Reason** is false.*

(*d*) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : In an AC circuit, potential difference across the capacitor may be greater than the applied voltage.

Reason : $V_C = IX_C$, whereas $V = IZ$ and X_C can be greater than Z also.

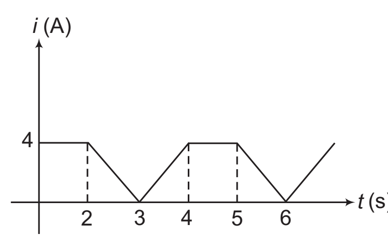
2. Assertion : In series L-C-R circuit, voltage will lead the current function for frequency greater than the resonance frequency.

Reason : At resonance frequency, phase difference between current function and voltage function is zero.

3. Assertion : Resonance frequency will decrease in L-C-R series circuit if a dielectric slab is inserted in between the plates of the capacitor.

Reason : By doing so, capacity of capacitor will increase.

4. Assertion : Average value of current in the given graph is 3 A.



Reason : Average value can't be greater than the peak value of any function.

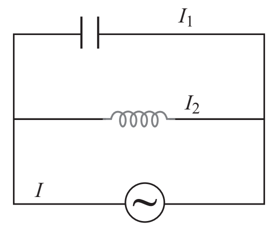
5. Assertion : In series L-C-R circuit, if a ferromagnetic rod is inserted inside an inductor, current in the circuit may increase or decrease.

Reason : By doing so X_L will increase.

6. Assertion : Potential difference across, resistor, capacitor and inductor each is 10 V. Then, voltage function and current functions should be in phase.

Reason : At this condition current in the circuit should be maximum.

7. Assertion : At some given instant I_1 and I_2 both are 2 A each. Then, I at this instant should be zero.



Reason : There is a phase difference of π between I_1 and I_2 functions.

8. Assertion : Peak value of current in AC through a resistance of $10\,\Omega$ is 2 A. Then, power consumed by the resistance should be 20 W.

Reason : Power in AC is $$P = I_{\text{rms}}^2 R$$

9. Assertion : An inductor coil normally produces more current with DC source compared to an AC source of same value of rms voltage.

Reason : In DC source, applied voltage remains constant with time.

10. Assertion : In an L-R series circuit in AC, current in the circuit will decrease with increase in frequency.

Reason : Phase difference between current function and voltage function will increase with increase in frequency.

11. Assertion : In series L-C-R, AC circuit, current and voltage are in same phase at resonance.

Reason : In series L-C-R, AC circuit, resonant frequency does not depend on the value of resistance. Hence, current at resonance does not depend on resistance.

Objective Questions

1. The term $\cos\phi$ in an AC circuit is called

(a) form factor (b) phase factor
(c) power factor (d) quality factor

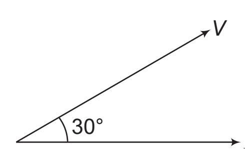
2. A DC ammeter cannot measure alternating current because

(a) AC changes its direction
(b) DC instruments will measure the average value
(c) AC can damage the DC instrument
(d) AC produces more heat

3. As the frequency of an alternating current increases, the impedance of the circuit

(a) increases continuously (b) decreases continuously
(c) remains constant (d) None of these

4. Phasor diagram of a series AC circuit is shown in figure. Then,



(a) The circuit must be containing resistor and capacitor only
(b) The circuit must be containing resistor and inductor only
(c) The circuit must be containing all three elements L, C and R
(d) The circuit cannot have only capacitor and inductor

5. The rms value of an alternating current

(a) is equal to 0.707 times peak value
(b) is equal to 0.636 times peak value
(c) is equal to $\sqrt{2}$ times the peak value
(d) None of the above

6. In an AC circuit, the applied potential difference and the current flowing are given by

$$V = 200 \sin 100\,t \text{ volt},\; I = 5 \sin\left(100\,t - \frac{\pi}{2}\right) \text{ amp}$$

The power consumption is equal to

(a) 1000 W (b) 40 W
(c) 20 W (d) zero

7. The impedance of a series L-C-R circuit in an AC circuit is

(a) $\sqrt{R + (X_L - X_C)}$ (b) $\sqrt{R^2 + (X_L^2 - X_C^2)}$
(c) R (d) None of these

8. If V_0 and I_0 are the peak current and voltage across the resistor in a series L-C-R circuit, then the power dissipated in the circuit is (Power factor = $\cos\theta$)

(a) $\frac{V_0 I_0}{2}$ (b) $\frac{V_0 I_0}{\sqrt{2}}$ (c) $V_0 I_0 \cos\theta$ (d) $\frac{V_0 I_0}{2}\cos\theta$

9. A generator produces a time varying voltage given by $V = 240 \sin 120\,t$, where t is in second. The rms voltage and frequency are

(a) 170 V and 19 Hz (b) 240 V and 60 Hz
(c) 170 V and 60 Hz (d) 120 V and 19 Hz

10. An L-C-R series circuit has a maximum current of 5 A. If $L = 0.5$ H and $C = 8\,\mu$F, then the angular frequency of AC voltage is

(a) 500 rad/s (b) 5000 rad/s
(c) 400 rad/s (d) 250 rad/s

11. The current and voltage functions in an AC circuit are

$$i = 100 \sin 100\,t \text{ mA},\; V = 100 \sin\left(100t + \frac{\pi}{3}\right) \text{V}$$

The power dissipated in the circuit is

(a) 10 W (b) 2.5 W (c) 5 W (d) 5 kW

12. A capacitor becomes a perfect insulator for

(a) alternating current (b) direct current (c) both (a) and (b) (d) None of these

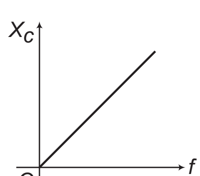
13. For an alternating voltage $V = 10 \cos 100\,\pi t$ volt, the instantaneous voltage at $t = \frac{1}{600}$ s is

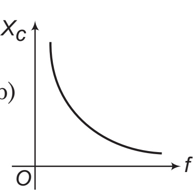
(a) 1 V (b) 5 V
(c) $5\sqrt{3}$ V (d) 10 V

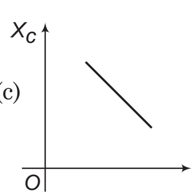
14. In a purely resistive AC circuit,

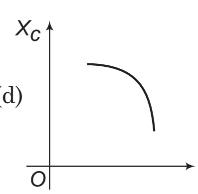
(a) voltage leads current (b) voltage lags current
(c) voltage and current are in same phase (d) nothing can be said

15. Identify the graph which correctly represents the variation of capacitive reactance X_C with frequency

(a) 

(b) 

(c) 

(d) 

16. In an AC circuit, the impedance is $\sqrt{3}$ times the reactance, then the phase angle is

(a) 60° (b) 30°

(c) zero (d) None of these

17. Voltage applied to an AC circuit and current flowing in it is given by

$$V = 200\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and } i = -\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

Then, power consumed in the circuit will be

(a) 200 W (b) 400 W

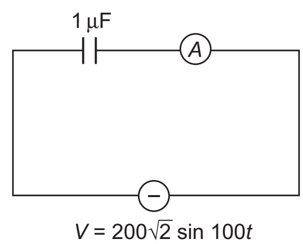
(c) $200\sqrt{2}$ W (d) None of these

18. When 100 volt DC source is applied across a coil, a current of 1 A flows through it. When 100 V AC source of 50 Hz is applied to the same coil, only 0.5 A current flows. Calculate the inductance of the coil.

(a) $(\pi/\sqrt{3})$ H (b) $(\sqrt{3}/\pi)$ H

(c) $(2/\pi)$ H (d) None of these

19. In the circuit shown in figure, the reading of the AC ammeter is



(a) $20\sqrt{2}$ mA (b) $40\sqrt{2}$ mA (c) 20 mA (d) 40 mA

20. An AC voltage is applied across a series combination of L and R. If the voltage drop across the resistor and inductor are 20 V and 15 V respectively, then applied peak voltage is

(a) 25 V (b) 35 V

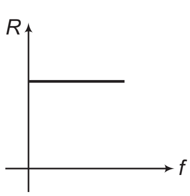
(c) $25\sqrt{2}$ V (d) $5\sqrt{7}$ V

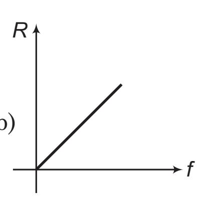
21. For wattless power in an AC circuit, the phase angle between the current and voltage is

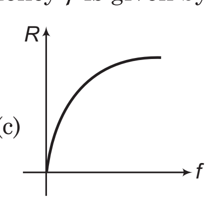
(a) 0° (b) 90°

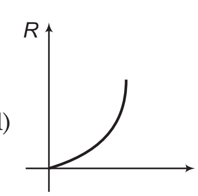
(c) 45° (d) Not possible

22. The correct variation of resistance R with frequency f is given by

(a) 

(b) 

(c) 

(d) 

23. If L and R be the inductance and resistance of the choke coil, then identify the correct statement.

(a) L is very high compared to R (b) R is very high compared to L

(c) Both L and R are high (d) Both L and R are low

24. When an AC signal of frequency 1 kHz is applied across a coil of resistance 100 Ω, then the applied voltage leads the current by 45°. The inductance of the coil is

(a) 16 mH (b) 12 mH
(c) 8 mH (d) 4 mH

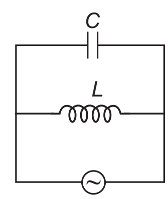
25. The frequency of an alternating current is 50 Hz. The minimum time taken by it in reaching from zero to peak value is

(a) 5 ms (b) 10 ms
(c) 20 ms (d) 50 ms

26. An alternating voltage is applied across the *R*-*L* combination. $V = 220 \sin 120\,t$ and the current $I = 4 \sin(120t - 60°)$ develops. The power consumption is

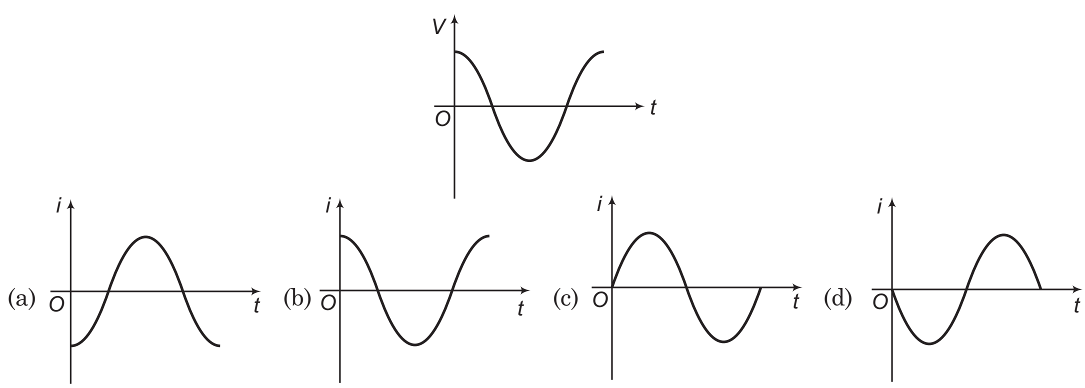
(a) zero (b) 100 W
(c) 220 W (d) 440 W

27. In the AC network shown in figure, the rms current flowing through the inductor and capacitor are 0.6 A and 0.8 A, respectively. Then, the current coming out of the source is



(a) 1.0 A
(b) 1.4 A
(c) 0.2 A
(d) None of the above

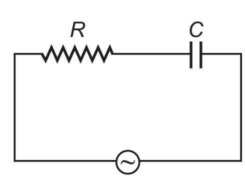
28. The figure represents the voltage applied across a pure inductor. The diagram which correctly represents the variation of current *i* with time *t* is given by



29. A steady current of magnitude *I* and an AC current of peak value *I* are allowed to pass through identical resistors for the same time. The ratio of heat produced in the two resistors will be

(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) None of these

30. A 50 Hz AC source of 20 V is connected across *R* and *C* as shown in figure. The voltage across *R* is 12 V. The voltage across *C* is



(a) 8 V
(b) 16 V
(c) 10 V
(d) Not possible to determine unless value of *R* and *C* are given

Subjective Questions

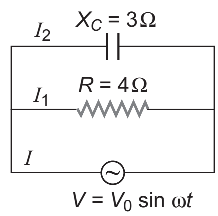
Note *You can take approximations in the answers.*

1. A 300 Ω resistor, a 0.250 H inductor, and a 8.00 μF capacitor are in series with an AC source with voltage amplitude 120 V and angular frequency 400 rad/s.
   (a) What is the current amplitude?
   (b) What is the phase angle of the source voltage with respect to the current? Does the source voltage lag or lead the current?
   (c) What are the voltage amplitudes across the resistor, inductor, and capacitor?
2. A series circuit has an impedance of 60.0 Ω and a power factor of 0.720 at 50.0 Hz. The source voltage lags the current.
   (a) What circuit element, an inductor or a capacitor, should be placed in series with the circuit to raise its power factor?
   (b) What size element will raise the power factor to unity?
3. Voltage and current for a circuit with two elements in series are expressed as
$$V(t) = 170 \sin(6280t + \pi/3) \text{ volt}$$
$$i(t) = 8.5 \sin(6280t + \pi/2) \text{ amp}$$
   (a) Plot the two waveforms.
   (b) Determine the frequency in Hz.
   (c) Determine the power factor stating its nature.
   (d) What are the values of the elements?
4. A 5.00 H inductor with negligible resistance is connected across an AC source. Voltage amplitude is kept constant at 60.0 V but whose frequency can be varied. Find the current amplitude when the angular frequency is
   (a) 100 rad/s
   (b) 1000 rad/s
   (c) 10000 rad/s
5. A 300 Ω resistor is connected in series with a 0.800 H inductor. The voltage across the resistor as a function of time is $V_R = (2.50\text{ V})\cos[(950\text{ rad/s})t]$.
   (a) Derive an expression for the circuit current.
   (b) Determine the inductive reactance of the inductor.
   (c) Derive an expression for the voltage V_L across the inductor.
6. An L-C-R series circuit with $L = 0.120$ H, $R = 240\ \Omega$, and $C = 7.30\ \mu$F carries an rms current of 0.450 A with a frequency of 400 Hz.
   (a) What are the phase angle and power factor for this circuit?
   (b) What is the impedance of the circuit?
   (c) What is the rms voltage of the source?
   (d) What average power is delivered by the source?
   (e) What is the average rate at which electrical energy is converted to thermal energy in the resistor?
   (f) What is the average rate at which electrical energy is dissipated (converted to other forms) in the capacitor?
   (g) In the inductor?

LEVEL 2

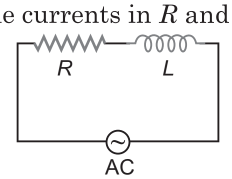
Single Correct Option

1. A capacitor and resistor are connected with an AC source as shown in figure. Reactance of capacitor is $X_C = 3\,\Omega$ and resistance of resistor is $4\,\Omega$. Phase difference between current I and I_1 is $\left[\tan^{-1}\left(\frac{3}{4}\right) = 37°\right]$



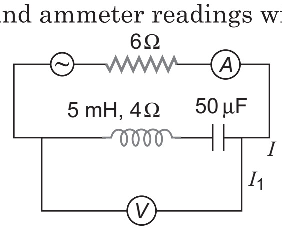
(a) 90° (b) zero (c) 53° (d) 37°

2. A circuit contains resistance R and an inductance L in series. An alternating voltage $V = V_0 \sin \omega t$ is applied across it. The currents in R and L respectively will be



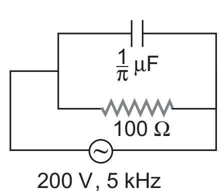
(a) $I_R = I_0 \cos \omega t$, $I_L = I_0 \cos \omega t$
(b) $I_R = -I_0 \sin \omega t$, $I_L = I_0 \cos \omega t$
(c) $I_R = I_0 \sin \omega t$, $I_L = -I_0 \cos \omega t$
(d) None of the above

3. In the circuit shown in figure, the AC source gives a voltage $V = 20 \cos (2000t)$. Neglecting source resistance, the voltmeter and ammeter readings will be



(a) 0 V, 2.0 A (b) 0 V, 1.4 A (c) 5.6 V, 1.4 A (d) 8 V, 2.0 A

4. A signal generator supplies a sine wave of 200 V, 5 kHz to the circuit shown in the figure. Then, choose the **wrong** statement.

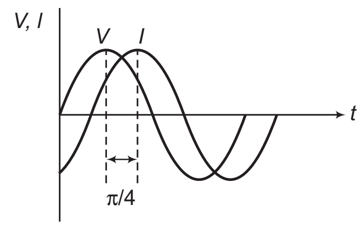


(a) The current in the resistive branch is 0.2 A
(b) The current in the capacitive branch is 0.126 A
(c) Total line current is ≈ 0.283 A
(d) Current in both the branches is same

5. A complex current wave is given by $i = (5 + 5 \sin 100\,\omega t)$ A. Its average value over one time period is given as

(a) 10 A
(b) 5 A
(c) $\sqrt{50}$ A
(d) 0

6. An AC voltage $V = V_0 \sin 100\,t$ is applied to the circuit, the phase difference between current and voltage is found to be $\pi/4$, then

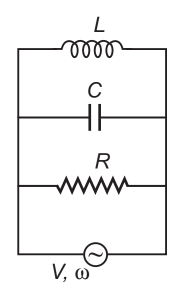


(a) $R = 100\ \Omega, C = 1\ \mu\text{F}$
(b) $R = 1\ \text{k}\Omega, C = 10\ \mu\text{F}$
(c) $R = 10\ \text{k}\Omega, L = 1$ H
(d) $R = 1\ \text{k}\Omega, L = 10$ H

7. In series *L-C-R* circuit, voltage drop across resistance is 8 V, across inductor is 6 V and across capacitor is 12 V. Then,

(a) voltage of the source will be leading in the circuit
(b) voltage drop across each element will be less than the applied voltage
(c) power factor of the circuit will be 3 /4
(d) None of the above

8. Consider an *L-C-R* circuit as shown in figure with an AC source of peak value V_0 and angular frequency ω. Then, the peak value of current through the AC source is



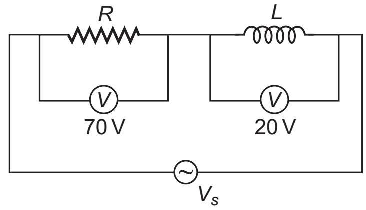
(a) $\dfrac{V_0}{\sqrt{\dfrac{1}{R^2} + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

(b) $V_0 \left[\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)\right]^2$

(c) $\dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

(d) None of these

9. The adjoining figure shows an AC circuit with resistance R, inductance L and source voltage V_s. Then,

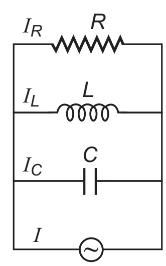


(a) the source voltage $V_s = 72.8$ V
(b) the phase angle between current and source voltage is $\tan^{-1}(7/2)$
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

10. When an alternating voltage of 220 V is applied across a device P, a current of 0.25 A flows through the circuit and it leads the applied voltage by an angle $\pi/2$ radian. When the same voltage source is connected across another device Q, the same current is observed in the circuit but in phase with the applied voltage. What is the current when the same source is connected across a series combination of P and Q?

(a) $\frac{1}{4\sqrt{2}}$ A lagging in phase by $\pi/4$ with voltage
(b) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\pi/4$ with voltage
(c) $\frac{1}{\sqrt{2}}$ A leading in phase by $\pi/4$ with voltage
(d) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\pi/2$ with voltage

11. In a parallel L-C-R circuit as shown in figure if I_R, I_L, I_C and I represent the rms values of current flowing through resistor, inductor, capacitor and the source, then choose the appropriate correct answer.

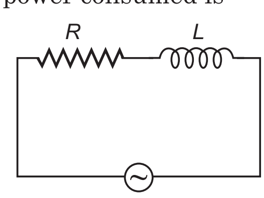


(a) $I = I_R + I_L + I_C$
(b) $I = I_R + I_L - I_C$
(c) I_L or I_C may be greater than I
(d) None of these

12. In a series L-C-R circuit, current in the circuit is 11 A when the applied voltage is 220 V. Voltage across the capacitor is 200 V. If the value of resistor is 20 Ω, then the voltage across the unknown inductor is

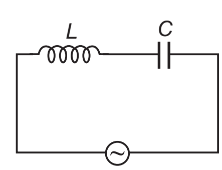
(a) zero
(b) 200 V
(c) 20 V
(d) None of these

13. In the circuit shown in figure, the power consumed is



(a) zero
(b) $\frac{V_0^2}{2R}$
(c) $\frac{V_0^2 R}{2(R^2 + \omega^2 L^2)}$
(d) None of these

14. In a series L-C circuit, the applied voltage is V_0. If ω is very low, then the voltage drop across the inductor V_L and capacitor V_C are

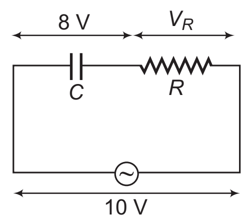


(a) $V_L = \frac{V_0}{2}; V_C = \frac{V_0}{2}$
(b) $V_L = 0; V_C = V_0$
(c) $V_L = V_0; V_C = 0$
(d) $V_L = -V_C = \frac{V_0}{2}$

15. A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If coil is connected to a DC battery of emf 12 volt and internal resistance 4 Ω, then current through it in steady state is

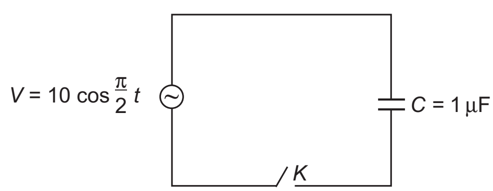
(a) 2.4 A (b) 1.8 A
(c) 1.5 A (d) 1.2 A

16. In a series *C*-*R* circuit shown in figure, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. The voltage across *R*, and the phase difference between current and the applied voltage will respectively be



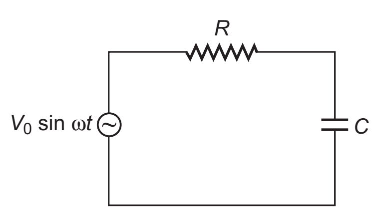
(a) 6 V, $\tan^{-1}\left(\frac{4}{3}\right)$ (b) 3 V, $\tan^{-1}\left(\frac{3}{4}\right)$
(c) 6 V, $\tan^{-1}\left(\frac{3}{4}\right)$ (d) None of these

17. An AC voltage source described by $V = 10\cos(\pi/2)t$ is connected to a 1 μF capacitor as shown in figure. The key *K* is closed at $t = 0$. The time ($t > 0$) after which the magnitude of current *I* reaches its maximum value for the first time is



(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

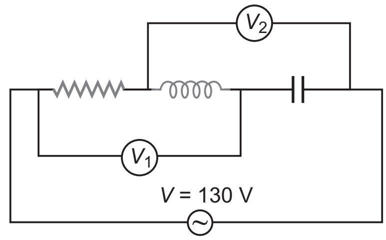
18. An AC voltage source $V = V_0 \sin \omega t$ is connected across resistance *R* and capacitance *C* as shown in figure. It is given that $R = 1/\omega C$. The peak current is I_0. If the angular frequency of the voltage source is changed to $\omega/\sqrt{3}$, then the new peak current in the circuit is



(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{\sqrt{2}}$
(c) $\frac{I_0}{\sqrt{3}}$ (d) $\frac{I_0}{3}$

More than One Correct Options

1. In a R-L-C series circuit shown, the readings of voltmeters V_1 and V_2 are 100 V and 120 V. Choose the correct statement(s).



(a) Voltage across resistor, inductor and capacitor are 50 V, 86.6 V and 206.6 V respectively
(b) Voltage across resistor, inductor and capacitor are 10 V, 90 V and 30 V respectively
(c) Power factor of the circuit is $\frac{5}{13}$
(d) Circuit is capacitive in nature

2. Current in an AC circuit is given by $i = 3 \sin \omega t + 4 \cos \omega t$, then
(a) rms value of current is 5 A
(b) mean value of this current in positive one-half period will be $\frac{6}{\pi}$
(c) if voltage applied is $V = V_m \sin \omega t$, then the circuit may contain resistance and capacitance
(d) if voltage applied is $V = V_m \cos \omega t$, then the circuit may contain resistance and inductance only

3. A tube light of 60 V, 60 W rating is connected across an AC source of 100 V and 50 Hz frequency. Then,
(a) an inductance of $\frac{2}{5\pi}$ H may be connected in series
(b) a capacitor of $\frac{250}{\pi}$ μF may be connected in series to it
(c) an inductor of $\frac{4}{5\pi}$ H may be connected in series
(d) a resistance of 40 Ω may be connected in series

4. In an AC circuit, the power factor
(a) is unity when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal inductance only
(c) is zero when the circuit contains an ideal resistance only
(d) is zero when the circuit contains an ideal inductance only

5. In an AC series circuit, $R = 10\ \Omega$, $X_L = 20\ \Omega$ and $X_C = 10\ \Omega$. Then, choose the correct options
(a) Voltage function will lead the current function
(b) Total impedance of the circuit is $10\sqrt{2}\ \Omega$
(c) Phase angle between voltage function and current function is 45°
(d) Power factor of circuit is $\frac{1}{\sqrt{2}}$

6. In the above problem further choose the correct options.

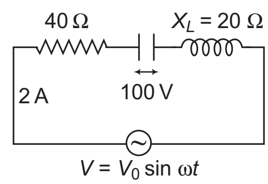
(a) The given values are at frequency less than the resonance frequency

(b) The given values are at frequency more than the resonance frequency

(c) If frequency is increased from the given value, impedance of the circuit will increase

(d) If frequency is decreased from the given value, current in the circuit may increase or decrease

7. In the circuit shown in figure,



(a) $V_R = 80$ V (b) $X_C = 50\ \Omega$

(c) $V_L = 40$ V (d) $V_0 = 100$ V

8. In *L-C-R* series AC circuit,

(a) If R is increased, then current will decrease (b) If L is increased, then current will decrease

(c) If C is increased, then current will increase (d) If C is increased, then current will decrease

Comprehension Based Questions

Passage I (Q. No. 1 to 3)

A student in a lab took a coil and connected it to a 12 V DC source. He measures the steady state current in the circuit to be 4 A. He then replaced the 12 V DC source by a 12 V, ($\omega = 50$ rad/s) AC source and observes that the reading in the AC ammeter is 2.4 A. He then decides to connect a 2500 μF capacitor in series with the coil and calculate the average power developed in the circuit. Further he also decides to study the variation in current in the circuit (with the capacitor and the battery in series).

Based on the readings taken by the student, answer the following questions.

1. The value of resistance of the coil calculated by the student is

(a) 3 Ω (b) 4 Ω

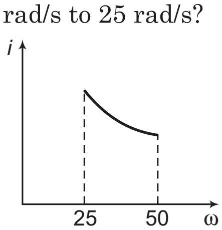
(c) 5 Ω (d) 8 Ω

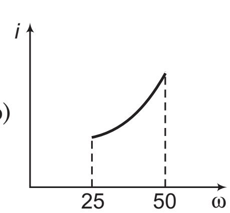
2. The power developed in the circuit when the capacitor of 2500 μF is connected in series with the coil is

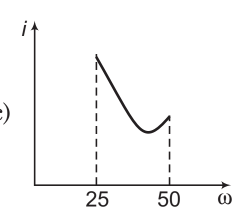
(a) 28.8 W (b) 23.04 W

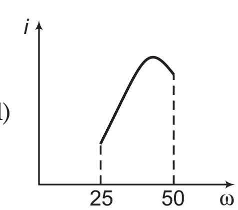
(c) 17.28 W (d) 9.6 W

3. Which of the following graph roughly matches the variations of current in the circuit (with the coil and capacitor connected in the series) when the angular frequency is decreased from 50 rad/s to 25 rad/s?

(a) 

(b) 

(c) 

(d) 

Passage II (Q. No. 4 to 6)

It is known to all of you that the impedance of a circuit is dependent on the frequency of source. In order to study the effect of frequency on the impedance, a student in a lab took 2 impedance boxes P and Q and connected them in series with an AC source of variable frequency. The emf of the source is constant at 10 V. Box P contains a capacitance of $1\,\mu F$ in series with a resistance of $32\,\Omega$. And the box Q has a coil of self-inductance 4.9 mH and a resistance of $68\,\Omega$ in series. He adjusted the frequency so that the maximum current flows in P and Q. Based on his experimental set up and the reading by him at various moment, answer the following questions.

4. The angular frequency for which he detects maximum current in the circuit is

(a) $10^5/7$ rad/s (b) 10^4 rad/s
(c) 10^5 rad/s (d) $10^4/7$ rad/s

5. Impedance of box P at the above frequency is

(a) $70\,\Omega$ (b) $77\,\Omega$
(c) $90\,\Omega$ (d) $100\,\Omega$

6. Power factor of the circuit at maximum current is

(a) 1/2 (b) 1
(c) 0 (d) $1/\sqrt{2}$

Match the Columns

1. Match the following two columns for a series AC circuit.

Column I	Column II
(a) Only C in the circuit	(p) current will lead
(b) Only L in the circuit	(q) voltage will lead
(c) Only R in the circuit	(r) $\phi = 90°$
(d) R and C in the circuit	(s) $\phi = 0°$

2. Applied AC voltage is given as

$$V = V_0 \sin \omega t$$

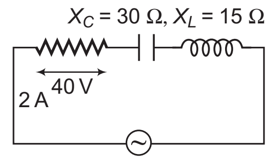
Corresponding to this voltage, match the following two columns.

Column I	Column II
(a) $I = I_0 \sin \omega t$	(p) only R circuit
(b) $I = -I_0 \cos \omega t$	(q) only L circuit
(c) $I = I_0 \sin(\omega t + \pi/6)$	(r) may be C-R circuit
(d) $I = I_0 \sin(\omega t - \pi/6)$	(s) may be L-C-R circuit

3. For an L-C-R series AC circuit, match the following two columns.

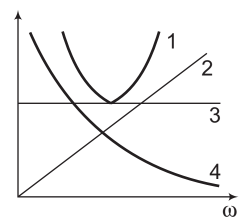
Column I	Column II
(a) If resistance is increased	(p) current will increase
(b) If capacitance is increased	(q) current will decrease
(c) If inductance is increased	(r) current may increase or decrease
(d) If frequency is increased	(s) power may decrease or increase

4. In the circuit shown in figure, match the following two columns. In Column II, quantities are given in SI units.



Column I	Column II
(a) Value of resistance R	(p) 60
(b) Potential difference across capacitor	(q) 20
(c) Potential difference across inductor	(r) 30
(d) Applied potential difference	(s) None of the above

5. Corresponding to the figure shown, match the two columns.



Column I	Column II
(a) Resistance	(p) 4
(b) Capacitive reactance	(q) 1
(c) Inductive reactance	(r) 2
(d) Impedance	(s) 3

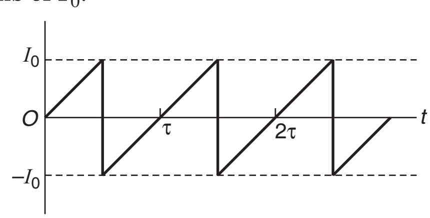
Subjective Questions

Note *Power factor leading means current is leading.*

1. A coil is in series with a 20 μF capacitor across a 230 V, 50 Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the current in the circuit is

(a) leading (b) lagging

2. The current in a certain circuit varies with time as shown in figure. Find the average current and the rms current in terms of I_0.

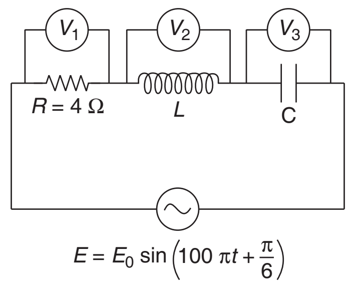


3. Two impedances Z_1 and Z_2 when connected separately across a 230 V, 50 Hz supply consume 100 W and 60 W at power factor of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply, find

(a) total power absorbed and overall power factor

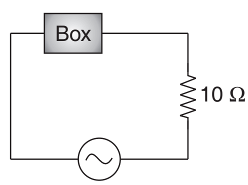
(b) the value of reactance to be added in series so as to raise the overall power factor to unity.

4. In the figure shown, the reading of voltmeters are $V_1 = 40$ V, $V_2 = 40$ V and $V_3 = 10$ V. Find

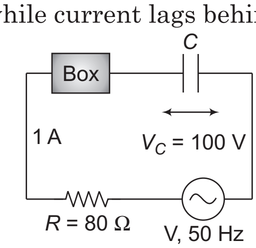


(a) the peak value of current
(b) the peak value of emf
(c) the value of L and C

5. In the circuit shown in figure power factor of box is 0.5 and power factor of circuit is $\sqrt{3}/2$. Current leading the voltage. Find the effective resistance of the box.



6. A circuit element shown in the figure as a box is having either a capacitor or an inductor. The power factor of the circuit is 0.8, while current lags behind the voltage. Find

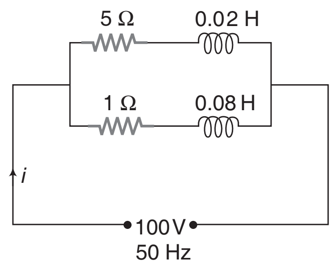


(a) the source voltage V,
(b) the nature of the element in box and find its value.

7. The maximum values of the alternating voltages and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are $200\sqrt{2}$ V and 10 A, respectively. At $t = 0$, both are increasing positively.
(a) Write down the expression for voltage and current at time t.
(b) Determine the power consumed in the circuit.

8. An L-C circuit consists of an inductor coil with $L = 5.00$ mH and a 20.0 μF capacitor. There is negligible resistance in the circuit. The circuit is driven by a voltage source with $V = V_0 \cos \omega t$. If $V_0 = 5.00$ mV and the frequency is twice the resonance frequency, determine
(a) the maximum charge on the capacitor
(b) the maximum current in the circuit
(c) the phase relationship between the voltages across the inductor, the capacitor and the source.

9. A coil having a resistance of 5 Ω and an inductance of 0.02 H is arranged in parallel with another coil having a resistance of 1 Ω and an inductance of 0.08 H. Calculate the power absorbed when a voltage of 100 V at 50 Hz is applied.



10. A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115 V – 50 Hz supply. Another circuit takes a current of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz supply, then calculate

(a) the current (b) the power consumed and (c) the power factor

Answers

Introductory Exercise 28.1

1. (a) 628 Ω (b) 6.37 mH (c) 1.59 kΩ (d) 1.59 mF **2.** 0.036 H, 111.8 V **3.** 7.7 H, 6 A

Introductory Exercise 28.2

1. 650 Hz, 0 **2.** 0.2

Exercises

LEVEL 1

Assertion and Reason

1. (a) **2.** (b) **3.** (a) **4.** (b) **5.** (a or b) **6.** (b) **7.** (a) **8.** (a,b) **9.** (b) **10.** (b)
11. (c)

Objective Questions

1. (c) **2.** (b) **3.** (d) **4.** (d) **5.** (a) **6.** (d) **7.** (d) **8.** (d) **9.** (a) **10.** (a)
11. (b) **12.** (b) **13.** (c) **14.** (c) **15.** (b) **16.** (d) **17.** (d) **18.** (b) **19.** (c) **20.** (c)
21. (b) **22.** (a) **23.** (a) **24.** (a) **25.** (a) **26.** (c) **27.** (c) **28.** (c) **29.** (a) **30.** (b)

Subjective Questions

1. (a) 0.326 A (b) 35.3°, lagging (c) 97.8 V, 32.6 V, 102 V
2. (a) Inductor (b) 0.133 H
3. (b) 1000 Hz (c) $\frac{\sqrt{3}}{2}$, leading (d) $R = 17.32\,\Omega$, $C = 15.92\,\mu F$
4. (a) 0.12 A (b) 1.2×10^{-2} A (c) 1.2×10^{-3} A
5. (a) (8.33 mA)cos (950 rad/s) t (b) 760 Ω (c) – (6.33 V) sin (950 rad/s)t
6. (a) 45.8°, voltage leads the curren,t 0.697 (b) 343 Ω (c) 155 V (d) 48.6 W (e) 48.6 W (f) 0 (g) 0

LEVEL 2

Single Correct Option

1.(c) 2.(d) 3.(c) 4.(b) 5.(b) 6.(b) 7.(d) 8.(b) 9.(a) 10.(b)
11.(c) 12.(b) 13.(c) 14.(b) 15.(c) 16.(a) 17.(a) 18.(b)

More than One Correct Options

1.(a,c,d) 2.(c,d) 3.(c,d) 4.(a,d) 5.(a,b,c,d) 6.(b,c,d) 7.(a,b,c) 8.(a)

Comprehension Based Questions

1.(a) 2.(c) 3.(b) 4.(a) 5.(b) 6.(b)

Match the Columns

1. (a) → p,r (b) → q,r (c) → s (d) → p
2. (a) → p,s (b) → q (c) → r,s (d) → s
3. (a) → q,s (b) → r,s (c) → r,s (d) → r,s
4. (a) → q (b) → p (c) → r (d) → s
5. (a) → s (b) → p (c) → r (d) → q

Subjective Questions

1. (a) 0.416 H (b) 0.597 H **2.** zero, $\frac{I_0}{\sqrt{3}}$ **3.** (a) 99 W, 0.92 leading (b) 194.2 Ω
4. (a) $10\sqrt{2}$A (b) $50\sqrt{2}$V (c) $\frac{1}{25\pi}$H, (d) $\frac{1}{100\pi}$F **5.** 5 Ω
6. (a) 100 V (b) inductor, $L = \frac{1.6}{\pi}$ H
7. (a) $V = 400\sin(100\pi t + \pi/4)$, $i = 20\sin(100\pi t + \pi/6)$ (b) $P = 3864$ W
8. (a) 33.4 nC (b) 0.211 mA
   (c) Source and inductor voltages in phase. Capacitor voltage lags by 180°.
9. 797 W **10.** (a) 5.5 A (b) 1.188 kW (c) 0.939 lag

Hints & Solutions

23. Current Electricity

INTRODUCTORY EXERCISE 23.1

1. $\because \quad i = \frac{q}{t} = \frac{ne}{t}$

$\therefore \quad n = \frac{it}{e} = \frac{(0.7)(1)}{1.6 \times 10^{-19}}$

$= 4.375 \times 10^{18}$

2. $\because \quad q = it$

$= (3.6)(3 \times 60 \times 60)$

$= 38880$ C

3. (a) $\because \quad q = it = (7.5)(45) = 337.5$ C

(b) $n = \frac{q}{e} = \frac{337.5}{1.6 \times 10^{-19}}$

$= 2.1 \times 10^{21}$

4. $f = \frac{v}{2\pi r} = \frac{2.2 \times 10^6}{(2\pi)(5.3 \times 10^{-11})}$

$= 6.6 \times 10^{15}$ Hz

$I = qf$

$= (1.6 \times 10^{-19})(6.6 \times 10^{15})$

$= 1.06 \times 10^{-3}$ A

$= 1.06$ mA

5. $\Delta q = \int_0^{10} i dt = \int_0^{10} (10 + 4t)\, dt$

$= 300$ C

6. Current due to both is from left to right. So, the two currents are additive.

INTRODUCTORY EXERCISE 23.2

1. False. Only under electrostatic conditions (when $i = 0$) all points of a conductor are at same potential.

INTRODUCTORY EXERCISE 23.3

1. $i = neAv_d$

i.e. $v_d \propto i$

When current has increased from $i = 1.2$ A to $i = 6.0$ A, i.e five times, then drift velocity will also increase to five times.

2. From $i = neAv_d$

We have $\quad v_d = \frac{i}{neA}$

or $\quad v_d = \frac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$

$= 0.735 \times 10^{-6}$ m/s

$= 0.735\ \mu$m/s

$t = \frac{l}{v_d}$

$= \frac{10 \times 10^3}{0.735 \times 10^{-6}}$ s

$= \frac{10 \times 10^3}{0.735 \times 10^{-6} \times 60 \times 60 \times 24 \times 365}$ yr

$= 431.4$ yr

INTRODUCTORY EXERCISE 23.4

1. $R = \rho \frac{l}{A} = \frac{(1.72 \times 10^{-8})(35)}{(\pi/4)(2.05 \times 10^{-3})^2} = 0.18\ \Omega$

2. $\rho = \frac{1}{\sigma} \quad \Rightarrow \quad \rho\sigma = \text{constant}$

3. $R = \frac{\rho l}{A}$

$\therefore \quad A = \frac{\rho l}{R}$

$m = (Vd)$, where V = volume and d = density

$\therefore \quad m = (Ald)$

$= \frac{\rho l^2}{R} d$

$= \frac{(1.72 \times 10^{-8})(3.5)^2 (8.9 \times 10^3)}{0.125}$

$= 15 \times 10^{-3}$ kg

$= 15$ g

4. $R = \frac{\rho\,(L)}{A} = \frac{\rho L}{tL} = \frac{\rho}{t}$

i.e. R is independent of L.

Hence, the correct option is (c).

INTRODUCTORY EXERCISE 23.5

1. Copper is metal and germanium is semiconductor. Resistance of a metal decreases and that of a semiconductor increases with decrease in temperature.

$\therefore$ Correct option is (d).

2. $4.1[1+4.0\times10^{-3}(\theta-20)]$
$$=3.9[1+5.0\times10^{-3}(\theta-20)]$$
Solving we get,
$$\theta\approx 85°C$$

INTRODUCTORY EXERCISE 23.6

1. PD across each resistance is 10 V.

$$\therefore \quad i_{2\Omega}=\frac{10}{2}=5A$$
$$i_{4\Omega}=\frac{10}{4}=2.5A$$

2. $V_A=0$ V (as it is earthed)

$$V_C-V_A=5\text{ V}$$
$$\therefore \quad V_C=5\text{ V}$$
$$V_B-V_A=2\text{ V}$$
$$\therefore \quad V_B=2\text{ V}$$
$$V_D-V_C=10\text{ V}$$
$$\therefore \quad V_D=10+V_C=15\text{ V}$$
$$i_{1\Omega}=\frac{V_C-V_B}{1}=3A \text{ from } C \text{ to } B \text{ as } V_C>V_B$$
$$i_{2\Omega}=\frac{V_D-V_A}{2}=7.5A \text{ from } D \text{ to } A \text{ as } V_D>V_A$$

3. $V_A=V_B$

$$\therefore \quad V_{AB}=0$$
or
$$E-ir=0$$
$$\therefore \quad E-\left(\frac{15+E}{8}\right)(2)=0$$

Solving this equation, we get
$$E=5\text{ V}$$

4. Net emf $=(n-2m)E$
$$=(10-2\times2)(1)$$
$$=6\text{ V}$$
$$i=\frac{\text{Net emf}}{\text{Net resistance}}$$
$$=\frac{6}{10+2}=0.5\text{ A}$$

5. $V_{R_1}=0$

$$\therefore \quad i_{R_1}=0$$
$$V_{R_2}=V_{R_3}=10\text{ V}$$
$$\therefore \quad i_{R_2}=i_{R_3}=\frac{10}{10}=1\text{ A}$$

INTRODUCTORY EXERCISE 23.7

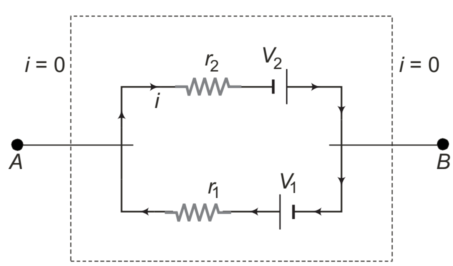
1. Applying loop law equation in upper loop, we have
$$E+12-ir-1=0 \qquad \ldots\text{(i)}$$
Applying loop law equation in lower loop, we have where
$$i=1+2=3\text{A}$$
$$E+6-1=0 \qquad \ldots\text{(ii)}$$
Solving these two equations, we get
$$E=-5\text{ V and } r=2\Omega$$

2. Power delivered by a battery $=Ei$
$$=12\times3$$
$$=36\text{ W}$$
Power dissipated in resistance
$$=i^2R=(3)(2)^2$$
$$=12\text{ W}$$

INTRODUCTORY EXERCISE 23.8

1. **(a) Equivalent emf (V) of the battery**

PD across the terminals of the battery is equal to its emf when current drawn from the battery is zero. In the given circuit,



Current in the internal circuit,
$$i=\frac{\text{Net emf}}{\text{Total resistance}}=\frac{V_1+V_2}{r_1+r_2}$$
Therefore, potential difference between A and B would be
$$V_A-V_B=V_1-ir_1$$
$$\therefore \quad V_A-V_B=V_1-\left(\frac{V_1+V_2}{r_1+r_2}\right)r_1=\frac{V_1r_2-V_2r_1}{r_1+r_2}$$
So, the equivalent emf of the battery is
$$V=\frac{V_1r_2-V_2r_1}{r_1+r_2}$$
Note that if $V_1r_2=V_2r_1 : V=0$

If $V_1 r_2 > V_2 r_1 : V_A - V_B$ = Positive i.e. A side of the equivalent battery will become the positive terminal and *vice-versa.*

(b) **Internal resistance (r) of the battery**

r_1 and r_2 are in parallel. Therefore, the internal resistance r will be given by

$$1/r = 1/r_1 + 1/r_2$$

or $$r = \frac{r_1 r_2}{r_1 + r_2}$$

2. 

$$E = \frac{\Sigma\,(E/r)}{\Sigma\,(1/r)} = \frac{(6/1) - (2/1)}{(1/1) + (1/1)} = 2\,\text{V}$$

Now, net emf of E and 4 V is 2V as they are oppositely connected.

3. $$E_{eq} = \frac{\Sigma\,(E/r)}{\Sigma\,(1/r)}$$

$$= \frac{(10/1) + (4/2) + (6/2)}{(1/1) + (1/2) + (1/2)}$$

$$= 7.5\,\text{V}$$

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} + \frac{1}{2}$$

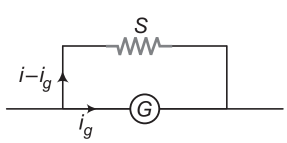
$\therefore$ $$r = 0.5\,\Omega$$

INTRODUCTORY EXERCISE 23.9

1. $V = i_g\,(G + R)$

$\therefore$ $R = \dfrac{V}{i_g} - G$ = series resistance connected with galvanometer

$$= \left(\frac{5}{5 \times 10^{-3}}\right) - 1 = 999\,\Omega$$

2. 

$$\frac{i_g}{i - i_g} = \frac{S}{G}$$

$\therefore$ $$S = \left(\frac{i_g}{i - i_g}\right) G$$

$$= \left[\frac{(50 \times 10^{-6})}{(5 \times 10^{-3} - 50 \times 10^{-6})}\right](100) \approx 1.0\,\Omega$$

3. $V = i_g\,G$

$\therefore$ $$i_g = \frac{V}{G}$$

Now, $$nV = i_g\,(G + R) = \frac{V}{G}\,(G + R)$$

$\therefore$ $$R = (n - 1)\,G$$

INTRODUCTORY EXERCISE 23.10

1. $$r = R\left(\frac{l_1}{l_2} - 1\right)$$

$$= 5\left(\frac{0.52}{0.4} - 1\right)$$

$$= 1.5\,\Omega$$

2. (a) $V_{AJ} = \dfrac{E}{2}$ or emf of lower battery

$\therefore$ $$i\,R_{AJ} = \frac{E}{2}$$

or $$\left(\frac{E}{15r + r}\right)\left(\frac{15r}{600}\right)(l) = \frac{E}{2}$$

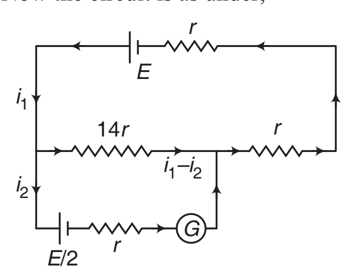
Solving this equation, we get

$$l = 320\ \text{cm}$$

(b) Resistance of 560 cm $= \left(\dfrac{15r}{600}\right)(560)$

$$= 14r$$

Now the circuit is as under,



Applying loop law in upper loop we have,

$$E - 14r\,(i_1 - i_2) - i_1 r - i_1 r = 0 \qquad \ldots\text{(i)}$$

Applying loop law in lower law loop we have,

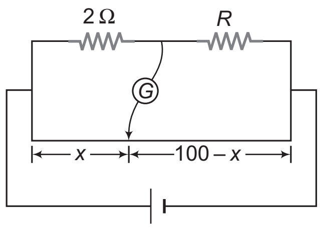
$$-\frac{E}{2} - i_2 r + (i_1 - i_2)\,(14r) = 0 \qquad \ldots\text{(ii)}$$

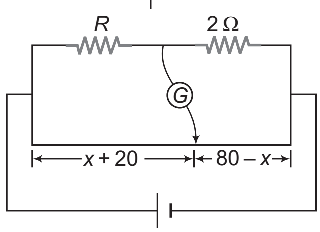
Solving these two equations

we get, $$i_2 = \frac{3E}{22r}$$

INTRODUCTORY EXERCISE 23.11

1. $R > 2\,\Omega \quad \Rightarrow \quad 100 - x > x$





Applying $\dfrac{P}{Q} = \dfrac{R}{S}$

We have $\dfrac{2}{R} = \dfrac{x}{100 - x}$ …(i)

$\dfrac{R}{2} = \dfrac{x + 20}{80 - x}$ …(ii)

Solving Eqs. (i) and (ii), we get $R = 3\,\Omega$

$\therefore$ Correct option is (a).

2. Using the concept of balanced, Wheatstone bridge, we have,

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{X}{(52 + 1)} = \frac{10}{(48 + 2)}$$

$$\therefore \qquad X = \frac{10 \times 53}{50} = 10.6\,\Omega$$

$\therefore$ Correct option is (b).

3. Slide wire bridge is most sensitive when the resistance of all the four arms of bridge is same. Hence, B is the most accurate answer.

INTRODUCTORY EXERCISE 23.12

1. $\dfrac{P}{Q} = \dfrac{R}{X} \Rightarrow X = \left(\dfrac{Q}{P}\right) R = \left(\dfrac{1}{10}\right) R$

R lies between 142 Ω and 143 Ω.

Therefore, the unknown resistance X lies between 14.2 Ω and 14.3 Ω.

2. Experiment can be done in similar manner but now K_2 should be pressed first then K_1.

3. BC, CD and BA are known resistances.

The unknown resistance is connected between A and D.

INTRODUCTORY EXERCISE 23.13

1. Yellow $\rightarrow$ 4

Red $\rightarrow$ 2

Orange $\rightarrow 10^3$

Gold $\rightarrow$ 5

$\therefore \quad R = (4.2 \times 10^3 \pm 5\%)\,\Omega$

2. 2 $\rightarrow$ Red

4 $\rightarrow$ Yellow

$10^6 \rightarrow$ Blue

5% $\rightarrow$ Gold

Exercises

LEVEL 1

Assertion and Reason

1. If PD between two terminals of a resistance is zero, then current through resistance is zero, this is confirmed. But PD between any two points of a circuit is zero, this does not mean current is zero.

2. In parallel, $V =$ constant

$\therefore$ From the equation

$$P = \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R}$$

3. $R = \dfrac{\rho l}{A}$ or $\dfrac{R}{l} =$ resistance per unit length $= \dfrac{\rho}{A} \propto \dfrac{1}{A}$

Near A, area of cross-section is less. Therefore, resistance per unit length will be more. Hence from the equation, $H = i^2Rt$, heat generation near A will be more.

Current density, $J = \dfrac{i}{A}$ or $J \propto \dfrac{1}{A}$ (as i is same)

4. Since net resistance decreases, therefore main current increases. Hence, net potential difference across voltmeter also increases.

5. Even if ammeter is non-ideal, its resistance should be small and net parallel resistance is less than the smallest individual resistance.

$\therefore$ R_{net} < resistance of ammeter in the changed situation. Hence, net resistance of the circuit will decrease. So, main current will increase. But maximum percentage of main current will pass through ammeter (in parallel combination) as its resistance is less. Hence, reading of ammeter will increase.

Initial voltmeter reading = emf of battery

Final voltmeter reading = emf of battery

− potential drop across shown resistance.

Hence, voltmeter reading will decrease.

6. If current flows from a to b, then equation will become

$$V_a - ir - E = V_b \quad \text{or} \quad V_a - V_b = E + ir$$

So, $V_a - V_b$ is always positive. Hence, V_a is always greater than V_b.

7. Current in the circuit will be maximum when $R = 0$.

8. Resistance will increase with temperature on heating. Hence current will decrease.

Further $\quad P = \dfrac{V^2}{R}$ or $P \propto \dfrac{1}{R}$

Resistance is increasing. Hence, power consumed across R should decrease.

$V = IR$ is just an equation between P D across a resistance current passing through it and its resistance. This is not Ohm's law.

9. Electrons get accelerated by the electric. Then, suddenly collision takes place. Then, again accelerated and so on.

11. $$E_{eq} = \frac{E_1/r_1 + E_2/r_2}{(1/r_1) + (1/r_2)}$$

$$= E_1\left[\frac{(1/r_1) + (E_2/E_1)(1/r_2)}{(1/r_1) + (1/r_2)}\right]$$

So, E_{eq} may be greater than E_1 also, if $E_2/E_1 > 1$

r_1 and r_2 are in parallel. Hence, r_{eq} is less than both r_1 and r_2 individually.

Objective Questions

3. $H = I^2 Rt$

$$\therefore \quad [R] = \left[\frac{H}{I^2 t}\right]$$

$$= \left[\frac{ML^2T^{-2}}{I^2T}\right] = [ML^2T^{-3}I^{-2}]$$

4. $$R = \frac{l}{\sigma A}$$

$$\therefore \quad \sigma = \frac{l}{RA} = \frac{\text{m}}{\text{ohm - m}^2} = \text{ohm}^{-1}\text{-m}^{-1}$$

6. $$0.5 = \frac{E}{r + 3.75} \quad \text{...(i)}$$

$$0.4 = \frac{E}{r + 4.75} \quad \text{...(ii)}$$

Solving these two equations, we get

$$E = 2\text{V}$$

7. In parallel current distributes in increase ratio of resistance

$$\therefore \quad \frac{I_G}{I_S} = \frac{S}{G}$$

$$\therefore \quad G = \left(\frac{I_S}{I_G}\right)(S)$$

$$= \left(\frac{50 - 20}{20}\right)(12)$$

$$= 18\ \Omega$$

8. $$\frac{I_G}{I_S} = \frac{S}{G}$$

$$\therefore \quad S = \left(\frac{I_G}{I_S}\right) G$$

$$= \left(\frac{2}{98}\right) G = \frac{G}{49}$$

9. $P = \dfrac{V^2}{R}$ or $P \propto \dfrac{1}{R}$

$$\frac{P_2}{P_1} = \frac{R_1}{R_2} = \frac{l_1}{l_2} \quad (\text{as } R \propto l)$$

$$\therefore \quad P_2 = \left(\frac{l_1}{l_2}\right) P_1 = \left(\frac{l}{0.9\, l}\right) P_1 = 1.11\, P_1$$

So, power will increase by 11%.

10. By symmetry, $V_A = V_B$

or $\quad V_{AB} = 0$

11. $$r = R\left(\frac{l_1}{l_2} - 1\right) = 10\left(\frac{75}{60} - 1\right)$$

$$= 2.5\ \Omega$$

12. Let $V_0 = V$

Now, $\quad I_{AO} + I_{BO} = I_{OC}$

$$\therefore \quad \frac{6 - V}{6} + \frac{3 - V}{3} = \frac{V - 2}{2}$$

Solving this equation, we get

$$V = 3\text{ V}$$

13. $v_d = \dfrac{i}{neA} = \dfrac{i}{ne\,(\pi r^2)} \quad \Rightarrow \quad v_d \propto \dfrac{i}{r^2}$

14. For making voltmeter of higher range, more resistance is required.

15. $V_{20\Omega} = V_{\text{Total}}$

$\therefore \qquad (20)\,(0.3) = (R_{\text{Total}})\,(0.8)$

$\therefore \qquad R_{\text{Total}} = \dfrac{30}{4}\,\Omega$

$\therefore \qquad \dfrac{4}{30} = \dfrac{1}{R_1} + \dfrac{1}{20} + \dfrac{1}{15}$

Solving we get $R_1 = 60\Omega$

16. Net resistance will decrease by increasing the parallel resistors. Therefore, main current i will increase, further,

$(\text{PD})_{\text{Voltmeter}} = V_{\text{Total}} - (\text{PD})_{\text{Ammeter}}$

$\qquad = V_{\text{Total}} - i \quad$ (resistance of ammeter)

Since, i has increased. Hence, PD across voltmeter will decrease.

17. $P = \dfrac{V^2}{R} \quad$ or $\quad P \propto \dfrac{1}{R} \qquad \ldots$(i)

Resistor is cut in n equal parts. Therefore, each resistance will become $\dfrac{R}{n}$. Now, these are connected in parallel. Therefore, net resistance will become $\dfrac{1}{n}$ times $\dfrac{R}{n}$. or $\dfrac{R}{n^2}$.

Now, from Eq. (i), power will become n^2 times.

18. If A is fused, then complete circuit is broken.

19. $E - ir = 0$

$\therefore \qquad 3 - \left(\dfrac{3+15}{1+2+R}\right)(1) = 0$

Solving this equation, we get

$R = 3\Omega$

20. $100 = (5)\left[\dfrac{(2500\,R)}{2500 + R}\right]$

Solving this equation, we get

$R = 20\Omega$

21. Five parallel combination, each of value

$\dfrac{R}{10} + \dfrac{R}{10} = \dfrac{R}{5}$

22. $\dfrac{P}{Q} = \dfrac{20}{100-20} \quad$ or $\quad Q = 4P \qquad \ldots$(i)

$\therefore \qquad P < Q$

Now, $\qquad \dfrac{P+15}{Q} = \dfrac{40}{100-40}$

or $\qquad \dfrac{P+15}{Q} = \dfrac{2}{3} \qquad \ldots$(ii)

Solving these two equations, we get

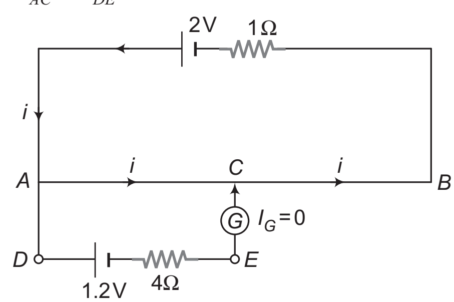
$P = 9\Omega$

23. Total potential of 10V equally distributes between 50 Ω and other parallel combination of 100 Ω and voltmeter. Hence, their net resistance should be same. Or

$\dfrac{100 \times R}{100 + R} = 50$

$\therefore \quad R = 100\ \Omega =$ resistance of voltmeter

24. $V_{AC} = V_{DE}$



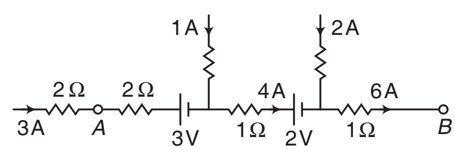
$i\,(R_{AC}) = E = 1.2$

$\therefore \qquad \left(\dfrac{2}{4+1}\right)\left(\dfrac{4}{100} \times l\right) = 1.2$

Solving this equation, we get

$l = 75$ cm

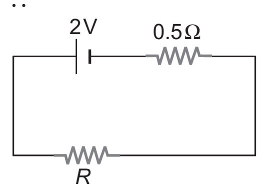
25.



$V_A - 3 \times 2 - 3 - 1 \times 4 + 2 - 1 \times 6 = V_B$

$\therefore \qquad V_A - V_B = 17$ V

26.



Equivalent simple circuit is given as maximum power across R is obtained

When $\qquad R = r = 0.5\ \Omega$

$i = \dfrac{2}{R + r} = 2\text{A}$

$\therefore \qquad = i^2R = (2)^2\,(0.5) = 2\text{W}$

27. $r = R\left(\frac{E}{V} - 1\right) = 5\left(\frac{2.2}{1.8} - 1\right) = \frac{10}{9}\ \Omega$

28. $i = \frac{10 - 5}{2.5 + 2.5 + 40} = \frac{1}{9}$ A (clockwise)

$$V_B - 15\,i - 25\,i = V_A$$

$$\therefore \quad V_A - V_B = 40\,i = -\frac{40}{9}\ \text{V}$$

29. Potential drop across potentiometer wire

$$= (0.2 \times 10^{-3})\,(100) = 0.02\ \text{V}$$

Now given resistance and potentiometer wire are in series with given battery. So, potential will drop in direct ratio of resistance.

$$\therefore \quad \frac{0.02}{2 - 0.02} = \frac{R}{490}$$

$$\therefore \quad R = 4.9\,\Omega$$

30. When K is open

$$R_{\text{net}} = 3R/2$$

$$\therefore \quad i_1 = E/(3R/2) = \frac{2E}{3R}$$

When K is closed

$$R_{\text{net}} = 2\left[\frac{R \times 2R}{R + 2R}\right] = \frac{4}{3}\,R$$

$$\therefore \quad i_2 = E\,/\,(4R/3) = \frac{3E}{4R}$$

$$\therefore \quad \frac{i_1}{i_2} = \frac{8}{9}$$

31. $\frac{I_G}{I_S} = \frac{S}{G}$

$$\therefore \quad S = \left(\frac{I_G}{I_S}\right) G$$

$$= \frac{(1/34)}{(33/34)} \times 3663 = 111\,\Omega$$

32. Simple series and parallel grouping of resistors.

33. Two balanced Wheatstone bridges in parallel.

34. $R_{ab} = \frac{(3 \times 15)}{3 + 15} = 2.5\ \Omega$

As $\quad R_{60°} = \left(\frac{18}{360°}\right)(60°) = 3\,\Omega$

35. $R_{AB} = 2$ [Net resistance of infinite series] + 1

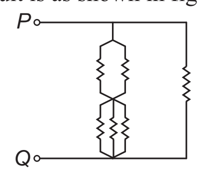
In parallel net resistance is always less than the smallest one. Hence, net resistance of infinite series is less than 1Ω.

$$\therefore \quad 1\Omega < R_{AB} < 3\Omega$$

36. $R_0 = R_{AB} = R + \frac{(R + R_0)\,R}{(R + R_0) + R}$

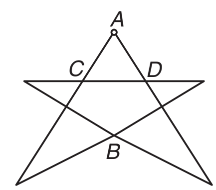
Solving this equation, we get $R = \frac{R_0}{\sqrt{3}}$

37. Simple circuit is as shown in figure,

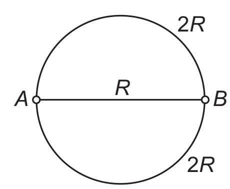


Each $\rightarrow R$

38. Wheatstone (balanced) between A and B. So, resistance between C and D can removed.



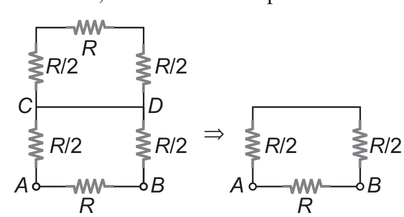
39.



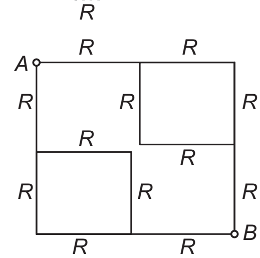
$$R = \left(\frac{4}{2\pi r}\right)(2r) = \frac{4}{\pi}$$

Now 2Ω, 2Ω and R are in parallel.

40.



41.



Connection can be removed from centre. $3R$ and $3R$ from two sides of AB are in parallel.

Subjective Questions

1. Under electrostatic conditions (when no current flows), $E = 0$. When current is non-zero, then electric field is also non-zero.

2. There is random or thermal motion of free electrons in the absence of potential difference.

3. $$i = qf = q\left(\frac{v}{2\pi R}\right)$$
$$= \frac{(1.6\times10^{-19})\,(2.2\times10^{6})}{(2\pi)\,(5\times10^{-11})}$$
$$= 1.12\times10^{-3}\ \text{A} = 1.12\ \text{mA}$$

4. $P = \dfrac{V^2}{R}$

$\therefore$ $$R = \frac{V^2}{P}$$
$$R_1 = \frac{(120)^2}{40} = 360\ \Omega$$
$$R_2 = \frac{(120)^2}{60} = 240\ \Omega$$
$$R_3 = \frac{(120)^2}{75} = 192\ \Omega$$

Now, all these resistors are in parallel.

5. (a) $i = \dfrac{12-6}{4+8} = 0.5$ A

(b) $P_{R_1} = i^2R_1 = 1\text{W}$

$\Rightarrow$ $P_{R_2} = i^2R_2 = 2\text{W}$

(c) Power supplied by $E_1 = E_1\ i = 6$ W

and power consumed by $E_2 = E_2 i = 3\text{W}$

6. $\dfrac{8}{12} = \dfrac{l}{40-l}$

Solving this equation, we get

$$l = 16\ \text{cm}$$

7. (a) Ideal voltmeter means infinite resistance.

$\therefore$ $i = 0$

(b) $V = E$ (if $i = 0$)

$= 5$ V

(c) Reading of voltmeter $= E = 5$ V

8. (a) $E_1 > E_2$

Therefore, net current is anti-clockwise or from B to A.

(b) Current through E_1 is normal. Hence, it is doing the positive work.

(c) Current flows from B to A

$\therefore$ $V_B > V_A$

9. $i = \dfrac{150-50}{2+3} = 20$ A (anti-clockwise)

$V_Q + 150 - 20\times2 = V_P$

$\therefore$ $V_Q = V_P - 110 = -10$ V

10. $\rho = 8.89\times10^3\ \text{kg/m}^3$

Mass of $1\text{m}^3 = 8.89\times10^3$ kg

$= 8.89\times10^3\ \text{kg} = 8.89\times10^6\text{g}$

$\therefore$ Number of gram moles $= \dfrac{8.89\times10^6}{63.54} = 1.4\times10^5$

Number of atoms $= 1.4\times10^5\times6.02\times10^{23}$

$= 8.42\times10^{28}$

One atom emits one conduction electron. Therefore, number of free electrons in unit volume (or 1m^3 volume)

$$n = 8.42\times10^{28}\ \text{per m}^3$$

Now, $i = neA\,v_d$

$\therefore$ $$v_d = \frac{i}{neA} = \frac{i}{ne\,\pi r^2}$$
$$= \frac{2.0}{(8.42\times10^{28})\,(1.6\times10^{-19})(\pi)\,(0.5\times10^{-3})^2}$$
$$= 1.9\times10^{-4}\ \text{m/s}$$

11. (a) In 1 m, potentials difference,

$V = 0.49$ $\therefore$ $V = iR$

$\therefore$ $$i = \frac{0.49}{R} = \frac{(0.49)\,A}{\rho l}$$
$$= \frac{(0.49)\,(\pi/4)\,(0.84\times10^{-3})^2}{(2.75\times10^{-8})\,(1)}$$
$$= 9.9\ \text{A}$$

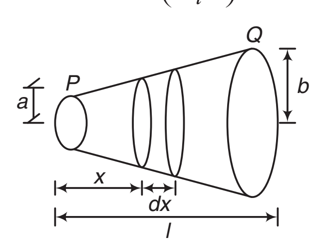
(b) PD between two points, 12 m apart

$= (0.49\ \text{V/m})\ (12\text{m}) = 5.88$ V

(c) $R = \dfrac{V}{i} = \dfrac{5.88}{9.9} = 0.6\ \Omega$

12. Radius at distance x from end P,

$$r = a + \left(\frac{b-a}{l}\right)x$$



Resistance of element of thickness dx is

$$dR = \frac{\rho\,(dx)}{\pi r^2} \qquad \left(\text{Using } R = \frac{\rho l}{A}\right)$$

$$\therefore \qquad R = \int_{X=0}^{X=l} dR$$

13. $i = \dfrac{E}{R+r} \quad \Rightarrow \quad P = \text{power across } R = i^2 R$

$$P = \left(\frac{E}{R+r}\right)^2 R \qquad \ldots\text{(i)}$$

For power to be maximum,

$$\frac{dP}{dR} = 0$$

By putting $\dfrac{dP}{dR} = 0$ we get, $R = r$

Further, by putting $R = r$ in Eq. (i)

We get, $\quad P_{\max} = \dfrac{E^2}{4r}$

14. As derived in the above question,

$$P_{\max} = \frac{E^2}{4r}$$

Here, $\quad E =$ net emf $= 2 + 2 = 4$ V

and $\quad r =$ net internal resistance

$$= 1 + 1 = 2\Omega$$

$$\therefore \quad P_{\max} = \frac{(4)^2}{(4)\,(2)} = 2\text{W}$$

15. In series,

$$\alpha_{eq} = \frac{R_{01}\,\alpha_1 + R_{02}\,\alpha_2}{R_{01} + R_{02}}$$

$$= \frac{(600)\,(0.001) + (300)\,(0.004)}{600 + 300}$$

$$= 0.002 \text{ per}^\circ\text{C}$$

Now, $R_t = R_0\,[1 + \alpha\,\Delta\theta]$

$$= (600 + 300)\,[1 + 0.002 \times 30] = 954\Omega$$

16. In parallel current distributes in inverse ratio of resistance $1 \rightarrow$ Aluminium $2 \rightarrow$ Copper

$$\frac{R_1}{R_2} = \frac{i_2}{i_1}$$

$$\frac{\rho_1 l_1 / A_1}{\rho_2 l_2 / A_2} = \frac{2}{3}$$

$$\therefore \quad \frac{\rho_1 l_1 d_2^2}{\rho_2 l_2 d_1^2} = \frac{2}{3}$$

$$\therefore \quad d_2 = \left(\sqrt{\frac{2\rho_2 l_2}{3\rho_1 l_1}}\right) d_1$$

$$= \left(\sqrt{\frac{2 \times 0.017 \times 6}{3 \times 0.028 \times 7.5}}\right) (1\text{ mm})$$

$$= 0.569 \text{ mm}$$

17. (a) $E = \dfrac{V}{l} = \dfrac{0.938}{0.75} = 1.25$ V/m

(b) $E = J\rho$

$$\therefore \quad \rho = \frac{E}{J} = \frac{1.25}{4.4 \times 10^7}$$

$$= 2.84 \times 10^{-8}\ \Omega\text{-m}$$

18. (a) $J = \dfrac{i}{A} = \dfrac{V}{RA} = \dfrac{V}{\left(\dfrac{\rho l}{A}\right) A}$

$$J = \frac{V}{\rho l} \qquad \ldots\text{(i)}$$

or $\quad J \propto \dfrac{1}{l}$

$l_{\min} = d$. So, J is maximum. Hence, potential difference should be applied across the face $(2d \times 3d)$

From Eq. (i),

$$J_{\max} = \frac{V}{\rho d}$$

(b) $i = \dfrac{V}{R} = \dfrac{V}{(\rho l / A)} = \dfrac{VA}{\rho l}$

or $\quad i \propto \dfrac{A}{l}$

Across face $(2d \times 3d)$, area of cross-section is maximum and l is minimum. Hence, current is maximum.

$$i_{\max} = \frac{V\,(2d \times 3d)}{\rho\,(d)} = \frac{6Vd}{\rho}$$

19. (a) $\rho = \dfrac{RA}{l} = \dfrac{(0.104)\,(\pi/4)\,(2.5 \times 10^{-3})^2}{14}$

$$= 3.65 \times 10^{-8}\ \Omega\text{-m}$$

(b) $V = El = 1.28 \times 14 = 17.92$ V

$$\therefore \quad i = \frac{V}{R} = \frac{17.92}{0.104} = 172.3 \text{ A}$$

(c) $v_d = \dfrac{i}{neA}$

$$= \frac{172.3}{(8.5 \times 10^{28})\,(1.6 \times 10^{-19})\left(\dfrac{\pi}{4}\right)(2.5 \times 10^{-3})^2}$$

$$= 2.58 \times 10^{-3} \text{ m/s}$$

20. $R_1 + R_2 = 20$ ...(i)

$$\alpha_{eq} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} \quad \text{(in series)}$$

$$0 = \frac{R_1(-0.5\times10^{-3}) + R_2(5.0\times10^{-3})}{20}$$

$\therefore \quad R_1 = 10R_2$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$R_2 = R_{Fe} = \frac{20}{11}\,\Omega$$
$$= 1.82\Omega$$
$$R_1 = R_{Cu} = 10R_2 = 18.18\ \Omega$$

21. $8\,\Omega$ and $12\,\Omega$ resistors are in parallel.

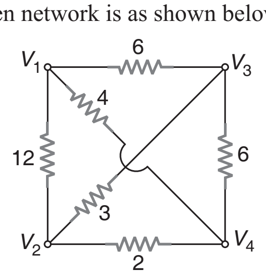
$$\therefore \quad R_{net} = \frac{8\times12}{8+12} = 4.8\ \Omega$$
$$\therefore \quad i = \frac{24}{4.8} = 5\text{A}$$

22. All four resistors are in parallel

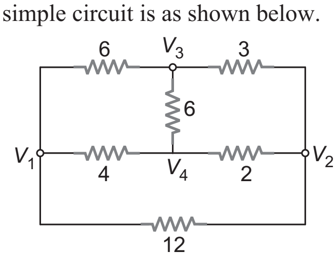
$$\therefore \quad \frac{1}{R} = \frac{1}{8} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$$
$$R = \frac{8}{5}\,\Omega$$
$$\therefore \quad i = \frac{24}{8/5} = 15\text{ A}$$

23. All these resistors are in parallel.

24. The given network is as shown below.



The simple circuit is as shown below.



Now, this is a balanced Wheatstone bridge in parallel with $12\ \Omega$ resistance.

25. First case

$$i = \frac{12+6}{1+2+3} = 3\text{A} \quad \text{(clockwise)}$$

Now, $V_A - V_G = 12$ V

$\therefore \quad V_A = 12$ V, as $V_G = 0$

$V_A - V_B = 1\times3 = 3$ V

$\therefore \quad V_B = V_A - 3 = 9$ V

$V_B - V_C = 2\times3 = 6$ V

$\therefore \quad V_C = V_B - 6 = 3$ V

$V_G - V_D = 6$ V

$\therefore \quad V_D = -6$ V, as $V_G = 0$

In the second case,

$$i = \frac{12-6}{1+2+3} = 1\text{ A}$$

Rest procedure is same.

26. $i = \dfrac{200}{5+10+25} = 5$ A (anti-clockwise)

$V_3 - V_G = 25\times5 = 125$

$\therefore \quad V_3 = 125$ V as $V_G = 0$

$V_G - V_2 = 10\times5 = 50$

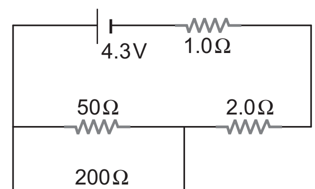
$\therefore \quad V_2 = -50$ V

$V_2 - V_1 = 5\times5 = 25$ V

$\therefore \quad V_1 = V_2 - 25 = -75$ V

Now, $V_{3-2} = V_3 - V_2$

27. (a)

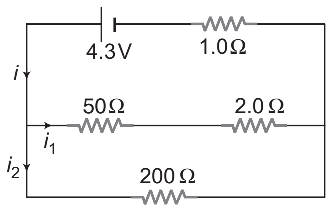


$$R_{net} = 1.0 + 2.0 + \frac{50\times200}{50+200}$$
$$= 43\ \Omega$$
$$\therefore \quad i = \frac{4.3}{43} = 0.1\text{ A}$$
$$= \text{Readings of ammeter}$$

Readings of voltmeter

$= (i)$ net resistance of $50\ \Omega$ and $200\ \Omega$

$$= (0.1)\left(\frac{50\times200}{50+200}\right)$$
$$= 4\text{ V}$$

(b) 

$$R_{net} = 1.0 + \frac{52 \times 200}{52 + 200} = 42.27\ \Omega$$

$\therefore \quad i = \dfrac{4.3}{42.27} \approx 0.1$ A

Now $\dfrac{i_1}{i_2} = \dfrac{200}{52}$

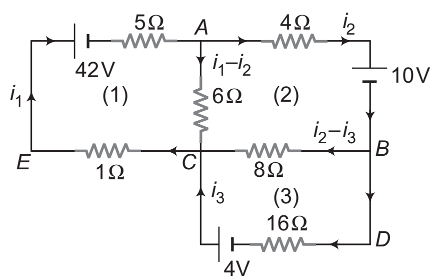
$\therefore \quad i_1 = \left(\dfrac{200}{252}\right)(0.1) = 0.08$ A

= Reading of ammeter

$\therefore$ Reading to voltmeter

= Potential difference across 50 Ω and 2.0 Ω

= $0.08 \times 52 \approx 4.2$ V

28. 

Loop 1

$$-42 - 6(i_1 - i_2) - 5i_1 - i_1 = 0 \qquad \ldots\text{(i)}$$

Loop 2

$$-4i_2 - 10 - 8(i_2 - i_3) + 6(i_1 - i_2) = 0 \qquad \ldots\text{(ii)}$$

Loop 3

$$8(i_2 - i_3) = -16\, i_3 + 4 = 0 \qquad \ldots\text{(iii)}$$

Solving these equations, we get

$i_1 = 4$ A, $i_2 = 1.0$ A and $i_3 = 0.5$ A

29. Net resistance of voltmeter ($R = 400\ \Omega$) and 400 Ω will be 200 Ω. Now, we are getting a balanced Wheatstone bridge with 100 Ω and 200 Ω resistors on each side. Potential difference across each side will be 10 V which will distribute in direct ratio of resistors 100 Ω and 200 Ω.

$\therefore \quad \dfrac{V_{100\,\Omega}}{V_{200\,\Omega}} = \dfrac{100}{200} = \dfrac{1}{2}$

or $\quad V_{200\,\Omega} = \left(\dfrac{2}{3}\right)(10) = \dfrac{20}{3}$ V

30. (a) (i) When switch S is open, V_1 and V_2 are in series, connected to 200 V battery. Potential will drop in direct ratio of their resistors.

$\therefore \quad V_1 : V_2 = R_{V_1} : R_{V_2} = 3000 : 2000$

$= 3 : 2$

$\therefore \quad V_1 = \dfrac{3}{5} \times 200 = 120$ V

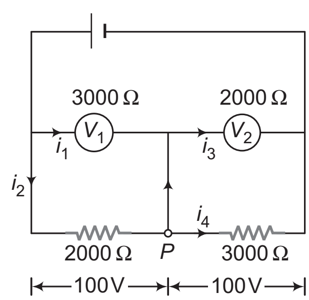
$V_2 = \dfrac{2}{5} \times 200 = 80$ V

(ii) When S is closed then V_1 and R_1 are in parallel. Similarly, V_2 and R_2 are also in parallel. Now, they are in series and they come out to be equal. So, 200 V will equally distribute between them.

$\therefore \quad V_1 = V_2 = \dfrac{200}{2} = 100$ V each

(b) $i_2 = \dfrac{100}{2000} = \dfrac{1}{20}$ A

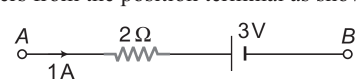
$i_4 = \dfrac{100}{3000} = \dfrac{1}{30}$ A



If we apply junction law at P, then current through switch

$= i_2 - i_4 = \dfrac{1}{60}$ A in upward direction.

31. Power absorbed by resistor is i^2R or 2W. Therefore, remaining 3W is absorbed by the battery (= Ei). Hence, E is 3 V and current of 1 A enters from the position terminal as shown below.



$V_{AB} = E + ir$

$= E + iR \qquad$ (Here, $r = R$)

$= 3 + (1)(2) = 5$ V

32. $8.4 = E - 1.5\, r \qquad \ldots$(i)

$9.4 = E + 3.5\, r \qquad \ldots$(ii)

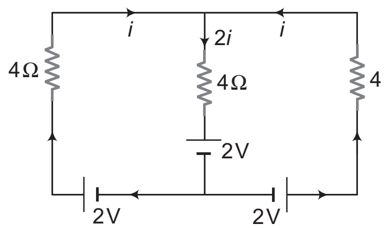
Solving these two equations, we get

$r = 0.2\ \Omega$ and $E = 8.7$ V

33. During charging,

$$V = E + ir = 2 + (5)(0.1) = 2.5 \text{ V}$$

34. Simple circuit is as shown below



By symmetry, currents on two sides will be same (let i)

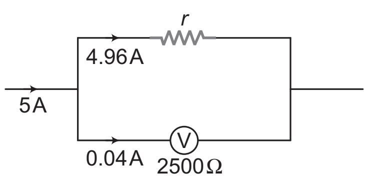
Now if we apply loop law in any of the closed loop, we will get $i = 0$.

35. Net resistance should remain unchanged.

$$\therefore \quad R + G = R' + \frac{GS}{G+S}$$

$$\therefore \quad R' - R = G - \frac{GS}{G+S} = \frac{G^2}{G+S}$$

36. Current through voltmeter



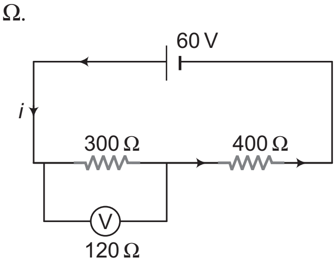
$$= \frac{V}{R} = \frac{100}{2500} = 0.04 \text{ A}$$

In parallel current distribution in inverse ratio of resistors. Hence,

$$\frac{4.96}{0.04} = \frac{2500}{r}$$

$$\therefore \quad r = 20.16 \ \Omega$$

37. Voltmeter reads 30 V, half of 60 V. Hence, resistance of 400 Ω and voltmeter is also equal to 300 Ω.



$$\therefore \quad 300 = \left(\frac{400 \times R}{400 + R}\right)$$

where, R = resistance of voltmeter.

Solving the above equation, we get

$$R = 1200 \ \Omega$$

In the new situation,

$$R_{\text{net}} = 400 + \frac{(300)(1200)}{300 + 1200} = 640 \ \Omega$$

$$\therefore \quad i = \frac{60}{640} = 0.09375 \text{ A}$$

Now voltage drop across

Voltmeter = 60 – potential drop across 400 Ω resistor

$= 60 - (400)\, i$

$= 60 - (400)(0.09375)$

$= 22.5$ V

38. $R_{\text{net}} = 60 + \dfrac{(60)(120)}{60 + 120} = 100 \ \Omega$

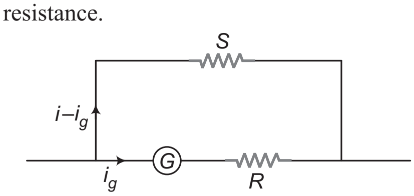
$$\therefore \quad i = \frac{120}{100} = 1.2 \text{ A}$$

Now, reading of voltmeter

$= 120$ – potential drop across R_1

$= 120 - (60)(1.2) = 48$ V

39. In parallel current distributes in inverse ratio of resistance.



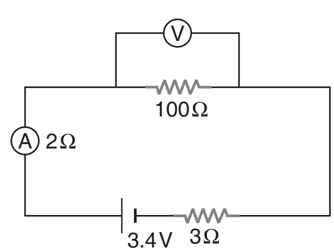
$$\therefore \quad \left(\frac{i - i_g}{i_g}\right) = \frac{G + R}{S}$$

$$\therefore \quad R = \left(\frac{i - i_g}{i_g}\right) S - G$$

$$= \left(\frac{20}{10^{-3}}\right)(0.005) - 20 = 80 \ \Omega$$

Note *In calculations, we have taken* $i - i_g \approx i$.

40.



Reading of voltmeter = 3.4 − Voltage drop across ammeter and 3Ω resistance

$$= (3.4) - 0.04 \times 2 - 0.04 \times 3$$
$$= 3.2 \text{ V}$$

Now, $\quad 3.2 \text{ V} = \dfrac{(0.04)(100\,R)}{(100 + R)}$

where, R = resistance of voltmeter

$\therefore \quad R = 400\ \Omega$

If voltmeter is ideal, then

$$i = \frac{3.4}{2 + 100 + 3} = 0.03238 \text{ A}$$

Reading of voltmeter = $100i$ = 3.238 V

41. (a) $V = E - ir$

$$= E - \left(\frac{E}{r + R_V}\right) r$$

$$V = \left(\frac{ER_V}{r + R_V}\right) \qquad \ldots\text{(i)}$$

(b) $V = \dfrac{E}{100}$

Substituting in Eq. (i), we get

$$R_V = 4.5 \times 10^{-3}\ \Omega$$

(c) $V = E\left(\dfrac{1}{1 + \dfrac{r}{R_V}}\right)$

If R_V is increased from this value, V will increase.

42. (a) $\qquad I_A = \dfrac{\varepsilon}{R + R_A + r}$

$\therefore \qquad \varepsilon = (R + R_A + r)\, I_A$

Now, $\qquad I'_A = \dfrac{\varepsilon}{R + r} \qquad \ldots\text{(i)}$

Substituting the value of ε, we get

$$I'_A = I_A\left(1 + \frac{R_A}{R_A + r}\right)$$

If $\qquad R_A \to 0,\ I'_A \to I_A$

(b) In Eq. (i) substituting

$I_A = 0.99\, I'_A$ and the given values, we get

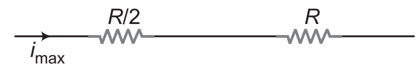
$R_A = 0.0045\ \Omega$

(c) $I_A = \dfrac{I'_A}{1 + \dfrac{R_A}{R_A + r}} = \dfrac{I'_A}{1 + \dfrac{1}{1 + \dfrac{r}{R_A}}}$

If R_A is decreased from this value, then I_A will increase from 99% of I'_A.

43. $P = i^2 R$

$\therefore \qquad i_{\max} = \sqrt{\dfrac{P_{\max}}{R}} = \sqrt{\dfrac{36}{2.4}} = \sqrt{15}$ A



Total maximum power = $(i_{\max})^2 \left(\dfrac{3R}{2}\right)$

$$= (15)(1.5)(2.4) = 54 \text{ W}$$

44. $V = E - ir$

$\therefore \qquad r = \dfrac{E - V}{i} = \dfrac{2.6 - 2}{1}$

$$= 0.6\ \Omega$$

Now, power generated in the battery

$$P = i^2 r$$
$$= (1)^2 (0.6) = 0.6 \text{ W}$$

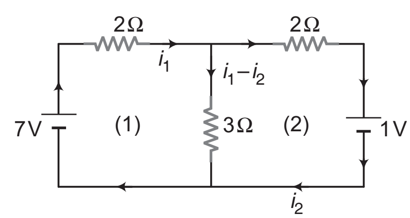
Power supplied by the battery = Ei

$$= 2.6 \text{ W}$$

$\therefore$ Net power supplied for external circuit

$$= 2.6 - 0.6 = 2.0 \text{ W}$$

45.



Loop equation in loop (1)

$$+ 7 - 2i_1 - 3(i_1 - i_2) = 0 \qquad \ldots\text{(i)}$$

Loop equation in loop (2)

$$- 1 + 3(i_1 - i_2) - 2i_2 = 0 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$i_1 = 2\text{A} \quad \text{and} \quad i_2 = 1 \text{ A}$$

Power supplied by $E_1 = E_1 i_1 = 14$ W

But power consumed by $E_2 = E_2 i_2 = 1$W

46. (a) $\qquad i = \dfrac{12}{5 + 1} = 2$ A

$\Rightarrow \qquad P_1 = Ei = 24$ W

(b) $P_2 = i^2 r = (2)^2 (1) = 4$W

(c) $P = P_1 - P_2 = 20$ W

47. (b) $i = \dfrac{V}{R}$ in each resistance

(e) $P = \dfrac{V^2}{R}$ in each resistance

(f) $$P = \frac{V^2}{R}$$

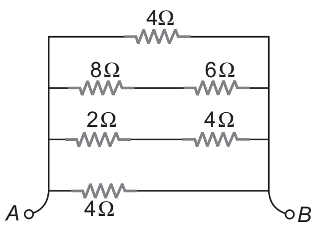
or $$P \propto \frac{1}{R} \qquad \text{(as } V \text{ is same)}$$

48. (a) $P = \dfrac{V^2}{R}$

$\therefore \quad V = \sqrt{PR} = \sqrt{5 \times 15 \times 10^3} = 273.8\ \text{V}$

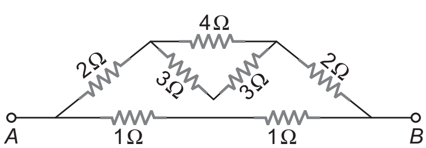
(b) $P = \dfrac{V^2}{R} = \dfrac{(120)^2}{9 \times 10^3} = 1.6\ \text{W}$

49. (a)

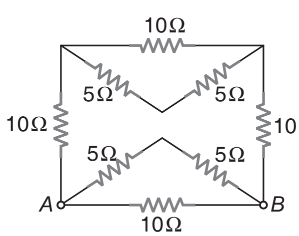


(b) A balanced Wheatstone bridge in parallel with R.

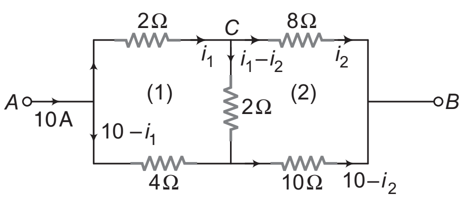
(c)



(d)



(e)



Let us take a current of 10 A between A and B

Loop equation in loop (1)

$$-2i_1 - 2(i_1 - i_2) + 4(10 - i_1) = 0 \qquad \text{...(i)}$$

Loop equation in loop (2)

$$-8i_2 + 10(10 - i_2) + 2(i_1 - i_2) = 0 \qquad \text{...(ii)}$$

Solving these two equations, we get

$$i_1 = 6.53\ \text{A} \quad \text{and} \quad i_2 = 6.11\ \text{A}$$

Now let us find V_{AB} across path ACB,

$$V_{AB} = 2i_1 + 8i_2 = 61.94\ \text{V}$$

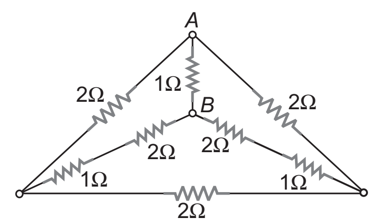
Now, $V_{AB} = i_{\text{net}}\ R_{\text{net}} = (10)\ R_{\text{net}}$

$\therefore \quad 61.94 = 10\ R_{\text{net}}$

or $\quad R_{\text{net}} = 6.194\ \Omega$

(f) Two resistors in vertical middle wire can be removed.

(g) Now balanced Wheatstone bridge, in parallel with 1Ω resistance between points A and B. The encircled resistance of 2Ω can be removed from the Wheatstone bridge.



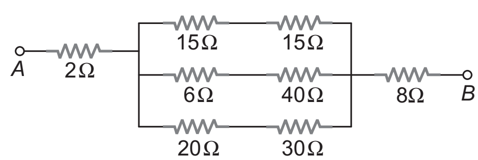
One resistor of 2Ω has already removed from the original circuit given the question. As its two ends will be the same potential (by symmetry).

50. $R_i + 5R$

If connected by two equal resistors between B and D and between C and E, the combination is a balanced Wheatstone bridge and two resistors in series.

$\therefore \quad R_f = R + R + R = 3R = 0.6\ R_i$

51.



52. (b) Three resistors are in parallel. Then, one resistor in series with this combination.

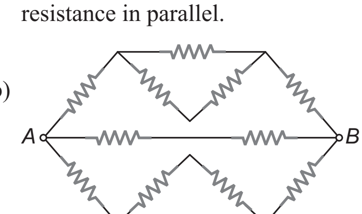
(c) Balanced Wheatstone bridge. Hence, two resistors in vertical wire can be removed.

(d) All four resistors are in parallel

(e) A balanced Wheatstone bridge.

53. (a) A balanced Wheatstone bridge with one resistance in parallel.

(b)



LEVEL 2

Single Correct Option

1. No current will flow through voltmeter. As it is ideal (infinite resistance). Current through two batteries

$$i = \frac{1.5 - 1.3}{r_1 + r_2} = \frac{0.2}{r_1 + r_2}$$

Now, $\quad V = E_2 - ir_2$

$\therefore \quad 1.45 = 1.5 - \left(\frac{0.2}{r_1 + r_2}\right)(r_2)$

Solving this equation, we get

$$r_1 = 3r_2$$

2. In series, PD distributes in direct ratio of resistance.
In first case,

$$\frac{198}{V_{AB} - 198} = \frac{900}{R_1} \quad \ldots\text{(i)}$$

In second case, $\quad \frac{180}{V_{AB} - 180} = \frac{900}{2R_1} \quad \ldots\text{(ii)}$

Solving these two equations, we get

$$V_{AB} = 220\text{ V}$$

3. Maximum current will pass through A.

$$P = i^2R$$

or $\quad P \propto i^2 \quad$ (R is same)

4. $\quad 4\,(R + R_A) = 20\text{ V}$

$\therefore \quad R = 5 - R_A$

where, R_A = resistance of ammeter

5. $r = R\left(\frac{l_1}{l_2} - 1\right) = 132.4\left(\frac{70}{60} - 1\right) \approx 22.1\,\Omega$

6. Initial current, $\quad i_1 = \frac{E_1 + E_2}{R + r_1 + r_2}$

Final current, when second battery is short circuited is

$$i_2 = \frac{E_1}{R + r_1}$$

$$i_2 > i_1 \text{ if } \frac{E_1}{R + r_1} > \frac{E_1 + E_2}{R + r_1 + r_2}$$

or $\quad E_1R + E_1r_1 + E_1r_2 > E_1R + E_1r_1 + E_2R + E_2\,r_1$

or $\quad E_1r_2 > E_2\,(R + r_1)$

7. B and C are in parallel

$\therefore \quad V_B = V_C$

Further $\quad R_A = R$

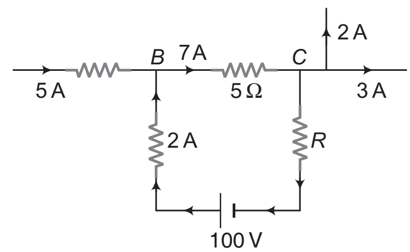
$$R_{BC} = \frac{(1.5R)\,(3R)}{1.5\,R + 3R} = R$$

or $\quad R_A = R_{BC}$

$\therefore \quad V_A = V_{BC}$

Because $\quad iR_A = iR_{BC}$

8.



Applying loop equation in closed loop we have,

$$+\,100 - 30 - 35 - 2R = 0$$

$\therefore \quad 2R = 35\text{ V} = V_R$

$$V_{5\Omega} = 7 \times 5 = 35\text{ V}$$

$\therefore \quad \frac{V_{5\Omega}}{V_R} = 1$

9. $r = R\left(\frac{l_1}{l_2} - 1\right)$

$$= R\left(\frac{y}{x} - 1\right) = \left(\frac{y - x}{x}\right)R$$

10. Let $R = at + b$

At $\quad t = 10\text{ s}, R = 20\,\Omega$

$\therefore \quad 20 = 10\,a + b \quad \ldots\text{(i)}$

At $\quad t = 30a + b \quad \ldots\text{(ii)}$

Solving these two equations, we get

$$a = 1.0\,\Omega/\text{s}$$

and $\quad b = 10\,\Omega$

$\therefore \quad R = (t + 10)$

$$i = \frac{E}{R} = \frac{10}{t + 10}$$

$$\Delta q = \int_{10}^{30} i\,dt$$

$$= \int_{10}^{30}\left(\frac{10}{t + 10}\right)dt$$

$$= 10\log_e(2)$$

11. Suppose n (< 1) fraction of length is stretched to m times.

Then, $\quad (1 - n)\,l + (nl)\,m = 1.5l$

or $\quad nm - n = 0.5 \quad \ldots\text{(i)}$

$$R = \frac{\rho l}{A} = \frac{\rho l}{(V/l)} \qquad (V = \text{volume})$$

$$= \frac{\rho l^2}{V}$$

or $\quad R \propto l^2 \qquad$ (if V = constant)

Now, the second condition is

$$(1 - n) R + (nR) m^3 = 4R$$

$\therefore \quad nm^2 - n = 3 \qquad \ldots$(iii)

Solving these two equations, we get

$$n = \frac{1}{8}$$

12. Initially, $\quad \dfrac{l_1}{100 - l_1} = \dfrac{X}{R} = \dfrac{2}{3}$

$\therefore \quad l_1 = \dfrac{2}{5} \times 100$

$= 40$ cm

Finally, $\quad \dfrac{l_2}{100 - l_2} = \dfrac{X}{R'} = \dfrac{12}{8} = \dfrac{3}{2}$

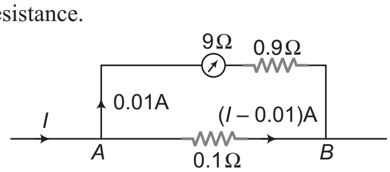
$\therefore \quad l_2 = \dfrac{3}{5} \times 100$

$= 60$ cm

$\therefore \quad J$ is displaced by

$$l_2 - l_1 = 20 \text{ cm}$$

13. In parallel, current distributes in inverse ratio of resistance.



$\therefore \quad \dfrac{0.01}{I - 0.01} = \dfrac{0.1}{9 + 0.9}$

Solving we get, $\quad I = 1$ A

14. Equivalent emf of two batteries ε_1 and ε_2 is

$$\varepsilon = \frac{\varepsilon_1/r_1 + \varepsilon_2/r_2}{1/r_1 + 1/r_2} = \frac{(2/2) + (4/6)}{(1/2) + (1/6)}$$

$= 2.5$ V

Now, $\quad V_{AN} = \varepsilon$

$\therefore \quad (I_{AN})(R_{AN}) = \varepsilon$

or $\quad \left(\dfrac{12}{4 + 4 \times 4}\right)(4)(l) = 2.5$

Solving this equation, we get

$$l = \frac{25}{24} \text{ m}$$

15. When K_1 and K_2 both are closed R_1 is short-circuited,

$$R_{\text{net}} = (50 + r)\ \Omega$$

When K_1 is open and K_2 is closed, current remains half.

Therefore, net resistance of the circuit becomes two times.

or $\quad (50 + r) + R_1 = 2(50 + r)$

Of the given options, the above equation is satisfied if

$$r = 0 \quad \text{and} \quad R_1 = 50\ \Omega$$

16. 100 Ω, 25 Ω and 20 Ω are in parallel.

Their, net resistance is 10 Ω

$\therefore \quad R_{\text{net}} = 4\ \Omega + 10\Omega + 6\Omega = 20\Omega$

$V = i R_{\text{net}} = 80$ V

17. All these resistors are in parallel.

$\therefore \quad R_{\text{net}} = \dfrac{R}{3} + r = 4\ \Omega$

Hence, the main current

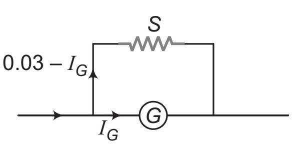
$$i = \frac{E}{R_{\text{net}}} = 1 \text{ A}$$

Current through either of the resistance

is $\quad \dfrac{i}{3} \quad$ or $\quad \dfrac{1}{3}$ A

$\therefore \quad V = iR = \left(\dfrac{1}{3}\right)(9) = 3$ V

18.



In parallel, current distributers in inverse ratio of resistance.

$$\frac{0.03 - I_G}{I_G} = \frac{G}{S} = \frac{r}{(r/4)} = 4$$

Solving this equation, we get

$$I_G = 0.006 \text{ A}$$

19. $\quad \dfrac{8\Omega}{6\Omega} = \dfrac{4\ \Omega}{3\Omega}$

$\therefore \quad V_A = V_B$

or $\quad V_{AB} = 0$

20. $V = iR$

$\therefore \quad V \propto R \qquad$ (as i = constant)

$\therefore \quad \dfrac{V_A}{V_B} = \left(\dfrac{\rho l_A}{\pi r_A^2}\right)\left(\dfrac{\pi r_B^2}{\rho l_B}\right)$

or $\quad \dfrac{r_B}{r_A} = \sqrt{\dfrac{V_A}{V_B} \times \dfrac{l_B}{l_A}}$

$$= \sqrt{\frac{3}{2} \times \frac{1}{6}} = \frac{1}{2}$$

21. Current decreases $\dfrac{20}{30}$ times or $\dfrac{2}{3}$ times. Therefore, net resistance should become $\dfrac{3}{2}$ times.

$\therefore \quad R + 50 = \dfrac{3}{2}(2950 + 50)$

Solving we get, $\quad R = 4450\,\Omega$

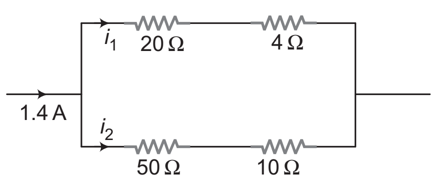
22.
$$E_0 = V_{AC} = (i)_{AC}\,(R)_{AC}$$
$$= \left(\frac{E}{10}\right)\left(\frac{10}{1} \times 0.2\right)$$
$$= \frac{E}{5} \qquad \ldots\text{(i)}$$

In second case,
$$E_0 = \left(\frac{E}{10 + x}\right)\left(\frac{10}{1} \times 0.3\right) \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get
$$x = 5\,\Omega$$

23. V and V_0 are oppositely connected.

24. Balanced Wheatstone bridge. Hence, 1.5 Ω resistance can be removed from the circuit.



$$\frac{i_1}{i_2} = \frac{50 + 10}{20 + 4} = \frac{2.5}{1}$$

$\therefore \quad i_1 = \left(\dfrac{2.5}{2.5 + 1}\right)(1.4) = 1\text{ A}$

25. Resistance between A and B can be removed due to balanced Wheatstone bridge concept. Now, R_{DE} and R_{GH} are in series and they are connected in parallel with 10 V battery.

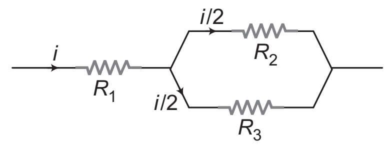
$\therefore \quad I_{DE} = \dfrac{10}{R_{DE} + R_{HG}} = \dfrac{10}{2 + 2}$

$$= 2.5\text{ A}$$

26. Net resistance of $3\,\text{k}\Omega$ and voltmeter is also $2\,\text{k}\Omega$. Now, the applied 10 V is equally distributed between $2\,\text{k}\Omega$ and $2\,\text{k}\Omega$. Hence, reading of voltmeter

$$= \frac{10}{2} = 5\text{ V}$$

27. $R_2 = R_3$ as $P = \dfrac{V^2}{R}$ and in parallel V is same.



Hence, $\quad P_{R_2} = P_{R_3}$

If $\quad R_2 = R_3$

Now current through R_1 is double so R_1 should be $\dfrac{1}{4}$th of R_2 or R_3 for same power. As $P = i^2R$.

More Than One Correct Options

1. $H = \dfrac{V^2}{R_1} t_1 \quad \Rightarrow \quad R_1 = \dfrac{V^2 t_1}{H}$

Similarly, $\quad R_2 = \dfrac{V^2 t_2}{H}$

In series, $\quad H = \left(\dfrac{V^2}{R_1 + R_2}\right) t$

$$t = \frac{H\,(R_1 + R_2)}{V^2}$$

Substituting the values of R_1 and R_2, we get

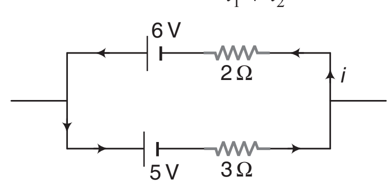
$$t = t_1 + t_2$$

In parallel, $\quad H = \dfrac{V^2}{R_{\text{net}}}\, t = V^2 t\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$$= V^2 t\left(\frac{H}{V^2 t_1} + \frac{H}{V^2 t_2}\right)$$

Solving we get, $\quad t = \dfrac{t_1 t_2}{t_1 + t_2}$

2.



$$i = \frac{6 - 5}{2 + 3} = 0.2\text{ A}$$

$$V_1 = E_1 - ir_1 = 6 - 0.2 \times 2$$
$$= 5.6\text{ V}$$

3. (a) In series, current is same.

$\therefore \quad I_A = I_B$

(b) $V_A + V_B = V_C$

$\therefore \quad I_A R_A + I_B R_B = I_C R_C$

(d) In parallel, current distributes in inverse ratio of resistance.

$$\therefore \quad \frac{I_B}{I_C} = \frac{I_A}{I_C} = \frac{R_C}{R_A + R_B}$$

4. Same as above.

5. (a) $V = iR$

In series i is same. Hence, V is also same as R is given same.

(b) $R = \dfrac{\rho l}{A}$

R is same. Hence, A should be smaller in first wire. Secondly, $v_d = \dfrac{i}{ne\,A}$ or $v_d \propto \dfrac{1}{A}$

A of first wire is less. Hence, its drift velocity should be more.

(c) $E = \dfrac{V}{l}$ or $E \propto \dfrac{1}{l}$ $\quad (V \to \text{same})$

7. If switch S is open,

$$i_1 \lambda l = E_2$$

where, i_1 = current in upper circuit and λ is resistance per unit length of potentiometer wire.

$$\therefore \quad \text{Null point length, } l = \frac{E_2}{i_1 \lambda}$$

(a) If jockey is shifted towards right, resistance in upper circuit will increase. So, current i_1 will decrease. Hence, l will increase.

(b) If E_1 is increased, i_1 will also increase. So, l will decrease.

(c) $l \propto E_2$

(d) If switch is closed, then null point will be obtained corresponding to

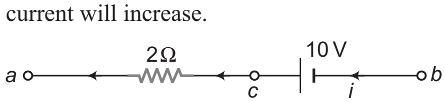
$$V_2 = E_2 - i_2 r_2$$

which is less than E_2. Hence, null point length will decrease.

8. By closing S_1, net external resistance will decrease. So, main current will increase.

By closing S_2, net emf will remain unchanged but net internal resistance will decrease. Hence, main current will increase.

9.



$$V_b + 10 - 2i = V_a$$

$$V_b - V_a = 2i - 10 = 2\text{ V}$$

$$\therefore \quad i = 6\text{A}$$

Now, $\quad V_C - V_a = 2 \times 6 = 12\text{ V}$

10. Between a and c, balanced Wheatstone bridge is formed. Across all other points simple series and parallel grouping of resistors.

Comprehension Based Questions

1 and **2.** $\quad r = R\left(\dfrac{l_1}{l_2} - 1\right)$

$$\therefore \quad 10 = R\left(\frac{500}{490} - 1\right)$$

Solving this equation, we get

$$R = 490\ \Omega$$

Further $\quad r = R\left(\dfrac{E}{V} - 1\right)$

or $\quad 10 = 490\left(\dfrac{2}{V} - 1\right)$

Solving, we get $\quad V = 1.96\text{ V}$

Match the Columns

1. Let potential of point e is V volts. Then,

$$I_{ae} + I_{be} + I_{ce} + I_{de} = 0$$

$$\therefore \quad \left(\frac{2-V}{1}\right) + \left(\frac{4-V}{2}\right) + \left(\frac{6-V}{1}\right) + \left(\frac{4-V}{2}\right) = 0$$

or $\quad V = 4\text{ V}$

Now current through any wire can be obtained by the equation,

$$I = \frac{\text{PD}}{R}$$

2. $i_1 = i_2$ or i is same at both sections.

$$A_1 < A_2$$

(a) Current density $= \dfrac{i}{A} \propto \dfrac{1}{A}$

(c) $\dfrac{\text{Resistance}}{\text{length}} = \dfrac{\rho}{A} \propto \dfrac{1}{A}$

(d) and (b) E or potential difference per unit length = (i) (Resistance per unit length)

$$= (i)\left(\frac{\rho}{A}\right) \propto \frac{1}{A}$$

3. By introducing parallel resistance R_3 in the circuit, total resistance of the circuit will decrease. Hence, main current i will increase.

Now, $\quad V_{R1} = E - V_{R_2} = E - iR_2$

Since, i is increasing, so V_{R_2} will increase. Hence, V_{R_1} or current passing through R_1 will decrease.

4. (a) $R = \dfrac{H}{i^2 t}$

$\therefore \quad [R] = [ML^2T^{-2}]/[A^2T] = [ML^2T^{-3}A^{-2}]$

(b) $V = iR$

$\therefore \quad [V] = [A][ML^2T^{-3}A^{-2}] = [ML^2T^{-3}A^{-1}]$

(c) $\rho = \dfrac{RA}{l}$

$\therefore \quad [\rho] = \dfrac{[ML^2T^{-3}A^{-2}][L^2]}{[L]} = [ML^3T^{-3}A^{-2}]$

(d) $[\sigma] = \left[\dfrac{1}{\rho}\right] = [M^{-1}L^{-3}T^3A^2]$

5. $i = \dfrac{4-1}{1+1+1} = 1A$ (anti-clockwise)

(a) $V_A = E - ir = 4 - 1 \times 1 = 3$ V

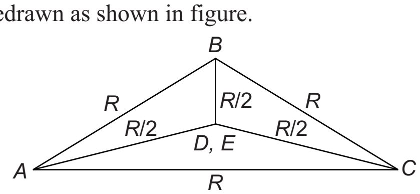
(b) $V_B = E + ir = 1 + 1 \times 1 = 2$ V

(c) $|P_A| = Ei - i^2 r = (4 \times 1) - (1)^2 (1) = 3$W

(d) $|P_B| = Ei + i^2 r = (1)(1) + (1)^2 (1) = 2$W

Subjective Questions

1. (a) Points D and E are symmetrically located with respect to points A and C. The circuit can be redrawn as shown in figure.

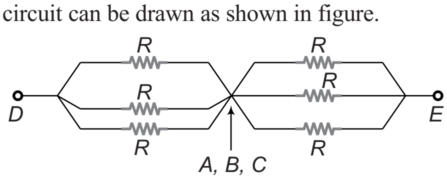


This is a combination of a balanced Wheatstone bridge in parallel with a resistance R. So, the resistance between B and D (or E) can be removed.

$$\frac{1}{R_{AC}} = \frac{1}{R} + \frac{1}{\dfrac{R}{2} + \dfrac{R}{2}} + \frac{1}{2R}$$

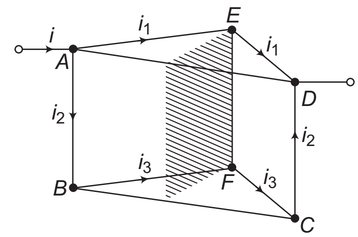
or $\quad R_{AC} = \dfrac{2R}{5}$ **Ans.**

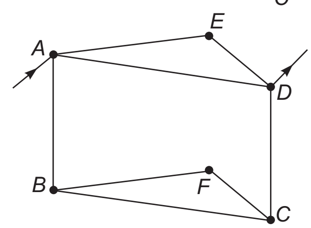
(b) With respect to D and E, points A, B and C all are symmetrically located. Hence, the simplified circuit can be drawn as shown in figure.



$\therefore \quad R_{DE} = \dfrac{R}{3} + \dfrac{R}{3} = \dfrac{2R}{3}$ **Ans.**

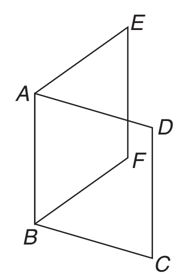
2. (a) Due to symmetry about the shaded plane, current distribution on either side of the plane will be identical and points E and F will be at same potential and no current will flow through it.





$$\therefore \quad R_{AD} = \frac{\dfrac{2}{3} r \times \dfrac{8}{3} r}{\dfrac{2}{3} r + \dfrac{8}{3} r} = \frac{8}{15} r$$ **Ans.**

(b) Redrawing the given arrangement for resistance across AB. Potentials $V_D = V_E$



$V_C = V_F$

$\therefore$ No current flows through DE and CF.

$$\therefore \quad R_{AB} = \frac{\dfrac{3}{2} r \times r}{\dfrac{3}{2} r + r} = \frac{3}{5} r$$ **Ans.**

3. (a) Current flowing through resistance 5Ω is 11 A

Power dissipated $= i^2 R$

$= (121)5 = 605$ W

(b) $V_B + 8V + 3V + 12V - 12V - 5V = V_C$

$V_B + 11V - 5V = V_C$

$6V = V_C - V_B$

(c) Both batteries are being charged. **Ans.**

4. (a) $V_A - V_B = 6\text{ V}$

Since, $V_B = 0$

$\therefore$ $V_A = 6\text{ V}$

$V_A - V_C = 4\text{ V} \Rightarrow V_C = V_A - 4 = 2\text{ V}$

(b) $V_A - V_D = V_A - V_C = 4\text{ V}$

From unitary method, we can find that,

$$AD = \left(\frac{100}{6}\right)(4) = 66.67\text{ cm}$$

(c) Since, they are at same potential, no current will flow through it.

(d) $V_A - V_B$ is still 6 V $\therefore$ $V_A = 6\text{ V}$

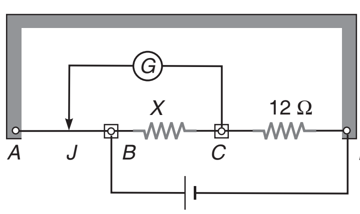
Further, $V_A - V_C = 7.5\text{ V}$

$\therefore$ $V_C = -1.5\text{ V}$

Since, EMF of the battery in lower circuit is more than the EMF of the battery in upper circuit. No such point will exist.

5. (a) There are no positive and negative terminals on the galvanometer because only zero deflection is needed.

(b)



(c) $AJ = 60\text{ cm} \Rightarrow BJ = 40\text{ cm}$

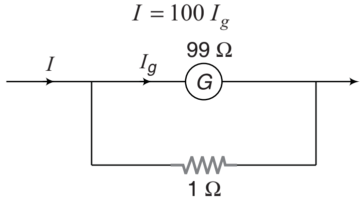
If no deflection is taking place. Then, the Wheatstone bridge is said to be balanced. Hence,

$$\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}} \quad \text{or} \quad \frac{X}{12} = \frac{40}{60} = \frac{2}{3}$$

or $X = 8\ \Omega$ **Ans.**

6. For ammeter $99I_g = (I - I_g)\,1$

or $I = 100\,I_g$ ...(i)



I_g is the full scale deflection current of the galvanometer and I the range of ammeter.

For the circuit in Fig.1, given in the question

$$\frac{12\text{V}}{2 + r + \dfrac{99 \times 1}{99 + 1}} = 3\text{ A} \Rightarrow r = 1.01\ \Omega \quad \textbf{Ans.}$$

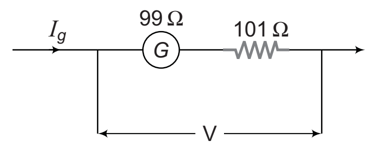
For voltmeter, range

$V = I_g\,(99 + 101)$

$V = 200\,I_g$ ...(ii)

Also resistance of the voltmeter

$= 99 + 101 = 200\ \Omega$



In Fig. 2, resistance across the terminals of the battery

$$R_1 = r + \frac{200 \times 2}{200 + 2} = 2.99\ \Omega$$

$\therefore$ Current drawn from the battery,

$$I_1 = \frac{12}{2.99} = 4.01\text{ A}$$

$\therefore$ Voltmeter reading

$$\frac{4}{5}V = 12 - I_1 r = 12 - 4.01 \times 1.01$$

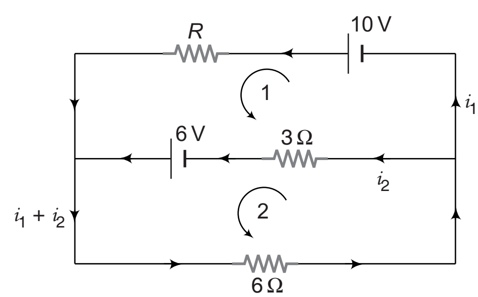
$$V = 7.96 \times \frac{5}{4} = 9.95\text{ V} \quad \textbf{Ans.}$$

Using Eq. (ii), $I_g = \dfrac{9.95}{200} = 0.05\text{ A}$

Using Eq.(i), range of the ammeter

$I = 100\,I_g = 5\text{ A}$ **Ans.**

7. Applying Kirchhoff's laws in two loops we have,



$10 - i_1R - 6 + 3i_2 = 0$ ...(i)

$6 - 6(i_1 + i_2) - 3i_2 = 0$ ...(ii)

Solving these two equations, we get

$$i_1 = \frac{6}{R + 2}$$

Power developed in R,

$$P = i_1^2 R = \frac{36}{(R + 2)^2}R \quad \text{...(iii)}$$

For power to be maximum,

$$\frac{dP}{dR} = 0$$

$\therefore$ $(R+2)^2(36) - (36R)(2)(R+2) = 0$

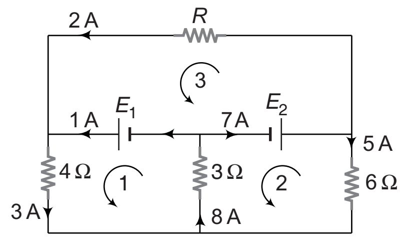
or $R + 2 - 2R = 0$

$\therefore$ $R = 2\,\Omega$ **Ans.**

For maximum power from Eq. (iii), we have

$P_{max} = 4.5$ W **Ans.**

8. Applying loop law in loops 1, 2 and 3, we have



$$E_1 - 12 - 24 = 0$$

$\therefore$ $E_1 = 36$ V

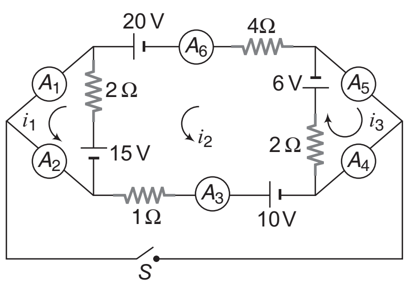
$-E_2 + 24 + 30 = 0$ or $E_2 = 54$ V

$-2R - E_1 + E_2 = 0$

or $R = \dfrac{E_2 - E_1}{2}$

$= 9\,\Omega$

9. Using the loop current method,



(a) $-2(i_1 - i_2) + 15 = 0$ ...(i)

$-4i_2 + 20 - 2(i_2 - i_1) - 15 - i_2$

$-10 - 2(i_2 + i_3) - 6 = 0$

or $2i_1 - 9i_2 - 2i_3 - 11 = 0$ ...(ii)

$-2(i_2 + i_3) - 6 = 0$

or $i_2 + i_3 + 3 = 0$ ...(iii)

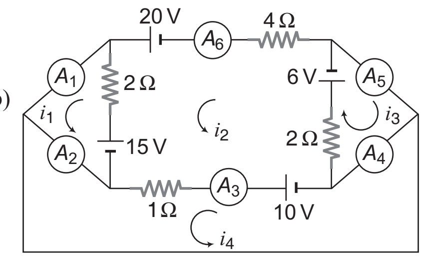
Solving these equations, we get

$i_1 = 9.5$ A, $i_2 = 2$ A and $i_3 = -5$ A

Ammeter	A_1	A_2	A_3	A_4
Reading (amp)	9.5	9.5	2	5

PD across switch $= 10 + (1)i_2 = 10 + 2 = 12$ V **Ans.**

(b)



When switch is closed

$-2(i_1 - i_2) + 15 = 0$ ...(i)

$2i_1 - 9i_2 - 2i_3 - 11 + i_4 = 0$ ...(ii)

$i_2 + i_3 + 3 = 0$ ...(iii)

$10 - (i_4 - i_2) = 0$ ...(iv)

Solving these four equation, we get

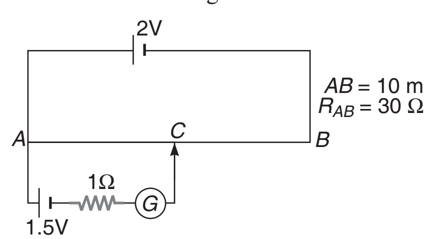
$i_1 = 12.5$ A, $i_2 = 5.0$ A , $i_3 = -8.0$ A

$i_4 = 15$ A

Ammeter	A_1	A_2	A_3	A_4
Reading (amp)	12.5	2.5	10	7

And the current through switch is 15 A. **Ans.**

10.



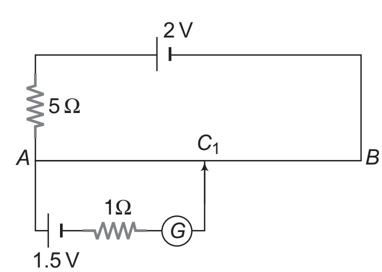
Potential gradient across wire,

$$AB = \frac{2}{10} = 0.2 \text{ V/m}$$

Now, $V_{AC} = 1.5$ V or $(0.2)(AC) = 1.5$

$\therefore$ $AC = 7.5$ m **Ans.**

(a) $V_{AB} = \left(\dfrac{R_{AB}}{R_{AB} + 5}\right) \times 2 = \left(\dfrac{30}{30+5}\right) \times 2 = \dfrac{12}{7}$ V



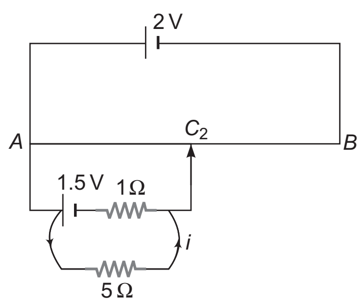
$\therefore$ Potential gradient across

$$AB = \frac{12}{70}\ \text{V/m}$$

Now, $\quad V_{AC} = 1.5\ \text{V}$

$$\therefore \quad \left(\frac{12}{70}\right)(AC_1) = 1.5$$

$$\therefore \quad AC_1 = 8.75\ \text{m} \qquad \textbf{Ans.}$$

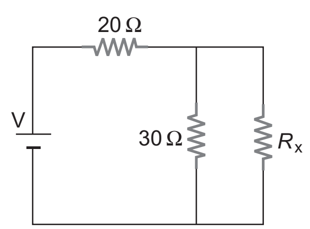


(b) $V_{AC_2} = V$

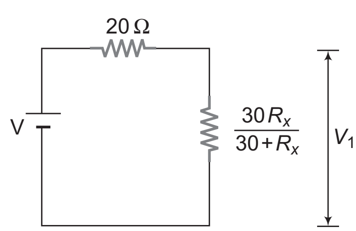
or $\quad (0.2)(AC_2) = \left(\dfrac{5}{5+1}\right)(1.5)$

or $\quad AC_2 = 6.25\ \text{m} \qquad$ **Ans.**

11. V = constant



$\Rightarrow$



$$V_1 = \left(\frac{\dfrac{30R_x}{30+R_x}}{\dfrac{30R_x}{30+R_x} + 20}\right)V$$

$$= \left(\frac{30R_x}{50R_x + 600}\right)V$$

$\therefore$ Power generated in R_x is

$$P = \frac{V_1^2}{R_x} = \frac{900 R_x V^2}{(50R_x + 600)^2}$$

For P to be constant,

$$\frac{dP}{dR_x} = 0$$

or $$\frac{(50R_x + 600)^2(900V^2) - 1800 \times 50 \times R_x V^2 (50R_x + 600)}{(50R_x + 600)^4} = 0$$

or $\quad 50R_x + 600 - 100R_x = 0$

$$\therefore \quad R_x = 12\ \Omega \qquad \textbf{Ans.}$$

12. The two batteries are in parallel. Thermal power generated in R will be maximum when,

total internal resistance = total external resistance

or $$R = \frac{R_1R_2}{R_1 + R_2}$$

$$E_{eq} = \frac{\left(\dfrac{E_1}{R_1} + \dfrac{E_2}{R_2}\right)}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)}$$

$$= \left(\frac{E_1R_2 + E_2R_1}{R_1 + R_2}\right)$$

$$R_{net} = \frac{2R_1R_2}{R_1 + R_2}$$

$$\therefore \quad i = \frac{E_{eq}}{R_{net}} = \frac{E_1R_2 + E_2R_1}{2R_1R_2}$$

Maximum power through R

$$P_{max} = i^2R = \frac{(E_1R_2 + E_2R_1)^2}{4R_1R_2(R_1 + R_2)} \qquad \textbf{Ans.}$$

13. $$\frac{V^2}{R} = k(T - T_0) + C\left(\frac{dT}{dt}\right)$$

or $$\frac{dT}{\dfrac{V^2}{R} - k(T - T_0)} = \frac{dt}{C}$$

or $$\int_{T_0}^{T} \frac{dT}{\dfrac{V^2}{R} - k(T - T_0)} = \int_0^t \frac{dt}{C}$$

(at $t = 0$, temperature of conductor $T = T_0$)

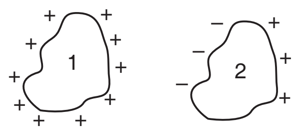
Solving this equation, we get

$$T = T_0 + \frac{V^2}{kR}(1 - e^{-kt/C}) \qquad \textbf{Ans.}$$

24. Electrostatics

INTRODUCTORY EXERCISE 24.1

1. Due to induction effect, a charged body can attract a neutral body as shown below.



Body-1 is positively charged and body-2 is neutral. But we can see that due to distance factor attraction is more than the repulsion.

4. Number of atoms in 3 gram-mole of hydrogen atom = number of electrons in it

$= 3\,N_0 = (3 \times 6.02 \times 10^{23})$

where, N_0 = Avogadro number

$\therefore$ Total charge

$$= -(1.6 \times 10^{-19})(3 \times 6.02 \times 10^{23})$$
$$= -2.89 \times 10^5 \text{C}$$

INTRODUCTORY EXERCISE 24.2

1. $F_e = \dfrac{1}{4\pi\,\varepsilon_0}\dfrac{q_1q_2}{r^2}$

and $\qquad F_g = G\,\dfrac{m_1\,m_2}{r^2}$

$\therefore \qquad \dfrac{F_e}{F_g} = \dfrac{(1/4\pi\,\varepsilon_0)\,q_1q_2}{G\,m_1m_2}$

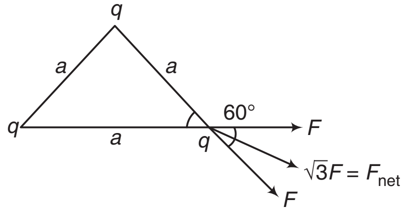
$$= \frac{(9 \times 10^{+9})(1.6 \times 10^{-19})^2}{(6.67 \times 10^{-11})(9.11 \times 10^{-31})(1.67 \times 10^{-27})}$$
$$= 2.27 \times 10^{39}$$

2. $$F = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2}$$

$\therefore \qquad \varepsilon_0 = \dfrac{q_1q_2}{4\pi F r^2}$

Units and dimensions can be found by above equation.

3.

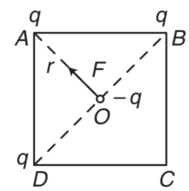


F = Force between two point charges

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q \times q}{a^2}\right)$$

$$F_{\text{net}} = \sqrt{F^2 + F^2 + 2FF\cos 60°}$$
$$= \sqrt{3}F$$
$$= \left(\frac{\sqrt{3}}{4\pi\varepsilon_0}\right)\left(\frac{q}{a}\right)^2$$

4.



Net force on $-q$ from the charges at B and D is zero.

So, net force on $-q$ is only due to the charge at A.

$$F = \left(\frac{1}{4\pi\,\varepsilon_0}\right)\frac{q \times q}{r^2}$$

where, $\qquad r = \dfrac{\sqrt{2}a}{2} = \dfrac{a}{\sqrt{2}}$

$\therefore \qquad F = \left(\dfrac{1}{4\pi\,\varepsilon_0}\right)\dfrac{q^2}{(a/\sqrt{2})^2}$

$$= \left(\frac{1}{2\pi\,\varepsilon_0}\right)\left(\frac{q}{a}\right)^2$$

5. The charged body attracts the natural body because attraction (due to the distance factor) is more than the repulsion.

7. $F = \dfrac{1}{4\pi\,\varepsilon_0}\cdot\dfrac{q_1q_2}{r^2}$

$$(q_1)_{\min} = (q_2)_{\min} = e_2$$

$\therefore \qquad F_{\min} = \dfrac{1}{4\pi\,\varepsilon_0}\dfrac{e^2}{r^2}$

9. Two forces are equal and opposite.

INTRODUCTORY EXERCISE 24.3

1. Electric field lines are not parallel and equidistant.

2. Electric lines flow higher potential to lower potential.

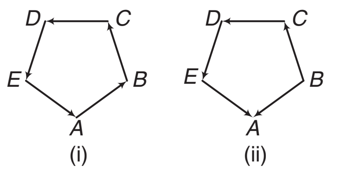
$\therefore \qquad V_A > V_B$

3. If charged particle is positive, and at rest. Electric field lines are straight then only it will move in the direction of electric field.

4. See the hint of above question.

5. Electric field lines start from positive charge and terminate on negative charge.

6. In case of five charges at five vertices of regular pentagon net electric field at centre is zero. Because five vectors of equal magnitudes from a closed regular pentagon as shown in Fig. (i).



Where one charge is removed. Then, one vector (let **AB**) is deceased hence the net resultant is equal to magnitude of one vector

$$= |\mathbf{BA}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$$

7. $\mathbf{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{r^3}\right)(\mathbf{r}_p - \mathbf{r}_q)$

Here, $r = \sqrt{(3)^2 + (4)^2} = 5\text{m}$

$$\therefore \quad \mathbf{E} = \frac{(9\times10^9)(-2\times10^{-6})}{(5)^3}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

$$= -(4.32\hat{\mathbf{i}} + 5.76\hat{\mathbf{j}})\times10^2 \text{ N/C}$$

INTRODUCTORY EXERCISE 24.4

1. $K_i + U_i = K_f + U_f$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_i} = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_f}$$

$$\therefore v = \sqrt{\frac{2}{m}\left(\frac{1}{4\pi\varepsilon_0}\right)(q_1q_2)\left(\frac{1}{r_i}-\frac{1}{r_f}\right)}$$

$$= \sqrt{\frac{2}{10^{-4}}(9\times10^9)(-2\times10^{-12})\left(\frac{1}{1.0}-\frac{1}{0.5}\right)}$$

$$= 18.97 \text{ m/s}$$

2. Work done by electrostatic forces

$$= -\Delta U$$
$$= U_i - U_f$$
$$= \left(\frac{1}{4\pi\varepsilon_0}\right)(q_1q_2)\left(\frac{1}{r_i}-\frac{1}{r_f}\right)$$
$$= (9\times10^9)(-2\times10^{-12})\left(\frac{1}{1.0}-\frac{1}{2.0}\right)$$
$$= -9\times10^{-3}\text{J}$$
$$= -9 \text{ mJ}$$

3. Work done by electrostatic forces

$$W = -\Delta U = U_i - U_f$$
$$\therefore \quad U_f = U_i - W$$
$$= (-6.4\times10^{-8}) - (4.2\times10^{-8})$$
$$= -10.6\times10^{-8}\text{ J}$$

4. $U_\infty = 0$

$$U_r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r} \qquad \text{(For two charges)}$$

$$U_r \neq U_\infty$$

For $\quad r \neq \infty$

$$U_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right]$$

Now, U_r can be equal to U_∞ for finite value of r.

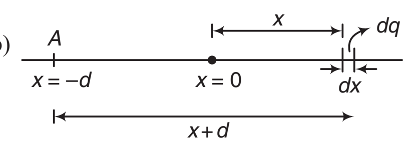
INTRODUCTORY EXERCISE 24.5

1. $W_{ab} = \Delta U_{a-b} = U_b - U_a = q(V_b - V_a) = qV_{ba}$

$$\therefore \; V_{ba} = \frac{W_{ab}}{q} = \frac{12}{10^{-2}} = 1200 \text{ V}$$

2. (a) $\alpha = \frac{\lambda}{x} = \frac{\text{C/m}}{\text{m}} = \frac{\text{C}}{\text{m}^2}$

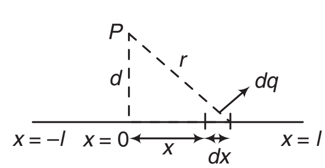
(b)



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{x+d}$$
$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\lambda dx}{x+d}\right)$$
$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\alpha x\,dx}{x+d}\right)$$

$$\therefore \quad V = \int_{x=0}^{x=L} dV = \frac{\alpha}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right]$$

3.



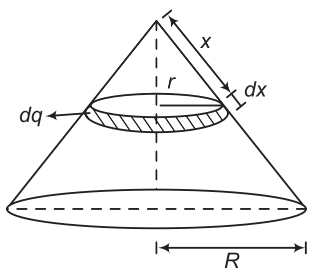
$$dq = \left(\frac{q}{2l}\right) dx$$

At point P,

$$dV = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{r}\right)$$

$$= \left(\frac{1}{4\pi\,\varepsilon_0}\right)\left(\frac{\frac{q}{2l}\cdot dx}{\sqrt{d^2 + x^2}}\right)$$

$\therefore$ $$V = 2\int_{x=0}^{x=l} dV$$

4. 

$$W = -\Delta U = U_{\text{apex}} - U_\infty$$

$\therefore$ $$W = qV_{\text{apex}} \qquad \ldots\text{(i)}$$

as $$U_\infty = 0$$

$\mathbf{V}_{apex}$ $$\frac{r}{x} = \frac{R}{L}$$

$\therefore$ $$r = \left(\frac{R}{L}\right) x$$

Surface charge density,

$$\sigma = \frac{Q}{\pi Rl}$$

$$dq = (\sigma)\,(dA)$$

$$= \left(\frac{Q}{\pi Rl}\right)(2\pi r)\,dx$$

$$= \left(\frac{\theta}{\pi RL}\right)(2\pi)\left(\frac{R}{L}x\right)dx$$

$$= \left(\frac{2Q}{L^2}\right) x\,dx$$

Now, $$dV = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{x}\right)$$

$$= \left(\frac{2Q}{4\pi\varepsilon_0 L^2}\right) dx$$

$\therefore$ $$V = \int_0^L dV = \frac{Q}{2\pi\varepsilon_0 L}$$

Substituting in Eq. (i), we have

$$W = \frac{Qq}{2\pi\varepsilon_0 L}$$

INTRODUCTORY EXERCISE 24.6

1. (a) $$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\,\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\,\hat{\mathbf{j}}\right]$$

$$= -a\,[(2x)\,\hat{\mathbf{i}} - (2y)\,\hat{\mathbf{j}}] = -2a\,[x\hat{\mathbf{i}} - y\hat{\mathbf{j}}]$$

(b) Again

$$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\,\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\,\hat{\mathbf{j}}\right]$$

$$= -a\,[\,y\hat{\mathbf{i}} + x\hat{\mathbf{j}}\,]$$

2. $E = -\dfrac{dV}{dx} = -$ Slope of V-x graph.

From $x = -2$m to $x = 0$, slope $= +5$ V/m

$\therefore$ $E = -5$ V/m

From $x = 0$ to $x = 2$m, slope $= 0$,

$\therefore$ $E = 0$

From $x = 2$m to $x = 4$m, slope $= +5$V/m,

$\therefore$ $E = -5$ V/m

From $x = 4$ m to $x = 8$m, slope $\equiv -5$ V/m

$\therefore$ $E = +5$ V/m

Corresponding E-x graph is as shown in answer.

3. $$\frac{\partial V}{\partial x} = \frac{-50}{5} = -10 \text{ V/m}$$

Now, $$|\mathbf{E}| = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}$$

No information is given about $\dfrac{\partial V}{\partial y}$ and $\dfrac{\partial V}{\partial z}$. Hence,

$$|\mathbf{E}| \ge \left|\frac{\partial V}{\partial x}\right| \quad \text{or} \quad |\mathbf{E}| \ge 10 \text{ V/m}$$

4. $V_A = V_D$ and $V_B = V_C$ as the points A and D or B and C are lying on same equipotential surface ($\perp$ to electric field lines). Further, V_A or $V_D > V_B$ or V_C as electric lines always flow from higher potential to lower potential

$$V_A - V_B = V_D - V_C = Ed$$

$$= (20)\,(1) = 20 \text{ V}$$

INTRODUCTORY EXERCISE 24.7

1. (a) Given surface is a closed surface. Therefore, we can directly apply the result.

$$\phi = \frac{q_{in}}{\varepsilon_0} = 0 \quad \text{as} \quad q_{in} = 0$$

(b) Again given surface is a closed surface. Hence, we can directly apply the result.

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0} \quad \text{as} \quad q_{in} = q$$

(c) Given surface is not closed surface. Hence, we cannot apply the direct result of Gauss's theorem. If we draw a complete sphere, then

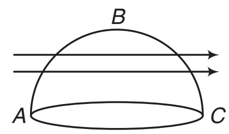
$$\phi \text{ through complete sphere} = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \phi \text{ through hemisphere} = \frac{1}{2}\left(\frac{q}{\varepsilon_0}\right)$$

2. Net charge from any closed surface in uniform electric field = 0

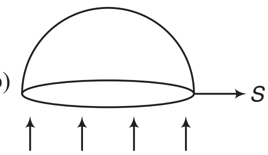
$\therefore$ Net charge inside any closed surface in uniform electric field = 0

3. (a)



Net flux entering from AB = net flux entering from BC.

(b)



$$\phi = ES = E\,(\pi R^2)$$

4. Given electric field is uniform electric field. Net flux from any closed surface in uniform electric field = 0.

Exercises

LEVEL 1

Assertion and Reason

1. An independent negative charge moves from lower potential to higher potential. In this process, electrostatic potential energy decreases and kinetic energy increases.

2. Two unlike charges come together when left freely.

3.
$$\mathbf{E} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$

$$\therefore \quad |\mathbf{E}| = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}$$

$$\therefore \quad |\mathbf{E}| \geq \frac{\partial V}{\partial x}$$

or $\quad = 10$ V/m

4. $V = \dfrac{kq}{R}$ or $kq = VR$

For inside points ($r \leq R$),

$$E = \frac{kq}{R^3} r$$

or $\quad E \propto r$

At distance $r = \dfrac{R}{2}$,

$$E = \frac{(VR)}{R^3}\left(\frac{R}{2}\right) = \frac{V}{2R}$$

6. $V_A - V_B = -\int_B^A \mathbf{E} \cdot d\mathbf{r}$

$$= -\int_{(0,4)}^{(4,0)} (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$$= -\int_{(0,\,4)}^{(4,\,0)} (4dx + 4dy) = 0$$

$\therefore \quad V_A = V_B$

7. At stable equilibrium position, potential energy is minimum.

8. In uniform electric field, net force on an electric dipole = 0

Therefore, no work is done in translational motion of the dipole.

Electric lines also flow from higher potential to lower potential. Electrostatical force on positive charge acts in the direction of electric field. Therefore, work done is positive.

9. Charge on shell does not contribute in electric field just inside the shell. But it contributes in the electric field just outside it. So, there is sudden change in electric field just inside and just outside it. Hence, it is discontinuous.

10. $|E| = 0$ minimum at centre and $|V| = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$ is maximum at centre.

Objective Questions

1. $\phi = ES \rightarrow \left(\frac{\text{V}}{\text{m}}\right)(\text{m}^2) \rightarrow$ volt-m

2. $g_e = g - \left(\frac{qE}{m}\right)$

or g_e will decrease. Hence, $T = 2\pi\sqrt{\frac{l}{g_e}}$ will increase.

3. Electric lines terminate on negative charge.

4.
$$W = \Delta U_i$$
$$= U_f - U_i$$
$$= 3\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{qq}{2l}\right] - 3\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{qq}{l}\right]$$
$$= \left(-\frac{3}{2}\right)\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}\right) l = -\frac{3}{2} Fl$$

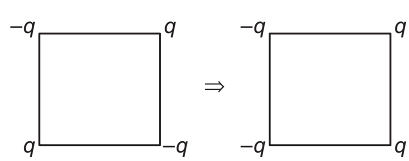
5.
$$v = \sqrt{\frac{2qV}{m}}$$
$$v_p = \sqrt{\frac{2eV}{m}}$$
$$v_d = \sqrt{\frac{2e\,(2V)}{2m}}$$
$$v_\alpha = \sqrt{\frac{2\,(2e)\,(4V)}{4m}}$$

The ratio is $1:1:\sqrt{2}$.

6. $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$

$$V' = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{2q}{4r}\right) = \frac{V}{2}$$

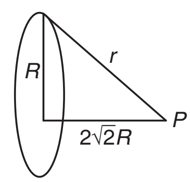
7.



$$W = \Delta U = U_f - U_i$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{-q^2}{a} - \frac{q^2}{\sqrt{2}\,a} + \frac{q^2}{a} + \frac{q^2}{a} - \frac{q^2}{\sqrt{2}\,a} - \frac{q^2}{a}\right) - \left(\frac{-q^2}{a} + \frac{q^2}{\sqrt{2}\,a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}\,a} - \frac{q^2}{a}\right)\right]$$
$$= \frac{q^2}{4\pi\varepsilon_0 a}[4 - 2\sqrt{2}]$$

8.
$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{3R} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{3R}$$
$$= \frac{q_{\text{net}}}{(4\pi\varepsilon_0)\,(3R)}$$
$$\therefore \quad R = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_{\text{net}}}{3V}\right)$$
$$= \frac{(9\times10^9)\,(3\times10^{-6})}{(3)\,(9000)}$$
$$= 1\text{ m}$$

9.

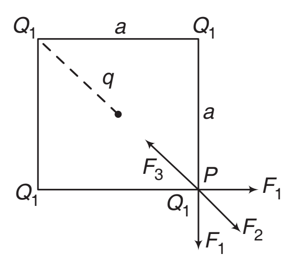


$$r = 3R$$
$$V_p = \frac{k\,q_{\text{net}}}{r} = \frac{k\,(3q)}{3R} = \frac{k\,q}{R}$$

10. $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = mg\sin\theta$

$$\therefore \quad r = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q^2}{mg\sin\theta}\right)}$$
$$= \sqrt{\frac{(9\times10^9)\,(2.0\times10^{-6})^2}{(0.1)\,(9.8)\sin 30°}}$$
$$= 27\times10^{-2}\text{m}$$
$$= 27\text{ cm}$$

11.



$Q_1 = (2\sqrt{2} - 1)\,Q$

F_1 = Force between Q_1 and Q_1 at distance a

F_2 = Force between Q_1 and Q_1 at distance $\sqrt{2}a$

F_3 = Force between Q_1 and q at distance $\frac{a}{\sqrt{2}}$

For F_3 to be in the shown direction, q and Q_1 should have opposite signs. For net charge to be zero on Q_1 placed at P.

$|F_3|$ = Resultant of F_1, F_2 and F_1

$$\therefore \quad \frac{k\,Q_1 q}{(a/\sqrt{2})^2} = \sqrt{2}F_1 + F_2$$

$$= \sqrt{2}\left[\frac{k\,Q_1 Q_1}{a^2}\right] + \frac{k\,Q_1 Q_1}{(\sqrt{2}a)^2}$$

$$\therefore \quad |q| = \frac{Q_1}{2}\left[\sqrt{2} + \frac{1}{2}\right] = \left(\frac{2\sqrt{2}+1}{4}\right) Q_1$$

$$= \left(\frac{2\sqrt{2}+1}{4}\right)(2\sqrt{2}-1)Q$$

$$= \frac{7}{4}Q \quad \text{or} \quad q = -\frac{7}{4}Q$$

12. $|a| = \left|\frac{qE}{m}\right| = \left|\frac{q\sigma}{2\varepsilon_0\, m}\right|$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{4s\,\varepsilon_0\, m}{|q|\,|\sigma|}}$$

$$= \sqrt{\frac{4\times 0.1\times 8.86\times 10^{-12}\times 1.67\times 10^{-27}}{1.6\times 10^{-19}\times 2.21\times 10^{-9}}}$$

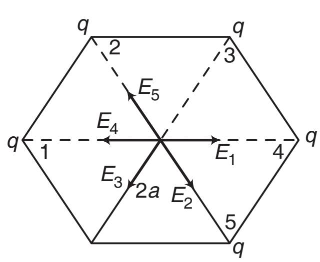
$$= 4\times 10^{-6}\ \text{s}$$

13. 

Between A and B two forces on third charge will act in same direction. So, this charge cannot remain in equilibrium.

To the right of B or left of A forces are in opposite directions but their magnitudes are different. Because charges have equals magnitudes but distances are different.

14. Between $2q$ and $-q$, two electric fields are in same direction. So their resultant can't be zero. To the right of $2q$ left of $-q$ they are in opposite directions. So, net field will be zero nearer to charge having small magnitude.

15. 

E_1 and E_4 are cancelled.

E_2 and E_5 are cancelled.

$$\therefore \quad E_{\text{net}} = E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(2a)^2} = \frac{q}{16\pi\varepsilon_0 a^2}$$

16. $F_1 = \dfrac{k\,q_1 q_2}{r^2}$ $\qquad \left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

$$F_2 = \frac{k\,q_1' q_2'}{r^2}$$

where, $\quad q_1' = q_2' = \dfrac{q_1+q_2}{2}$

$$\therefore \quad F_2 = \frac{k\,(q_1+q_2/2)^2}{r^2}$$

$$\left(\frac{q_1+q_2}{2}\right)^2 > q_1 q_2$$

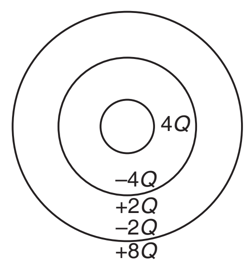
This is because,

$$(q_1+q_2)^2 = (q_1-q_2)^2 + 4q_1q_2$$

or $\quad (q_1+q_2)^2 > 4q_1q_2$

or $\quad \left(\dfrac{q_1+q_2}{2}\right)^2 > q_1q_2$

$\therefore \quad F_2 > F_1$

17. 

18. $1000\left(\dfrac{4}{3}\pi r^3\right) = \dfrac{4}{3}\pi R^3 \Rightarrow R = 10r$

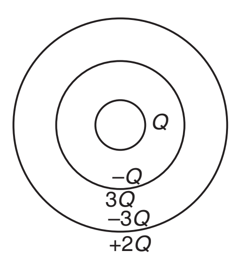
i.e. radius has become 10 times.

Charge will become 1000 times.

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(\text{Charge})}{(\text{Radius})} \quad \text{or} \quad V \propto \frac{\text{Charge}}{\text{Radius}}$$

Hence, potential will become 100 times.

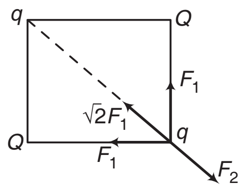
19. $\text{PD} = \dfrac{q_{\text{in}}}{4\pi\varepsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{2R}\right)$ or $\text{PD} \propto q_{\text{in}}$

20. 

The desired ratio is $-\dfrac{3Q}{2Q} = -\dfrac{3}{2}$

21. $\mathbf{S} = (1)\,\hat{\mathbf{i}} \;\Rightarrow\; \phi = \mathbf{B}\cdot\mathbf{S} = 5$ V-m

22.



$F_2 =$ Force between q and q

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{q\times q}{(\sqrt{2}\,a)^2}$$

$F_1 =$ Force between Q and q. For net force on q to be zero .

$$F_2 = \sqrt{2}\,F_1$$

$$\therefore \qquad \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{2a^2} = \sqrt{2}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2}\right]$$

$$\therefore \qquad |q| = 2\sqrt{2}Q$$

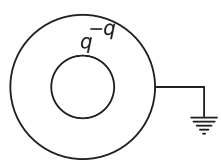
with sign, $\qquad q = -\,2\sqrt{2}\,Q$

23. $V_B = 0$

$$\therefore \qquad \frac{kq_A}{r_B} + \frac{kq_B}{r_B} = 0$$

$$\therefore \qquad q_B = -\,q_A = -\,q$$

Charge distribution is as shown below.



From Gauss's theorem, electric field at any point is given by

$$E = \frac{kq_{\text{in}}}{r^2}$$

q_{in} inside A and outside B is zero. Therefore, $E = 0$

24. $\dfrac{kq}{r} = \dfrac{3}{2}\left(\dfrac{kq}{R}\right)\Big/2 \quad$ or $\quad r = \dfrac{4}{3}R$

$\therefore$ Distance from surface $= r = R$

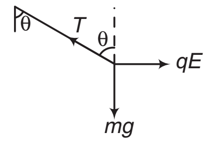
$$= \frac{R}{3}$$

25. $\phi = \dfrac{q_{\text{in}}}{\varepsilon_0}$

$$q_{\text{in}} = 0$$

$$\therefore \qquad \phi = 0$$

26.



$$T\,\sin\theta = qE$$

or $\qquad T = \dfrac{qE}{\sin\theta}$

Similarly, $\quad T\,\cos\theta = mg$

27. $E = \dfrac{\sigma}{\varepsilon_0}$

$$E_1 = E_2$$

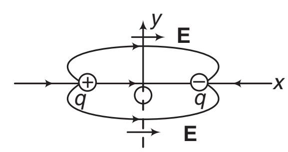
$$\therefore \qquad \sigma_1 = \sigma_2$$

$$V = \frac{\sigma R}{\varepsilon_0}$$

or $\qquad V \propto R \qquad$ (as $\sigma \rightarrow$ same)

$$\therefore \qquad \frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{a}{b}$$

28.



Potential is zero at infinite and at origin. Therefore, PD = 0. Hence, the work done asked in part (c) is also zero.

29. According to principle of generator PD in this case only depends on the charge on inner shell.

30.
$$500 = \frac{k\,|q|}{r^2} \qquad \text{...(i)}$$

$$-\,3000 = \frac{k\,(-\,q)}{r} \qquad \text{...(ii)}$$

Solving these two equations, we get

$$r = 6 \text{ m}$$

$$\therefore \qquad |q| = \frac{500\,r^2}{k}$$

$$= \frac{(500)\,(6)^2}{9\times 10^9}$$

$$= 2\times 10^{-6}\text{C}$$

$$= 2\,\mu\text{C}$$

31. $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1q_2}{r_1^2} = \dfrac{1}{4\pi\varepsilon_0 k}\cdot\dfrac{q_1q_2}{r_2^2}$

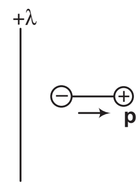
$$\therefore \qquad r_2 = \frac{r_1}{\sqrt{k}} = \frac{50}{\sqrt{5}}$$

$$= 22.36 \text{ m}$$

32. $W_{A \to B} = q\,(V_B - V_A)$

$$= q\left[-\int_A^B E dr\right] = q\int_B^A E\,dr$$

$$= q\int_{2a}^{a} \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{1}{2}\right)$$

33. 

Negative charge of dipole is near to positively charge line charge. Hence, attraction is more.

34. $r = \sqrt{(4-1)^2 + (2-2)^2 + (0-4)^2}$

$= 5$ m

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

$$= \frac{(9\times10^9)\,(2\times10^{-8})}{5} = 36\text{ V}$$

Field is in the direction of $\mathbf{r} = \mathbf{r}_p - \mathbf{r}_q$

35. $V = \dfrac{kq}{R}$

$\therefore \qquad kq = VR$

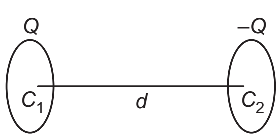
$$E = \frac{Kq}{r^2} = \frac{VR}{r^2}$$

36. $W = Fs\cos\theta = qEs\cos\theta$

$$\therefore \qquad E = \frac{W}{qs\cos\theta}$$

$$= \frac{4}{0.2\times2\times\cos 60°}$$

$= 20$ N/C

37. 

$k = \dfrac{1}{4\pi\varepsilon_0}$

$$V_{C_1} = \frac{kQ}{R} - \frac{kQ}{\sqrt{R^2+d^2}}$$

$$V_{C_2} = \frac{kQ}{\sqrt{R^2+d^2}} - \frac{kQ}{R}$$

$$\therefore \quad V_{C_1} - V_{C_2} = \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

38. $E = 0$, inside a hallow charged spherical conducting shell.

39. $W = \mathbf{F}\cdot\mathbf{r}$

$= (Q\mathbf{E})\cdot\mathbf{r} = Q\,(\mathbf{E}\cdot\mathbf{r})$

$= Q\,(E_1 a + E_2 b)$

Subjective Questions

1. $F = \dfrac{kq\,(Q-q)}{r^2}$ $\qquad\left(\text{where, } k = \dfrac{1}{4\pi\varepsilon_0}\right)$

For F to be maximum, $\dfrac{dF}{dq} = 0$

By putting $\dfrac{dF}{dq} = 0$, we get

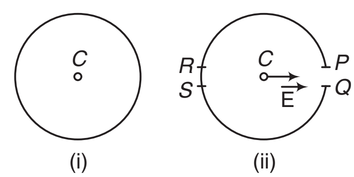
$$q = \frac{Q}{2}$$

3. $E = \dfrac{\sigma}{2\varepsilon_0}$

$\therefore \qquad \sigma = (2E\,\varepsilon_0)$

$= 2\times3.0\times8.86\times10^{-12}$

$= 5.31\times10^{-11}$ C/m^2

4. 

If loop is complete, then net electric field at centre C is zero. Because equal and opposite pair of electric field vectors are cancelled.

If PQ portion is removed as shown in figure, then electric field due to portion RS is not cancelled. Hence, electric field is only due to the option RS.

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{q_{RS}}{R^2} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(q/2\pi R)\,x}{R^2}$$

$$= \frac{qx}{8\pi^2\varepsilon_0 R^3}$$

5. Let q_1 and q_2 are the initial charges. After they are connected by a conducting wire, final charge on them become.

$$q'_1 = q'_2 = \left(\frac{q_1+q_2}{2}\right)$$

Now, given that

$$0.108 = \frac{(9\times10^9)\,(q_1)\,(q_2)}{(0.5)^2} \qquad \ldots\text{(i)}$$

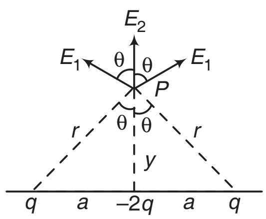
$$0.036 = \frac{(9\times 10^9)\left(\frac{q_1+q_2}{2}\right)^2}{(0.5)^2} \qquad \ldots\text{(ii)}$$

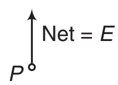
Solving Eqs. (i) and (ii), we can find q_1 and q_2.

6. Since, net force on electric dipole in uniform electric field is zero. Hence, torque can be calculated about point. This comes out to be a constant quantity given by

$$\tau = \mathbf{p}\times\mathbf{E}$$

7.





$E_1 =$ Electric field at P due to q

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}$$

$E_2 =$ Electric field at P due to $-2q$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2q}{y^2}$$

$\therefore$ Net electric field at P,

$$E = 2E_1\cos\theta + E_2$$

$$= 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{r^2}\right)\left(\frac{y}{r}\right)+\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{2q}{y^2}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{y}{r^3}\right]+\left[\frac{1}{y^2}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{y}{(a^2+y^2)^{3/2}}+\frac{1}{y^2}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{y}{y^3}+\left(1+\frac{a^2}{y^2}\right)^{-3/2}+\frac{1}{y^2}\right]$$

Applying binomial expression for $\frac{a^2}{y^2} << 1$ we get,

$$E = \frac{2q}{4\pi\varepsilon_0}\left(-\frac{3}{2}\frac{a^3}{y^4}\right)$$

$$= -\frac{3qa^2}{4\pi\varepsilon_0 y^4}$$

or $$\mathbf{E} = -\frac{3qa^2}{4\pi\varepsilon_0 y^4}\,\hat{\mathbf{j}}$$

8. If we make a bigger cube comprising of eight small cubes of size given in the question with charge at centre (or at D).

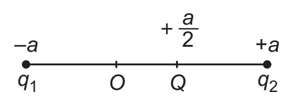
Then, total flux through large closed cube $= \frac{q}{\varepsilon_0}$.

There are 24 symmetrical faces like $EFGH$ on outermost surface of this bigger cube.

Total flux from these 24 faces is $\frac{q}{\varepsilon_0}$. Hence, flux from anyone force $= \frac{1}{24}\left(\frac{q}{\varepsilon_0}\right)$.

Electric lines are tangential to face $AEHD$. Hence, flux is zero.

9. (a)

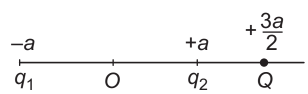


q_1 and q_2 should be of same sign.

Further, $$\frac{Kq_1Q}{(3a/2)^2} = \frac{Kq_2Q}{(a/2)^2}$$

$\therefore$ $$q_1 = 9q_2$$

(b)



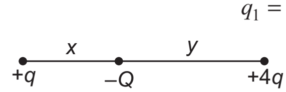
Therefore, q_1 and q_2 should be of opposite signs. Further,

$$\frac{kq_1Q}{(5a/2)^2} = \frac{kq_2Q}{(a/2)^2}$$

Or magnitude wise $q_1 = 25\,q_2$ with sign

$$q_1 = -25q_2$$

10.



$$x + y = L \qquad \ldots\text{(i)}$$

Let force on $-Q$ charge should be zero.

$\therefore$ $$\frac{kQ.\,q}{x^2} = \frac{kQ\,(4q)}{y^2}$$

$\therefore$ $$\frac{x}{y} = \frac{1}{2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$x = \frac{L}{3} \text{ and } y = \frac{2L}{3}$$

Net force on $4q$ should be zero.

$\therefore$ $$\frac{k\,|Q|\,(4q)}{(2L/3)^2} = \frac{kq\,(4q)}{L^2}$$

$\therefore \quad |Q| = \frac{4}{9}q$

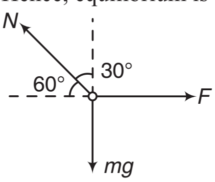
With sign,

$$Q = -\frac{4}{9}q$$

Similarly net force on q should be zero.

$\therefore \quad \frac{k|Q|q}{(L/3)^2} = \frac{kq(4q)}{L^2} \quad \text{or} \quad |Q| = \frac{4}{9}q$

If we, sightly displace $-Q$ towards $4q$, attraction between these charges will increase, hence $-Q$ will move towards $+4q$ and it will not return back. Hence, equilibrium is unstable.

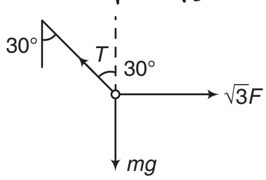
11. 

Using Lami's theorem, we have

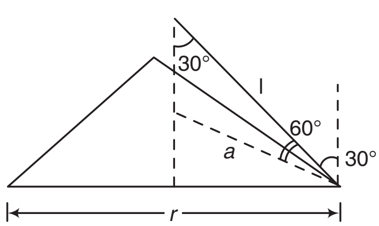
$$\frac{mg}{\sin(90° + 30°)} = \frac{F}{\sin(90° + 60°)}$$

$\therefore \quad F = \frac{mg}{\sqrt{3}} \quad \text{or} \quad \frac{1}{4\pi\varepsilon_0}\frac{(q)(q)}{R^2} = \frac{mg}{\sqrt{3}}$

$\therefore \quad q = \sqrt{\frac{4\pi\,\varepsilon_0\, mg\, R^2}{\sqrt{3}}}$

12. 

F = Electrostatic force between two charged balls.
$\sqrt{3}F$ = Resultant of electrostatic force on any one ball from rest two balls.



$$a = l\cos 60° = 10 \text{ cm}$$
$$r = 2a\cos 30°$$
$$= (2)(10)\left(\frac{\sqrt{3}}{2}\right) = 10\sqrt{3}\text{ cm}$$

Now, applying Lami's theorem for the equilibrium of ball we have

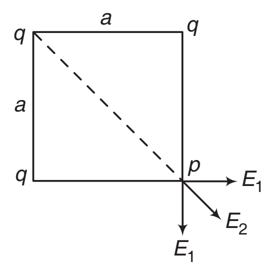
$$\frac{mg}{\sin(90° + 30°)} = \frac{\sqrt{3}F}{\sin(90° + 60°)}$$

$$F = \frac{mg}{3} = \frac{1}{4\pi\varepsilon_0}\frac{(q)(q)}{r^2} = \frac{mg}{3}$$

$\therefore \quad q = \sqrt{\frac{mgr^2}{3(1/4\,\pi\varepsilon_0)}}$

$$= \sqrt{\frac{(0.1\times10^{-3})(9.8)(10\sqrt{3}\times10^{-2})^2}{3\times9\times10^9}}$$

$$= 3.3\times10^{-8}\text{ C}$$

13. 

E_1 = Electric field due to charge q at distance a

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{a^2}\right)$$

E_2 = Electric field due to charge at distance $\sqrt{2}a$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{(\sqrt{2}\,a)^2} = \frac{E_1}{2}$$

Net electric field at P,

$$E_{net} = \sqrt{2}\,E_1 + \frac{E_2}{2} \quad \text{(In the direction of } E_2\text{)}$$

$$= \sqrt{2}E_1 + \frac{E_1}{2}$$

$$= \left(\sqrt{2} + \frac{1}{2}\right)E_1$$

$$= \frac{(2\sqrt{2}+1)q}{8\pi\varepsilon_0 a^2}$$

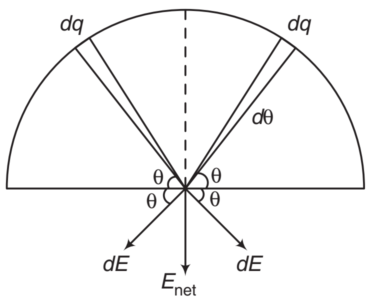
14. $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}(\mathbf{r}_p - \mathbf{r}_q)$

$$r = \sqrt{(1.2)^2 + (1.6)^2} = 2\text{m}$$

$\therefore \quad \mathbf{E} = \frac{(9\times10^9)(-8\times10^{-9})}{(2)^3}(1.2\hat{\mathbf{i}} - 10.8\hat{\mathbf{i}})$

$$= (14.4\hat{\mathbf{j}} - 10.8\hat{\mathbf{j}})\text{ N/C}$$

15.

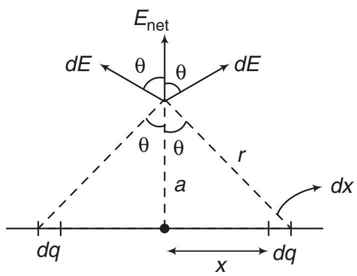


$$E_{\text{net}} = 2\int_0^{90^\circ} dE \sin\theta$$

$$= 2\int_0^{90^\circ}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{R^2}\right)\sin\theta$$

$$= 2\int_0^{90^\circ}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\lambda R d\theta}{R^2}\right)\sin\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

16.



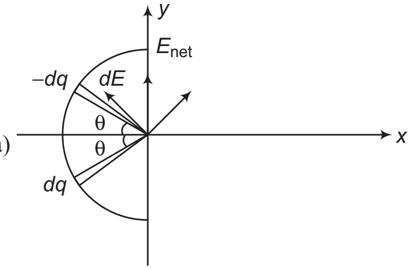
$$E_{\text{net}} = 2\int_{x=0}^{x=L/2} dE \cos\theta s$$

$$= 2\int_0^{L/2}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{r^2}\right)\cdot\left(\frac{a}{r}\right)$$

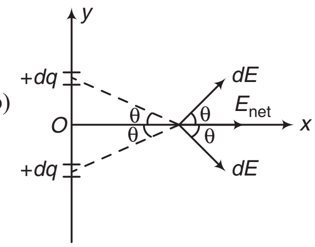
$$= 2\int_0^{L/2}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\frac{Q}{L}\cdot dx}{a^2+x^2}\right)\frac{a}{\sqrt{a^2+x^2}}$$

Solving this integration, we find the result.

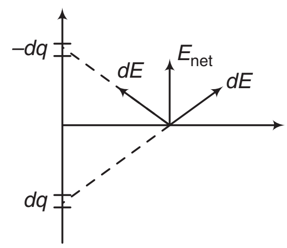
17. (a)



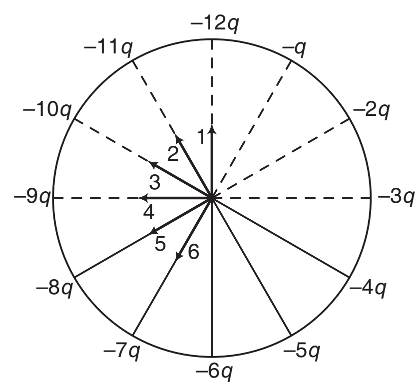
(b)



(c)



18. Six vectors of equal magnitudes are as shown in figure.



Now, resultant of two vectors of equal magnitudes (= E say) at 120° is also E and passing through their bisector line.

So, resultant of 1 and 5 is also E in the direction of 3. Similarly, resultant of 2 and 6 is also E in the direction of 4.

Finally, resultant of $2E$ in the direction of 3 and $2E$ in the direction of 4 passes through the bisector line of 3 and 4 (or 9.30)

19. $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 25\times\sin 45^\circ}{10} = \dfrac{5}{\sqrt{2}}\,\text{s}$

$$R = S_x = u_x T + \frac{1}{2}a_x T^2$$

$$= (u\cos 45^\circ)(T) + \frac{1}{2}\left(\frac{qE}{m}\right)T^2$$

$$= (25)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}}\right) + \left(\frac{2\times10^{-6}\times2\times10^{7}}{1}\right)\left(\frac{5}{\sqrt{2}}\right)^2$$

$$= 62.5 + 250 = 312.5 \text{ m}$$

20. (a) $$g_e = g + \frac{qE}{m}$$

$$= 10 + \frac{(1.6\times10^{-19})(720)}{1.67\times10^{-27}}$$

$$\approx 6.9\times10^{10} \text{ m/s}^2$$

Now, $$R = \frac{u^2 \sin 2\theta}{g_e}$$

$$\therefore \quad (1.27\times10^{-3}) = \frac{(9.55\times10^{3})^2 \sin 2\theta}{6.9\times10^{10}}$$

Solving this equation, we get

$$\theta = 37° \text{ and } 53°$$

(b) Apply $T = \dfrac{2u\sin\theta}{g_e}$

21. (a) $$\mathbf{a} = \frac{q\mathbf{E}}{m} = -\frac{(1.6\times10^{-19})(120\hat{\mathbf{j}})}{(9.1\times10^{-31})}$$

$$= -(2.1\times10^{13}\hat{\mathbf{j}}) \text{ m/s}^2$$

(b) $$t = \frac{X}{v_x} = \frac{2\times10^{-2}}{1.5\times10^{5}} = \frac{4}{3}\times10^{-7}\text{ s}$$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$= (1.5\times10^5\,\hat{\mathbf{i}} + 3.0\times10^6\,\hat{\mathbf{j}}) - \left(2.1\times10^{13}\times\frac{4}{3}\times10^{-7}\right)\hat{\mathbf{j}}$$

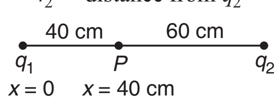
$$= (1.5\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\times10^5 \text{ m/s}$$

22. $$\frac{kq_1}{r_1} = \frac{k\,|q_2|}{r_2}$$

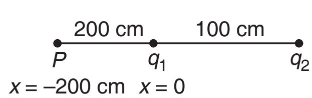
$$\therefore \quad \frac{r_1}{r_2} = \frac{q_1}{|q_2|} = \frac{2}{3}$$

where, r_1 = distance from q_1 and

r_2 = distance from q_2

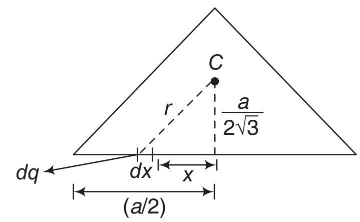


First point is at $x = 40$ cm where, $\dfrac{r_1}{r_2} = \dfrac{2}{3}$



Second point is at $x = -200$ cm, where $\dfrac{r_1}{r_2} = \dfrac{2}{3}$.

23.



$$dq = \lambda dx$$

Potential at C, due to charge dq is

$$dV = \frac{k(dq)}{r} \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

$\therefore$ Total potential $= 6\displaystyle\int_{x=0}^{x=a/2} dV$

24. (a) $$\Delta U = -q(\Delta V)$$

$$= -(12\times10^{-6})(50)$$

$$= -6.0\times10^{-4}\text{J}$$

(b) $$\Delta V = Ed = E(\Delta x)$$

$$= (250)(0.2)$$

$$= 50 \text{ V}$$

25. $k_A + U_A = k_B + U_B$

$$\therefore \quad \frac{1}{2}mv_A^2 + qV_A = \frac{1}{2}mv_B^2 + qV_B$$

$$\therefore \quad v_B = \sqrt{v_A^2 + \frac{2}{m}q(V_A - V_B)}$$

$$= \sqrt{(5)^2 + \frac{2\times(-5\times10^{-6})}{2\times10^{-4}}(200-800)}$$

$$= 7.42 \text{ m/s}$$

$v_B - v_A$ as the negative charge is moving (freely) from lower potential at A to higher potential at B. So, its electrostatic PE will decrease and kinetic energy will increase.

26. (a) $$V_C = \frac{k.\,q_{\text{net}}}{R} = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{-5Q}{R}\right)$$

(b) $$V_p = \frac{k\,q_{\text{net}}}{r}$$

where, r = distance of P from any point on circumference

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{-5Q}{\sqrt{R^2+z^2}}\right)$$

27. $$W = -\Delta U = U_i - U_f = q_2V_i - q_2V_f$$

$$= q_2\left(\frac{kq_1}{r_i}\right) - q_2\left(\frac{kq_1}{r_f}\right)$$

$$= kq_1 q_2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= (9 \times 10^9)(2.4 \times 10^{-6})(-4.3 \times 10^6) \left(\frac{1}{0.15} - \frac{1}{0.25\sqrt{2}}\right)$$

$$= -0.356 \text{ J}$$

28. (a) $U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$ ...(i)

where, $k = \frac{1}{4\pi\varepsilon_0}$

(b) Suppose q_3 is placed at coordinate x (> 0.2 m or 20 cm), then in Eq. (i) of part (a), put

$U = 0, r_{12} = 0.2$ m, $r_{13} = x$

and $r_{23} = (x - 0.2)$

Now, solving Eq. (i) we get the desired value of x.

29. $U = 0$

$\therefore \quad k\left(\frac{q \times q}{a} + \frac{q \times Q}{a} + \frac{q \times Q}{a}\right) = 0$

or $\quad Q = -\frac{q}{2}$

30. Apply $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r}$ ...(i)

$\mathbf{E}$ is given in the question.

and $\quad d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$

$\therefore \quad \mathbf{E} \cdot d\mathbf{r} = (5dx - 3dy)$

$\therefore \quad -\int \mathbf{E} \cdot d\mathbf{r} = (3y - 5x)$

With limits answer comes out to be

$V_B - V_A = 3(y_f - y_i) - 5(x_f - x_i)$

31. Procedure is same as work done in the above question. The only difference is, electric field is

$\mathbf{E} = (400\hat{\mathbf{j}})$ V/m

$\therefore \quad V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r} = -400(y_f - y_i)$

32. Similar to above two problems. But electric field here is

$\mathbf{E} = (20\hat{\mathbf{j}})$ N/C

$\therefore \quad V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r} = -20(x_f - x_i)$

33. (a) $[A] = \frac{[V]}{[xy]} = \frac{[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]}{[\text{L}][\text{L}]}$

$= [\text{MT}^{-3}\text{ A}^{-1}]$

(b) $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$

(c) Substituting, $A = 10, x = 1, y = 1$ and $z = 1$ in the expression of part (b) we have

$\mathbf{E} = (-20\hat{\mathbf{i}} - 20\hat{\mathbf{j}} - 20\hat{\mathbf{k}})$ N/C

$\therefore \quad |\mathbf{E}| = \sqrt{(-20)^2 + (-20)^2 + (-20)^2}$

$= 20\sqrt{3}$ N/C

34. See the hints of Q No. 31 to Q No. 33.

$V_f - V_i = V_B - V_A = -20(x_f - x_i) - 30(y_f - y_i)$

or $\quad V_f = -20(2 - 0) - 30(2 - 0)$

$= -100$ V (as $V_i = 0$)

35. (a) $\quad \mathbf{E} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$

$= -[(Ay - 2Bx)\hat{\mathbf{i}} + (Ax + C)\hat{\mathbf{j}}]$

$\therefore \quad E_x = (-Ay + 2Bx)$

and $\quad E_y = (-Ax - C)$

(b) $\mathbf{E} = 0$ if E_x and E_y are separately zero.

$\because \quad -Ay + 2Bx = 0$ ...(i)

and $\quad -Ax - C = 0$ ...(ii)

Solving these two equations

We get

$x = -\frac{C}{A}$ and $y = -\frac{2BC}{A^2}$

36. Sphere is a closed surface. Therefore, Gauss's theorem can be applied directly on this

$\therefore \quad \phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0}$

or $\quad q_{\text{in}} = (\phi_{\text{total}})(\varepsilon_0)$

$= (360)(8.86 \times 10^{-12})$

$= 3.19 \times 10^{-9}$ C

$= 3.19$ nC

37. (a) $\phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0}$

(b) $\quad q_{\text{in}} = (\phi_{\text{total}})(\varepsilon_0)$

38. $\mathbf{S} = (0.2)\hat{\mathbf{i}}$

$\therefore \quad \phi = \mathbf{B} \cdot \mathbf{S} = 0.2\left(\frac{3}{5}E_0\right)$

$= 0.2 \times 0.6 \times 2 \times 10^3$

$= 240$ N-m^2/C

39. Electric flux enters from the surface parallel to y - z plane at $x = 0$. But $\mathbf{E} = 0$ at $x = 0$.

Hence, flux entering the cube = 0.

Flux leaves the cube from the surface parallel to y - z plane at $x = a$.

Flux leaving the cube $= ES$

$$= \left(\frac{E_0 x}{l}\right)(a^2) = \left(\frac{E_0 a}{l}\right)(a^2) \qquad (\text{at } x = a)$$

Substituting the values, we get

$$\phi = \frac{(5 \times 10^3)(10^{-2})^3}{(2 \times 10^{-2})}$$
$$= 2.5 \times 10^{-1}$$
$$= 0.25 \text{ N-m}^2/\text{C}$$

At all other four surfaces, electric lines are tangential. Hence, flux is zero.

$$\therefore \qquad \phi_{\text{net}} = (0.25) \text{ N - m}^2/\text{C} = \frac{q_{\text{in}}}{\varepsilon_0}$$

$$\therefore \qquad q_{\text{in}} = (0.25)(\varepsilon_0)$$
$$= (0.25)(8.86 \times 10^{-12})$$
$$= 2.2 \times 10^{-12} \text{ C}$$

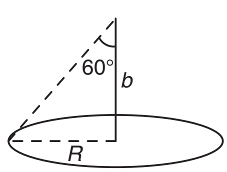
40. See the hint of Example-1 of section solved examples for miscellaneous examples. We have

$$\phi = \frac{(1 - \cos\theta) Q}{2\varepsilon_0}$$

But, $\dfrac{Q}{\varepsilon_0} = \phi_{\text{total}}$

$$\therefore \qquad \phi = (\phi_{\text{total}})\left(\frac{1 - \cos\theta}{2}\right) \qquad \text{...(i)}$$

Given that $\qquad \phi = \dfrac{1}{4}(\phi_{\text{total}})$



Substituting in Eq. (i),

we get, $\qquad \theta = 60°$

$$\frac{R}{b} = \tan 60° = \sqrt{3}$$

$$\therefore \qquad R = \sqrt{3} b$$

41. (a) $\qquad \mathbf{S_j} = (L^2)(-\hat{\mathbf{j}})$

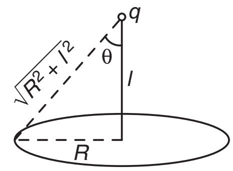
$$\therefore \qquad \phi s_1 = \mathbf{E} \cdot \mathbf{S_j} = -CL^2$$

Similarly, we can find flux from other surfaces.

Note *Take area vector in outward direction of the cube.*

(b) Total flux from any closed surface in uniform electric field is zero.

42. We have

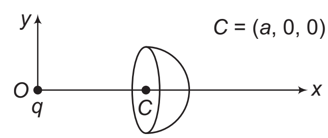


$$\phi = \frac{q(1 - \cos\theta)}{2\varepsilon_0}$$

Here, flux due to $+q$ and $-q$ are in same direction.

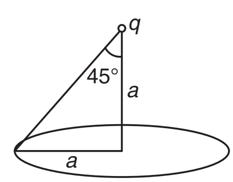
$$\therefore \qquad \phi_{\text{total}} = 2\phi = \frac{q(1 - \cos\theta)}{\varepsilon_0}$$
$$= \frac{q}{\varepsilon_0}\left[1 - \frac{l}{\sqrt{R^2 + l^2}}\right]$$

43.



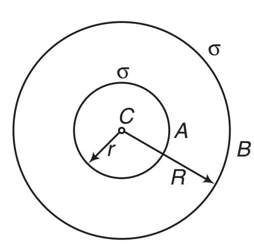
Flux passing through hemisphere = flux passing through circular surface of the hemisphere.

For finding flux through circular surface of hemisphere we can again use the concept used in above problem.



$$\phi = \frac{q(1 - \cos\theta)}{2\varepsilon_0}$$
$$= \frac{q}{2\varepsilon_0}(1 - \cos 45°)$$
$$= \frac{q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

44.



$$\sigma(4\pi r^2 + 4\pi R^2) = Q$$

$\therefore \qquad \sigma = \dfrac{Q}{4\pi\,(r^2 + R^2)}$

Potential at centre

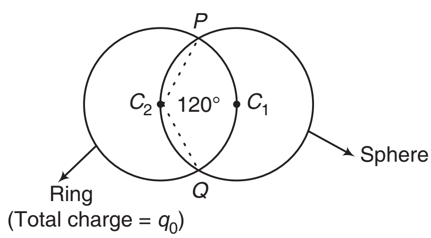
= potential due to A + potential due to B

$$= \frac{\sigma r}{\varepsilon_0} + \frac{\sigma R}{\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0}(r + R)$$

$$= \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{r + R}{r^2 + R^2}\right)$$

45.



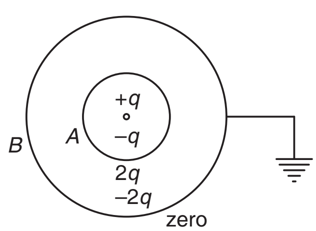
Charge on arc PQ of ring $= \dfrac{q_0}{3}$

This is also the charge lying inside the closed sphere.

$\therefore \quad \phi$ through closed sphere $= \dfrac{q_{\text{in}}}{\varepsilon_0}$

$$= \frac{(q_0/3)}{\varepsilon_0} = \frac{q_0}{3\varepsilon_0}$$

46.



If outermost shell is earthed. Then, charge on outer surface of outermost shell in this case is always zero.

47. (a) $V_A = \dfrac{\sigma a}{\varepsilon_0} - \dfrac{\sigma b}{\varepsilon_0} + \dfrac{\sigma c}{\varepsilon_0}$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

$$V_B = \left(\frac{\sigma a}{\varepsilon_0}\right)\left(\frac{a}{b}\right) - \frac{\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

$$V_C = \left(\frac{\sigma a}{\varepsilon_0}\right)\left(\frac{a}{c}\right) - \left(\frac{\sigma b}{\varepsilon_0}\right)\left(\frac{b}{c}\right) + \frac{\sigma c}{\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

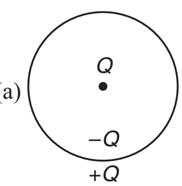
(c) $W_{A \to C} = 0$

$\therefore \qquad V_A = V_C$

or $\quad (a - b + c) = \left(\dfrac{a^2 - b^2}{c}\right) + c$

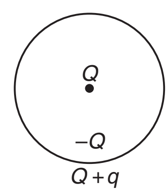
or $\qquad a + b = c$

48. (a)



$\sigma_i = \dfrac{-Q}{4\pi a^2}$ and $\sigma_0 = \dfrac{Q}{4\pi a^2}$

(b)



$$\sigma_i = \frac{-Q}{4\pi a^2}$$

and $\qquad \sigma_0 = \dfrac{(Q + q)}{4\pi a^2}$

(c) According to Gauss's theorem,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{in}}}{r^2}$$

For $x \le R$, $q_{\text{in}} = Q$

and $r = x$ both cases.

$\therefore \qquad E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x^2}$

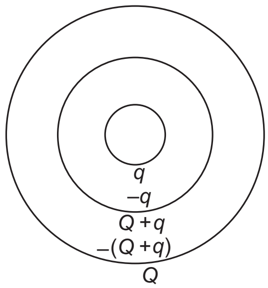
49. Let Q is the charge on shell B (which comes from earth)

$$V_B = 0$$

$\therefore \qquad \dfrac{kq}{b} + \dfrac{kQ}{b} - \dfrac{kq}{c} = 0$

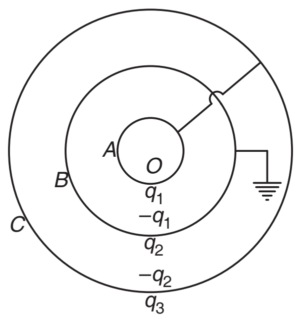
$\therefore \qquad Q = \left(\dfrac{b}{c} - 1\right)q$

Charges appearing on different faces are as shown below.



$$Q + q = \left(\frac{b}{c}\right) q$$

50.



Total charge on $A + C$ is $3q$

$$\therefore \quad q_1 - q_2 + q_3 = 3q \qquad \ldots\text{(i)}$$

$$V_B = 0$$

$$\therefore \quad \frac{kq_1}{2R} + k\left(\frac{q_2 - q_1}{2R}\right) + k\left(\frac{q_3 - q_2}{3R}\right) = 0 \qquad \ldots\text{(ii)}$$

$$V_A = V_C$$

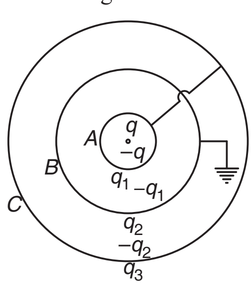
$$\therefore \quad \frac{kq_1}{R} + k\left(\frac{q_2 - q_1}{2R}\right) + k\left(\frac{q_3 - q_2}{3R}\right)$$

$$= \frac{kq_1}{3R} + k\left(\frac{q_2 - q_1}{3R}\right) + k\left(\frac{q_3 - q_2}{3R}\right) \qquad \ldots\text{(iii)}$$

In the above equations, $k = \dfrac{1}{4\pi\varepsilon_0}$.

Solving these three equations, we can find the asked charges.

51.



Total charge on $A + C$ is $3q$. Therefore,

$$- q + q_1 - q_2 + q_3 = 3q \qquad \ldots\text{(i)}$$

$$V_B = 0$$

$$\therefore \quad k\left(\frac{q_1 - q}{2R}\right) + k\left(\frac{q_2 - q_1}{2R}\right) + k\left(\frac{q_3 - q_2}{3R}\right) = 0 \qquad \ldots\text{(ii)}$$

$$V_A = V_C$$

$$\therefore \quad k\left(\frac{q_1 - q}{R}\right) + k\left(\frac{q_2 - q_1}{2R}\right) + k\left(\frac{q_3 - q_2}{3R}\right)$$

$$= k\left(\frac{q_1 - q}{3R}\right) + k\left(\frac{q_2 - q_1}{3R}\right) + k\left(\frac{q_3 - q_2}{3R}\right) \qquad \ldots\text{(iii)}$$

In the above equations,

$$k = \frac{1}{4\pi\varepsilon_0}$$

Solving above there equations, we can find q_1, q_2 and q_3.

52. (a) From Gauss's theorem

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_{\text{in}}}{r^2}\right) = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{2Q}{r^2}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 r^2}$$

(b) According to principle of generator, potential difference depends only on q_{in}.

$$\therefore \quad \text{PD} = \frac{2\theta}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{3R}\right) = \frac{Q}{3\pi\varepsilon_0 R}$$

(c) According to principle of generator, whole inner charge transfers to outer sphere.

(d) $V_{\text{in}} = 0$

$$\therefore \quad \frac{k\, q_{\text{in}}}{R} - \frac{kQ}{3R} = 0 \quad \Rightarrow \quad q_{\text{in}} = \frac{Q}{3}$$

53. (a) At $r = R$,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} - \frac{2Q}{2R} + \frac{3Q}{3R}\right]$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q}{R}\right)$$

At $r = 3R$

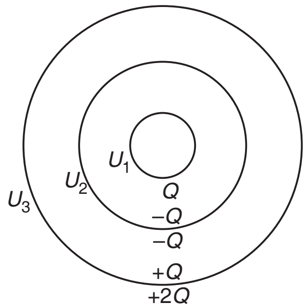
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{3R} - \frac{2Q}{3R} + \frac{3Q}{3R}\right] = \frac{Q}{6\pi\varepsilon_0 R}$$

(b) $E = \left(\dfrac{1}{4\pi\varepsilon_0}\right)\left(\dfrac{q_{\text{in}}}{r^2}\right)$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{Q - 2Q}{(5R/2)}\right] = \frac{-Q}{25\pi\varepsilon_0 R^2}$$

Minus sign implies that this electric field is radially towards centre.

(c)



$$U_T = U_1 + U_2 + U_3$$

where, $U_1 = \frac{1}{2}\frac{Q^2}{C_1}$

where, $C_1 = 4\pi\varepsilon_0 \Big/ \left(\frac{1}{R} - \frac{1}{2R}\right)$

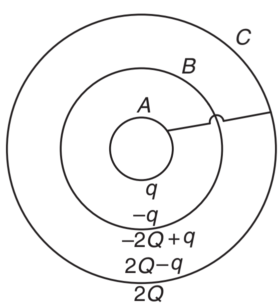
$$U_2 = \frac{1}{2}\frac{Q^2}{C_2}$$

where, $C_2 = 4\pi\varepsilon_0 \Big/ \left(\frac{1}{2R} - \frac{1}{3R}\right)$

and $U_3 = \frac{1}{2}\frac{(2Q)^2}{C_3}$

where, $C_3 = 4\pi\varepsilon_0 \Big/ \left(\frac{1}{3R} - \frac{1}{\infty}\right)$

(d)



Total charge on $(A + C)$ is $3Q + Q = 4Q$.

Now, $V_A = V_C$

$\therefore$ $\frac{kq}{R} - \frac{k\,(2Q)}{2R} + \frac{k\,(4Q - q)}{3R}$

$$= \frac{k\,(Q - 2Q + 3Q)}{3R} \qquad \ldots\text{(i)}$$

Solving this equation, we get

$$q = \frac{Q}{2}$$

Now, $q_A = q = \frac{Q}{2}$

$$q_C = 4Q - q = \frac{7}{2}Q$$

(e) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_{\text{in}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q - 2Q}{(5R/2)^2}\right] = \frac{-3Q}{50\pi\varepsilon_0 R^2}$

Minus sign indicates that electric field is radially inwards.

LEVEL 2

Single Correct Option

1. Let us conserve angular momentum of $+2q$ about the point at $+Q$.

$$mv_1 r_1 \sin\theta_1 = mv_2 r_2 \sin\theta_2$$

$$(m)(v)(R)\sin 150° = m\left(\frac{v}{\sqrt{3}}\right) r_{\min} \sin 90°$$

$\therefore$ $r_{\min} = \frac{\sqrt{3}}{2}R$

2. $(v_A)_y = v \Rightarrow (v_B)_y = 2v \sin 30° = v$

Since, y-component of velocity remains unchanged. Hence electric field is along $(-\hat{\mathbf{i}})$ direction. Work done by electrostatic force in moving from A to B = change in its kinetic energy

$\therefore$ $(eE)\,(2a - a) = \frac{1}{2}m\,(4v^2 - v^2)$

$$E = \frac{mv^2}{2ea}$$

or $\mathbf{E} = -\frac{3mv^2}{2ea}\hat{\mathbf{i}}$

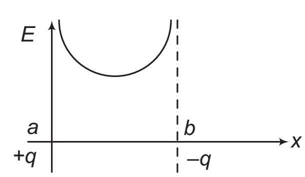
Rate of doing work done = power

$$= Fv\cos\theta$$

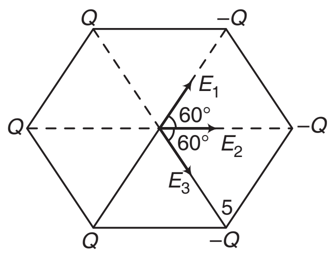
$$= \left(\frac{3mv^2}{2ea}\right)^2 (e)\,(2v)\cos 30°$$

$$= \frac{3\sqrt{3}}{2}\frac{mv^2}{a}$$

3.



Just to the right of a, electric field is along ab ($\therefore$positive) and tending to infinite. Similarly, electric field just to the left of b electric field is again along ab ($\therefore$ positive) and tending to infinite.

4. 

$$E = E_1 = E_2 = E_3 = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{P}{x^3}\right)$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2Qa}{x^3}$$

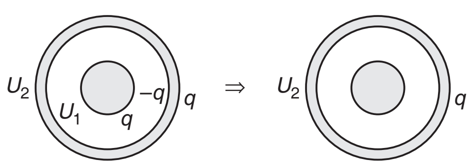
Resultant of E_1 and E_3 is also equal to E along E_2

$\therefore$ $E_{\text{net}} = 2E$ (along E_2)

$$= \frac{Qa}{\pi\varepsilon_0 x^3}$$

5. From centre to the surface of inner shell, potential will remain constant = 10 V (given).

6. By closing the switch whole inner charge transfers to outer shell.



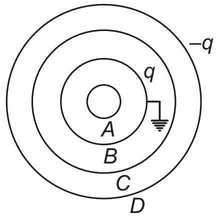
Heat produced $= U_i - U_f$

$= (U_1 + U_2) - U_2$

$= U_1 = \frac{1}{2}\frac{q^2}{C}$

where, $C = \frac{4\pi\varepsilon_0}{(1/a - 1/2)a} = 8\pi\varepsilon_0 a$

$\therefore$ $\text{Heat} = \frac{q^2}{16\pi\varepsilon_0 a} = \frac{kq^2}{4a}$

7. 

Let Q charge comes on shell-C from earth. Then,

$$V_C = 0$$

$\therefore$ $\frac{kq}{3a} + \frac{kQ}{3a} - \frac{kq}{4a} = 0$

Solving, we get

$$Q = -\frac{q}{4}$$

Now, $V_A = \frac{kq}{2a} - \frac{kq/4}{3a} - \frac{kq}{4a} = \frac{kq}{6a}$

and $V_C = 0$

$\therefore$ $V_A - V_C = \frac{kq}{6a}$

8. $\rho = \frac{q}{(4/3)\,\pi R^3}$

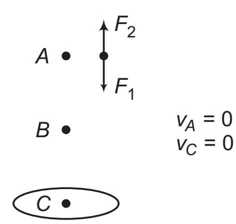
$\therefore$ $q = \frac{4}{3}\pi\rho R^3$

$$V_C - V_S = \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}\right) - \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

$$= \frac{q}{8\pi\varepsilon_0 R}$$

Substituting the value of q, we have

$$V_C - V_S = \frac{\rho R^3}{6\varepsilon_0}$$

9. 

Hence, in between A and C there is a point B, where speed of the particle should be maximum.

$$F_1 = mg = \text{constant}$$

F_2 = electrostatic repulsion (which increases as the particle moves down)

From A to B kinetic energy of the particle increases and potential energy decreases. Then, from B to C kinetic energy decreases and potential energy increases.

10. Over Q_1, potential is $+\,\alpha$. Hence, Q_1 is positive.

$V_A = 0$ and A point is nearer to Q_2. Therefore, Q_2 should be negative and $|Q_1| > |Q_2|$.

At A and B, potential is zero, not the force.

Equilibrium at C will depend on the nature of charge which is kept at C.

11. V_1 is positive and V_2 is negative. Hence at all points,

$$V_1 > V_2$$

12. Just to the right of q_1, electric field is $+\alpha$ or in positive direction (away from q_1). Hence, q_1 is positive. Just to the left of q_2, electric field is $-\alpha$ or towards left (or away from q_2). Hence, q_2 is also positive.

Further $E = 0$ near q_1. Hence,

$$q_1 < q_2$$

13. Electric lines of forces of q will not penetrate the conductor.

14. $\mathbf{E} = 400 \cos 45° \,\hat{\mathbf{i}} + 400 \sin 45° \,\hat{\mathbf{j}}$

$$V_A - V_B = -\int_B^A \mathbf{E} \cdot d\mathbf{r}$$

where, $\quad d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$

15. q_A will remain unchanged.

Hence, according to principle of generator potential difference will remain unchanged.

$$V'_A - V'_B = V_A - V_B$$

or $\quad V'_A = V_A - V_B \qquad$ (as $V'_B = 0$)

16. $W_T = 0$

$W_{F_e} = (F_e)$ (displacement in the direction of force)

$\quad$ = Kinetic energy of the particle.

$$\therefore \quad \frac{1}{2}mv^2 = qE\left[\frac{l}{2} - \frac{l}{2}\cos 60°\right]$$

$$\therefore \quad v = \sqrt{\frac{qEl}{m}}$$

17. $L = mv\, r_\perp = m\,(at)\,(x_0)$

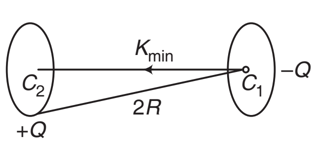
$$= m\left(\frac{qE_0}{m}\right) t\,(x_0) \quad \text{or} \quad L \propto t$$

18. $U_i + K_i = U_f + K_f$

or $\quad qV_i + \frac{1}{2}mv_{\min}^2 = qV_f + 0$

or $\quad q\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q}{R}\right) + \frac{1}{2}mv_{\min}^2 + q\left[\frac{3}{2} \times \frac{Q}{4\pi\varepsilon_0 R}\right]$

From here, we can find $v_{\min}$.

19. 

$$k_{C_1} + U_{C_1} = k_{C_2} + U_{C_2}$$

$$k_{\min} + qV_{C_1} = 0 + qV_{C_2} \qquad \text{...(i)}$$

$$V_{C_1} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2R} - \frac{Q}{R}\right)$$

$$V_{C_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{Q}{2R}\right)$$

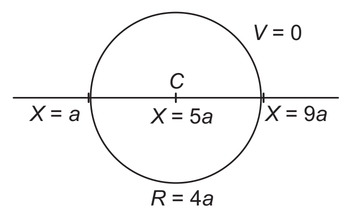
Substituting these values in Eq. (i), we can find $K_{\min}$.

20. $\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}}$

Now we can use, $\quad \int dV = -\int \mathbf{E} \cdot d\mathbf{r}$

two times and can find values of E_x and E_y.

21. Let $P = (x, y)$



$$r_1 = \sqrt{(x + 3a)^2 + y^2}$$

$$r_2 = \sqrt{(x - 3a)^2 + y^2}$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_2} - \frac{2Q}{r_1}\right] = 0 \qquad \text{...(i)}$$

Substituting values of r_1 and r_2 in Eq. (i), we can see that equation is of a circle of radius $4a$ and centre at $5a$.

22. $F_x = 0$

$$\therefore \quad a_x = 0$$

$$F_y = qE$$

$$\therefore \quad a_y = \frac{qE}{m}$$

$x = vt$ and

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

Substituting $t = \frac{x}{v}$ in expression of y, we get

$$y = \frac{1}{2}\left(\frac{qEx^2}{mv^2}\right)$$

$$\text{KE} = \frac{1}{2}m\,(v_x^2 + v_y^2)$$

where, $v_x = v$ and $v_y = a_y t = \frac{qE}{m}t$

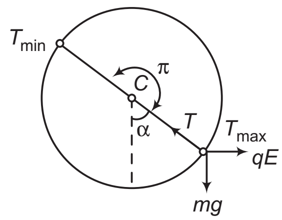
23. $qE = \dfrac{mv^2}{r}$

$$q\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right) = \frac{mv^2}{r}$$

$\therefore \qquad v = \sqrt{\dfrac{g\lambda}{2\pi\varepsilon_0 m}} = \sqrt{\dfrac{2kq\lambda}{m}}$

Now, $\quad T = \dfrac{2\pi r}{v}$

24.



$$T\sin\alpha = qE$$
$$T\cos\alpha = mg$$

$\therefore \qquad \alpha = \tan^{-1}\left(\dfrac{qE}{mg}\right)$

Minimum tension will be obtained at $\alpha + \pi$.

25. Energy required $= \Delta U = U_f - U_i$

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{q^2}{a}\right) - \left(\frac{q^2}{a} - \frac{q}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{\sqrt{2}a} + \frac{q^2}{a}\right)\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0 a}[\sqrt{2} + 1]$$

26. On both sides of the positive charge $V = +\infty$ just over the charge.

27. $U = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{a} \qquad$ (a = side of triangle)

$$W = U_f - U_i = 3\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right) - U$$

$$= 3U - U = 2U$$

28. $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r} + 0 \quad \text{or} \quad r \propto \frac{1}{v^2}$$

If v is doubled, the minimum distance r will remain $\dfrac{1}{4}$ th.

29. See the hint of Sample Example 24.9

$$K = \frac{\rho}{\rho - \sigma} = \frac{1.6}{1.6 - 0.8} = 2$$

30. $V_p = 2\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a}\right) - 2\left[\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{\sqrt{b^2 + a^2}}\right]$

$$= \frac{q}{4\pi\varepsilon_0 a}\left[2 - 2\left(1 + \frac{b^2}{a^2}\right)^{-1/2}\right]$$

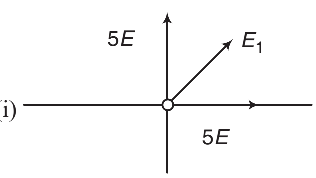
Since, $b << a$, we can apply binomial expansion

$\therefore \qquad V_p = \dfrac{q}{4\pi\varepsilon_0 a}\left[2 - 2\left(1 - \dfrac{b^2}{2a^2}\right)\right]$

$$= \frac{qb^2}{4\pi\varepsilon_0 a^3}$$

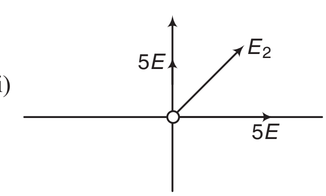
31. Let E = magnitude of electric field at origin due to charge $\pm q$. Then,

(i)



$$E_1 = \sqrt{(5E)^2 + (5E)^2} = 5\sqrt{2}E$$

(ii)



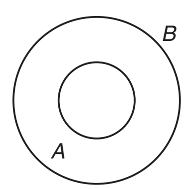
E_2 is again $5\sqrt{2}E$.

Similarly, we can find E_3 and E_4 also.

32. $U = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R^2}\right)^2$

$$= \frac{1}{2}\varepsilon_0\left[\frac{9\times 10^9 \times \frac{1}{9}\times 10^{-9}}{(1)^2}\right]^2 = \frac{\varepsilon_0}{2}\ \text{J/m}^3$$

33.



They have a common potential in the beginning. This implies that only B has the charge in the beginning.

$$\therefore \qquad V = \frac{kq_B}{b} \quad \text{or} \quad kq_B = Vb$$

Now, suppose q_A charge is given to A. Then,

$$V_A = \frac{kq_A}{a} + \frac{kq_B}{b} = 0$$

$$\text{or} \qquad kq_A = -a\left(\frac{kq_B}{b}\right) = -aV$$

$$\text{Now,} \qquad V_B = \frac{kq_A}{b} + \frac{kq_B}{b}$$

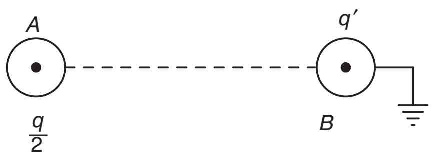
$$= -\frac{a}{b}V + V = V\left(1 - \frac{a}{b}\right)$$

34. Let $\mathbf{E} = E_x\,\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$

Apply $\int dV = -\int \mathbf{E}\cdot d\mathbf{v}$

three times and find values of E_x, E_y and E_z. Then, again apply the same equation for given point.

35.



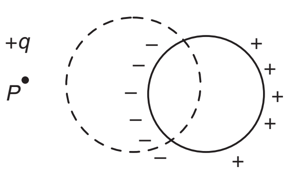
Let charge on B is q'

$$V_B = 0$$

$$\therefore \qquad \frac{(k(q/2))}{d} + \frac{kq'}{r} = 0$$

$$\therefore \qquad q' = -\frac{qr}{2d}$$

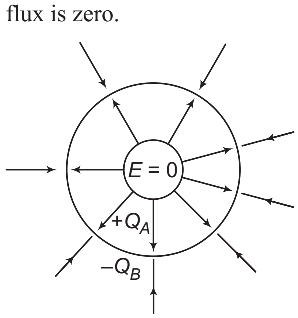
36.



The induced charges on conducting sphere due to $+q$ charge at P are as shown in figure.

Now, net charge inside the closed dotted surface is negative. Hence, according to Gauss's theorem net flux is zero.

37.



Since $|Q_B| > Q_A$, electric field outside sphere B is inwards (say negative). From A to B enclosed charge is positive. Hence, electic field is radially outwards (positive).

38. $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}}\right] = -[(-ky)\,\hat{\mathbf{i}} + (-kx)\,\hat{\mathbf{j}}]$

$$\therefore \qquad |\mathbf{E}| = \sqrt{(ky)^2 + (kx)^2}$$

$$= k\sqrt{x^2 + y^2} = kr$$

$$\therefore \qquad |\mathbf{E}| \propto r$$

More than One Correct Options

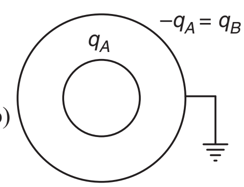
1. (a) $V_A = 2V = \frac{kq_A}{R} + \frac{kq_B}{2R}$

$$V_B = \frac{3}{2}V = \frac{kq_A}{2R} + \frac{kq_B}{2R}$$

Solving these two equations, we get

$$\frac{q_A}{q_B} = \frac{1}{2}$$

(b)



$$\frac{q'_A}{q'_B} = \frac{q_A}{-q_A} = -1$$

(c) & (d) Potential difference between A and B will remain unchanged as by earthing B, charge on will not changed.

$$\therefore \qquad V'_A - V'_B = V_A - V_B$$

$$= 2V - \frac{3}{2}V = \frac{V}{2}$$

$$\therefore \qquad V'_A = \frac{V}{2}$$

$$\text{as} \qquad V'_B = 0$$

2. $T = \frac{2u_y}{g} = \frac{2 \times 10}{10} = 2\text{ s}$

$$H = \frac{u_y^2}{2g} = \frac{(10)^2}{20} = 5\text{ m}$$

$$R = \frac{1}{2}a_xT^2 = \frac{1}{2}\left(\frac{qE}{m}\right)T^2$$

$$= \frac{1}{2}\left(\frac{10^{-3}\times 10^{4}}{2}\right)(2)^2$$

$$= 10 \text{ m}$$

3. $100 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(R+0.05)}$ ...(i)

$75 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(R+0.1)}$ ...(ii)

Solving these equations, we get

$$q = \frac{5}{3}\times 10^{-9}\text{ C}$$

and $R = 0.1$ m

(a) $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$

$$= \frac{(9\times 10^9)\left(\frac{5}{3}\times 10^{-9}\right)}{0.1}$$

$= 150$ V

(c) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{V}{R} = \frac{150}{0.1}$

$= 1500$ V/m

(d) $V_{\text{centre}} = 1.5\, V_{\text{surface}}$

5. Electric field at any point depends on both charges Q_1 and Q_2. But electric flux passing from any closed surface depends on the charged enclosed by that closed surface only.

6. Flux from any closed surface $= \frac{q_{\text{in}}}{\varepsilon_0}$,

$q_{\text{in}} = 0$, due to a dipole.

8. $E = \frac{k\, q_{\text{in}}}{r^2}$ $\left(\text{Here, } k = \frac{1}{4\pi\varepsilon_0}\right)$

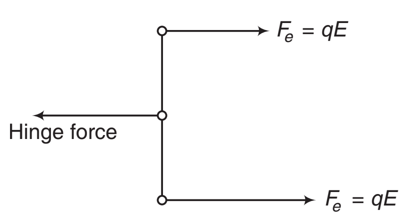
$\therefore$ $E_A = E_c = 0$

but, $E_B \neq 0$

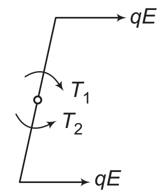
$V = \frac{kq}{R}$ $(r \le R)$

$V = \frac{kq}{r}$ $(r \ge R)$

9.



Higher force $= 2qE$ (towards left)



If we displace the rod, $\tau_1 = \tau_2$ or $\tau_{\text{net}} = 0$ in displaced position too. Hence, equilibrium is neutral.

10. Along the line AB, charge q is at unstable equilibrium position at B (When displaced from B along AB, net force on it is away from B, whereas force at B is zero). Hence, potential energy at B is maximum.

Along CD equilibrium of q is stable. Hence, potential energy at B is minimum along CD.

Comprehension Based Questions

1. $V_{\text{outer}} = 0$

$\therefore$ $\frac{kQ}{2r} + \frac{kQ_1}{2r} = 0$

$\therefore$ $Q_1 = -Q =$ charge on outer shell

2. $V_{\text{inner}} = 0$

$\therefore$ $\frac{kQ_2}{r} + \frac{kQ_1}{2r} = 0$

$\therefore$ $Q_2 = -\frac{Q_1}{2} = \frac{Q}{2} =$ charge on inner shell

Charge flown through $S_2 =$ initial charge on inner shell – final charge on it

$$= Q - Q_2 = \frac{Q}{2}$$

3. After two steps charge on inner shell remains $\frac{Q}{2}$ or half.

So, after n-times

$$q_{\text{in}} = \frac{Q}{(2)^n}$$

Now, according to the principle of generator, potential difference depends on the inner charge only.

$\therefore$ $\text{PD} = \frac{q_{\text{in}}}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2r}\right]$

$$= \frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$$

4. According to Gauss's theorem,

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_{in}}{r^2}\right) \quad \ldots\text{(i)}$$

For $r \le R$

$$q_{in} = \int_0^r (4\pi r^2) \cdot dr \cdot \rho$$

$$= \int_0^r (4\pi r^2)(\rho_0)\left(1 - \frac{r}{R}\right) dr$$

$$= 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

Substituting in Eq. (i), we get

$$E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{3} - \frac{r^2}{4r}\right]$$

5. For outside the ball,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{total}}{r^2} \quad \ldots\text{(i)}$$

where, $\quad q_{total} = \int_0^R (4\pi r^2)(\rho_0)\left(1 - \frac{r}{R}\right) dr$

Substituting this value in Eq. (i), we get

$$E = \frac{\rho_0 R^3}{12\,\varepsilon r^2}$$

6. For outside the ball, electric field will continuously decrease.

Hence, it will be maximum somewhere inside the ball. For maximum value,

$$\frac{dE}{dr} = 0$$

$$\therefore \quad \frac{d}{dr}\left[\frac{\rho_0}{\varepsilon}\left(\frac{r}{3} - \frac{r^2}{4R}\right)\right] = 0$$

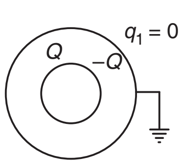
Solving, we get $r = \dfrac{2R}{3}$

7. Submitting $r = \dfrac{2R}{3}$ in the same expression of electric field, we get its maximum value.

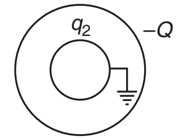
8. Potential difference in such situation depends on inner charge only. So, potential difference will remain unchanged. Hence,

$$\Delta V = V_a - V_b$$

9.



(i)



(ii)

$V_{inner} = 0$ when solid sphere is earthed

$$\because \quad \frac{kq_2}{a} - \frac{kQ}{b} = 0$$

$$\because \quad q_2 = Q\left(\frac{a}{b}\right)$$

10. Whole inner charge transfers to shell.

$\therefore$ Total charge on shell $= q_2 - Q$

$$= Q\left(\frac{a}{b} - 1\right)$$

Match the Columns

1. (a) $\mathbf{E_C}$ and $\mathbf{E_F}$ are cancelled. $\mathbf{E_E}$ and $\mathbf{E_D}$ at 60°

(b) $\mathbf{E_B}$ and $\mathbf{E_E}$ are cancelled. $\mathbf{E_F}$ and $\mathbf{E_D}$ at 120°.

(c) $\mathbf{E_B}$ and $\mathbf{E_E}$ are cancelled. Similar, $\mathbf{E_F}$ and $\mathbf{E_C}$ are cancelled.

(d) $\mathbf{E_F}$ and $\mathbf{E_D}$ at 120°. So, their resultant is E in the direction of $\mathbf{E_E}$. Hence, net is $2E$.

2.

$$\int dV = -\int \mathbf{E} \cdot d\mathbf{r}$$

$$\therefore \quad V_A - V_B = -\int_B^A \mathbf{E} \cdot d\mathbf{r}$$

3. $V = \dfrac{kq}{R} \Rightarrow kq = VR$

(a) $$V = \frac{kq}{R^3}(1.5R^2 - 0.5r^2)$$

$$= \frac{VR}{R^3}\left[\frac{3}{2}R^2 - \frac{1}{2}\left(\frac{R}{2}\right)^2\right]$$

$$= \frac{11}{8}V$$

(b) $V = \dfrac{kq}{r} = \dfrac{VR}{2R} = \dfrac{V}{2}$

(c) $E = \dfrac{kq}{R^3} \cdot r$

$$= \frac{(V_R)}{(R^3)}\left(\frac{R}{2}\right) = \frac{V}{2R} = \frac{V}{2} \qquad \text{(if } R = 1\text{ m)}$$

(d) $$E = \frac{kq}{r^2} = \frac{VR}{(2R)^2}$$

$$= \frac{V}{4R} = \frac{V}{4} \text{ for } R = 1\text{ m}$$

Subjective Questions

1. (a) By comparing this problem with spring-block system problem suspended vertically.

Here, $mg \equiv qE = 50 \times 10^{-6} \times 5 \times 10^{5} = 25$ N

$$X_{max} = 2\text{ mg/K}$$

Here, $X_{max} = 2\, qE/K = \dfrac{2 \times 25}{100} = 0.5$ m

or $= 50$ cm **Ans.**

(b) Equilibrium position will be at $x = mg/K$.

Here, it will be at $x = qE/K = \dfrac{25}{100} = 0.25$ m

or 25 cm **Ans.**

(c) Force $Q\mathbf{E}$ is constant force, which does not affect the period of oscillation of SHM.

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{4}{100}}$$

$$= \frac{2\pi}{5}\text{ s} = 1.26\text{ s}$$ **Ans.**

(d) $\mu mg = 0.2 \times 4 \times 10 = 8$ N

Therefore, here constant force will be

$qE - \mu mg = 25 - 8 = 17$ N $= F$ (say)

$$X_{max} = \frac{2F}{K} = \frac{2 \times 7}{100}$$

$$= 0.34\text{ m}$$ **Ans.**

2. Total charge on ring $= \lambda(2\pi a) = q$ (say)

Electric field at distance x from the centre of ring.

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qx}{(a^2 + x^2)^{3/2}} = \frac{\lambda ax}{2\varepsilon_0(a^2 + x^2)^{3/2}}$$

Restoring force on $-Q$ charge in this position would be

$$F = -QE = -\left[\frac{\lambda aQx}{2\varepsilon_0(a^2 + x^2)^{3/2}}\right]$$

For $x << a$,

$$F = -\left[\frac{\lambda aQ}{2\varepsilon_0 a^3}\right]x = -\left[\frac{\lambda Q}{2\varepsilon_0 a^2}\right]x$$

Comparing with $F = -kx$,

$$k = \frac{\lambda Q}{2\varepsilon_0 a^2}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2\varepsilon_0 ma^2}{\lambda Q}}$$ **Ans.**

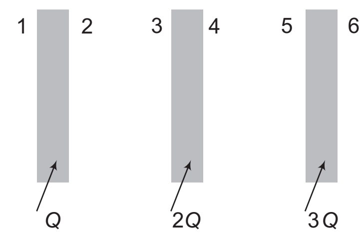
3. $q_1 = q_6 = \dfrac{q_{net}}{2} = 3Q$

$$q_2 = Q - q_1 = -2Q$$

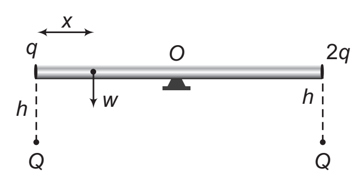
$$q_3 = -q_2 = +2Q$$

$$q_4 = 2Q - q_3 = 0$$

$$q_5 = -q_4 = 0$$



4.



(a) Net torque on the rod about $O = 0$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(2q)}{h^2}\left(\frac{L}{2}\right) + w\left(\frac{L}{2} - x\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{h^2}\left(\frac{L}{2}\right)$$

$$x = \frac{L}{2}\left[1 + \frac{Qq}{(4\pi\varepsilon_0)wh^2}\right]$$ **Ans.**

(b) There will be no force from the bearing it,

$w =$ net electrostatic repulsion from both the charges.

$$\therefore \quad w = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(3q)}{h^2}$$

or $$h = \sqrt{\frac{3Qq}{4\pi\varepsilon_0 w}}$$ **Ans.**

5. $\mathbf{E} = -\left(\dfrac{\partial V}{\partial x}\hat{\mathbf{i}} + \dfrac{\partial V}{\partial y}\hat{\mathbf{j}}\right) = (-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N/C

$$\mathbf{a} = \frac{q\mathbf{E}}{m}$$

$$= \frac{10^{-6}}{10}(-3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$$

$$= (-3 \times 10^{-7}\,\hat{\mathbf{i}} - 4 \times 10^{-7}\,\hat{\mathbf{j}})\text{ m/s}^2$$

When particle crosses x-axis, $y = 0$.

Initial y-coordinate was 3.2 m.

and $a_y = -4 \times 10^{-7}$ m/s^2

$\therefore \quad y = 0$ at time $t = \sqrt{\dfrac{2 \times 3.2}{4 \times 10^{-7}}} = 4000$ s

At this instant x-coordinate will be

$$x = x_i + \frac{1}{2} a_x t^2$$

$$= 2 + \frac{1}{2}(-3 \times 10^{-7})(4000)^2 = -0.4 \text{ m}$$

Now, $V_i = (3 \times 2) + (4 \times 3.2) = 18.8$ V

$V_f = (3)(-0.4) = -1.2$ V

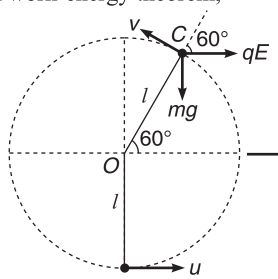
$\Delta V = 20$ V

$\therefore$ Speed, $v = \sqrt{\dfrac{2q\Delta V}{m}}$

$$= \sqrt{\frac{2 \times 10^{-6} \times 20}{10}}$$

$= 2.0 \times 10^{-3}$ m/s **Ans.**

6. From work-energy theorem,



$$\frac{1}{2} m (v^2 - u^2) = - mgl (1 + \sin 60°) + qE\, l \cos 60°$$

Substituting the values, we get

$$u^2 - v^2 = 32.32 \quad \ldots\text{(i)}$$

Further, at C tension in the string is zero.

Hence, $\dfrac{mv^2}{l} = mg \sin 60° - qE \cos 60°$

or $\quad v^2 = 3.66 \quad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$u = 6$ m/s **Ans.**

7. There are total 28 pairs of charges.

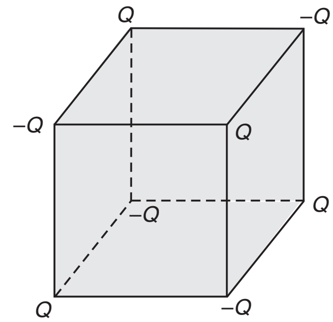
12 pairs $\to$ Q and $-Q$ $\to$ distance L

12 pairs $\to$ (Q and Q) or ($-Q$ and $-Q$) $\to \sqrt{2}L$

4 pairs $\to$ Q and $-Q$ $\to \sqrt{3}L$

$$\therefore \quad U = 12\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{-Q^2}{L}\right) + 12\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{\sqrt{2}L}\right) + 4\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{-Q^2}{\sqrt{3}L}\right)$$

$$= -\frac{Q^2}{\pi\varepsilon_0 L}\left(\frac{3\sqrt{6} + \sqrt{2} - 3\sqrt{3}}{\sqrt{6}}\right)$$



with decrease in L, potential energy will decrease. Therefore, cube should shrink as the conservative forces act in the direction of decreasing potential energy.

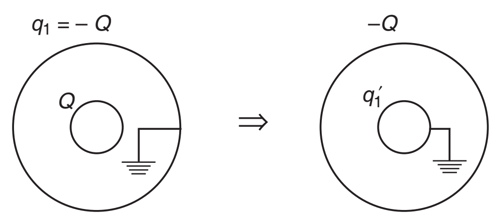
Increase in KE of the system = decrease in PE

or $\quad 8\left(\dfrac{1}{2}mv^2\right) = U_i - U_f$

$$= \frac{Q^2}{\pi\varepsilon_0}\left(\frac{3\sqrt{6} + \sqrt{2} - 3\sqrt{3}}{\sqrt{6}}\right)\left(\frac{1}{nL} - \frac{1}{L}\right)$$

or $\quad v = \sqrt{\dfrac{Q^2(1-n)(3\sqrt{6} + \sqrt{2} - 3\sqrt{3})}{4nm\pi L\varepsilon_0\sqrt{6}}}$ **Ans.**

8. Let charge q_1 comes from the earth on outer shell



$$V_{\text{outer}} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{2r} + \frac{q_1}{2r}\right] = 0$$

or $\quad q_1 = -Q$

When S_2 is closed and opened,

$$V_{\text{inner}} = 0$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1'}{r} - \frac{Q}{2r}\right] = 0$$

or $\quad q_1' = \dfrac{Q}{2}$

Proceeding in the similar manner after n such operations we get,

Charge on the inner shell,

$$q_n' = \frac{Q}{(2)^n}$$

and the potential difference between the shells,

$$\Delta V = \frac{q_n'}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{2r}\right)$$

$$\Delta V = \frac{1}{(2)^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right] \qquad \textbf{Ans.}$$

9. (a) Over charge Q_2, field intensity is infinite along negative x-axis. Therefore, Q_2 is negative.

Beyond $x > (l + a)$, field intensity is positive. Therefore, Q_1 is positive.

(b) At $x = l + a$, field intensity is zero.

$$\therefore \quad \frac{kQ_1}{(l+a)^2} = \frac{kQ_2}{a^2} \quad \text{or} \quad \left|\frac{Q_1}{Q_2}\right| = \left(\frac{l+a}{a}\right)^2$$

(c) Intensity at distance x from charge 2 would be

$$E = \frac{kQ_1}{(x+l)^2} - \frac{kQ_2}{x^2}$$

For E to be maximum $\frac{dE}{dx} = 0$

$$\text{or} \quad -\frac{2kQ_1}{(x+l)^3} + \frac{2kQ_2}{x^3} = 0$$

$$\text{or} \quad \left(1 + \frac{l}{x}\right)^3 = \frac{Q_1}{Q_2} = \left(\frac{l+a}{a}\right)^2$$

$$\text{or} \quad 1 + \frac{l}{x} = \left(\frac{l+a}{a}\right)^{2/3}$$

$$\text{or} \quad x = \frac{l}{\left(\frac{l+a}{a}\right)^{2/3} - 1}$$

$$\text{or} \quad b = \frac{l}{\left(\frac{l+a}{a}\right)^{2/3} - 1} \qquad \textbf{Ans.}$$

10. Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2, is q_1. Therefore, charge on S_1 will be $Q - q_1$. Say it is q_1'.

$$\therefore \quad \frac{q_1}{q_1'} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$$

It implies that Q charge is to be distributed in S_2 and S_1 in the ratio of R/r.

$$\therefore \quad q_1 = Q\left(\frac{R}{R+r}\right) \qquad \ldots\text{(i)}$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R+r}\right)$$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1 + \frac{R}{R+r}\right)\left(\frac{R}{R+r}\right)$$

$$= Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

Similarly,

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

and

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots + \left(\frac{R}{R+r}\right)^n\right]$$

$$\text{or} \quad q_n = Q\frac{R}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right] \qquad \ldots\text{(ii)}$$

$$\left[S_n = \frac{a(1-r^n)}{(1-r)}\right]$$

Therefore, electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^2}{2C} = \frac{q_n^2}{2(4\pi\varepsilon_0 R)} \quad \text{or} \quad U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

where, q_n can be written from Eq. (ii).

(b) $$q_n = \frac{QR}{R+r}\left[1 + \frac{R}{R+r} + \ldots + \ldots + \left(\frac{R}{R+r}\right)^{n-1}\right]$$

as $n \to \infty$

$$q_\infty = \frac{QR}{R+r}\left(\frac{1}{1 - \frac{R}{R+r}}\right)$$

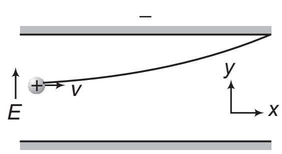
$$= \frac{QR}{R+r}\left(\frac{R+r}{r}\right) = Q\frac{R}{r} \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

$$\therefore \quad U_\infty = \frac{q_\infty^2}{2C} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_0 R}$$

$$\text{or} \quad U_\infty = \frac{Q^2 R}{8\pi\varepsilon_0 r^2} \qquad \textbf{Ans.}$$

11. $v_x = v = \sqrt{\left(\dfrac{2qV}{m}\right)},\ t = \dfrac{l}{v_x} = l\sqrt{\dfrac{m}{2qV}}$

$$a_y = \frac{qE}{m} = \frac{qat}{m} = \frac{dv_y}{dt}$$



Integrating both sides, we get

$$v_y = \frac{qat^2}{2m}$$

or $$v_y = \left(\frac{qa}{2m}\right) l^2 \left(\frac{m}{2qV}\right) = \frac{al^2}{4V}$$

Now, angle of deviation

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{al^2}{4V} \Big/ \sqrt{\frac{2qV}{m}}\right)$$

$$= \tan^{-1}\left(\frac{al^2}{4V}\sqrt{\frac{m}{2eV}}\right)$$ **Ans.**

12. From energy conservation,

$$U_C + K_C = U_D + K_D$$

or $$2\left[\frac{9\times10^9 \times (5\times10^{-5})(-5\times10^{-5})}{5}\right] + 4$$

$$= 2\left[\frac{9\times10^9 \times (5\times10^{-5})(-5\times10^{-5})}{AD}\right] + 0$$

Solving we get $AD = 9$ m

$\therefore$ Maximum distance,

$$OD = \sqrt{(9)^2 - (3)^2}$$

$$= \sqrt{72}\ \text{m}$$ **Ans.**

13. From conservation of energy,

$$U_i + K_i = U_f + K_f$$

or $$qV_i + K_i = qV_f + K_f$$

or $$q\left(\frac{Q}{4\pi\varepsilon_0 r}\right) + \frac{1}{2}mv^2$$

$$= q\left(\frac{Q}{4\pi\varepsilon_0 R^3}\right)\left(1.5R^2 - 0.5\frac{R^2}{4}\right) + 0$$

or $$\frac{1}{2}mv^2 = \frac{11\,Qq}{3\pi\varepsilon_0 R} - \frac{Qq}{4\pi\varepsilon_0 r}$$

or $$v = \sqrt{\frac{Qq}{2\pi\varepsilon_0 mR}\left(\frac{r-R}{r} + \frac{3}{8}\right)}$$ **Ans.**

14. From energy conservation principle,

$$K_i + U_i = K_f + U_f$$

or $$\frac{1}{2}mv^2 + (+q)V_i = 0 + (+q)V_f$$

or $$v = \sqrt{\frac{2q}{m}(-V_i + V_f)}$$

$$= \sqrt{\frac{2q}{m}\left[\frac{-Q}{\sqrt{10}R} + \frac{8Q}{5R} + \frac{Q}{R} - \frac{8Q}{4R}\right]\frac{1}{4\pi\varepsilon_0}}$$

$$= \sqrt{\frac{Qq}{2\pi\varepsilon_0 mR}\left(\frac{3\sqrt{10}-5}{5\sqrt{10}}\right)}$$ **Ans.**

15. (a) $E = \dfrac{kQx}{(R^2+x^2)^{3/2}}$,

$$\frac{dE}{dx} = kQ\left[\frac{(R^2+x^2)^{3/2} - x.\frac{3}{2}(R^2+x^2)^{1/2}(2x)}{(R^2+x^2)^3}\right]$$

or $$\frac{dE}{dx} = kQ\left[\frac{R^2+x^2-3x^2}{(R^2+x^2)^{5/2}}\right]$$

or $$dE = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}\right]dx$$

$$\therefore\ |\Delta E| = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}\right]\Delta x$$ **Ans.**

Here, $\Delta x = 2a$

$$\therefore\ F = |q\Delta E| = \frac{Qqa}{2\pi\varepsilon_0}\left[\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}\right]$$

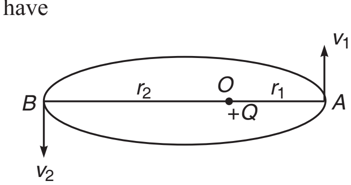
(b) $W = U_f - U_i$

$$= -pE\cos 180° + pE\cos 0° = 2pE$$

$$= 2(q)(2a)\left[\frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}\right]$$

$$= \frac{aqQx}{\pi\varepsilon_0(R^2+x^2)^{3/2}}$$ **Ans.**

16. From conservation of mechanical energy and conservation of angular momentum about point O, we have



$$\frac{1}{2}mv_1^2 - \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{r_1} = \frac{1}{2}mv_2^2 - \frac{1}{4\pi\varepsilon_0}.\frac{Q.q}{r_2} \quad \text{...(i)}$$

and $\qquad mv_1 r_1 \sin 90° = mv_2 r_2 \sin 90°$

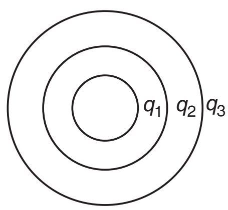
or $\qquad v_1 r_1 = v_2 r_2 \qquad$ ...(ii)

Solving these two equations, we have

$$v_1 = \sqrt{\frac{Qqr_2}{2\pi\varepsilon_0 mr_1(r_1 + r_2)}}$$

and $$\sqrt{\frac{Qqr_1}{2\pi\varepsilon_0 mr_2(r_1 + r_2)}} = v_2 \qquad \textbf{Ans.}$$

17. Let $q_1 : q_2$ and q_3 be the respective charges. Then,



$$10 = \frac{9 \times 10^9}{10^{-2}} \left[\frac{q_1}{1} + \frac{q_2}{2} + \frac{q_3}{4}\right]$$

$$0 = \frac{9 \times 10^9}{10^{-2}} \left[\frac{q_1}{2} + \frac{q_2}{2} + \frac{q_3}{4}\right]$$

and $$40 = \frac{9 \times 10^9}{10^{-2}} \left[\frac{q_1}{4} + \frac{q_2}{4} + \frac{q_3}{4}\right]$$

Solving these equations, we get

$q_1 = +\frac{200}{9} \times 10^{-12}$ C, $q_2 = -200 \times 10^{-12}$ C and

$q_3 = \frac{3200}{9} \times 10^{-12}$ C

(a) **At $r = 1.25$ cm**

$$V = \frac{9 \times 10^9}{10^{-2}} \left[\frac{(200/9) \times 10^{-12}}{1.25} - \frac{200 \times 10^{-12}}{2} + \frac{(3200/9) \times 10^{-12}}{4}\right]$$

$= 6$ V $\qquad$ **Ans.**

(b) **Potential at $r = 2.5$ cm**

$$V = \frac{9 \times 10^9}{10^{-2}} \left[\frac{(200/9) \times 10^{-12}}{2.5} - \frac{200 \times 10^{-12}}{2.5} + \frac{(3200/9) \times 10^{-12}}{4}\right]$$

$= 16$ V $\qquad$ **Ans.**

(c) Electric field at $r = 1.25$ cm will be due to charge q_1 only.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r^2}$$

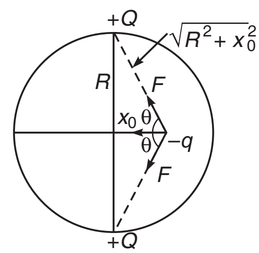
$$= \frac{9 \times 10^9 \times (200/9) \times 10^{-12}}{(1.25 \times 10^{-2})^2}$$

$= 1.28 \times 10^3$ V/m $\qquad$ **Ans.**

18. (a) $F_{net} = 2F \cos\theta$

$$= \frac{2kQ.q}{(\sqrt{R^2 + x_0^2})^2} \cdot \frac{x_0}{\sqrt{R^2 + x_0^2}} \qquad \left(\text{Here, } k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$= \frac{2kQqx_0}{(R^2 + x_0^2)^{3/2}}$$



We can generalised the force by putting $x_0 = x$, we have

$$F = -\frac{2kQqx}{(R^2 + x^2)^{3/2}} \qquad \textbf{Ans.}$$

(b) Motion of bead will be periodic between

$x = \pm x_0$ $\qquad$ **Ans.**

(c) For $\frac{x}{R} << 1, R^2 + x^2 \approx R^2$

or $\quad F = -\left(\frac{2kQq}{R^3}\right)x$ or $a = \frac{F}{m} = -\left(\frac{2kQq}{mR^3}\right)x$

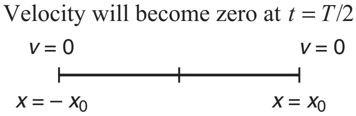
Since $a \propto -x$, motion will be simple harmonic in nature.

Comparing with $a = -\omega^2 x$, $\quad \omega = \sqrt{\frac{2kQq}{mR^3}}$

$x = x_0 \cos\omega t$ (as the particle starts from extreme position)

$$v = \frac{dx}{dt} = -\omega\, x_0 \sin\omega t \qquad \textbf{Ans.}$$

(d) Velocity will become zero at $t = T/2 = \pi/\omega$



or $$t = \pi\sqrt{\frac{mR^3}{2kQq}} \qquad \textbf{Ans.}$$

19. Net charge between $r = a$ to $r = r$ would be

$$Q = \int_a^r \rho(4\pi r^2)\, dr = \int_a^r \frac{C}{r}(4\pi r^2)\, dr$$

$$Q = 2\pi C(r^2 - a^2)$$

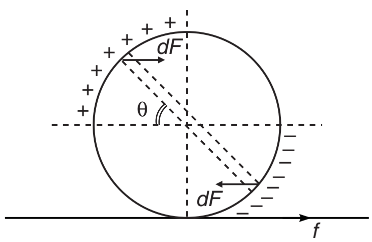
$$E_r = \frac{k(Q+q)}{r^2} = \frac{k[2\pi C(r^2 - a^2) + q]}{r^2}$$

From this expression, we can see that it we put

$$C = \frac{q}{2\pi a^2}$$

$$E_r = \frac{kq}{a^2} = \text{constant}$$ **Ans.**

20. $dF = \lambda(Rd\theta)E_0$



Perpendicular distance between two equal and opposite pairs of dF will be

$$r_\perp = 2R \sin\theta$$

$\therefore \quad d\tau = dFr_\perp = 2\lambda R^2 E_0 \sin\theta d\theta$

$\therefore \quad \tau = \int_0^{\pi/2} d\tau = 2\lambda R^2 E_0$ (clockwise)

These pairs of forces will not provide net force.

Let force of friction on ring is f in forward direction.

For pure rolling to take place,

$$a = R\alpha$$

or $\quad \frac{f}{m} = R\left[\frac{\tau - fR}{mR^2}\right]$

or $\quad f = \frac{\tau}{R} - f$

or $\quad f = \frac{\tau}{2R} = \lambda RE_0$ **Ans.**

21. Two forces will act on the tank.

(a) Electrostatic force,

(b) Thrust force.

Let v be the velocity at any instant. Then,

$$F_{\text{net}} = QE - mnv$$

or $\quad (m_0 + mnt)\frac{dv}{dt} = QE - mnv$

or $\quad \int_0^v \frac{dv}{QE - mnv} = \int_0^t \frac{dt}{m_0 + mnt}$

or $\quad \ln\left(\frac{QE}{QE - mnv}\right) = \ln\left(\frac{m_0 + mnt}{m_0}\right)$

or $\quad \frac{QE}{QE - mnv} = \frac{m_0 + mnt}{m_0}$

or $\quad v = QE\left(\frac{t}{m_0 + mnt}\right)$ **Ans.**

22. $qE = 30$ N, vertical component of electric force $= 30 \sin 30° = 15$ N and horizontal component of electric force $= 30 \cos 30° = 15\sqrt{3}$ N

$$a_y = \frac{mg - 15}{m} = \frac{30 - 15}{3} = 5 \text{ m/s}^2 \text{ (downwards)}$$

$$a_x = \frac{15\sqrt{3}}{3} = 5\sqrt{3} \text{ m/s}^2$$

$$T_1 = \frac{2u_y}{a_y} = \frac{2 \times 20 \sin 30°}{5} = 4 \text{ s}$$

$$T_2 = eT_1 = 2 \text{ s}$$

Horizontal velocity after first drop

$$= (20 \cos 30°) + a_x T_1$$
$$= (10\sqrt{3}) + (5\sqrt{3})4$$
$$= 30\sqrt{3} \text{ m/s}$$

$\therefore$ Horizontal distance travelled between first drop and second drop

$$= (30\sqrt{3})T_2 + \frac{1}{2}a_x T_2^2$$
$$= (30\sqrt{3})(2) + \frac{1}{2}(5\sqrt{3})(2)^2$$
$$= 70\sqrt{3} \text{ m}$$ **Ans.**

25. Capacitors

INTRODUCTORY EXERCISE 25.1

1. $U = \frac{1}{2}\frac{q^2}{C}$

$\therefore \quad [C] = \left[\frac{q^2}{U}\right] = \left[\frac{A^2T^2}{ML^2T^{-2}}\right]$

$= [M^{-1}L^{-2}T^4A^2]$

2. Charge does not flow if their potentials are same.

3. $q_1 = C_1V_1 = 10\,\mu C$

$q_2 = C_2V_2 = -40\,\mu C$

(a) $V = \frac{q_{\text{Total}}}{C_{\text{Total}}} = \frac{-30\,\mu C}{3\,\mu F} = -10$ volt

(b) $q_1' = C_1V$ and $q_2' = C_2V$

(c) $\Delta U = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

INTRODUCTORY EXERCISE 25.2

1. $q = CV$

2. (a) $V = \frac{q}{C}$

(b) $C = \frac{\varepsilon_0 A}{d}$

$\therefore \quad A = \frac{Cd}{\varepsilon_0}$

(c) $\sigma = \frac{q}{A}$

3. (a) $K = \frac{E_0}{E} = \frac{3.20 \times 10^5}{2.50 \times 10^5}$

$= 1.28$

(b) $\sigma_i = \sigma_0\left(1 - \frac{1}{K}\right)$

$= (E_0\varepsilon_0)\left(1 - \frac{1}{K}\right)$

$= (3.20 \times 10^5)(8.86 \times 10^{-12})\left(1 - \frac{1}{1.28}\right)$

$= 6.2 \times 10^{-7}$ C/m^2

INTRODUCTORY EXERCISE 25.3

1. $C_{\text{net}} = 2\,\mu F$, $q = CV = 2 \times 15 = 30\,\mu C$

Now, this q will be distributed between 4 μF and 2 μF in direct ratio of their capacities.

2. $C_{\text{net}} = 3\,\mu F$, $q = CV = 3 \times 40 = 120\,\mu C$.

Now, this q will be distributed between 9 μF and 3 μF in the direct ratio of their capacities.

Exercises

LEVEL 1

Assertion and Reason

1. Capacitance of conductor depends on the dimension of the conductor and the medium in which this conductor is kept.

3. Energy supplied by the battery is

$$\Delta qV = (CV)(V) = CV^2.$$

Energy stored in the capacitor is $\frac{1}{2}CV^2$.

4. In graph-1, discharging is slow.

Hence, $\tau_{C_1} > \tau_{C_2}$

Further,

$$\tau_C = CR$$

$\therefore \quad \tau_C \propto R \qquad$ (as C = constant)

5. Charges are same, if initially the capacitors are uncharged.

Further, $\quad V = \frac{q}{C}$

Hence, $\quad V \propto \frac{1}{C}$

if q is same.

6. Charge (or current) will not flow in the circuit as they have already the same potential, which is a condition of parallel grouping.

Further,

$$\frac{q_1}{q_2} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2} = \frac{1}{2}$$

7. Capacitor and R_2 are short-circuited. Hence, current through R_2 is zero and capacitor is not charged.

8. Capacitor and resistance in its own wire are directly connected with the battery. Hence, time constant during charging is CR.

9. $U = \frac{1}{2}\frac{q^2}{C}$ or $U \propto \frac{1}{C}$ as q is same in capacitors (if initially they are uncharged)

10. By inserting dielectric slab, value of C_2 will increase. In series, potential difference distributes in inverse ratio of capacitance. If capacitance C_2 is increased PD across C_2 will decrease. If C_2 is increased, charge on capacitors will also increase. So, positive charge or current flows in clockwise direction.

Objective Questions

1. $F = qE = q\left(\frac{\sigma}{2\varepsilon_0}\right) = q\left(\frac{q}{2A\varepsilon_0}\right)$ q will not change.

$\therefore \quad F = \text{constant}$

3. $C = C_1 + C_2$

$$= (4\pi\varepsilon_0 a) + (4\pi\varepsilon_0 b)$$
$$= 4\pi\varepsilon_0 (a + b)$$

4. $V_{\text{net}} = V_1 + V_2 + \ldots$ (in series)

$$= V + V + \ldots$$
$$= nV$$

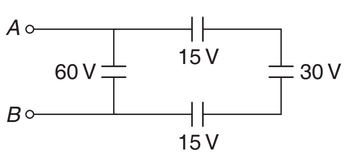
5. $V_{32} = V_5 = 6\text{ V}$

$\therefore \quad q_5 = CV = 30\,\mu\text{C}$

$$q_{32} = \left(\frac{3 \times 2}{3 + 2}\right) \times 6 = 7.2\,\mu\text{C}$$

$\therefore \quad \frac{q_5}{q_2} = \frac{30}{7.2}$

6.

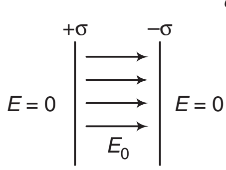


7. $\because \quad qE = mg$

$$q\left(\frac{V}{d}\right) = \left(\frac{4}{3}\pi r^3 \rho\right)g$$

$$V \propto \frac{r^3}{q}$$

8.



$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0} = \text{constant}$

9. At $t = 0$, when capacitor is under charged, equivalent resistance of capacitor $= 0$

In this case, $6\,\Omega$ and $3\,\Omega$ are parallel (equivalent $= 2\,\Omega$)

$\therefore \quad R_{\text{net}}\ (1 + 2)\,\Omega = 3\,\Omega$

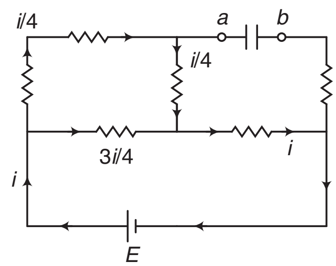
$\therefore \quad$ Current from battery $= \frac{12}{3} = 4\text{A}$

$= $ Current through $1\,\Omega$ resistor

10. Final potential difference $= E$

$\therefore \quad$ Final charge $= EC$

11.



$$V_a - \frac{i}{4}R - iR = V_b$$

$\therefore \quad V_a - V_b = \frac{5}{4}\,iR = 10 \quad \ldots\text{(i)}$

$$R_{\text{net}} = R + \frac{(3R)(R)}{(3R + R)}$$
$$= \frac{7}{4}R$$

$\therefore \quad i = \frac{E}{(7/4)R} = \left(\frac{4E}{7R}\right)$

Substituting in Eq. (i), we have

$$\left(\frac{5}{4}\right)\left(\frac{4E}{7R}\right) \times R = 10$$

$\therefore \quad E = 14\text{ V}$

12. All capacitors have equal capacitance. Hence, equal potential drop (= 2.5 V) will take place across all capacitors.

$$V_N - V_B = 2.5\text{ V}$$
$$0 - V_B = 2.5\text{ V}$$

$\therefore \quad V_B = -\,2.5\text{ V}$

Further, $\quad V_A - V_N = 3\,(2.5)\text{ V}$

$$= 7.5\text{ V}$$

$\therefore \quad V_A = +\,7.5\text{ V} \quad (\text{as } V_N = 0)$

13. $q = CV = 200\,\mu\text{C}$

In parallel, the common potential is given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{200\,\mu\text{C}}{(2+2)\,\mu\text{F}} = 50\text{ V}$$

Heat loss $= U_i - U_f$

$$= \frac{1}{2}(2\times10^{-6})(100)^2 - \frac{1}{2}(4\times10^{-6})(50)^2$$

$$= 5\times10^{-3}\text{J}$$

$$= 5\text{ mJ}$$

14. $$P = i^2R = (i_0e^{-t/\eta})^2 R$$

$$= (i_0^2R)\,e^{-2t/\eta}$$

$$= P_0e^{-t/(\eta/2)}$$

Hence, the time constant is $\frac{\eta}{2}$.

15. Common potential in parallel grouping

$$= \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{EC}{2C} = \frac{E}{2}$$

16. $V_A - 6 - 3\times2 + \frac{9}{1} - 3\times3 = V_B$

$\therefore \quad V_A - V_B = 12\text{ V}$

17. In steady state condition, current flows from outermost loop.

$$i = \frac{12}{6+2} = 1.5\text{ A}$$

Now, $V_C = V_{6\Omega} = iR$

$$= 1.5\times6 = 9\text{ V}$$

$\therefore \quad q = CV_C = 18\,\mu\text{C}$

19. Horizontal range,

$$R = \frac{2u_x\times u_y}{g} = l \quad \dots\text{(i)}$$

Maximum height, $H = \frac{u_y^2}{2g} = d \quad \dots\text{(ii)}$

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{1}{4}\left(\frac{u_y}{u_x}\right) = \frac{d}{l}$$

or $$\frac{u_y}{u_x} = \frac{u\sin\theta}{u\cos\theta}$$

$$= \tan\theta = \frac{4d}{l}$$

20. $V = Ed = \left(\frac{\sigma}{2\varepsilon_0}\right)d$

$\therefore \quad d = \frac{2\varepsilon_0V}{\sigma}$

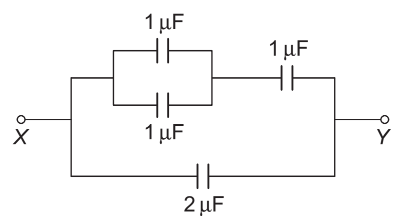
$$= 2\frac{(8.86\times10^{-12})(5)}{10^{-7}}$$

$$= 0.88\times10^{-3}\text{m}$$

$$= 0.88\text{ mm}$$

21. Three capacitors (consisting of two loops are short-circuited).

22. The equivalent circuit is as shown below.



23. $C_1 = C_{\text{RHS}} + C_{\text{LHS}}$

$$= \frac{K_2\,\varepsilon_0(A/2)}{d} + \frac{K_1\varepsilon_0(A/2)}{d}$$

$$= \frac{\varepsilon_0A}{2d}(K_1+K_2) = \frac{5\varepsilon_0A}{2d}$$

$$C_2 = \frac{\varepsilon_0A}{d - d/2 - d/2 + \frac{d/2}{K_1} + \frac{d/2}{K_2}}$$

$$= \frac{2\varepsilon_0A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$$

$$= \frac{12\,\varepsilon_0A}{5d}$$

$\therefore \quad \frac{C_1}{C_2} = \frac{25}{24}$

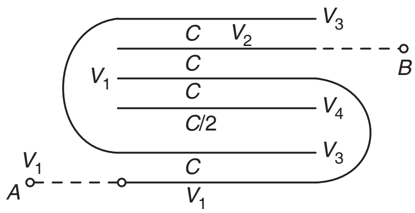
24. A balanced Wheatstone bridge is parallel with C.

25. First three circuits are balanced Wheatstone bridge circuits.

26. $C = C_{\text{LHS}} + C_{\text{RHS}}$

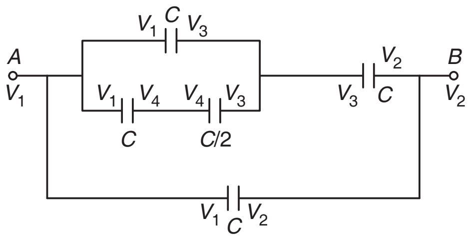
$$= \frac{K_1\varepsilon_0\,(A/2)}{d} + \frac{\varepsilon_0(A/2)}{d - d/2 - d/2 + \frac{d/2}{K_2} + \frac{d/2}{K_3}}$$

$$= \frac{\varepsilon_0A}{d}\left[\frac{K_1}{2} + \frac{K_2K_3}{K_2+K_3}\right]$$

27.


$$C = \frac{\varepsilon_0 A}{d} = 7\,\mu\text{F}$$

The equivalent circuit is as shown in figure.



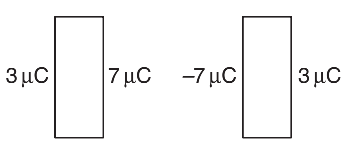
$$C_{AB} = \frac{11}{7}C = \frac{11}{7}(7\,\mu\text{F})$$
$$= 11\,\mu\text{F}$$

Subjective Questions

1. Charge on outermost surfaces

$$= \frac{q_{\text{total}}}{2} = \frac{(10-4)\,\mu\text{C}}{2}$$
$$= 3\,\mu\text{C}$$

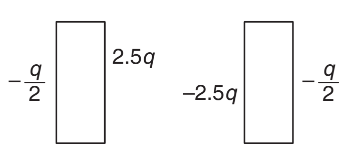
Hence, charges are as shown below.



2. Charge on outermost surfaces

$$= \frac{q_{\text{total}}}{2} = \frac{2q-3q}{2} = -\frac{q}{2}$$

Hence, charge on different faces are as shown below.



Electric field and hence potential difference between the two plates is due to ± 2.5 q.

$$\text{PD} = Ed$$

$$= \left(\frac{\sigma}{\varepsilon_0}\right)d = \left(\frac{2.5\,q\,d}{\varepsilon_0 A}\right)$$

Capacitance, $C = \dfrac{\varepsilon_0 A}{d}$

3. All three capacitors are in parallel with the battery. PD across each of them is 10 V. So, apply $q = CV$ for all of them.

4. Capacitor and resistor both are in parallel with the battery. PD across capacitor is 10 V. Now, apply $q = CV$.

5. In steady state, current flows in lower loop of the circuit.

$$i = \frac{30}{6+4} = 3\text{A}$$

Now, potential difference across capacitor = potential difference across 4 Ω resistance.

$$= iR$$
$$= (3)(4) = 12\text{ V}$$
$$\therefore \quad q = CV = (2\,\mu\text{F})(12\text{ V})$$
$$= 24\,\mu\text{C}$$

6. (a) $$C_{\text{net}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{2}{3}\,\mu\text{F}$$

$$q_{\text{net}} = C_{\text{net}}V$$
$$= \left(\frac{2}{3}\,\mu\text{F}\right)(1200\text{ V})$$
$$= 800\,\mu\text{C}$$

In series, q remains same.

$$\therefore \quad q_1 = q_2 = 800\,\mu\text{C}$$
$$V_1 = \frac{q_1}{C_1} = 800\text{ V}$$

and $$V_2 = \frac{q_2}{C_2} = 400\text{ V}$$

(b) Now, total charge will become 1600 μC. This will now distribute in direct ratio of capacity.

$$\therefore \quad \frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{1}{2}$$
$$q_1 = \left(\frac{1}{3}\right)(1600) = \frac{1600}{3}\,\mu\text{C}$$
$$q_2 = \left(\frac{2}{3}\right)(1600) = \left(\frac{3200}{3}\right)\mu\text{C}$$

They will have a common potential (in parallel) given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{1600\,\mu C}{3\,\mu F}$$

$$= \frac{1600}{3}\,V$$

7. Charge, $q = CV = 10^4\,\mu C$

In parallel, common potential is given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$20 = \frac{(10^4\,\mu C)}{(C + 100)\,\mu C}$$

Solving this equation, we get

$$C = 400\,\mu F$$

8. Charge supplied by the battery,

$$q = CV$$

Energy supplied by the battery,

$$E = qV = CV^2$$

Energy stored in the capacitor,

$$U = \frac{1}{2}CV^2$$

$\therefore$ Energy dissipated across R in the form of heat

$$= E - U = \frac{1}{2}CV^2 = U$$

9. $i = i_0\, e^{-t/\tau_C}$

Putting $i = \frac{i_0}{2}$, we get

$$t = (\ln 2)\,\tau_C = (0.693)\,\tau_C$$

10. Both capacitors have equal capacitance. Hence, half-half charge distribute over both the capacitors.

$$q_1 = q_2 = \frac{q_0}{2}$$

q_1 decreases exponentially from q_0 to $\frac{q_0}{2}$ while q_2 increases exponentially from 0 to $\frac{q_0}{2}$.

Corresponding graphs and equation are given in the answer.

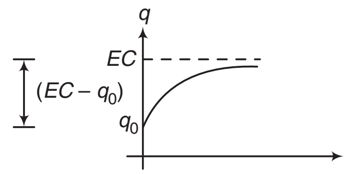
Time constant of two exponential equations will be

$$\tau_C = (C_{net})\,R = \left(\frac{C}{2}\right)R = \frac{CR}{2}$$

11. $q_i = q_0$

$q_f = EC$

Now, charge on capacitor changes from q_i to q_f exponentially.



$$\therefore \quad q = q_0 + (EC - q_0)(1 - e^{-t/\tau_C})$$

$$= EC\,(1 - e^{-t/\tau_C}) + q_0 e^{-t/\tau_C}$$

Here, $\tau_C = CR$

12. (a) Immediately after the switch is closed whole current passes through C_1.

$$\therefore \quad i = E/R_1$$

(b) Long after switch is closed no current will pass through C_1 and C_2.

$$\therefore \quad i = \frac{E}{R_1 + R_3}$$

13. (a) At $t = 0$ equivalent resistance of capacitor is zero. R_1 and R_2 are in parallel across the battery PD across each is E.

$$\therefore \quad i_{R_1} = E/R_1$$

$$i_{R_2} = E/R_2$$

(b) In steady state, no current flow through capacitor wire. PD across R_1 is E.

$$\therefore \quad i_{R1} = E/R_1 \quad \text{and} \quad i_{R2} = 0$$

(c) In steady state, potential difference across capacitor is E.

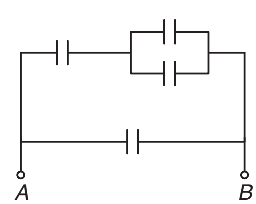
$$\therefore \quad U = \frac{1}{2}CV^2 = \frac{1}{2}CE^2$$

(d) When switch is opened, capacitor is discharged through resistors R_1 and R_2.

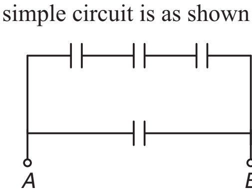
$$\tau_C = CR_{net}$$

$$= C\,(R_1 + R_2)$$

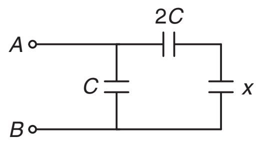
14. (a) Simple circuit is as shown below.



(b) The simple circuit is as shown below.



(c) Let $C_{AB} = x$. Then,



Now, $C_{AB} = C + \dfrac{(2C)(x)}{2C + x}$

or $x = C + \dfrac{2Cx}{2C + x}$

Solving this equation, we get

$$x = 2C$$

15. (a) $V = 660$ V across each capacitor

Now, $q = CV$ for both

(b) $q_{net} = q_1 + q_2$

$= (3.96 - 2.64) \times 10^3$ C

$= 1.32 \times 10^{-3}$ C

Now, common potential

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{13.2 \times 10^{-3}}{10 \times 10^{-6}}$$

$= 132$ V

Now, apply $q = CV$ for both capacitors.

16. $u = \dfrac{1}{2} \varepsilon_0 E^2$

$= \dfrac{1}{2} \varepsilon_0 \left(\dfrac{V}{d}\right)^2$

17. $d = \dfrac{V_{max}}{E_{max}}$

$C = \dfrac{K\varepsilon_0 A}{d}$

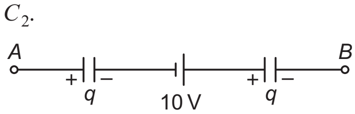
$\therefore \quad A = \dfrac{dC}{K\,\varepsilon_0} = \dfrac{(V_{max})(C)}{(K)(E_{max})(\varepsilon_0)}$

18. $0.1 = \dfrac{1}{2}(C_1 + C_2)(2)^2$ …(i)

$1.6 \times 10^{-2} = \dfrac{1}{2}\left(\dfrac{C_1 C_2}{C_1 + C_2}\right)(2)^2$ …(ii)

Solving these two equations, we can find C_1 and C_2.

19.



q shown in figure is in μC.

Now, $V_A - \dfrac{q}{1} + 10 - \dfrac{q}{2} = V_B$

or $V_A - V_B = \dfrac{3q}{2} - 10 = 5$

$\therefore \quad q = 10$ μC

Now, $V = \dfrac{q}{C}$ across each capacitor.

20. See the answer.

21. In series, potential difference distributes in inverse ratio of capacitance.

$\therefore \quad \dfrac{V_A}{V_B} = \dfrac{C_B}{C_A} = \dfrac{C_2}{C_1} = \dfrac{60}{40} = \dfrac{3}{2}$

$\therefore \quad C_2 = 1.5\, C_1$ …(i)

Now, $\dfrac{V'_A}{V'_B} = \dfrac{C'_B}{C'_A}$

or $\dfrac{10}{90} = \dfrac{C_2}{(C_1 + 2)}$

or $C_1 + 2 = 9C_2$ …(ii)

Solving Eqs. (i) and (ii), we get

$C_1 = 0.16$ μF

and $C_2 = 0.24$ μF

22. (a) $q = CV$

(b) $C = \dfrac{\varepsilon_0 A}{d}$ or $C \propto \dfrac{1}{d}$

If d is doubled, C will remain half. Hence, q will also remain half.

(c) $q = CV = \left(\dfrac{\varepsilon_0 A}{d}\right)V = \dfrac{\varepsilon_0 (\pi R^3)V}{d}$

or $q \propto R^2$

R is doubled. Hence, q will become four times or 480 μC.

23. Energy lost = energy stored $= \dfrac{1}{2}CV^2$

24. (a) $C = \dfrac{\varepsilon_0 A}{d}$

(b) $q = CV$

(c) $E = \dfrac{V}{d}$

25. (a) $\dfrac{1}{C_{net}} = \dfrac{1}{8.4} + \dfrac{1}{8.2} + \dfrac{1}{4.2}$

$\therefore \quad C_{net} = 2.09$ μF

$q_{net} = C_{net} V$

$= (2.09)(36)$

$= 75.14$ μC ≈ 76 μC

In series, charge remains same in all capacitors.

(b) $U_{total} = \frac{1}{2} C_{net} V^2$

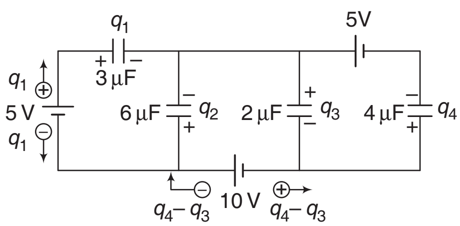
(c) $q_{total} = (3)(76)\,\mu C = 288\,\mu C$

Now common potential in parallel,

$$V = \frac{q_{total}}{C_{total}} = \frac{228\,\mu C}{(8.4 + 8.2 + 4.2)\,\mu F}$$

(d) $U_{total} = \frac{1}{2} C_{net} V^2$

26.



If we see the charge on positive plate of 6 μF capacitor, then

$$q_2 = -q_1 - (q_4 - q_3) \qquad \ldots\text{(i)}$$

Now, applying three loop equations, we have

$$5 - \frac{q_1}{3} + \frac{q_2}{6} = 0 \qquad \ldots\text{(ii)}$$

$$10 - \frac{q_2}{6} - \frac{q_3}{2} = 0 \qquad \ldots\text{(iii)}$$

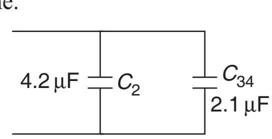
$$5 - \frac{q_3}{2} - \frac{q_4}{4} = 0 \qquad \ldots\text{(iv)}$$

Solving these four equations, we can find q_1, q_2, q_3 and q_4.

27. (a) Simple series and parallel grouping of capacitors.

(b)
$$q_{net} = C_{net} V$$
$$= (2.5 \times 10^{-6})(220)$$
$$= 5.5 \times 10^{-4}\,C$$

C_1, C_5 and equivalent of other three capacitors are in series. Hence, charges across them are same.



q_{total} will distribute between C_2 and C_{34} in direct ratio of capacitance.

$$\therefore \quad \frac{q_2}{q_{34}} = \frac{4.2}{2.1} = \frac{2}{1}$$

$$\therefore \quad q_2 = \frac{2}{3}(5.5 \times 10^{-4}) = 3.7 \times 10^{-4}\,C$$

$$q_{34} = \frac{1}{3}(5.5 \times 10^{-4}) = 1.8 \times 10^{-4}\,C$$

For finding PD across any capacitor, use the equation

$$C = \frac{q}{V}$$

28. In series, potential difference distributes in inverse ratio of capacitance.

$$\therefore \quad \frac{V_A}{V_B} = \frac{C_2}{C_1} \quad \text{or} \quad \frac{130}{100} = \frac{C_2}{C_1}$$

or
$$C_1 = \frac{C_2}{1.3} = C_A$$

K is made 2.5 times. Therefore, C_1 will also become 2.5 times.

$$C_1' = 2.5\,C_1 = \frac{2.5\,C_2}{1.3}$$

or
$$\frac{C_1'}{C_2} = \frac{25}{13}$$

Now,
$$\frac{V_A'}{V_B'} = \frac{C_2}{C_1'} = \frac{13}{25}$$

or
$$V_A' = \left(\frac{13}{13 + 25}\right)(230) = 78.68\,V$$

$$V_B' = 230 - V_B' = 151.32\,V$$

29. $C_{23} = \frac{2 \times 3}{2 + 3} = 1.2\,\mu F$

$$q_{total} = C_1 V = 110\,\mu C$$

Common potential in parallel is given by

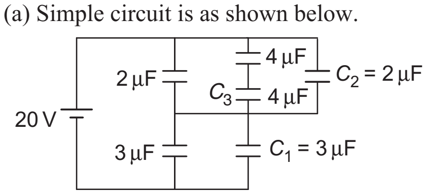
$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{110}{1 + 1.2} = 50\,V$$

$$q_{23} = (C_{23})V = 60\,\mu C$$

So, this much charge flows through the switch.

30. (a) Simple circuit is as shown below.



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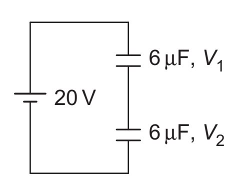


(b) $q_{net} = (C_{net})V = (34\,\mu F)(20\,V) = 60\,\mu C$

(c) Upper network and lower network both have same capacitance = 6 μF

$$V_1 - V_2 = \frac{20}{2} = 10\,V$$

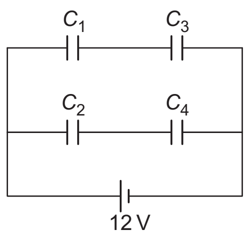
$$V_{C_1} = 10\,V$$

$$\therefore \quad q_{C_1} = (C_1)(V_{C_1}) = 30\,\mu C$$

(d) $V_{C_2} = 10\,V$, $\therefore$ $q_{C_2} = (C_2)(V_{C_2}) = 20\,\mu C$

(e) $V_{C_3} = 5\,V$, $\therefore$ $q_{C_3} = (C_3)(V_{C_3}) = 20\,\mu C$

31. (a)



$$C_{13} = \frac{1 \times 3}{1 + 3} = \frac{3}{4}\,\mu F$$

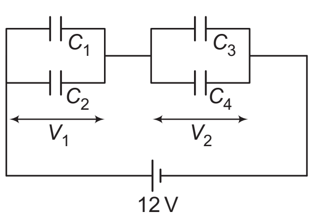
$$C_{24} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\,\mu F$$

$$V_{C_1C_3} = V_{C_2C_4} = 12\,V$$

$$\therefore \quad q_1 = q_3 = (C_{13})\,(V_{C_1C_3}) = \frac{3}{4} \times 12 = 9\,\mu C$$

$$q_2 = q_4 = (C_{24})\,(V_{C_2C_4}) = \frac{4}{3} \times 12 = 16\,\mu C$$

(b)



$$C_{12} = \frac{1 \times 2}{1 + 2} = \frac{2}{3}\,\mu F$$

$$C_{34} = \frac{3 \times 4}{3 + 4} = \frac{12}{7}\,\mu F$$

$$\frac{V_1}{V_2} = \frac{C_{34}}{C_{12}} = \frac{12/7}{2/3} = \frac{18}{7}$$

$$\therefore \quad V_1 = \left(\frac{18}{25}\right)(12) = 8.64\ V$$

$$V_2 = 12 - 8.64 = 3.36\ V$$

Now, we can apply $q = CV$ for finding charge on different capacitors.

32. $q_{total} = C_1V_0$

After switch is thrown towards right, C_{23} and C_1 are in parallel. The common potential is

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{C_1V_0}{C_1 + \left(\dfrac{C_2C_3}{C_2 + C_3}\right)}$$

$$\text{Now,} \quad q_{C_1} = C_1V = \frac{C_1^2V_0}{C_1 + \left(\dfrac{C_2C_3}{C_2 + C_3}\right)}$$

This is the same result as given in the answer.

$$q_{C_2} = q_{C_3} = C_{23}V$$

$$= \left(\frac{C_2C_3}{C_2 + C_3}\right)\left[\frac{C_1V_0}{C_1 + \dfrac{C_2C_3}{C_2 + C_3}}\right]$$

33. (a)

$$q = C_iV = \left(\frac{\varepsilon_0 A}{d}\right)V$$

$$V_f = \frac{q}{C_f} = \frac{(\varepsilon_0 AV/d)}{(\varepsilon_0 A/2d)} = 2V$$

(b)

$$U_i = \frac{1}{2}C_iV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$$

$$U_f = \frac{1}{2}C_fV_f^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{2d}\right)(2V)^2$$

$$= \left(\frac{\varepsilon_0 A}{d}\right)V^2$$

(c) $W = U_f - U_i = \dfrac{1}{2}\left(\dfrac{\varepsilon_0 A}{d}\right)V^2$

34. (a) After long time, capacitor gets fully charged by E_1.

$$\therefore \quad i_C = 0$$

$$\text{and} \quad i_{R_1} = i_{R_2} = \frac{E_1}{R_1 + R_2}$$

$$= \frac{20}{20 \times 10^3}$$

$$= 10^{-3}\ A = 1\ mA$$

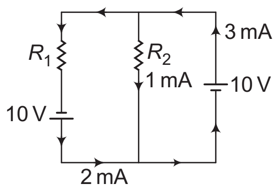
(b) In steady state (with E_1).

$$V_C = V_{R_2} = V_{R_1}$$

$$= \frac{E_1}{2} = 10\ V$$

Now, when the switch is shifted to position B, capacitor (at $t = 0$) behaves like a battery of 10 V.

The circuit in that case is as shown below.



Now, with the help of Kirchhoff's laws we can find different currents. Final currents are shown in the diagram.

35. (a) $V_a = 18$ V and $V_b = 0$ as no current flow through the resistors.

$\therefore \quad V_a - V_b = 10$ V

(b) $V_a - V_b = +$ ve.

Hence, $V_a > V_b$

(c) Current flows through two resistors,

$$i = \frac{18 - 0}{6 + 3} = 2\text{A}$$

$\therefore \quad V_b - 0 = iR = 2 \times 3$

or $\quad V_b = 6$V

(d) Initially, $\quad V_{3\mu F} = V_{6\mu F} = 18$V

$\therefore \quad q_{3\mu F} = 54\ \mu$C $\quad (q = CV)$

and $\quad q_{6\mu F} = 108\ \mu$C

Finally,

$$V_{6\mu F} = V_{6\Omega} = iR$$
$$= 2 \times 6 = 12 \text{ V}$$

$\therefore \quad q_{6\mu F} = 72\ \mu$C

$V_{3\mu F} = V_{3\Omega} = 6$V

$\therefore \quad q_{3\mu F} = 18\ \mu$C

$\Delta q = q_f - q_i = -36\ \mu$C on both capacitors.

36. (a) In resistors (in series) potential drops in direct ratio of resistance and in capacitors (in series) potential drops in inverse ratio of capacitance.

$$18 - V_a = \left(\frac{6}{6+3}\right)(18)$$

$\therefore \quad V_a = 6$ V

$$18 - V_b = \left(\frac{3}{6+3}\right)(18)$$

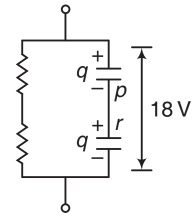
$V_b = 12$ V

(c) $V_{3\mu F} = V_{3\Omega} = iR = \left(\dfrac{18-0}{6+3}\right)(3) = 6$V

$\therefore \quad V_b - 0 = 6$ V

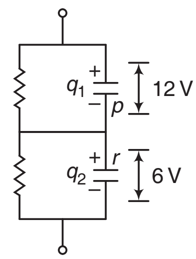
$\therefore \quad V_b = 6$ V

(d) Initially,



$$q = C_{\text{net}} V = (2\ \mu\text{F})\ (18)$$
$$= 36\ \mu\text{C}$$

Finally,



$$q_1 = 72\ \mu\text{C}$$
$$q_2 = 18\ \mu\text{C}$$

Charge flow from S = (Final charge on plates p and r) – (Initial charges on plates p and r)

$$= (-72 + 18) - (-36 + 36)$$
$$= -54\ \mu\text{C}$$

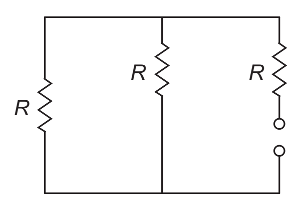
37. (a) In steady state,

$$V_C = \frac{V}{2}$$

$\therefore$ Steady state charge,

$$q_0 = CV_C = \frac{CV}{2}$$

For equivalent value of τ_C : We short circuit the battery and find the value of R_{net} across capacitors and then



$$R_{\text{net}} = \frac{3R}{2}$$

$\therefore \quad \tau_C = CR_{\text{net}} = \dfrac{3RC}{2}$

Now, $\quad q = q_0\,(1 - e^{-t/\tau_C})$

(b) **At $t = 0$,** capacitor offers zero resistance.

$$R_{\text{net}} = \frac{3R}{2}$$

$$\therefore \quad i = \frac{V}{3R/2} = \frac{2V}{3R}$$

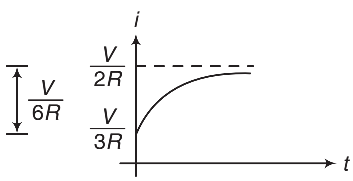
$$i_C = i_{AB} = \frac{1}{2} = \frac{V}{3R}$$

At t $= \infty$, capacitor offers infinite resistance. So, $i_C = 0$.

$$\therefore \quad i_{\text{battery}} = i_{AB} = \frac{V}{2R}$$

Now, current through AB increases exponentially from $\frac{V}{3R}$ to $\frac{V}{2R}$ with same time constant.

$(i\text{-}t)$ graph is as shown below.



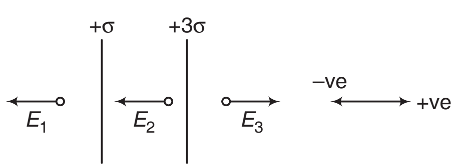
$(i\text{-}t)$ equation corresponding to this graph is

$$i = \frac{V}{3R} + \frac{V}{6R}(1 - e^{-t/\tau_C})$$

LEVEL 2

Single Correct Option

1.



$$E_1 = \frac{\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = \frac{2\sigma}{\varepsilon_0} = E_3$$

$$E_2 = \frac{3\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

E_1 and E_2 are in the negative direction and E_3 in positive direction.

2. Let E be the external field (toward right). Then,

$$E - \frac{\sigma}{2\varepsilon_0} = 8 \qquad \ldots\text{(i)}$$

$$E + \frac{\sigma}{2\varepsilon_0} = 12 \qquad \ldots\text{(ii)}$$

Solving these equations, we get $\sigma = 4\varepsilon_0$

3. At $t = 0$ when capacitors are initially uncharged, their equivalent resistance is zero. Hence, whole current passes through these capacitors.

4. Changing current is given by

$$i = i_0 e^{-t/\tau_C}$$

or

$$i = \frac{V}{R} e^{-t/CR}$$

If we have take log on both sides, we have

$$\ln(i) = \ln\left(\frac{V}{R}\right) - \left(\frac{1}{CR}\right) t$$

Hence, $\ln(i)$ *versus* t graph is a straight line with slope $\left(-\frac{1}{CR}\right)$ and intercept $+\ln\left(\frac{V}{R}\right)$.

Intercepts are same, but $|\text{slope}|_1 > |\text{slope}|_2$.

5. During charging of a capacitor 50% of the energy supplied by the battery is lost and only 50% is stored.

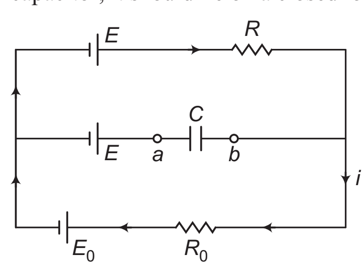
$$\therefore \quad \text{Total energy lost} = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(EC/2)^2}{C} = \frac{E^2C}{8}$$

Now, this total loss is in direct ratio $r : 2r$ or $1 : 2$

$$\therefore \quad \text{Energy lost in battery is } \frac{1}{3} \text{ rd of } \frac{E^2C}{8}.$$

6. Equal and opposite charges should transfer from two terminals of a battery. For charging of a capacitor, it should lie on a closed loop.

7.



$$i = \frac{[E - E_0]}{R + R_0}$$

Now, $V_a - E + E_0 + iR_0 = V_b$

$$\therefore \quad V_a - V_b = (E - E_0) - iR_0$$

$$= (E - E_0)\left[1 - \frac{R_0}{R + R_0}\right]$$

$$= \frac{R\,(E - E)}{R + R_0}$$

$$\therefore \quad q = C\,(V_a - V_b) = \frac{CR\,(E - E_0)}{R + R_0}$$

8. **Initially**

$$C_{net} = \frac{(C_0)(C_0)}{C_0 + C_0} = \frac{C_0}{2}$$

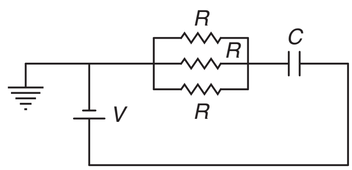
$$= 0.5\,C_0$$

Finally

$$C_{net} = \frac{(C_0/2)(2C_0)}{(C_0/2) + 2C_0}$$

$$= 0.4\,C_0$$

9. The simple circuit is as shown below.



$$R_{net} = \frac{R}{3}$$

$$\therefore \quad \tau_C = CR_{net} = \frac{CR}{3}$$

$$q = q_0\,(1 - e^{-t/\tau_C}),$$

where, $q_0 = CV$

10. Common potential in parallel grouping,

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{(2 \times 100) + (4 \times 50)}{2 + 4}$$

$$= \frac{200}{3}\text{ V}$$

$$\text{Loss} = U_i - U_f$$

$$= 10^{-6}\left[\left(\frac{1}{2} \times 2 \times 100 \times 100 + \frac{1}{2} \times 4 \times 50 \times 50\right) - \left(\frac{1}{2} \times 6 \times \frac{200}{3} \times \frac{200}{3}\right)\right]$$

$$= 1.7 \times 10^{-3}\text{J}$$

11. $V_0 = i_0 R = (10)(10) = 100$ V

After 2 s, current becomes $\frac{1}{4}$th. Therefore, after 1 s, current will remain half also called half-life.

$$t_{1/2} = (\ln 2)\,\tau_C = (\ln 2)\,CR$$

$$\therefore \quad C = \frac{(t_{1/2})}{(\ln 2)R} = \frac{1}{10 \ln 2}\text{ F}$$

$$\text{Total heat} = \frac{1}{2}CV_0^2$$

$$= \frac{1}{2} \times \frac{1}{10 \ln 2}(100)^2$$

$$= \frac{500}{\ln 2}\text{J}$$

12. Net capacitance between points A and P will be equal to the net capacitance between points P and B.

13. Total charge $= (2C)(4V) - CV$

$$= 7CV$$

Common potential after they are connected is

$$V_C = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$= \frac{7\,CV}{2C + C} = \frac{7}{3}V$$

$$\text{Heat} = U_i - U_f$$

$$= \frac{1}{2}CV^2 + \frac{1}{2}(2C)(4V)^2 - \frac{1}{2} \times 3C \times \left(\frac{7}{3}V\right)^2$$

$$= \frac{25}{3}CV^2$$

14. Total heat produced $= \frac{1}{2}CV^2$

$$= \frac{1}{2}(2\,\mu\text{F})(5)^2$$

$$= 25\,\mu\text{J}$$

Now, this should distribute in inverse ratio of resistors, as they are in parallel.

$$\therefore \quad \frac{H_{5\Omega}}{H_R} = \frac{R}{5}$$

or $\quad H_{5\Omega} = \left(\frac{R}{R+5}\right)$ (Total heat)

or $\quad 10 = \left(\frac{R}{R+5}\right)(25)$

Solving this equation, we get

$$R = \left(\frac{10}{3}\right)\Omega$$

15. In position-1, initial maximum current is

$$i_0 = \frac{V}{R} = \frac{10}{5} = 2\text{A}$$

At the given time, given current is 1A or half of the above value. Hence, at this is instant capacitor is also charged to half of the final value of 5 V.

Now, it is shifted to position-2 wherein steady state it is again charged to 5 V but with opposite polarity.

$$U_i = U_f = \frac{1}{2}CV^2 \qquad (\because\ V = 5\text{ V})$$

$\therefore$ Total energy supplied by the lower battery is converted into heat. But double charge transfer (from the normal) takes place from this battery.

$\therefore$ Heat produced = Energy supplied by the battery

$$= (\Delta q)V = (2CV)(V) = 2CV^2$$
$$= 2 \times 2 \times 10^{-6} \times (5)^2$$
$$= 100 \times 10^{-6}\text{ J}$$
$$= 100\ \mu\text{J}$$

16. Equivalent capacitance of 6 μF and 3 μF is also 2 μF and charge across it is also q or circuit is balanced. Hence, there is no flow of charge.

17. Two capacitors are in parallel.

$$\therefore \quad U = \frac{1}{2}C_{\text{net}}V^2$$
$$= \frac{1}{2}(2C)V^2 = CV^2$$
$$= \left(\frac{\varepsilon_0 A}{d}\right)V^2$$

18. Initially, the rate of charging is fast.

19. $V_{1\Omega} = 5 + 2 = 7\text{ V}$

$$\therefore \quad i_{1\Omega} = \frac{V}{R} = 7\text{A}$$
$$V_{2\mu\text{F}} = 6\text{ V}$$
$$\therefore \quad q_{2\mu\text{F}} = CV = 12\ \mu\text{C}$$

20. During charging capacitor and resistance of its wire are independently connected with the battery. Hence,

$$\tau_C = CR$$

During discharging capacitor is discharged through both resistors (in series). Hence,

$$\tau_C = C(2R) = 2CR$$

21. Total charge = 3 × 100 − 1 × 100 = 200 μC

Common potential (in parallel) after S is closed, is

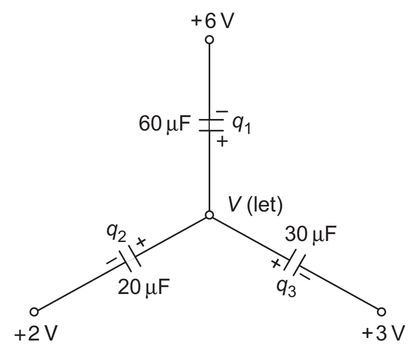
$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$
$$= \frac{200\ \mu\text{C}}{4\ \mu\text{F}}$$
$$= 50\text{ V}$$

22. $\dfrac{V_1}{V_{1.5}} = \dfrac{1.5}{1}$

$$\therefore \quad V_1 = \left(\frac{1.5}{1.5+1}\right)(30)$$
$$= 18\text{ V}$$
$$\frac{V_{2.5}}{V_{0.5}} = \frac{0.5}{2.5} = \frac{1}{5}$$
$$\therefore \quad V_{2.5} = \left(\frac{1}{1+5}\right)(30)$$
$$= 5\text{ V}$$

Now, $|V_{ab}| = V_1 - V_{2.5}$
$$= 13\text{ V}$$

23. In the figure,

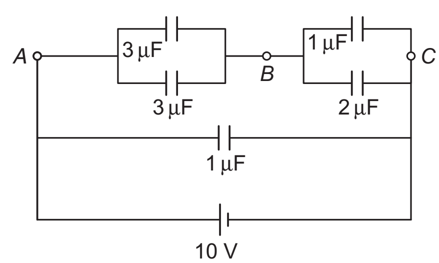


$$q_1 + q_2 + q_3 = 0$$
$$60(V-6) + 20(V-2) + 30(V-3) = 0$$

Solving this equation, we get

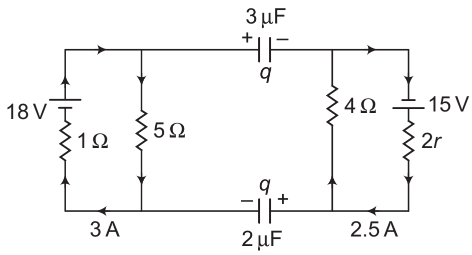
$$V = \frac{49}{11}\text{ V}$$

24.



$$\frac{V_{AB}}{V_{BC}} = \frac{(C)_{BC}}{(C)_{AB}} = \frac{3}{6} = \frac{1}{2}$$
$$\therefore \quad V_{AB} = \left(\frac{1}{1+2}\right)(10)\text{ V}$$
$$= \frac{10}{3}\text{ V}$$

25. Applying Kirchhoff's loop law in outermost loop, we have



$$-\frac{q}{3}+15-2\times 2.5-\frac{q}{2}-3\times 1+18=0$$

Solving this equation, we get

$$q=30\,\mu\text{C}$$

26. $\tau_C = CR = 6$ s

$$q_0 = CV = 10\,\mu\text{C}$$

Now, $q = q_0 e^{-t/\tau_C} = (10\,\mu\text{C})\,e^{-12/6}$

$$=\left(\frac{1}{e}\right)^2 (10\,\mu\text{C})$$

$$=(0.37)^2\,(10\,\mu\text{C})$$

27. $q=(E_1+E_2)\,C_{\text{net}}$

$$=(E_1+E_2)\frac{C_1C_2}{C_2+C_2}$$

$$V_{ab}=\frac{q}{C_2}=\left(\frac{E_1+E_2}{C_1+C_2}\right)C_1$$

28. $H_1 = H_2 = U_i - U_f$

The only change is by increasing the resistance τ_C increase. Hence, process of redistribution of charge slows down.

29. Just after the switch is closed C_1 is short-circuited and current passes through R_1 and C_1 only.

30.
$$i_1=\left(\frac{V}{2R}\right)e^{-\frac{t}{6\,CR}}$$

$\therefore$
$$i_2=\left(\frac{V}{R}\right)e^{-\frac{t}{CR}}$$

$\therefore$
$$\frac{i_1}{i_2}=\frac{e^{\frac{5t}{6\,CR}}}{2}$$

We can see that this ratio is increasing with time.

31. $\tau_C = CR$

$$=\left(\frac{K\,\varepsilon_0 A}{d}\right)\left(\frac{d}{A\sigma}\right)\qquad\left(R=\frac{l}{\sigma A}\right)$$

$$=\frac{K\varepsilon_0}{\sigma}$$

$$=\frac{5\times 8.86\times 10^{-12}}{7.4\times 10^{-12}}$$

$$=6\text{ s}$$

32. The given time is the half-life time of the exponentially decreasing equation.

$\therefore \quad t = t_{1/2} = (\ln 2)\,\tau_C = (\ln 2)\,CR_{\text{net}}$

$\therefore \quad R_{\text{net}} = \dfrac{t}{(\ln 2)\,C}$

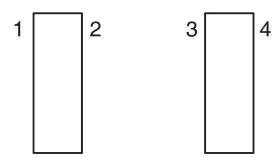
$$=\frac{2\,(\ln 2)\,\mu\text{s}}{(\ln 2)\,(0.5\,\mu\text{F})}=4\Omega$$

$\therefore$ Resistance of ammeter $= 2\Omega$

33. Four capacitors are in parallel charge across each is $q = CV$. Two surfaces of plate C marks two capacitors, one with B and other with D and C is connected to positive terminal of the battery. Hence,

$$q_C = 2CV = +\,40\,\mu\text{C}$$

34.



$$q_1=q_4=\frac{q_{\text{total}}}{2}=\frac{CV-CV+Q}{2}=\frac{Q}{2}$$

$$q_2=(Q+CV)-\frac{Q}{2}=\left(\frac{Q}{2}+CV\right)$$

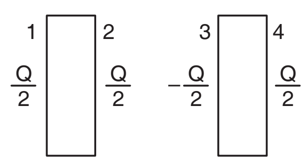
$$q_3=-\,q_2=-\left(\frac{Q}{2}+CV\right)$$

Electric field between two plates and hence the potential difference is due to q_2 and q_3 only.

$$\text{PD}=\frac{q_2}{C}=V+\frac{Q}{2C}$$

More than One Correct Options

1.



$$q_1=q_4=\frac{q_{\text{total}}}{2}=\frac{Q}{2}$$

$$q_2=Q-q_1=\frac{Q}{2}$$

$$q_3 = -q_2 = \frac{-Q}{2}$$

$$E_A = E_1 + E_4 \qquad \text{(towards left)}$$

$$= \frac{Q}{4A\varepsilon_0} + \frac{Q}{4A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$$E_C = -E_A$$

$$= \frac{Q}{2A\varepsilon_0} \qquad \text{(towards right)}$$

$$E_B = E_2 + E_3 \qquad \text{(towards right)}$$

$$= \frac{Q}{4A\varepsilon_0} + \frac{Q}{4A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

2. In steady state,

$$q_C = EC \text{ and } q_{2C} = 2EC$$

$$\tau_C = 2CR \text{ of both circuits}$$

At time t,

$$q_C = EC\ (1 - e^{t/\tau_C})$$

$$q_{2C} = 2EC\ (1 - e^{-t/\tau_C})$$

$$\therefore \quad \frac{q_C}{q_{2C}} = \frac{1}{2}$$

3. In steady state, current through capacitor wire is zero. Current flows through 200 Ω, 900 Ω and A_2.

$$V_C = \frac{q}{C} = \frac{4 \times 10^{-3}}{100 \times 10^{-6}}$$

$$= 40 \text{ V}$$

This is also potential drop across 900 Ω resistance and 100 Ω ammeter A_2 (Total resistance = 1000 Ω). Now, this 1000 Ω and 200 Ω are in series. Therefore,

$$V_2 = V_{200\,\Omega} = \frac{V_{1000\,\Omega}}{5}$$

$$= \frac{40}{5} = 8 \text{ V}$$

$$\text{Emf} = V_{1000\,\Omega} + V_{200\,\Omega} = 48 \text{ V}$$

$$i = \frac{\text{Emf}}{\text{Net resistance}}$$

$$= \frac{48}{1200} = \frac{1}{25} \text{ A}$$

4. Current through A is the main current passing through the battery. So, this current is more than the current passing through B. Hence, during charging more heat is produced in A.

In steady state,

$$i_C = 0$$

and

$$i_A = i_B$$

Hence, heat is produced at the same rate in A and B.

Further, in steady state

$$V_C = V_B = \frac{\varepsilon}{2}$$

$$\therefore \quad U = \frac{1}{2} CV_C^2 = \frac{1}{8} C\varepsilon^2$$

5. $F = qE = (q)\left(\dfrac{\sigma}{2\varepsilon_0}\right) = \dfrac{q^2}{2\varepsilon_0 A}$

q remains unchanged. Hence, F remains unchanged.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

q remains unchanged. Hence, E also remains unchanged.

$$U = \frac{q^2}{2C} \quad \text{or} \quad U \propto \frac{1}{C}$$

C will decrease. Hence, U will increase.

$$V = Ed \quad \text{or} \quad V \propto d$$

d is increasing. Hence, V will increase.

6. $C_i = \dfrac{(C)(2C)}{C + 2C} = \dfrac{2}{3}C$

$$q_i = C_i E = \frac{2}{3} EC$$

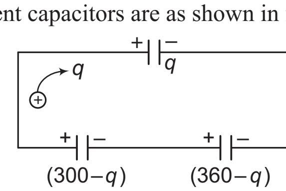
$$C_f = 2C$$

$$\therefore \quad q_f = 2EC$$

$$\Delta q = q_f - q_i$$

$$= \frac{4}{3} CE$$

7. Let $(+q)$ μC charge flows in the closed loop in clockwise direction. Then, final charges on different capacitors are as shown in figure.



Now, applying Kirchhoff's loop law

$$\frac{360 - q}{3} + \frac{300 - q}{2} = \frac{q}{1.5}$$

Solving the above equation, we get

$$q = 180\ \mu\text{C}$$

8. If the battery is disconnected, then q = constant

$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad C \propto \frac{1}{d}$$

d is decreased. Hence, C will increase.

$$U = \frac{1}{2}\frac{q^2}{C} \quad \text{or} \quad U \propto \frac{1}{C}$$

C is increasing. Hence, U will decrease.

$$V = \frac{q}{C} \quad \text{or} \quad V \infty \frac{1}{C}$$

C is increasing. Hence, V will decrease.

9. (a) At $t = 0$, emf of the circuit = PD across the capacitor = 6 V.

$$\therefore \quad i = \frac{6}{1+2} = 2\,\text{A}$$

Half-life of the circuit

$= (\ln 2)\,\tau_C\,(\ln 2)\,CR = (6 \ln 2)$ s.

In half-life time, all values get halved.

For example

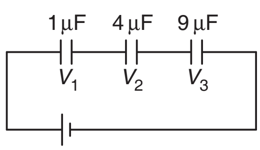
$$V_C = \frac{6}{2} = 3\,\text{V}$$

$$i = \frac{2}{1} = 1\,\text{A}$$

$$\therefore \quad V_{1\Omega} = iR = 1\,\text{V}$$

$$V_{2\Omega} = iR = 2\,\text{V}$$

10.



In series, $\quad V \propto \frac{1}{C} \qquad$ (as q = constant)

$$\therefore \quad \frac{V_2}{V_1} = \frac{1}{4}$$

or $$V_2 = \frac{V_1}{4} = \frac{10}{4} = 2.5\ \text{V}$$

$$\frac{V_3}{V_1} = \frac{1}{9}$$

$$\therefore \quad V_3 = \frac{V_1}{9} = \frac{10}{9}\ \text{V}$$

Now, $$E = \left(10 + 2.5 + \frac{10}{9}\right)\text{V}$$

Comprehension Based Questions

1. Finally, the capacitors are in parallel and total charge ($= q_0$) distributes between them in direct ratio of capacity.

$$\therefore \quad q_{C_2} = \left(\frac{C_2}{C_1 + C_2}\right) q_0 \rightarrow \text{in steady state.}$$

But this charge increases exponentially.

Hence, charge on C_2 at any time t is

$$q_{C_2} = \left(\frac{C_2\, q_0}{C_1 + C_2}\right)(1 - e^{-t/\tau_C})$$

Initially, C_2 is uncharged so, whatever is the charge on C_2, it is charge flown through switches.

2. Common potential in steady state when they finally come in parallel is

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{q_0}{C_1 + C_2}$$

Total heat dissipated $= U_i - U_f$

$$= \frac{q_0^2}{2C_1} - \frac{1}{2}(C_1 + C_2)\left(\frac{q_0}{C_1 + C_2}\right)^2$$

$$= \left(\frac{q_0^2}{2C_1}\right)\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

3. $E_{\text{air}} = E_0 = \dfrac{V}{d}$

4. $E_{\text{dielectric}} = \dfrac{E_0}{K} = \dfrac{V}{Kd}$

Match the Columns

1. (a) $C_i = \dfrac{4 \times 4}{4 + 4} = 2\,\mu\text{F}$

$$C_f = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 2}{8 + 2}$$

$$= 1.6\,\mu\text{F}$$

$$q = CV$$

Since, total capacity is decreasing. Hence, charge on both capacitors will decrease.

(b) $U_2 = \dfrac{1}{2}\dfrac{q^2}{C}$

q has become $\dfrac{1.6}{2}$ or 0.8 times but C is halved.

Hence, U_2 will increase.

(c) $V_2 = \dfrac{q}{C}$

q has become 0.8 times and C is halved.

Hence, V_2 will increase.

(d) $E_2 = \dfrac{V_2}{d}$ or $E_2 \propto V_2$

2. (a) $C_i = 2\,\mu\text{F}$

$$\therefore \quad q_i = 60\,\mu\text{C}$$

$$C_f = 6\,\mu\text{F}$$

$$\therefore \quad q_f = 180\,\mu\text{C}$$

$\therefore \quad \Delta q$ from the battery $= q_f - q_i = 120\,\mu\text{C}$.

(b) Between 4 μF and 2 μF charge distributes indirect ratio of capacity. Hence, on 2 μF

$$q_i = \left(\frac{2}{2+4}\right)(60\,\mu\text{C}) = 20\,\mu\text{C}$$

$$q_f = \left(\frac{2}{2+4}\right)(180)\;60\,\mu\text{C}$$

$\therefore \quad \Delta q = \Delta f - q_i = 40\,\mu\text{C}$

(c) On 3 μF, initial charge is 60 μF and final charge is zero.

$\therefore \qquad \Delta q = 60\,\mu\text{C}$

(d) On 4 μF

$$q_i = \left(\frac{4}{2+4}\right)(60\,\mu\text{C}) = 40\,\mu\text{C}$$

$$q_f = \left(\frac{4}{2+4}\right)(180\,\mu\text{C}) = 120\,\mu\text{C}$$

$\therefore \quad \Delta q = q_f - q_i = 80\,\mu\text{C}$

3. (a) In second figure, $V_{C_1} = V =$ maximum

Hence, q_{C_1} is maximum.

(b) In first figure, $V_{C_2} = \dfrac{V}{3}$

In second figure, $V_{C_2} = V$

In third figure, $V_{C_2} = \left(\dfrac{C}{2C + C}\right)V = \dfrac{V}{3}$

In fourth figure, $V_{C_2} = \left(\dfrac{2C}{C + 2C}\right)V = \dfrac{2V}{3}$

Now, $q = CV$

Hence, q_{C_2} is minimum in first and third figures.

(c) In second figure, $V_{C_1} = V =$ maximum

(d) Similar to option (b)

4. After closing the switch, the common potential is parallel.

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{CV}{3C} = \frac{V}{3}$$

$$U_C = \frac{1}{2}C\left(\frac{V}{3}\right)^2 = \frac{1}{18}CV^2$$

$$U_{2C} = \frac{1}{2}(2C)\left(\frac{V}{3}\right)^2 = \frac{1}{9}CV^2$$

Loss of energy $= U_i - U_f$

$$= \frac{1}{2}CV^2 - \frac{1}{2} \times 3C \times \left(\frac{V}{3}\right)^2$$

$$= \frac{1}{3}CV^2$$

5. Let $C_0 = \dfrac{\varepsilon_0 A}{d}$

$$C_1 = \frac{\varepsilon_0 (A/2)}{d} + \frac{K\,\varepsilon_0 (A/2)}{d}$$

$$= \frac{C_0}{2} + C_0 = \frac{3C_0}{2}$$

$$C_2 = \frac{\varepsilon_0 A}{d - d/2 + \dfrac{d/2}{K}} = \frac{2\,\varepsilon_0 A}{d\left(1 + \dfrac{1}{K}\right)}$$

$$= \frac{4}{3}\frac{\varepsilon_0 A}{d} = \frac{4}{3}C_0$$

$$\therefore \quad \frac{C_1}{C_2} = \frac{9}{8}$$

Capacitors are in series.

Hence, $\qquad q_1 = q_2$

or $\qquad \dfrac{q_1}{q_2} = 1$

$$U = \frac{1}{2}\frac{q^2}{C}$$

or $\qquad U \propto \dfrac{1}{C} \qquad$ (as q is same)

$$\therefore \quad \frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{8}{9}$$

6.

$$q_1 = q_8 = \frac{q_{\text{total}}}{2} = 7Q$$

$$q_2 = 4Q - q_1 = -3Q$$

$$q_3 = -q_2 = +3Q$$

$$q_4 = Q - q_3 = -2Q$$

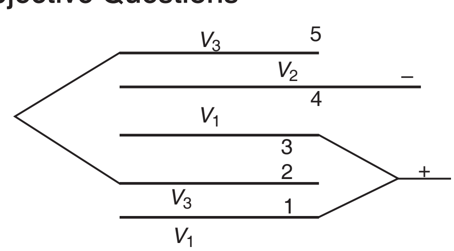
$$q_5 = -q_4 = +2Q$$

$$q_6 = 2Q - q_5 = 0$$

$$q_7 = -q_6 = 0$$

Subjective Questions

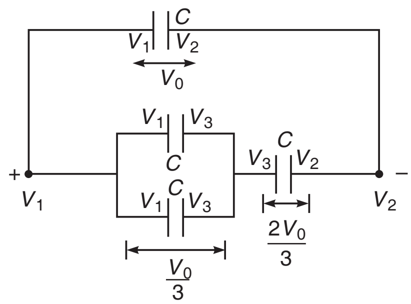
1.



$$C = \frac{\varepsilon_0 A}{d}$$

(between two successive plates).

The effective capacity has to be found between V_1 and V_2.



$$C_{net} = C + \frac{(2C)(C)}{2C + C} = \frac{5}{3}C = \frac{5}{3}\frac{\varepsilon_0 A}{d}$$

$$q_3 = +[C(V_1 - V_2) + C(V_1 - V_3)]$$

$$= \frac{\varepsilon_0 A}{d}\left[V_0 + \frac{V_0}{3}\right] = \frac{4\varepsilon_0 A V_0}{3d}$$

$$q_5 = C(V_3 - V_2) = \left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{2V_0}{3}\right)$$

$$= \frac{2\varepsilon_0 A V_0}{3d}$$

2. (a) $V = \frac{q_{net}}{C_{net}} = \frac{C_1 V_0}{C_1 + C_2} = \frac{V_0}{1 + C_2/C_1}$

$$= \frac{120}{1 + 4/8} = 80 \text{ V}$$

(b) $U_i = \frac{1}{2}C_1 V_0^2 = \frac{1}{2} \times 8 \times 10^{-6} \times (120)^2$

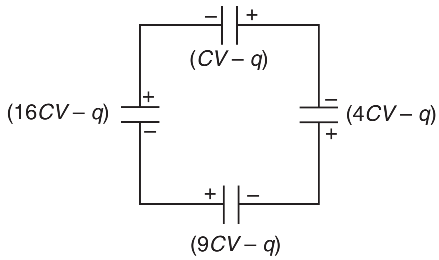
$$= 5.76 \times 10^{-2} \text{ J}$$

$$U_f = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times (80)^2$$

$$= 3.84 \times 10^{-2} \text{ J}$$

3. Let $+q$ charge rotates in the loop in clockwise direction for achieving equilibrium state. In final steady state, charges on the capacitors will be as shown below



Now, applying Kirchhoff's loop law we have

$$\left(\frac{CV - q}{C}\right) + \left(\frac{4CV - q}{2C}\right) + \left(\frac{9CV - q}{3C}\right) + \left(\frac{16CV - q}{4C}\right) = 0$$

or $\quad q = 4.8\,CV = \frac{24}{5}CV$

Now, $V_1 = V - \frac{q}{C} = V - \frac{24}{5}V = -\frac{19}{5}$ V

$$V_2 = 2V - \frac{q}{2C} = 2V - \frac{24}{10}V = -\frac{2}{5}V$$

$$V_3 = 3V - \frac{q}{3C} = 3V - \frac{24}{15}V = \frac{7}{5}V \text{ and}$$

$$V_4 = 4V - \frac{q}{4C} = 4V - \frac{6}{5}V = \frac{14}{5}V$$

4. At $t = 5$ ms, $V = 10$ V

$\therefore \quad i_R = \frac{V}{R} = \frac{10}{4} = 2.5$ A **Ans.**

Further, $q = CV = (300 \times 10^{-6})(2000t) = 0.6t$

$i_C = \frac{dq}{dt} = 0.6$ A = constant **Ans.**

5. Potential energy stored in the capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (200)^2 = 0.1 \text{ J}$$

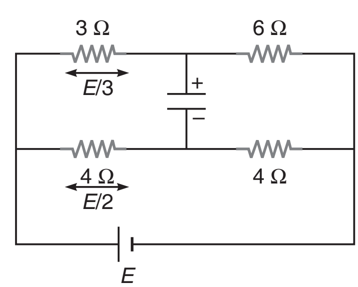
During discharging this 0.1 J will distribute in direct ratio of resistance,

$$\therefore \quad H_{400} = \frac{400}{400 + 500} \times 0.1$$

$$= 44.4 \times 10^{-3} \text{ J}$$

$= 44.4$ mJ **Ans.**

6.



(a) Current in lower branch = $E/8 = 3$ A

Current in upper branch = $E/9 = 24/9 = 2.67$ A

(b) PD across the capacitor = $E/2 - E/3 = E/6$

From $q = CV$, we have $16 = (4)\frac{E}{6}$

$\therefore \quad E = 24$ V

(c) After short-circuiting the battery, we will have to find net resistance across capacitor to calculate equivalent value of τ_C in discharging. $3\,\Omega$ and $6\,\Omega$ are in parallel. Similarly, $4\,\Omega$ and $4\,\Omega$ are in parallel. They are then in series.

$\therefore \quad R_{\text{net}} = 4\Omega,$

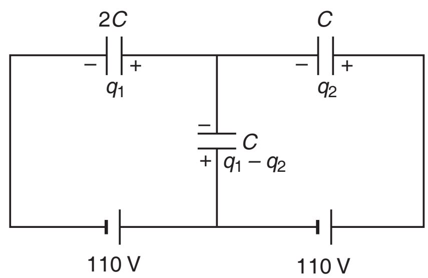
$$\tau_C = CR_{\text{net}} = (4 \times 4)\,\mu\text{s} = 16\,\mu\text{s}$$

During discharging $q = q_0 e^{-t/\tau_C}$

or $\quad 8 = 16 e^{-t/16}$

Solving this equation, we get $t = 11.1\,\mu\text{s}$ **Ans.**

7.



Applying loop law in two closed loops, we have

$$110 - \frac{q_2}{C} + \frac{q_1 - q_2}{C} = 0 \quad \text{or} \quad q_2 = (110C)$$

and $\quad -110 + \dfrac{q_1}{2C} + \dfrac{q_1 - q_2}{C} = 0$

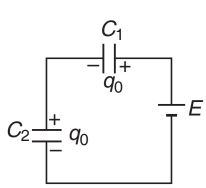
or $\quad q_1 = \left(\dfrac{440C}{3}\right)$

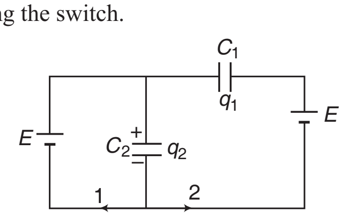
Potential difference between points M and N is

$$V_N - V_M = \frac{q_1 - q_2}{C}$$

$$= \frac{110}{3} \text{ volt}$$ **Ans.**

8. Let us first find charges on both capacitors before and after closing the switch.



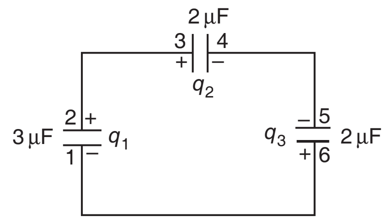


$$q_2 = EC_2 \quad \text{and} \quad q_1 = 0$$

$$q_0 = E\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

From 2, $-q_0$ charge will flow, so that charge on right hand side plate of C_1 becomes zero. From 1, q_2 charge will flow.

9. (i) Charge on capacitor A, before joining with an uncharged capacitor.



$$q_A = CV = (100)(3)\,\mu\text{C} = 300\,\mu\text{C}$$

Similarly, charge on capacitor B

$$q_B = (180)(2)\,\mu\text{C} = 360\,\mu\text{C}$$

Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in figure.

(q_1, q_2 and q_3 are in microcoulombs)

From conservation of charge

Net charge on plates 2 and 3 before joining = net charge after joining

$\therefore \quad 300 = q_1 + q_2 \quad \ldots\text{(i)}$

Similarly, net charge on plates 4 and 5 before joining = net charge after joining

$$-360 = -q_2 - q_3$$

or $\quad 360 = q_2 + q_3 \quad \ldots\text{(ii)}$

Applying Kirchhoff's second law in closed loop

$$\frac{q_1}{3} - \frac{q_2}{2} + \frac{q_3}{2} = 0$$

or $\quad 2q_1 - 3q_2 + 3q_3 = 0 \quad \ldots\text{(iii)}$

Solving Eqs. (i), (ii) and (iii), we get

$$q_1 = 90\,\mu\text{C}$$

$$q_2 = 210\,\mu\text{C}$$

and $\quad q_3 = 150\,\mu\text{C}$

(ii) (a) Electrostatic energy stored before completing the circuit,

$$U_i = \frac{1}{2}(3 \times 10^{-6})(100)^2 + \frac{1}{2}(2 \times 10^{-6})(180)^2 \qquad \left(U = \frac{1}{2}CV^2\right)$$

$$= 4.74 \times 10^{-2} \text{ J} \quad \text{or} \quad U_i = 47.4 \text{ mJ}$$

(b) Electrostatic energy stored after completing the circuit,

$$U_f = \frac{1}{2}\frac{(90 \times 10^{-6})^2}{(3 \times 10^{-6})} + \frac{1}{2}\frac{(210 \times 10^{-6})^2}{(2 \times 10^{-6})} + \frac{1}{2}\frac{(150 \times 10^{-6})^2}{(2 \times 10^{-6})} \qquad \left[U = \frac{1}{2}\frac{q^2}{C}\right]$$

$$= 1.8 \times 10^{-2} \text{ J} \quad \text{or} \quad U_f = 18 \text{ mJ}$$ **Ans.**

10. (a) In steady state, capacitors will be in parallel. Charge will distribute in direct ratio of their capacity.

$$\therefore \qquad q_1 = \left(\frac{C_1}{C_1 + C_2}\right) q_0$$

and $$q_2 = \left(\frac{C_2}{C_1 + C_2}\right) q_0$$

Initial emf in the circuit is potential difference across capacitor C_1 or q_0/C_1.

Therefore, initial current would be

$$i_0 = \frac{q_0/C_1}{R} = \frac{q_0}{C_1 R}$$

Current as function of time will be $i = i_0 e^{-t/\tau_C}$

Here, $$\tau_C = \left(\frac{C_1 C_2}{C_1 + C_2}\right) R$$ **Ans.**

(b) $U_i = \dfrac{1}{2}\dfrac{q_0^2}{C_1}$ and $U_f = \dfrac{1}{2}\dfrac{q_0^2}{C_1 + C_2}$

Heat lost in the resistor

$$= U_i - U_f = \frac{q_0^2}{2}\left[\frac{C_2}{C_1(C_1 + C_2)}\right]$$ **Ans.**

11. $$\tau_C = CR = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{\rho d}{A}\right) = K\varepsilon_0 \rho$$

$$= 5 \times 8.86 \times \frac{10^{-12}}{7.4 \times 10^{-12}} \approx 6 \text{ s}$$

Initial current,

$$i_0 = \frac{q_0/C}{R} = \frac{q_0}{CR} = \frac{q_0}{\tau_C} = \frac{8.55}{6} = 1.425\ \mu\text{A}$$

Now, current as function of time $i = i_0 e^{-t/\tau_C}$

or $$i = (1.425)e^{-12/6} = 0.193\ \mu\text{A}$$ **Ans.**

12. (a) $C = \dfrac{\varepsilon_0 A}{x}$, $U = \dfrac{Q^2}{2C} = \dfrac{Q^2 x}{2\varepsilon_0 A}$

(b) $$\frac{dU}{dx} = \frac{Q^2}{2\varepsilon_0 A}$$

$$\therefore \qquad dU = \left(\frac{Q^2}{2\varepsilon_0 A}\right) dx$$

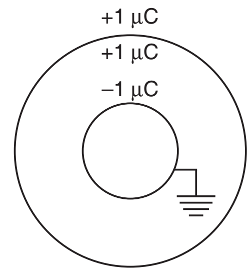
(c) $$\left(\frac{Q^2}{2\varepsilon_0 A}\right) dx = dW = F dx$$

$$\therefore \qquad F = \frac{Q^2}{2\varepsilon_0 A}$$

(d) Because E between the plates is due to both the plates.

While $F = (Q)$ (field due to other plate) **Ans.**

13. (a) Let q be the charge on smaller sphere. Then,



$$V_{\text{inner}} = 0$$

$$\therefore \qquad \frac{Kq}{2} + \frac{K(2)}{4} = 0 \quad \text{or} \quad q = -1\ \mu\text{C}$$

Now, $$V_{\text{outer}} = \frac{K(2-1) \times 10^{-6}}{4 \times 10^{-2}}$$

$$= \frac{9 \times 10^9 \times 10^{-6}}{4 \times 10^{-2}}$$

$$= 2.25 \times 10^5 \text{ V}$$ **Ans.**

(b) Charge distribution is as shown in above figure.

14. (a) **Fig. (a)** $V_6 = \dfrac{90}{3} = 30$ V, $q_6 = 6 \times 30 = 180\ \mu$C

$$V_3 = \frac{90}{3} = 30 \text{ V}$$

$$q_3 = 30 \times 3 = 90\ \mu\text{C}$$ **Ans.**

Fig. (b) Capacitor 1 μF is short-circuited.

Therefore, $q_1 = 0$.

$$V_{26} = \left(\frac{20}{20 + 20 + 10}\right) \times 100 = 40 \text{ V}$$

This 40 V will distribute in inverse ratio of capacity.

$$\therefore \qquad V_6 = \frac{2}{8} \times 40 = 10 \text{ V}$$

$$V_2 = \frac{6}{8} \times 40 = 30 \text{ V}$$

$$\therefore \qquad q_6 = 60\ \mu\text{C}, q_2 = 60\ \mu\text{C}$$ **Ans.**

(b) **Fig. (a)** When S is open, 6 μF is short-circuited

or $V_6 = 0, q_6 = 0$

and $V_3 = 90$ V, $q_3 = 270\ \mu$C **Ans.**

Fig. (b) When S is open, $V_1 = 100$ V

$$q_1 = 100\ \mu\text{C}$$

$$V_{26} = 100 \text{ V}$$

$$\therefore \qquad V_6 = \frac{2}{8} \times 100 = 25 \text{ V}$$

and $\quad V_2 = \frac{6}{8} \times 100 = 75$ V

$\therefore \quad q_6 = 150\,\mu\text{C},\ q_2 = 150\,\mu\text{C}$

Further, $V_A - 0 = V_2$

$\therefore \quad V_A = V_2 = 75$ V **Ans.**

15. Current in the circuit, $i = \frac{10}{4+1+2+3} = 1$ A

Now, $\quad V_{5\,\mu\text{F}} = V_{1,2\Omega} = 3$ V

$\therefore \quad q_5 = 15\,\mu\text{C}$

Further, $\quad V_{3\mu\text{F}} = V_{2,\,3\Omega} = 5$ V

$\therefore \quad q_3 = 15\,\mu\text{C}$ **Ans.**

16. (a) At $t = 0$, when capacitor is uncharged, its equivalent resistance is zero.

$$\therefore \quad R_{\text{net}} = 4 + \frac{6 \times 3}{6+3} = 6\ \text{M}\Omega$$

or $$i_1 = \frac{18 \times 10^3}{6 \times 10^6}\ \text{A} = 3\ \text{mA}$$

This will distribute in inverse ratio of resistances.

$$\therefore \quad i_2 = \frac{3}{6+3} i_1 = 1\ \text{mA and } i_3 = 2\ \text{mA}$$

At $t = \infty$, when capacitor is completely charged, equivalent resistance of capacitor is infinite.

$$i_3 = 0,\ i_1 = i_2 = \frac{18 \times 10^3}{(4+6) \times 10^6} = 1.8\ \text{mA}$$ **Ans.**

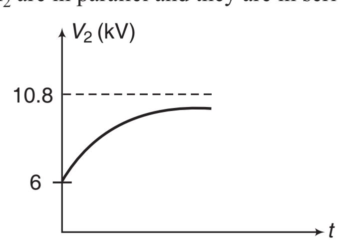
(b) At $\quad t = 0$,

$$V_2 = i_2 R_2 = (1 \times 10^{-3})(6 \times 10^6)\ \text{V}$$
$$= 6\ \text{kV}$$

At $\quad t = \infty$,

$$V_2 = i_2 R_2 = (1.8 \times 10^{-3})(6 \times 10^6)\ \text{V}$$
$$= 10.8\ \text{kV}$$ **Ans.**

(c) To find time constant of the circuit we will have to short-circuit the battery and find resistance across capacitor. In that case, R_1 and R_2 are in parallel and they are in series with R_3.



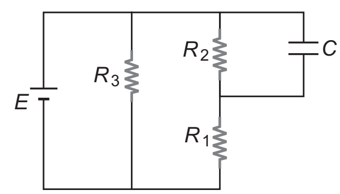
$$\therefore \quad R_{\text{net}} = 3 + \frac{4 \times 6}{4+6} = 5.4\ \text{M}\Omega$$

$$\therefore \quad \tau_C = CR_{\text{net}} = (10 \times 10^{-6})(5.4 \times 10^6)$$
$$= 54\ \text{s}$$

$$\therefore \quad V_2 = 6 + (10.8 - 6)(1 - e^{-t/C})$$
$$= 6 + 4.8(1 - e^{t/54})$$

Here, V_2 is in kV and t is second.

17. Circuit can be drawn as shown in figure.

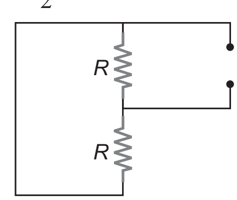


In charging of capacitor, R_3 has no role.
In steady state, potential difference across capacitor = potential difference across $R_2 = E/2$
Therefore, steady state charge across capacitor

$$q_0 = \frac{CE}{2}$$

To find time constant of circuit we will have to short circuit the battery, then we will find net resistance across capacitor.

$$R_{\text{net}} = \frac{R}{2} \quad \Rightarrow \quad \tau_C = CR_{\text{net}} = \frac{CR}{2}$$



$\therefore$ Charge in the capacitor at time t would be

$$q = q_0(1 - e^{-t/\tau_C}) = \frac{CE}{2}(1 - e^{-\frac{2t}{CR}})$$ **Ans.**

18. $q_2 = 20\,\mu\text{C}$

$\therefore \quad q_1 = 10\,\mu\text{C}$ (as they are in parallel)

Energy stored at this instant,

$$U = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$
$$= \frac{1}{2} \times \frac{(10^{-5})^2}{10^{-6}} + \frac{1}{2} \times \frac{(2 \times 10^{-5})^2}{2 \times 10^{-6}}$$
$$= 1.5 \times 10^{-4}\ \text{J}$$
$$= 0.15\ \text{mJ}$$

In charging of a capacitor 50% of the energy is stored and rest 50% is dissipated in the form of heat.

Therefore, 0.15 mJ will be dissipated in the form of heat across all the resistors. In series in direct ratio of resistance ($H = i^2Rt$) and in parallel in inverse ratio of resistance.

$\therefore \quad H_2 = 0.075$ mJ, $H_3 = 0.05$ mJ

and $\quad H_6 = 0.025$ mJ **Ans.**

19. (a) At $t = 0$, capacitor is equivalent to a battery of emf $\frac{E}{2}$.

Net emf of the circuit $= E - E/2 = E/2$

Total resistance is R.

Therefore, current in the circuit at $t = 0$ would be

$$i = \frac{E/2}{R} = \frac{E}{2R}$$ **Ans.**

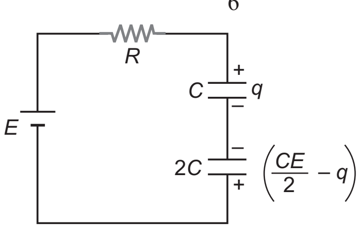
(b) Let in steady state there is total q charge on C.

Initial charge on C was $CE/2$. Therefore, charge on $2C$ in steady state would be $\left(\frac{CE}{2} - q\right)$ with polarities as shown. This is because net charge on lower plate of C and of upper plate on $2C$ should remain constant.

Applying loop law in the circuit in steady state, we have

$$E - \frac{q}{C} + \frac{CE/2 - q}{2C} = 0$$

$$\therefore \quad q = \frac{5}{6}CE$$



Therefore, charge on C increases from $q_i = \frac{CE}{2}$ to $q_f = \frac{5CE}{6}$ exponentially.

Equivalent time constant would be

$$\tau_C = \left(\frac{C \times 2C}{C + 2C}\right)R = \frac{2}{3}CR$$

Therefore, charge as function of time would be

$$q = q_i + (q_f - q_i)(1 - e^{-t/\tau_C})$$

$$= \frac{CE}{2} + \frac{CE}{3}\left(1 - e^{-\frac{3t}{2CR}}\right)$$ **Ans.**

20. When S_1 is closed and S_2 open, capacitor will discharge. At time $t = R_1C$, one time constant, charge will remain $q_1 = \left(\frac{1}{e}\right)$ times of CV or

$$q_1 = \frac{CV}{e}$$

When S_1 is open and S_2 closed, charge will increase (or may decrease also) from $\frac{CV}{e}$ to CE exponentially. Time constant for this would be $(R_1C + R_2C)$. Charge as function of time would be

$$q = q_i + (q_f - q_i)(1 - e^{-t/\tau_C})$$

$$q = \frac{CV}{e} + \left(CE - \frac{CV}{e}\right)(1 - e^{-t/\tau_C})$$

After total time $2R_1C + R_2C$ or $t = R_1C + R_2C$, one time constant in above equation, charge will remain

$$q = \frac{CV}{e} + \left(CE - \frac{CV}{e}\right)\left(1 - \frac{1}{e}\right)$$

$$= EC\left(1 - \frac{1}{e}\right) + \frac{VC}{e^2}$$

21. At $t = 0$, capacitor C_0 is like a battery of

$$\text{emf} = \frac{Q_0}{C_0} = 1\ \text{V}$$

Net emf of the circuit $= 4 - 1 = 3$ V

Total resistance is $R = 100\ \Omega$

$\therefore$ Initial current $= \frac{3}{100} = 0.03$ A

This current will decrease exponentially to zero.

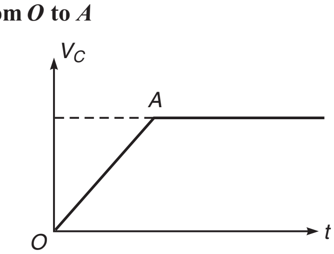
$$\therefore \quad i = 0.03\ e^{-t/\tau_C}$$

Here, $\quad \tau_C = C_{\text{net}}R = (1 \times 10^{-6})(100)$

$$= 10^{-4}\ \text{s}$$

$$\therefore \quad i = 0.03\ e^{-10^4 t}$$ **Ans.**

22. **From O to A**



$$V_C = at \quad (a = \text{constant})$$

$$\therefore \quad q_C = CV_C = Cat$$

$\therefore \quad i = \frac{dq_C}{dt} = aC$

$V_R = iR = aCR = \text{constant}$

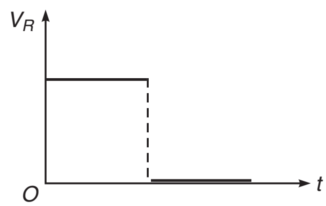
From A onwards $V_C =$ constant

$\therefore \quad q_C = \text{constant}$

$\therefore \quad i = \frac{dq_C}{dt} = 0$

or $\quad V_R = 0$

Therefore, V_R *versus* t graph is as shown in figure.



23. **From O to A** $\quad V = at$

Here, a is a positive constant.

$\therefore \quad at = \frac{q}{C} + iR$

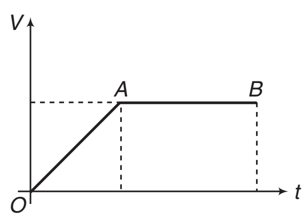
Differentiating w.r.t. time, we have

$$a = \frac{1}{C}\left(\frac{dq}{dt}\right) + \left(\frac{di}{dt}\right)R$$

or $\quad \left(\frac{di}{dt}\right)R = a - \frac{i}{C} \qquad \left(\text{as } i = \frac{dq}{dt}\right)$

$\therefore \quad \int_0^i \frac{di}{a - i/C} = \int_0^t \frac{dt}{R}$

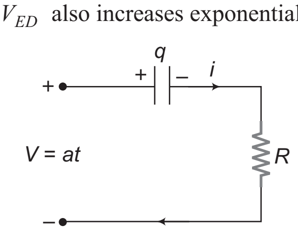
$\therefore \quad i = aC(1 - e^{-t/CR})$



i.e. current in the circuit increases exponentially

$\therefore \quad V_{ED} = iR = aCR\,(1 - e^{-t/CR})$

or $\quad V_{ED}$ also increases exponentially.



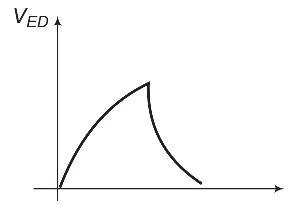
From A onwards When $V =$ constant $\qquad$ (say V_0)

$V_0 = at \quad \text{or} \quad t = \frac{V_0}{a}$

$\therefore \quad V_{ED} = aCR\,(1 - e^{-V_0/aCR})$

After this V_{ED} will decrease exponentially.

Hence, a rough graph is as shown in figure.



24. $q_i = CV_i = 100\,\mu\text{C},\ q_f = CV_f = -50\,\mu\text{C}$

Therefore, charge will vary from $100\,\mu$C to $-50\,\mu$C exponentially.

$\therefore \quad q = -50 + 150\, e^{-t/\tau_C}$, Here q is in μC

$\tau_C = CR = (10^{-6})(5 \times 10^3) = 5 \times 10^{-3}$

$\therefore \quad q = -50 + 150\, e^{-t/\tau_C}$

$V_C = \frac{q}{C} = (-50 + 150 e^{-t/\tau_C})\ \text{V}$

or $\quad V_C = 50\,(3e^{-200t} - 1)$

$i = -\frac{dq}{dt} = \frac{150 \times 10^{-6}}{\tau_C}\, e^{-200t}$

$= \frac{150 \times 10^{-6}}{5 \times 10^{-3}}\, e^{-200t} = 30 \times 10^{-3} e^{-200t}$

$V_R = iR = 150 e^{-200t}$ **Ans.**

25. At $t = 0$, equivalent resistance of an uncharged capacitor is zero and a charged capacitor is like a battery of emf = potential difference across the capacitor.

(a) $V_{C_1} = q_1/C_1 = 2$ V

$\therefore$ Net emf of the circuit $= 9 - 2 = 7$ V

or $\quad 9 + 2 = 11$ V

Net resistance $= 30 + \frac{30 \times 60}{30 + 60} = 50\ \Omega$

$\therefore$ Current at $t = 0$ would be $i_0 = \frac{7}{50}$ A

or $\quad \frac{11}{50}$ A **Ans.**

(b) In steady state, no current will flow through the circuit. C_2 will therefore be short-circuited, while PD across C_1 will be 9 V.

$\therefore \quad Q_2 = 0 \quad \text{and} \quad Q_1 = 9\,\mu\text{C}$ **Ans.**

26. Time constant of the circuit is

$$\tau_C = CR = (0.5 \times 10^{-6})(500) = 2.5 \times 10^{-4} \text{ s}$$

For $t \le 250\,\mu$s $\quad i = i_0 e^{-t/\tau_C}$

Here, $\quad i_0 = \dfrac{20}{500} = 0.04 \text{ A}$

$\therefore \quad i = (0.04\, e^{-4000t})$ amp

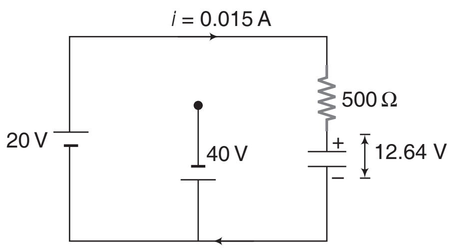
At $\quad t = 250\,\mu\text{s} = 2.5 \times 10^{-4}$ s

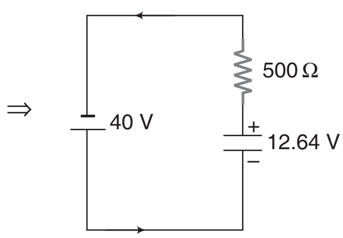
$$i = 0.04\, e^{-1} = 0.015 \text{ amp}$$

At this moment PD across the capacitor,

$$V_C = 20\,(1 - e^{-1}) = 12.64 \text{ V}$$

So when the switch is shifted to position 2, the current in the circuit is 0.015 A (clockwise) and PD across capacitor is 12.64 V





As soon as the switch is shifted to position 2 current will reverse its direction with maximum current.

$$i_0' = \frac{40 + 12.64}{500}$$

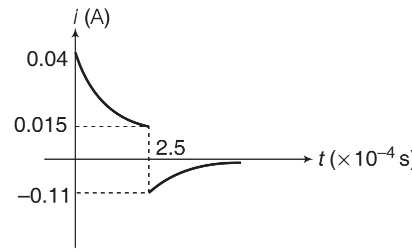
$$= 0.11 \text{ A}$$

Now, it will decrease exponentially to zero.

For $t \ge 250\,\mu$s

$$i = -\,i_0'\, e^{-t/\tau_C} = -\,0.11\, e^{-4000\,t}$$

The $(i - t)$ graph is as shown in figure.



27. Capacitor C_1 will discharge according to the equation,

$$q = q_0 e^{-t/\tau_C} \qquad \ldots\text{(i)}$$

Here, $\quad \tau_C = C_1 R$

and discharging current :

$$i = \frac{-\,dq}{dt} = \frac{q_0}{\tau_C}.e^{-t/\tau_C} = \frac{q_0 e^{-t/\tau_C}}{C_1 R} \qquad \ldots\text{(ii)}$$

At the given instant $i = i_0$

Therefore, from Eq. (ii)

$q_0 e^{-t/\tau_C} = i_0 C_1 R$ at this instant

or charge C_1 at this instant will be

$$q = (i_0 C_1 R) \qquad \text{[From Eq. (ii)]}$$

Now, this charge q will later on distribute in C_1 and C_2.

$$\therefore \qquad U_i = \frac{q^2}{2C_1}$$

and $\quad \dfrac{q^2}{2(C_1 + C_2)} = U_f$

$\therefore$ Heat generated in resistance,

$$H = U_i - U_f$$

Substituting values of $q : U_i$ and U_f, we get

$$H = \frac{(I_0 R)^2 C_1 C_2}{2(C_1 + C_2)} \qquad \textbf{Ans.}$$

26. Magnetics

INTRODUCTORY EXERCISE 26.1

1. $\because \quad qE = Bqv \sin\theta$

$\therefore \quad [E/B] = [v] = [LT^{-1}]$

2. From the property of cross product, **F** is always perpendicular to both **v** and **B**.

3. May be possible that $\theta = 0°$ or $180°$ between **v** and **B**, so that $\mathbf{F}_m = 0$

4. $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

Here, q has to be substituted with sign.

5. Apply Fleming's left hand rule.

6. $\because \quad F = Bqv\sin\theta$

$$\therefore \quad v = \frac{F}{Bq\sin\theta}$$

$$= \frac{4.6 \times 10^{-15}}{3.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times \sin 60°}$$

$$= 9.47 \times 10^{6} \text{ m/s}$$

7. $\because \quad F = Bqv \sin 90°$

$$= 0.8 \times 2 \times 1.6 \times 10^{-19} \times 10^{5}$$

$$= 2.56 \times 10^{-14} \text{ N}$$

INTRODUCTORY EXERCISE 26.2

1. Path C is undeviated. Therefore, it is of neutron's path. From Fleming's left hand rule magnetic force on positive charge will be leftwards and on negative charge is rightwards. Therefore, track D is of electron. Among A and B one is of proton and other of α-particle.

Further, $\quad r = \dfrac{mv}{Bq} \quad$ or $\quad r \propto \dfrac{m}{q}$

Since, $\quad \left(\dfrac{m}{q}\right)_\alpha > \left(\dfrac{m}{q}\right)_P$

$\therefore \quad r_\alpha > r_P$

or track B is of α-particle.

2. $r = \dfrac{\sqrt{2km}}{Bq}$ or $r \propto \sqrt{m}$ (k, q and B are same)

$$m_p > m_e \quad \Rightarrow r_p > r_e$$

3. The path will be a helix. Path is circle when it enters normal to the magnetic field.

4. Magnetic force may be non-zero. Hence, acceleration due to magnetic force may be non-zero.

Magnetic force is always perpendicular to velocity. Hence, its power is always zero or work done by magnetic force is always zero. Hence, it can be change the speed of charged particle.

5. $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

F is along position y-direction. q is negative and **v** is along positive x-direction. Therefore, **B** should be along positive z-direction.

6. (a) $r = \dfrac{mv}{Bq}$

or $r \propto m$ as other factors are same.

(b) $\quad f = \dfrac{Bq}{2\pi m}$

or $\quad f \propto \dfrac{1}{m}$

7. $\because \quad r = \dfrac{\sqrt{2qVm}}{Bq}$

INTRODUCTORY EXERCISE 26.3

1. $\because \quad \mathbf{l} = l\hat{\mathbf{i}}$

Now, $\quad \mathbf{F} = i(\mathbf{l} \times \mathbf{B})$

$$= i\,[(l\hat{\mathbf{i}}) \times (B_0\hat{\mathbf{j}} + B_0\hat{\mathbf{k}})]$$

$$= ilB_0(\hat{\mathbf{i}} - \hat{\mathbf{j}}) = |\mathbf{F}| = \sqrt{2}ilB_0$$

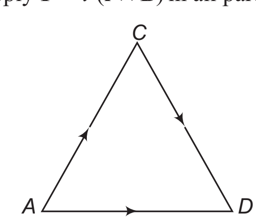
2. No, it will not change, as the new $\hat{\mathbf{i}}$ component of **B** is in the direction of **l**.

3. $\because \quad i = 3.5 \text{ A}$

$$\mathbf{l} = (-10^{-2}\hat{\mathbf{j}})$$

Now, apply $\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})$ in all parts.

4.



$\mathbf{F}_{ACD} = \mathbf{F}_{AD}$

$\therefore \quad |\mathbf{F}_{\text{net}}| = 2|\mathbf{F}_{AD}| = 2ilB$

$$= 2 \times 2 \times 4 \times 2 = 32 \text{ N}$$

INTRODUCTORY EXERCISE 26.4

1. $\because \quad \dfrac{M}{L} = \dfrac{q}{2m}$

$\therefore \quad M = \left(\dfrac{q}{2m}\right)L = \left(\dfrac{q}{2m}\right)(I\omega)$

$= \left(\dfrac{q}{2m}\right)\left(\dfrac{1}{2}mR^2\right)(\omega) = \dfrac{qR^2\omega}{4}$

2. $\because \quad M = iA = i(\pi R^2)$

$= (0.2)(\pi)(8 \times 10^{-2})^2$

$= (4.0 \times 10^{-3})\ \text{A-m}^2$

Now, $\quad \mathbf{M} = M\hat{\mathbf{M}} = (4.0 \times 10^{-3})(0.6\hat{\mathbf{i}} - 0.8\hat{\mathbf{j}})$

(a) $\tau = \mathbf{M} \times \mathbf{B}$

(b) $U = -\mathbf{M} \cdot \mathbf{B}$

3. $\because \quad L = 2\pi R \Rightarrow R = \dfrac{L}{2\pi}$

$M = iA = i(\pi R^2)$

$= (i)(\pi)\left(\dfrac{L}{2\pi}\right)^2 = \dfrac{iL^2}{4\pi}$

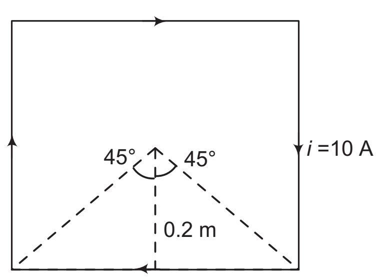
$\tau_{max} = MB \sin 90° = \dfrac{iBL^2}{4\pi}$

4. $\because \quad \Delta U = U_{0°} - U_{180°}$

$= -MB\cos 0° + MB\cos 180° = -2MB$

INTRODUCTORY EXERCISE 26.5

1. (a) From screw law, we can see that direction of magnetic field at centre of the square is inwards as the current is clockwise.



$$B = 4\left(\frac{\mu_0}{4\pi}\frac{i}{r}\right)(\sin\alpha + \sin\beta)$$

$$= \frac{4 \times 10^{-7} \times 10}{0.2}(\sin 45° + \sin 45°)$$

$$= 2.83 \times 10^{-5}\ \text{T}$$

(b) $2\pi R = 4(0.4)$

$2R = \dfrac{1.6}{\pi}\ \text{m}$

$B = \dfrac{\mu_0 i}{2R} = \dfrac{(4\pi \times 10^{-7})(10)}{(1.6/\pi)}$

$= 24.7 \times 10^{-6}\ \text{T}$

2. Magnetic field due to horizontal wire is zero. Magnetic field due to vertical is

$$B = \frac{\mu_0}{4\pi}\frac{i}{x}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0}{4\pi}\frac{i}{x} \qquad \text{(inwards)}$$

3. Both straight and circular wires will produce magnetic fields inwards.

$\therefore \quad B = \dfrac{\mu_0}{2\pi}\dfrac{i}{R} + \dfrac{\mu_0 i}{2R}$

$= \dfrac{(2 \times 10^{-7})(7)}{0.1} + \dfrac{(4\pi \times 10^{-7})(7)}{2 \times 0.1}$

$= 5.8 \times 10^{-5}\ \text{T}$

4. Magnetic field at O due to two straight wires = 0

Magnetic field due to circular wire,

$B = \dfrac{1}{4}$ (due to whole circle)

$= \dfrac{1}{4}\left(\dfrac{\mu_0 i}{2R}\right) \qquad$ (inwards)

$= \dfrac{1}{4} \times \dfrac{(4\pi \times 10^{-7})(5)}{2 \times 0.03}$

$= 2.62 \times 10^{-5}\ \text{T}$

5. Magnetic field at P due to straight wires = 0

Due to circular wires one is outwards (of radius a) and other is inwards. 60° means $\dfrac{1}{6}$ th of whole circle.

$\therefore \quad B = \dfrac{1}{6}\left[\dfrac{\mu_0 i}{2a} - \dfrac{\mu_0 i}{2b}\right] \qquad$ (outwards)

INTRODUCTORY EXERCISE 26.6

1. Applying Ampere's circuital law,

$B_A = \dfrac{\mu_0}{2\pi}\dfrac{i_{in}}{r}$

$= \dfrac{(2 \times 10^{-7})(1)}{10^{-3}}$

$= 2 \times 10^{-4}\ \text{T}$

This is due to (·) current of 1 A. Hence, magnetic lines are circular and anti-clockwise. Hence, magnetic field is upwards.

$$B_b = \frac{\mu_0}{2\pi}\frac{i_{\text{in}}}{r}$$
$$= \frac{(2\times10^{-7})(3-1)}{3\times10^{-3}}$$
$$= 1.33\times10^{-4}\text{ T}$$

This is due to net ⊗ current. Hence, magnetic lines are clockwise.

So, magnetic field at B is downwards.

2. $\int \mathbf{B}\cdot d\mathbf{l} = \mu_0(i_{\text{net}})$

Along path (a), net current enclosed by this path is zero.

Hence, line integral = 0

Along path (b), i_{net} is ⊗. So, magnetic lines along this current is clockwise. But, we have to take line integral in counter clockwise direction. Hence, line integral will be negative.

3. Using Ampere's circuital law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.

INTRODUCTORY EXERCISE 26.7

1. $$i = \left(\frac{k}{NBA}\right)\phi$$

$$\therefore \quad B = \frac{k\phi}{NiA} = \frac{(10^{-8})(90)}{100\times10^{-6}\times10^{-4}}$$
$$= 90\text{ T}$$

2. $$i = \left(\frac{k}{NBA}\right)\phi$$
$$= \frac{(0.125\times10^{-7})(6)(\pi/180)}{200\times5\times10^{-2}\times5\times2\times10^{-4}}$$
$$= 1.3\times10^{-7}\text{ A}$$

Exercises

LEVEL 1

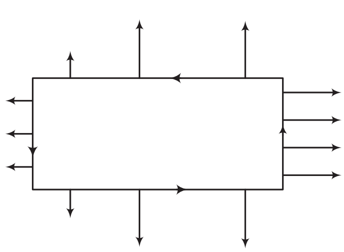
Assertion and Reason

2. By changing the direction of velocity direction of magnetic force will change. So, it is not a constant force.

3. To balance the weight, force on upper wire should be upwards (repulsion). Further equilibrium can be checked by displacing the wire from equilibrium position.

4. $\tau = MB\sin 90° \neq 0$

5. Magnetic field is outwards and increasing with x. So, magnetic force will also increase with x. The force on different sections are as shown in figure.



Force will act in positive x-direction. But, no torque will act.

6. $\because \quad r = \frac{\sqrt{2qVm}}{Bq} \Rightarrow r \propto \sqrt{\frac{m}{q}}$

m and q both are different. Ratio $\frac{m}{q}$ is not same for both.

7. $\because \quad \mathbf{F}_e + \mathbf{F}_m = 0$

$\therefore \quad q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) = 0$

$\therefore \quad \mathbf{E} = -(\mathbf{v}\times\mathbf{B}) = (\mathbf{B}\times\mathbf{v})$

8. $\because \quad \mathbf{F}_m = q(\mathbf{v}\times\mathbf{B})$

$\mathbf{F}_m \perp \mathbf{v}$ (always)

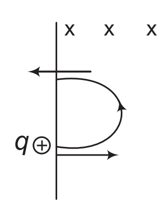
and $P = \mathbf{F}\cdot\mathbf{v}$

$\therefore$ Power of magnetic force is always zero.

$\mathbf{F}_e = q\mathbf{E}$

If $\mathbf{F}_e$ is also perpendicular to $\mathbf{v}$, then its power is also zero.

9.



10. $|\mathbf{v}| = \sqrt{2}v_0$ = speed, which always remains constant.

11. $R \propto v$, by increasing the speed two times radius also becomes two times. Hence, acceleration $(= v^2/R)$ will also become only two times.

Objective Questions

2. $T = \dfrac{2\pi m}{Bq}$, independent of v.

3. $B = \dfrac{\mu_0}{2\pi}\dfrac{i}{r}$, independent of diameter of wire.

8. In uniform $\mathbf{B}$ is force on any current carrying loop is always zero.

9. $M = NiA$

10. $\mathbf{B} \cdot \mathbf{F} = 0$ as $\mathbf{B} \perp \mathbf{F}$

$$\therefore \quad \mathbf{B} \cdot \alpha = 0$$

or $$2x + 3 - 4 = 0$$

$$\therefore \quad x = 0.5$$

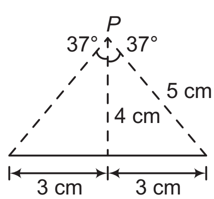
12. In uniform magnetic field, force on any current carrying loop is always zero.

13. $r = \dfrac{P}{Bq}$ or $r \propto \dfrac{1}{q}$ (as P = constant)

14. $\theta_i = 180°,\ \theta_f = 180° - \theta$

$$W = U_f - U_i$$
$$= -MB\cos(180° - \theta) - (-MB\cos 180°)$$
$$= MB\cos\theta - MB$$

15.



$r = 5$ cm

$$B = \frac{\mu_0}{4\pi}\frac{i}{r}(\sin 37° + \sin 37°)$$

16. $B_x = \dfrac{\mu_0 NiR^2}{2(R^2 + x^2)}$ and $B_c = \dfrac{\mu_0 Ni}{2R}$ $c \rightarrow$ centre.

17. $\mathbf{F} = I\ (\mathbf{l} \times \mathbf{B}) = I\ (\mathbf{ba} \times \mathbf{B})$

We can see that all (a), (b) and (c) options are same.

18. $F = Bqv$ or $F \propto v$

Now, $$v = \sqrt{\frac{2qV}{m}}$$

$$\therefore \quad F \propto \sqrt{V}$$

19. Two fields are additive.

$$\therefore \quad B_{\text{net}} = 2\left[\frac{\mu_0 I}{2\pi\,(R/2)}\right] = \frac{2\mu_0 I}{\pi R}$$

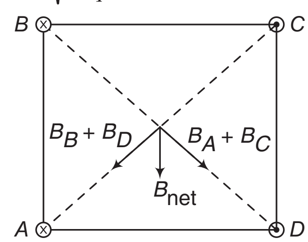
20. $$B_x = \frac{1}{8}B_c$$

$$\therefore \quad \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0 Ni}{2R}\right]$$

21. Electric field (acting along $\hat{\mathbf{j}}$ direction) will change the velocity component which is parallel to $\mathbf{B}$ (which is also along $\hat{\mathbf{j}}$ direction).
$\mathbf{B} = B_0\hat{\mathbf{j}}$ and $\mathbf{v} = v_0\mathbf{j}$ will rotate the particle in a circle. Hence, the net path is helical with variable pitch.

22. $r = \sqrt{\dfrac{2Km}{Bq}} \Rightarrow r \propto \dfrac{\sqrt{K}}{B}$

23.



24. $r = \sqrt{\dfrac{2qVm}{Bq}} = \left(\sqrt{\dfrac{2Vm}{q}}\right)\left(\dfrac{1}{B}\right)$

26. In (c), two wires are producing ⊙ magnetic field and two wires are producing ⊗ magnetic field.

27. I_1 produces circular magnetic lines current I_2 is each small circular element is parallel to $(\theta = 0°)$ magnetic field. Hence, force is zero.

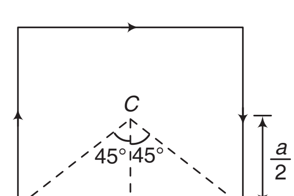
28. Arc of radius $a\left(\dfrac{1}{4}\text{th of circle}\right)$ produces magnetic field in $\hat{\mathbf{k}}$ direction or outwards, while arc of radius b produces magnetic field in $-\hat{\mathbf{k}}$ direction.

$$\therefore \quad \mathbf{B} = \frac{1}{4}\left(\frac{\mu_0 I}{2a}\right)\hat{\mathbf{k}} + \frac{1}{4}\left(\frac{\mu_0 I}{2a}\right)(-\hat{\mathbf{k}})$$
$$= \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)\hat{\mathbf{k}}$$

29. Equivalent current,

$$i = qf = ef$$
$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R}$$

30.



$$B_C = 4\left[\frac{\mu_0}{4\pi}\frac{I}{a/2}(\sin 45° + \sin 45°)\right]$$
$$= \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

31. At distance r from centre,

$B = \frac{\mu_0}{2\pi}\frac{(i_{in})}{r}$ (From Ampere's circuital law)

For path-1, $i_{in} \neq 0$

$\therefore$ $B_1 \neq 0$

For path-2, $i_{in} = 0$

$\therefore$ $B_2 = 0$

32. $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i}{r^3}(d\mathbf{l} \times \mathbf{r})$

$\therefore$ $d\mathbf{B}$ is in the direction $d\mathbf{l} \times \mathbf{r}$. Hence, $d\mathbf{l}$ is outwards and $\mathbf{r}$ is from $d\mathbf{l}$ towards P.

33. Magnetic field due the straight portions is zero. It is only due to arc of circle.

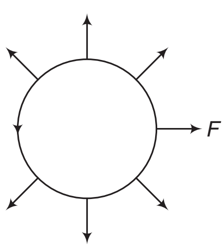
$\therefore$ $B = \frac{\phi}{2\pi}\left(\frac{\mu_0 I}{2x}\right)$ (Radius = x)

$= \frac{\mu_0 I \phi}{4\pi x}$

34. From centre, $r = (R - x)$

$$B = \frac{\mu_0}{2\pi}\frac{I}{R^2}r = \frac{\mu_0}{2\pi}\frac{I}{R^2}(R - x)$$

35. From Fleming's left hand rule, we can see that magnetic force is outwards on the loop.



So, it tends to expand.

Subjective Questions

1. $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, where $q = -1.6 \times 10^{-19}$ C for an electron and $q = +1.6 \times 10^{-19}$ C for a proton

2. $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

3. (a) $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

or $[(7.6 \times 10^{-3})\hat{\mathbf{i}} - (5.2 \times 10^{-3})\hat{\mathbf{k}}]$

$= (7.8 \times 10^{-6})[(-3.8 \times 10^3)\hat{\mathbf{j}}(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})]$

$\therefore$ $(7.8 \times 10^{-6})(3.8 \times 10^{-3})(B_x)$
$= (-5.2 \times 10^{-3})$

$\therefore$ $B_x = -0.175$ T

Similarly,

$(7.8 \times 10^{-6})(-3.8 \times 10^3)(B_z)$
$= 7.6 \times 10^{-3}$

or $B_z = -0.256$ T

(c) From the property of cross product.
$\mathbf{F}$ is always perpendicular to $\mathbf{B}$.
Hence, $\mathbf{F} \cdot \mathbf{B} = 0$

4. Apply $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

For example, let us apply for charged particle at e.

$$\mathbf{F}_e = q\left[\left(\frac{v}{\sqrt{2}}\hat{\mathbf{j}} - \frac{v}{\sqrt{2}}\hat{\mathbf{k}}\right) \times (B\hat{\mathbf{i}})\right]$$
$$= \frac{qvB}{\sqrt{2}}(-\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

5. $r = \frac{\sqrt{2qVm}}{Bq}$

$\therefore$ $B = \frac{1}{r}\sqrt{\frac{2Vm}{q}}$

$= \frac{1}{0.18}\sqrt{\frac{2 \times 2 \times 10^3 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}}$

$= 8.38 \times 10^{-4}$ T

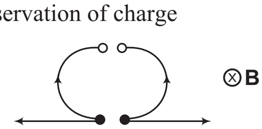
6. (a) $r = \frac{mv}{Bq} \Rightarrow v = \frac{Bqr}{m}$

(b) $t = \frac{T}{2} = \frac{\pi m}{Bq}$

(c) $r = \frac{\sqrt{2qVm}}{Bq}$

$\therefore$ $V = \frac{r^2B^2q^2}{2qm} = \frac{r^2B^2q}{2m}$

7. (a) Conservation of charge



They will collide after time,

$$t = \frac{T}{2} = \frac{\pi m}{Bq}$$

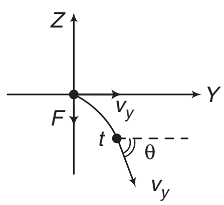
8. (a) At A, magnetic force should be towards right. From Fleming's left hand rule, magnetic field should be inwards.

Further, $r = \frac{mv}{Bq}$

$\therefore \quad B = \frac{mv}{qr} \qquad (r = 5\text{ cm})$

(b) $t_{AB} = \frac{T}{2} = \frac{\pi m}{Bq}$

9. Component of velocity parallel to **B**, i.e. v_x will remain unchanged $v_y\hat{\mathbf{j}}$ and $B\hat{\mathbf{j}}$ will rotate the particle in yz-plane ($\perp$ to **B**).



At the beginning direction to magnetic force is $-\hat{\mathbf{k}}$ [from the relation, $\mathbf{F} = q\,(v_y\hat{\mathbf{j}} \times B\hat{\mathbf{i}})$

In the time t, particle rotates an angle $\theta = \omega t = \left(\frac{Bq}{m}\right)t$ from its original path.

In the figure, we can set that, y- component of velocity at time t is $v_y \cos\theta$ and z- component is $-v_y \sin\theta$.

10. $\mathbf{F}_e + \mathbf{F}_m = 0$

or $\quad q\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B}) = 0$

or $\quad \mathbf{E} = -(\mathbf{v} \times \mathbf{B})$

or $\quad \mathbf{E} = (\mathbf{B} \times \mathbf{v})$

11. Work is done only by electrostatic force. Hence, from work-energy theorem

$= \frac{1}{2}mv^2 =$ work done by electrostatic force only

$= (qE_0)z \quad$ or $\quad$ Speed $v = \sqrt{\frac{2qE_0z}{m}}$

Particle rotates in a plane perpendicular to **B**, i.e. in xz-plane only. Hence, $v_y = 0$

12. When they are moving rectilinearly, net force is zero.

$\therefore \quad qE = Bqv \sin(90° - \theta)$

$\therefore \quad v = \frac{E}{B\cos\theta}$

When electric field is switched off,

$$p = \left(\frac{2\pi m}{Bq}\right) v\cos(90° - \theta)$$

$$= \frac{2\pi m\, E\tan\theta}{qB^2}$$

13. $W = F_m \quad$ or $\quad mg = ilB\sin 90°$

$\therefore \quad i = \frac{mg}{Bl} = \frac{(13 \times 10^{-3})\,(10)}{0.44 \times 0.62}$

$= 0.47$ A

Magnetic force should be upwards to balance the weight. Hence, from Fleming's left hand rule we can see that direction of current should be from left to right.

14. (a) $ilB = mg$

or $\quad \frac{V}{R}\,lB = mg \quad$ or $\quad V = \frac{mgR}{lB}$

$= \frac{(0.75)\,(9.8)\,(25)}{0.5 \times 0.45}$

≈ 817 V

(b) $a = \frac{ilB - mg}{m} = \frac{VlB}{mR} - g \qquad \left(\text{as } i = \frac{V}{R}\right)$

$= \frac{(817)\,(0.5)\,(0.45)}{(0.75)\,(2.0)} - 9.8$

$\approx 112.8\text{ m/s}^2$

15. $i = 5$ A $\quad\Rightarrow\quad \mathbf{B} = (0.02\hat{\mathbf{j}})$ T

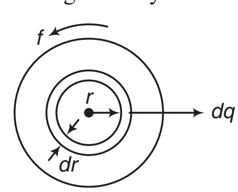
Now, applying $\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})$ in all parts. Let us find **l** for anyone parts.

$\mathbf{l}_{cd} = \mathbf{r}_d - \mathbf{r}_c$

$= (0.4\,\hat{\mathbf{j}} + 0.4\hat{\mathbf{k}}) - (0.4\,\hat{\mathbf{i}} + 0.4\hat{\mathbf{k}})$

$= (0.4\,\hat{\mathbf{j}} - 0.4\,\hat{\mathbf{i}})$

16. Let surface charge density is σ.



$dq = [(2\pi r)dr]\sigma$

Equivalent current,

$i = (dq)f$

$dM = iA = [(dq)f][\pi r^2]$

$\therefore \quad M = \int_0^R dM$

$dB = \frac{\mu_0 i}{2r} = \frac{\mu_0 (dq) f}{2r}$

$\therefore \quad B = \int_0^R dB$

Now, we can find the ratio $\frac{M}{B}$.

17. (a) From energy conservation,

$$U_\theta = K_\theta = U_{0^\circ} + K_{0^\circ}$$

or $(-MB\cos\theta) + 0 = (-MB\cos 0^\circ) + K_{0^\circ}$

Substituting the given values, we can calculate.

(b) To other side also it rotates upto the same angle.

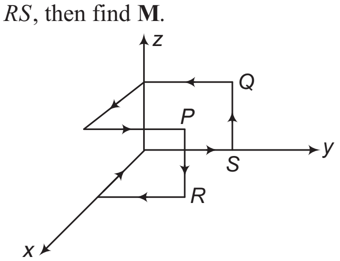
18. (a) $T = \dfrac{2\pi r}{v}$

(b) $I = qf = (e)\left(\dfrac{v}{2\pi r}\right)$

(c) $M = IA = \left(\dfrac{ev}{2\pi r}\right)(\pi r^2) = \dfrac{evr}{2}$

19. Assume equal and opposite currents in wires *cf* and *eh*.

20. Assume equal and opposite currents in wires *PQ* and *RS*, then find **M**.

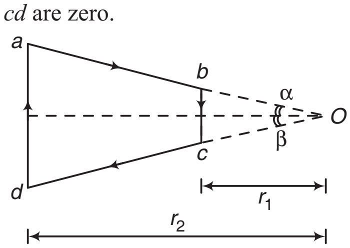


Now, $\mathbf{B}(2\hat{\mathbf{j}}) \Rightarrow \tau = \mathbf{M}\times\mathbf{B}$

21. $B_{net} = \sqrt{B^2 + B^2} = \sqrt{2}B$

where, $B = \dfrac{\mu_0}{4\pi}\dfrac{i}{r}(\sin 0^\circ + \sin 90^\circ)$

22. Magnetic fields at O due to currents in wires *ab* and *cd* are zero.



Magnetic field due to current in wire *da* (say B_2) is inwards due to current in wire *bc* (say B_1) is outwards.

$$B_1 = \frac{\mu_0}{4\pi}\frac{i}{r_1}(\sin\alpha + \sin\beta)$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{i}{r_2}(\sin\alpha + \sin\beta)$$

$B_1 > B_2$ as $r_1 < r_2$

$\therefore \quad B_{net} = B_1 - B_2$ (outwards)

23. In first quadrant magnetic field due to I_1 is outwards and due to I_2 is inwards. So, net magnetic field may be zero.

Similarly, in third quadrant magnetic field due to I_1 is inwards and due to I_2 magnetic field is outwards. Hence, only in first and third quadrants magnetic field may be zero.

Let magnetic field is zero at point $P(xy)$, then

$$B_{I_1} = B_{I_2}$$

$$\therefore \quad \frac{\mu_0}{2\pi}\frac{I_1}{y} = \frac{\mu_0}{2\pi}\frac{I_2}{x}$$

$$\therefore \quad y = \frac{I_1}{I_2}x$$

24. Two straight wires produces outward magnetic field by arc of circle produces inward magnetic field.

Due to straight wires,

$$B_1 = 2\left[\frac{\mu_0}{4\pi}\frac{i}{R}(\sin\theta + \sin 90^\circ)\right]$$

$$= \frac{\mu_0 i}{2\pi R} \quad \text{(outwards)}$$

Due to circular arc, $B_2 = \dfrac{\theta}{2\pi}\left(\dfrac{\mu_0 i}{2R}\right)$ (inwards)

For net field to be zero,

$$B_1 = B_2$$

or $\theta = 2\,\text{rad}$

25. (a) If currents are in the same direction, then above and below the wires magnetic fields are in the same direction. Hence, they can't produce zero magnetic field.

In between the wires, let $B = 0$ at a distance (r) cm from the wire carrying 75 A current. Then,

$$\frac{\mu_0}{2\pi}\left(\frac{75}{r}\right) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{25}{40-\pi}\right)$$

Solving, we get $r = 30$ cm.

(b) If currents are in opposite direction, then in between the wire magnetic field are in the same direction. So, they cannot produce zero magnetic field. The points should be above or below the wires, nearer to wire having smaller current. Let it is at a distance r from the wire having 25 A current. Then,

$$\frac{\mu_0}{2\pi}\left(\frac{25}{r}\right) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{75}{40+r}\right)$$

Solving this equation, we get $r = 20$ cm

26. Apply $B = \dfrac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$

27. I_2 produces inwards magnetic field at centre. Hence, I_1 should produce outward magnetic field. Or current should be towards right. Further,

$$\frac{\mu_0 I_2}{2R} = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I_1}{D}\right)$$

$$\therefore \qquad I_1 = \left(\frac{\pi D}{R}\right) I_2$$

28. (a) $B_1 = \frac{\mu_0 Ni}{2R}$

(b) $B_2 = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$, given $B_2 = \frac{B_1}{2}$

29. 10 A and 8 A current produce inward magnetic field. While 20 A current produces outward magnetic field. Hence, current in fourth wire should be $(20 - 10 - 8)$ A or 2 A and it should produce inward magnetic field. So, it should be downwards toward the bottom.

30. (a) $\mathbf{B}$ at origin $\mathbf{B}_{KLM} = \mathbf{B}_{KNM}$

$$= \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

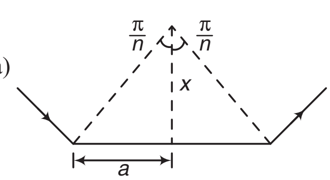
Now, we can apply $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ for finding force on it.

(b) In uniform magnetic field,

$$\mathbf{F}_{KLM} = \mathbf{F}_{KNM} = \mathbf{F}_{KM}$$
$$= I(\mathbf{l} \times \mathbf{B})$$
$$= I[\{2R(-\hat{\mathbf{k}})\} \times \{B_0 \hat{\mathbf{j}}\}]$$
$$= (2B_0 IR)\hat{\mathbf{i}}$$

$\therefore$ Net force is two times of the above value.

31. (a)



$$a = \frac{1}{2}\left(\frac{2\pi r}{n}\right) = \frac{\pi r}{n}$$

$$x = a\cot\frac{\pi}{n} = \left(\frac{\pi r}{n}\right)\cot\left(\frac{\pi}{n}\right)$$

$$B = n\left[\frac{\mu_0}{4\pi}\frac{i}{x}\left(\sin\frac{\pi}{n} + \sin\frac{\pi}{n}\right)\right]$$

$$= n\left[\frac{\mu_0}{4\pi}\frac{i}{(\pi r/n)\cot\pi/n}(2\sin\pi/n)\right]$$

$$= \frac{\mu_0 i n^2 \sin\left(\frac{\pi}{n}\right)\tan\left(\frac{\pi}{n}\right)}{2\pi^2 r}$$

(b) The above calculated magnetic field can be written as

$$B = \frac{\mu_0 i}{2r}\frac{\left(\frac{\sin\pi/n}{\pi/n}\right)^2}{\cos\pi/n}$$

As, $\qquad n \to \infty, \frac{\pi}{n} \to 0$

Hence, $\lim_{\pi/n \to 0}\left(\frac{\sin\pi/n}{\pi/n}\right) \to 1$

and $\qquad \lim_{\pi/n \to 0}(\cot\pi/n) \to 1$

or $\qquad B \to \frac{\mu_0 i}{2r}$

32. Current per unit area,

$$\sigma = \frac{I}{\pi a^2 - \pi(a/2)^2 - (a/2)^2}$$
$$= \frac{2I}{\pi a^2}$$

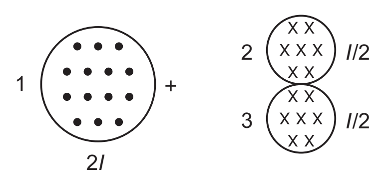
Total area is (πa^2). Therefore, the total current is

$$I_1 = (\sigma)(\pi a^2) = 2I$$

Cavity area is $\pi(a/2)^2$. Therefore, cavity current is

$$I_2 = (\sigma)(\pi a^2/4) = \frac{I}{2}$$

Now, the given current system can be assumed as shown below.



(a) **At P_1,** $\quad B_1 = \frac{\mu_0}{2\pi}\frac{2I}{5} = \frac{\mu_0 I}{\pi r}$ (towards left)

$$B_2 = \frac{\mu_0}{2\pi}\frac{I/2}{(r - a/2)} \quad \text{(towards right)}$$

and $\quad B_2 = \frac{\mu_0}{2\pi}\frac{I/2}{(r + a/2)}$ (towards right)

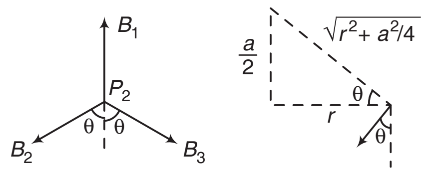
$\therefore \quad B_{net} = B_1 - B_2 - B_2$ (towards left)

$$= \frac{\mu_0 I}{\pi}\left[\frac{1}{r} - \frac{1}{4r - 2a} - \frac{1}{4r + 2a}\right]$$

$$= \frac{\mu_0 I}{\pi}\left[\frac{16r^3 - 4a^2 - 4r^2 - 2ar - 4r^2 + 2ar}{r(16r^2 - 4a^2)}\right]$$

$$= \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right] \quad \text{(towards left)}$$

(b)

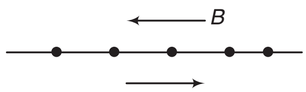


$$B_{net} = B_1 - 2B_2 \cos\theta] \qquad \text{(towards the top)}$$

$$= \frac{\mu_0}{2\pi}\frac{2I}{R} - 2\left[\frac{\mu_0}{2\pi}\frac{I/2}{\sqrt{r^2 + a^2/4}}\right]\frac{r}{\sqrt{r^2 + a^2/4}}$$

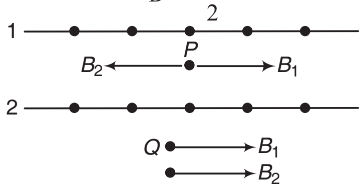
$$= \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right] \qquad \text{(towards the top)}$$

33.



B in the above situation is given by

$$B = \frac{\mu_0 \lambda}{2}$$



At point P, B_1 and B_2 are in opposite directions.

Hence, $\quad B_P = 0$

At point, Q, B_1 and B_2 are in same direction.

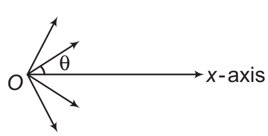
Hence, $\quad B_Q = 2\left(\frac{\mu_0 \lambda}{2}\right) = \mu_0 \lambda$

34. $$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a}$$

$\therefore \quad F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a} L \quad$ (Repulsion or upwards)

M of the loop is inwards and magnetic field to I_1 on the plane of loop is outwards. Hence, $\tau = 0$, as $\tau = \mathbf{M} \times \mathbf{B}$ and angle between two is 180°.

35.



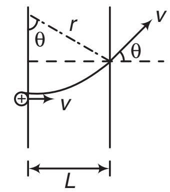
Electrons touch the x-axis again after every pitch. Therefore, the asked distance is

$$d = p = v_{11} T = (v \cos\theta)\left(\frac{2\pi m}{Bq}\right)$$

For paraxial electron $\theta \approx 0°$ and $q = e$,

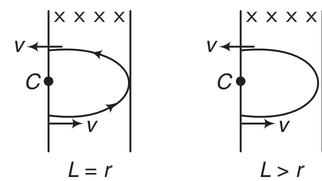
$$\therefore \quad d = \frac{2\pi m v}{Be}$$

36.



Deviation, $\theta = \sin^{-1}\left(\frac{L}{r}\right)$ for $L < r$

where, $\quad r = \frac{mv}{Bq}$



and $\quad \theta = \pi \quad$ if $\quad L \le r$

37. $a_E = \frac{qE_0}{m}$ (along negative z-direction)

Electric field will make z-component of velocity zero. At that time speed of the particle will be minimum and that minimum speed is the other component, i.e. v_0.

This is minimum when,

$$v_z = u_z + a_z t$$

or $\quad 0 = v_0 - \frac{qE_0}{m} t$

or $\quad t = \frac{m v_0}{qE_0}$

38. Path is helix and after one rotation only x-coordinate will change by a distance equal to pitch.

$$\therefore \quad x = p = (v_0 \cos\theta)\left(\frac{2\pi m}{Bq}\right)$$

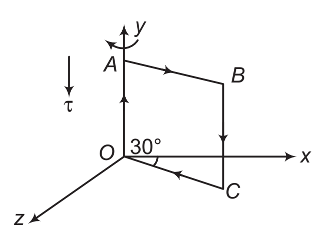
39. $\mathbf{M} = i(\mathbf{CO} \times \mathbf{OA})$

$= i(\mathbf{CO} \times \mathbf{CB})$

$= 4[(-0.1\,\hat{\mathbf{i}}) \times (0.2 \cos 30°\,\hat{\mathbf{j}} + 0.2 \sin 30°\,\hat{\mathbf{k}})$

$= (0.04\,\hat{\mathbf{j}} - 0.07\,\hat{\mathbf{k}})$ A-m^2

40.

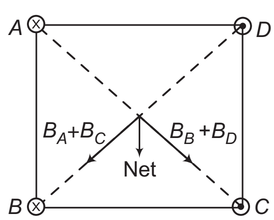


$$\mathbf{M} = Ni\,(\mathbf{OA} \times \mathbf{AB})$$
$$= Ni\,(\mathbf{OA} \times \mathbf{OC})$$
$$= (100)\,(1.2)\,[(0.4\,\hat{\mathbf{j}}) \times (0.03\cos 30°\,\hat{\mathbf{i}} + 0.3\sin 30°\,\hat{\mathbf{k}})]$$
$$= (7.2\,\hat{\mathbf{i}} - 12.47\,\hat{\mathbf{k}})\ \text{A-m}^2$$
$$\tau = \mathbf{M} \times \mathbf{B}$$
$$= [(7.2\,\hat{\mathbf{i}} - 12.47\,\hat{\mathbf{k}}) \times (0.8\,\hat{\mathbf{i}})]$$
$$= (-9.98\,\hat{\mathbf{j}})\ \text{N-m}$$
$$\therefore \quad |\tau| = 9.98\ \text{N-m}$$

Torque vector and expected direction of rotation is shown in figure.

41. $B_A = B_B = B_C = B_D$

$$= B = \frac{\mu_0}{2\pi}\frac{i}{r} = \frac{(2\times 10^{-7})\,(5)}{(0.2/\sqrt{2})}$$
$$= (5.0\sqrt{2}) \times 10^{-6}\ \text{T}$$



Net magnetic field

$$= \sqrt{(B_A + B_C)^2 + (B_B + B_D)^2}$$
$$= 2\sqrt{2}B$$
$$= 2.0 \times 10^{-5}\ \text{T} \quad \text{(towards bottom as shown)}$$

42. $i = \int_0^r (2\pi r\,dr)\,j = \int_0^r (2\pi r\,dr)\,(br) = \dfrac{2\pi\,br^3}{3}$

(a) **For $r_1 < R$**

$$B = \frac{\mu_0}{2\pi}\frac{i_{\text{in}}}{r_1}$$
$$= \frac{\mu_0}{2\pi}\left(\frac{2\pi b r_1^3/3}{r_1}\right)$$
$$= \frac{\mu_0 b r_1^2}{3}$$

(b) **For $r_2 > R$**

$$B = \frac{\mu_0}{2\pi}\frac{i_{\text{in}}}{r_2}$$
$$= \left(\frac{\mu_0}{2\pi}\right)\left(\frac{2\pi bR^3/b}{r_2}\right) = \frac{\mu_0 bR^3}{3r_2}$$

LEVEL 2

Single Correct Option

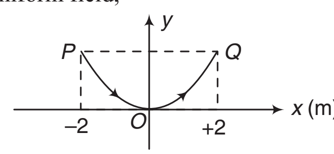
1. τ_{mg} about the left end (from where string is connected)

$$= |\mathbf{M} \times \mathbf{B}| = MB\sin 90°$$

or $\quad (mgR) = (NiA)B_0 = i\,(\pi R^2)B_0$

or $\quad i = \dfrac{mg}{\pi R B_0}$

2. In uniform field,



Magnetic force on POQ = magnetic force on straight wire PQ having the same current. Hence,

$$\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B}) = i\,(\mathbf{PQ} \times \mathbf{B})$$
$$= 2\,[(4\,\hat{\mathbf{i}}) \times (-0.02\,\hat{\mathbf{k}})]$$
$$= (0.16\,\hat{\mathbf{j}})$$
$$\therefore \quad \mathbf{a} = \frac{\mathbf{F}}{m} = \frac{(0.16\,\hat{\mathbf{j}})}{0.1} = (1.6\,\hat{\mathbf{j}})\ \text{m/s}^2$$

3. Linear impulse = mv

or $\quad F\Delta t = m\sqrt{2gh} \quad$ or $\quad (ilB)\,\Delta t = m\sqrt{2gh}$

But, $\quad i\,\Delta t = \Delta q$

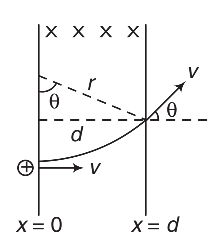
$\therefore \quad (\Delta q)\,(lB) = m\sqrt{2gh}$

Hence, $\quad \Delta q = \dfrac{m\sqrt{2gh}}{Bl}$

4. $\dfrac{M}{L} = \dfrac{q}{2m}$

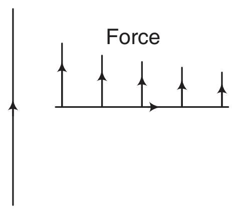
$$\therefore \quad M = \left(\frac{q}{2m}\right)L = \left(\frac{q}{2m}\right)(I\omega)$$
$$= \left(\frac{q}{2m}\right)\left(\frac{2}{5}mR^2\omega\right) = \frac{1}{5}qR^2\omega$$

5.



$$\sin\theta = \frac{d}{r} = \frac{d}{(mv/Bq)} \Rightarrow \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

6.



Variation of magnetic force on wire ACB is as shown in figure. Point of application of net force lies some where between A and C.

7. At a distance X from current I_2,

$$B = \frac{\mu_0}{2\pi}\frac{I_2}{X}$$

Magnetic force of small element dX of wire AB

$dF = I_2\,(dX)B \sin 90°$

$$\therefore \quad F = \int_{x=a}^{x=2a} dF$$

8. Apply screw law for finding magnetic field around a straight current carrying wire.

9. $$B_1 = \frac{\mu_0 IR^3}{2\,(R^2 + x^2)^{3/2}}$$

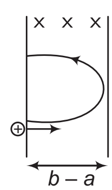
$$B_2 = \frac{\mu_0 I(2R)^2}{2\,[(2R)^2 + (2x)^2]^{3/2}}$$

$$\frac{B_1}{B_2} = 2$$

10. $(b - a) =$ radius of circular part

$$= \frac{mv}{Bq}$$

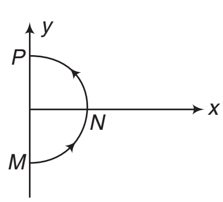
$$\therefore \quad V = \frac{Bq\,(b-a)}{m}$$



11. $$\frac{M}{L} = \frac{q}{2m}$$

$$M = \left(\frac{q}{2m}\right)(I\omega) = \left(\frac{q}{2m}\right)\left(\frac{ml^2}{3}\right)(2\pi f) = \frac{\pi q f l^2}{3}$$

12. At $x = 0$,



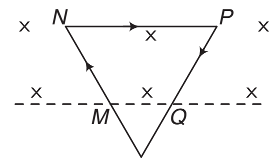
$$y = \pm\, 2m$$

$$\mathbf{F}_{MNP} = \mathbf{F}_{MP} = i\,[\mathbf{MP} \times \mathbf{B}\,]$$

$$= 3\,[(4\hat{\mathbf{j}}) \times (5\hat{\mathbf{k}})]$$

$$= 60\,\hat{\mathbf{i}}$$

13. $\mathbf{F}_{MNPQ} = \mathbf{F}_{MQ}$ and this force should be upwards to balance the weight.



$$\therefore \quad ilB = mg, \quad \text{where}$$

$$l = MQ = \frac{a}{2}$$

$$\therefore \quad i\,(a/2)\,B = mg \quad \text{or} \quad i = \frac{2Mg}{aB}$$

Force is upwards if current is clockwise or current in MQ is towards right.

14. $\frac{r}{2} =$ radius of circular path

$$= \frac{mv}{Bq} = \frac{mv}{(\mu_0 ni)\,(q)}$$

$$\therefore \quad v = \frac{\mu_0 qr\, ni}{2m}$$

15. (a) $\mathbf{B} \perp \mathbf{v}$, so it may along y-axis

(b) $\mathbf{F} \perp \mathbf{v}$,

$\therefore \quad \mathbf{a} \perp \mathbf{v} = 0$

or $\quad \mathbf{a} \cdot \mathbf{v} = 0$

or $\quad a_1 b_1 = a_2 b_2$

(c) See the logic of option (a).

(d) Magnetic force cannot change the kinetic energy of a particle.

16. Magnetic force is always perpendicular to velocity. So, it will always act in radial direction which will change tension at different points. But, time period and θ will remain unchanged.

17. Force on the wires parallel to x-axis will be obtained by integration (as $B \propto x$ and x coordinates vary along these wires). But on a loop there are two such wires. Force on them will be equal and opposite.

Forces on two wires parallel to y-axis can be obtained directly (without integration) as value of B is same along these wires. But their values will be different (as x-coordinate and therefore B is different).

$$F_{\text{net}} = \Delta F \qquad \text{(on two wires)}$$

$$= Ia\,(\Delta B) = Ia\,(B_0)\,(\Delta x)$$

$$= Ia\,B_0(a) = IB_0 a^2$$

This is indecent of x

$$\therefore \quad F_1 = F_2 = IBla^2 \neq 0$$

18. $B_x = \frac{\mu_0}{2\pi} \frac{I_{in}}{x}$

where, $I_{in} = I - \left[\frac{I}{\pi (c^2 - b^2)}\right][\pi (x^2 - b^2)]$

$= \frac{I (c^2 - x^2)}{(c^2 - b^2)}$

19. $\mathbf{F}_e = q\mathbf{E} = (1.6 \times 10^{-19})(-102.4 \times 10^3)\,\hat{\mathbf{k}}$

$= (-1.6384 \times 10^{-14}\,\hat{\mathbf{k}})\text{N}$

$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

$= (1.6 \times 10^{-19})[(1.28 \times 10^6\hat{\mathbf{i}}) \times (8 \times 10^{-2}\hat{\mathbf{j}})]$

$= (1.6384 \times 10^{-14}\hat{\mathbf{k}})\text{N}$

Now, we can see that

$$\mathbf{F}_e + \mathbf{F}_m = 0$$

20. Due to $\hat{\mathbf{j}}$ component of $\mathbf{B}$, magnetic force is zero. It is only due to $\hat{\mathbf{i}}$ component.
$\mathbf{l}$ is towards $-y$ direction, $\mathbf{B}$ is towards $\hat{\mathbf{i}}$ direction.
Hence, $\mathbf{l} \times \mathbf{B}$ (the direction of magnetic force is $+\hat{\mathbf{k}}$)

$$F = \int dF = \int_0^1 (i)(dy)(B)$$

$$= \int_0^1 (2 \times 10^{-3})(dy)(0.3\,y)$$

$$= 3 \times 10^{-4}\ \text{N}$$

21. $r = \frac{mv}{Bq}$ (during circular path)

$\therefore \quad v = \frac{Bqr}{m}$

Now, $qE = Bqv$

$\therefore \quad E = Bv = \frac{B^2qr}{m}$

$\therefore \quad E = \frac{(0.1)^2 (20 \times 10^{-6})(5 \times 10^{-2})}{(20 \times 10^{-9})}$

$= 0.5\ \text{V/m}$

22. $B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi \times 10^{-7})(5)}{(2)\left(\frac{5}{\sqrt{2}} \times 10^{-2}\right)}$

$= 2\sqrt{2}\pi \times 10^{-5}\,\text{T}$

$B_2 = \frac{\mu_0 i_2}{2R_2} = \frac{(4\pi \times 10^{-7})(5\sqrt{2})}{(2)(5 \times 10^{-2})}$

$= 2\sqrt{2}\pi \times 10^{-5}\,\text{T}$

$B_{net} = \sqrt{B_1^2 + B_2^2}$

23. $qE = Bqv$

$\therefore \quad v = \frac{E}{B}$

$r = \frac{mv}{Bq} = \frac{m(E/B)}{B(q)}$

$= \frac{E}{B^2S}$ (where, $S = q/m$)

24. $\frac{F}{l} =$ weight per unit length

$\therefore \quad \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r} = 0.01 \times 10$

or $\quad \frac{(2 \times 10^{-7})(100 \times 50)}{r} = 0.01 \times 10$

or $\quad r = 0.1\ \text{m}$

When B wire is displaced downwards from equilibrium position, magnetic attraction from A wire will decrease (which is upwards). But, weight (which is downwards). So, net force is downwards, in the direction of displacement from the mean position or away from the mean position. Hence, equilibrium is unstable.

25. Magnetic field due to current in the wire along z-axis is zero. Magnetic field due to wire along x-axis is along $\hat{\mathbf{j}}$ direction and magnetic field due to wire along y-axis is along $-\hat{\mathbf{i}}$ direction. Both wires produce

$$B = \frac{\mu_0}{2\pi}\frac{i}{a}$$

26. $|\mathbf{F}_{ABC}| + = |\mathbf{F}_{AC}| = ilB$

$= (2)(5)(2) = 20\ \text{N}$

27. $E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2 + x^2)^{3/2}}$

$B = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$

where, $\quad i = qf = q\left(\frac{V}{2\pi R}\right)$

$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

More than One Correct Options

1. $B_1 = \frac{\mu_0 N_1 i_1}{2R_1}$

$= \frac{(4\pi \times 10^{-7})(50)(2)}{2(5 \times 10^{-2})}$

$= 4\pi \times 10^{-4}\ \text{T}$

$$B_2 = \frac{\mu_0 N_2 i_2}{2R_2}$$

$$= \frac{(4\pi \times 10^{-7})(100)(2)}{(2)(10 \times 10^{-2})}$$

$$= 4\pi \times 10^{-4}\,\text{T}$$

When currents are in the same direction, then

$$B_{\text{net}} = B_1 + B_2$$

When currents are in the opposite directions, then

$$B_{\text{net}} = B_1 - B_2$$

2. (a) **v** is parallel on anti-parallel to **B**.

(c) $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

or $\qquad \mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$

3. $r = \dfrac{mV}{Bq} = \dfrac{(1)(10)}{(2)(1)} = 5\ \text{m}$

$T = \dfrac{2\pi m}{Bq} = \dfrac{(2)(\pi)(1)}{(2)(1)} = \pi$

$= 3.14\ \text{s}$

Plane of circle is perpendicular to **B**, i.e. xy-plane.

4. $\theta = 180°$

$\tau = MB \sin\theta = 0$

$U = -MB\cos\theta = +MB = \text{maximum}$

5. If current flows in a conductor, then

$E \neq 0$ (for inside points)

$E = 0$ (for outside points)

$B = \dfrac{\mu_0}{2\pi} \dfrac{i}{R^2} r$ (for inside points)

$B = 0$ at $r = 0$, i.e. at centre

$B = \dfrac{\mu_0}{2\pi} \dfrac{i}{r}$ for outside points.

6. $\mathbf{F}_{ab}$ = upwards

$\mathbf{F}_{abc}$ = leftwards

∴ Net force on loop is neither purely leftwards or rightwards or upwards or downwards.

7. $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

Depending on sign of q, $\mathbf{F}_m$ may be along positive z-axis or along negative z-axis.

$$\mathbf{F}_e = q\mathbf{E}$$

Again, depending on the value of q it may be along positive z-axis or along negative z-axis.

If q is positive, $\mathbf{v} \times \mathbf{B}$ and $\mathbf{F}_m$ comes along negative z-axis also. But, $\mathbf{F}_e$ comes along positive z-axis. So, it may also pass undeflected.

8. $\text{KE} = qV$ or $\text{KE} \propto V$

$r = \dfrac{\sqrt{2qVm}}{Bq}$ or $r \propto \sqrt{V}$

$T = \dfrac{2\pi m}{Bq}$ or T is independent of V.

9. (a) Point a lies to the right hand side of ef and fg. Hence, both wires produce inward magnetic field. Hence, net magnetic field is inwards. Same logic can be applied for other points also.

10. See the hint of Q.No-3 of Assertion & Reason section for Level 1

Match the Columns

1. (a) $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

q is negative, **v** is along $+\hat{\mathbf{i}}$ and **B** along $+\hat{\mathbf{j}}$.

Therefore, **F** is along negative z.

(b) Same logic is given in (a).

(c) **B** is parallel to **v**. So, magnetic force is zero. Charge is negative so, electric force is opposite to **E**.

(d) Charge is negative. So, electrostate force is in opposite direction of **E**.

2. For direction of magnetic force apply Fleming's left hand rule. According to that w and x are positively charged particles and y and z negatively charged particle.

Secondly,

$$r = \frac{\sqrt{2Km}}{Bq}$$

$$\therefore \qquad r \propto \frac{\sqrt{m}}{q} \qquad (K \rightarrow \text{same})$$

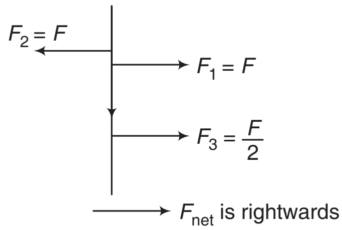
3. Force on a current carrying loop is zero for all angles. τ is maximum when Q, then angle between **M** and **B** is 90° minimum potential energy is at $\theta = 0°$. Positive potential energy is for obtuse angle. Direction of **M** is obtained by screw law.

4. Two currents are lying in the plane of paper. Its point is lying to the right hand side of the current carrying wire, magnetic field is inward ($-\hat{\mathbf{k}}$ direction).

If point lies to left hand side, field is outward ($\hat{\mathbf{k}}$ direction).

5. Let us take $F = \dfrac{\mu_0}{2\pi} \cdot \dfrac{i.\,i}{r}$

Current in same direction means attraction and current in opposite direction means repulsion. Let us find force on wire-2 by other three wires 1,3 and 4.



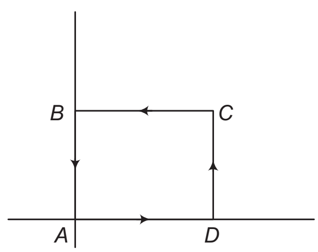
6. $\mathbf{F}_{ABC} = \mathbf{F}_{ADC} = \mathbf{F}_{AC}$

$\therefore \quad \mathbf{F}_{\text{loop}} = 2\,\mathbf{F}_{AC}$

$$= \left[\frac{I}{2}\,(\mathbf{l} \times \mathbf{B})\right]$$

$$= I\,(\mathbf{AC} \times \mathbf{B})$$

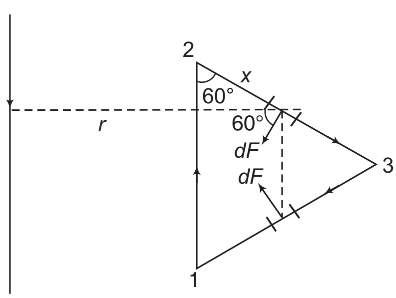
Hence, $\quad \mathbf{AC} = (l\,\hat{\mathbf{i}} + l\,\hat{\mathbf{j}})$



Now, putting value of **B** we can find net force in different cases.

Subjective Questions

1. Force on wire 12 will be $\left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)(Ia)$ in positive x-direction.



Force on 2-3 and 3-1 :

$$r = a + x\sin 60° = a + \sqrt{3}\,\frac{x}{2}$$

$$B_r = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{\mu_0}{2\pi}\frac{I}{a + \sqrt{3}\,\dfrac{x}{2}}$$

$$= \frac{\mu_0}{\pi}\frac{I}{2a + \sqrt{3}x}$$

$$\therefore \quad \mathbf{F}_{231} = \left[2\int_{x=0}^{x=a}(dF\cos 60°)\right](-\hat{\mathbf{i}})$$

$$= \left[2\int_0^a \frac{\mu_0}{\pi}\cdot\frac{I}{2a + \sqrt{3}x}.I.(dx)\frac{1}{2}\right](-\hat{\mathbf{i}})$$

$$= \left(\frac{\mu_0 I^2}{\pi}\right).\frac{1}{\sqrt{3}}[\ln\,(2a + \sqrt{3}x)]_0^a\,(-\hat{\mathbf{i}})$$

$$= \frac{\mu_0 I^2}{\sqrt{3}\pi}\ln\left(\frac{2 + \sqrt{3}}{2}\right)(-\hat{\mathbf{i}})$$

and $\quad \mathbf{F}_{12} = \dfrac{\mu_0 I^2}{2\pi}\,(\hat{\mathbf{i}})$

$$\therefore \quad \mathbf{F}_{\text{Total}} = \frac{\mu_0 I^2}{\pi}\left[\frac{1}{2} - \frac{1}{\sqrt{3}}\ln\left(\frac{2 + \sqrt{3}}{2}\right)\right](\hat{\mathbf{i}}) \quad \textbf{Ans.}$$

2. **Magnetic moment** According to Bohr's hypothesis, angular momentum in nth orbit is $L = n\left(\dfrac{h}{2\pi}\right)$.

Further, $\quad \dfrac{M}{L} = \dfrac{q}{2m}$ or $\dfrac{e}{2m}$

$\therefore$ Magnetic moment,

$$M = \left(\frac{e}{2m}\right)(L) = \left(\frac{e}{2m}\right)\left(n\frac{h}{2\pi}\right) = \left(\frac{neh}{4\pi m}\right)$$

Magnetic field induction

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{r^2} \qquad \ldots\text{(i)}$$

From Bohr's hypothesis :

$$mvr = \frac{nh}{2\pi} \qquad \ldots\text{(ii)}$$

Solving these two equations, we find

$$v = \frac{e^2}{2\varepsilon_0 nh} \quad \text{and} \quad r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$$

Now, magnetic field induction at centre

$$B = \frac{\mu_0 i}{2r}$$

Here, $\quad i = qf = (e)\left(\dfrac{v}{2\pi r}\right)$

$$\therefore \quad B = \left(\frac{\mu_0}{2r}\right)\left(\frac{ev}{2\pi r}\right) = \frac{\mu_0 e}{4\pi}\left(\frac{v}{r^2}\right)$$

$$= \left(\frac{\mu_0 e}{4\pi}\right)\left(\frac{e^2}{2\varepsilon_0 nh}\right)\left(\frac{\pi^2 m^2 e^4}{\varepsilon_0^2 n^4 h^4}\right)$$

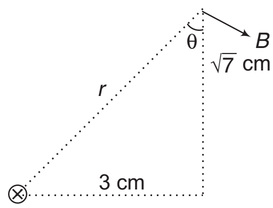
$$= \frac{\mu_0 \pi m^2 e^7}{8\varepsilon_0^3 h^5 n^5} \qquad \textbf{Ans.}$$

3. $r=\sqrt{9+7}=4$ cm

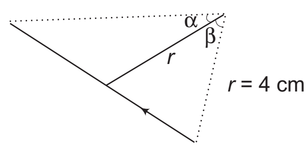
$$B=\frac{\mu_0}{4\pi}\frac{i}{r}(2\sin\alpha)$$

$$=(10^{-7})\left(\frac{30}{4\times10^{-2}}\right)\left(2\times\frac{3}{5}\right)$$

$$=9.0\times10^{-5}\text{ T}$$



(a)



(b)

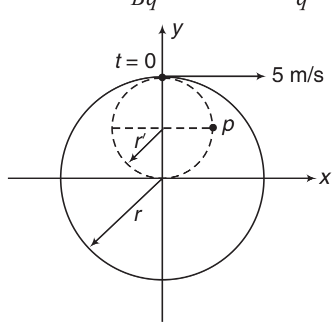
Net field $=4B\sin\theta$

$$=4\times9.0\times10^{-5}\times\frac{3}{4}$$

$$=2.7\times10^{-4}\text{ T}$$ **Ans.**

4. ***xy*-plane** $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(0.4)}{5}=\dfrac{8}{50}\pi$

Since, $T=\dfrac{2\pi m}{Bq}$ or $T\propto\dfrac{m}{q}$



After collision mass has become $\dfrac{5}{4}$ times and charge two times.

$\therefore$ $T'=\left(\dfrac{5}{4}\times\dfrac{1}{2}\right)T=\dfrac{5}{8}\times\dfrac{8}{50}\pi=\dfrac{\pi}{10}$ s

Given time $t=\dfrac{T'}{4}$, i.e. combined mass will complete one-quarter circle.

Further $r=\dfrac{P}{Bq}$

or $r\propto\dfrac{1}{q}$ (as P = constant)

Since, charge has become two times

$\therefore$ $r'=\dfrac{r}{2}=0.2$ m

At $t=(\pi/40)$ second, particle will be at P in xy-plane.

$\therefore$ $x=r'=0.2$ m

$y=r'=0.2$ m **Ans.**

***z*-coordinate** Mass of combined body has become 5 times of the colliding particle. Therefore, from conservation of linear momentum, velocity component in z-direction will become $\dfrac{1}{5}$ times. Or

$$v_z=\frac{1}{5}\times\frac{40}{\pi}\text{ m/s}=\frac{8}{\pi}\text{ m/s}$$

$\therefore$ $z=v_zt=\dfrac{8}{\pi}\times\dfrac{\pi}{40}=0.2$ m **Ans.**

5. $F_e=F_m$ or $eE=eBv$

$\therefore$ $v=\dfrac{E}{B}=\dfrac{120\times10^3}{50\times10^{-3}}$

$=2.4\times10^6$ m/s

Let n be the number of protons striking per second. Then,

$$ne=0.8\times10^{-3}$$

or $n=\dfrac{0.8\times10^{-3}}{1.6\times10^{-19}}$

$=5\times10^{15}$ m/s

Force imparted = Rate of change of momentum

$=nmv$

$=5\times10^{15}\times1.67\times10^{-27}\times2.4\times10^6$

$=2.0\times10^{-5}$ N **Ans.**

6. (a) Speed of particle at origin, $v=\sqrt{\dfrac{2qV}{m}}=v_x$

$$t=\frac{x}{v_x}=\frac{a}{\sqrt{\dfrac{2qV}{m}}}=a\sqrt{\frac{m}{2qV}}$$

$$y=\frac{1}{2}a_yt^2=\frac{1}{2}\left[\frac{qE}{m}\right]\left[a^2\times\frac{m}{2qV}\right]=\frac{a^2E}{4V}$$ **Ans.**

(b) Component parallel to **B** is

$$v_y=a_yt=\left(\frac{qE}{m}\right)\left(a\sqrt{\frac{m}{2qV}}\right)=Ea\sqrt{\frac{q}{2mV}}$$

Now, pitch = component parallel to **B** × time period

$$= v_y T = \left(Ea \sqrt{\frac{q}{2mV}} \right) \left(\frac{2\pi m}{Bq} \right)$$

$$= \frac{\pi Ea}{B} \sqrt{\frac{2m}{qV}} \qquad \textbf{Ans.}$$

7. **To graze at C** Using equation of trajectory of parabola,

$$y = x \tan\theta - \frac{ax^2}{2v^2 \cos^2\theta} \qquad \ldots\text{(i)}$$

Her , $a = \dfrac{qE}{m} = \dfrac{10^{-6} \times 10^{-3}}{10^{-10}}$

$$= 10 \text{ m/s}^2$$

Substituting in Eq. (i), we have

$$0.05 = 0.17 \tan 30° - \frac{10 \times (0.17)^2}{2v^2 \times (\sqrt{3}/2)^2}$$

Solving this equation, we have

$$v = 2 \text{ m /s}$$

In magnetic field, $AC = 2r$

or $0.1 = 2r$

or $r = 0.05 \text{ m} = \dfrac{mv \cos 30°}{Bq}$

$\therefore$ $B = \dfrac{mv \cos 30°}{(0.05)q}$

$$= \frac{(10^{-10})(2)(\sqrt{3}/2)}{(0.05)(10^{-6})}$$

$$= 3.46 \times 10^{-3} \text{ T}$$

$$= 3.46 \text{ mT} \qquad \textbf{Ans.}$$

8. Magnetic moment of the loop, $\mathbf{M} = (iA)\hat{\mathbf{j}}$

$$= (I_0 L^2)\hat{\mathbf{k}}$$

Magnetic field, $\mathbf{B} = (B \cos 45°)\,\hat{\mathbf{i}} + (B \sin 45°)\,\hat{\mathbf{j}}$

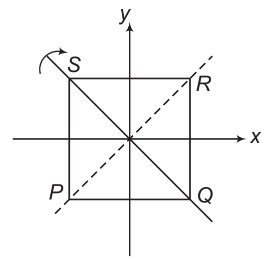
$$= \frac{B}{\sqrt{2}} (\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

(a) Torque acting on the loop, $\tau = \mathbf{M} \times \mathbf{B}$

$$= (I_0 L^2 \hat{\mathbf{k}}) \times \left[\frac{B}{\sqrt{2}} (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \right]$$

$$\therefore \quad \tau = \frac{I_0 L^2 B}{\sqrt{2}} (\hat{\mathbf{j}} - \hat{\mathbf{i}}) \quad \text{or} \quad |\tau| = I_0 L^2 B \qquad \textbf{Ans.}$$

(b) Axis of rotation coincides with the torque and since torque is along $\hat{\mathbf{j}} - \hat{\mathbf{i}}$ direction or parallel to QS. Therefore, the loop will rotate about an axis passing through Q and S as shown in the figure.



Angular acceleration, $\alpha = \dfrac{|\tau|}{I}$

where, I = moment of inertia of loop about QS.

$$I_{QS} = I_{PR} = I_{ZZ}$$

(From the theorem of perpendicular axis)

But, $I_{QS} = I_{PR}$

$$\therefore \quad 2I_{QS} = I_{ZZ} = \frac{4}{3} ML^2$$

$$I_{QS} = \frac{2}{3} ML^2$$

$$\therefore \quad \alpha = \frac{|\tau|}{I} = \frac{I_0 L^2 B}{2/3 ML^2} = \frac{3}{2} \frac{I_0 B}{M}$$

$\therefore$ Angle by which the frame rotates in time Δt is

$$\theta = \frac{1}{2} \alpha (\Delta t)^2$$

or $$\theta = \frac{3}{4} \frac{I_0 B}{M} \cdot (\Delta t)^2 \qquad \textbf{Ans.}$$

9. In equilibrium,

$$2T_0 = mg \quad \text{or} \quad T_0 = \frac{mg}{2} \qquad \ldots\text{(i)}$$

Magnetic moment, $M = iA = \left(\dfrac{\omega}{2\pi} Q \right) (\pi R^2)$

$$\tau = MB \sin 90° = \frac{\omega B Q R^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$).

For translational equilibrium of ring in vertical direction,

$$T_1 + T_2 = mg \qquad \ldots\text{(ii)}$$

For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega B Q R^2}{2}$$

or $$T_1 - T_2 = \frac{\omega B Q R^2}{2} \qquad \ldots\text{(iii)}$$

Solving Eqs. (ii) and (iii), we have

$$T_1 = \frac{mg}{2} + \frac{\omega B Q R^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_0}{2}$,

We have

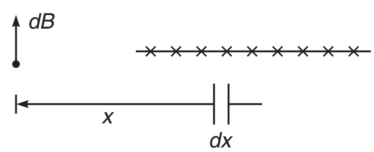
$$\frac{3T_0}{2} = T_0 + \frac{\omega_{max} BQR^2}{2D}$$

$$\therefore \quad \omega_{max} = \frac{DT_0}{BQR^2}$$ **Ans.**

10. $dB = \frac{\mu_0}{2\pi} \frac{(i/\omega).\, dx}{x}$

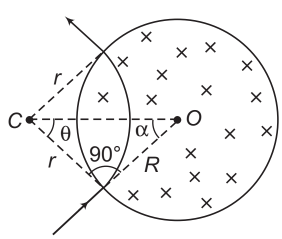
$$\therefore \quad B = \frac{\mu_0 i}{2\pi\omega} \int_d^{d+\omega} \frac{dx}{x}$$

$$B = \frac{\mu_0 i}{2\pi\omega} \ln\left(\frac{d+\omega}{d}\right)$$ (upwards) **Ans.**



11. $\theta = \tan^{-1}\left(\frac{R}{r}\right) = \tan^{-1}\left(\frac{R}{mv/Bq}\right) = \tan^{-1}\left(\frac{BqR}{mv}\right)$

Deviation $= 2\theta = 2\tan^{-1}\left(\frac{BqR}{mv}\right)$



12. (a) Yes, magnetic force for calculation of torque can be assumed at centre. Since, variation of torque about P from one end of the rod to the other end comes out to be linear.

$$\therefore \quad \tau = (IlB)\left(\frac{l}{2}\right) = \frac{Il^2B}{2}$$

$$= \frac{(6.5)(0.2)^2(0.34)}{2}$$

$$= 0.0442 \text{ N-m}$$ **Ans.**

(b) Magnetic torque on rod will come out to be clockwise. Therefore, torque of spring force should be anti-clockwise or spring should be stretched.

(c) In equilibrium,

Clockwise torque of magnetic force

= anti-clockwise torque of spring force

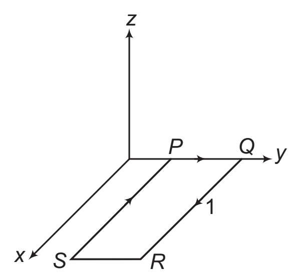
$$\therefore \quad 0.0442 = (kx)(l \sin 53°) = (4.8)(x)(0.2)\left(\frac{4}{5}\right)$$

or $\quad x = 0.057$ m

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times (4.8)(0.057)^2$$

$$= 7.8 \times 10^{-3} \text{J}$$ **Ans.**

13. Let the direction of current in wire PQ is from P to Q and its magnitude be I.



The magnetic moment of the given loop is

$$\mathbf{M} = -Iab\hat{\mathbf{k}}$$

Torque on the loop due to magnetic force is

$$\tau_1 = \mathbf{M} \times \mathbf{B}$$

$$= (-Iab\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\hat{\mathbf{i}}$$

$$= -3IabB_0\hat{\mathbf{j}}$$

Torque on weight of the loop about axis PQ is

$$\tau_2 = \mathbf{r} \times \mathbf{F} = \left(\frac{a}{2}\hat{\mathbf{i}}\right) \times (-mg\hat{\mathbf{k}})$$

$$= \frac{mga}{2}\hat{\mathbf{j}}$$

We see that when the current in the wire PQ is from P to Q, τ_1 and τ_2 are in opposite directions, so they can cancel each other and the loop may remain in equilibrium. So, the direction of current I in wire PQ is from P to Q. Further for equilibrium of the loop

$$|\tau_1| = |\tau_2|$$

or $$3IabB_0 = \frac{mga}{2}$$

$$I = \frac{mg}{6bB_0}$$ **Ans.**

Magnetic force on wire RS is

$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$$

$$= I[(-b\hat{\mathbf{j}}) \times \{(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\}]$$

$$\mathbf{F} = IbB_0(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$$ **Ans.**

27. Electromagnetic Induction

INTRODUCTORY EXERCISE 27.1

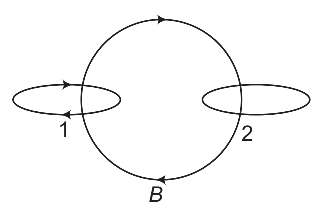
1. ⊗ magnetic field passing through loop is increasing. Hence, induced current will produce magnetic field. So, induced current should be anti-clockwise.

2. It is true that magnetic flux passing through the loop is calculated by integration. But, it remains constant.

3. $$\left|\frac{d\phi_B}{dt}\right| = [\text{Potential or EMF}]$$
$$= [\text{ML}^2\text{A}^{-1}\text{T}^{-3}]$$

4. ⊙ is increasing. Hence, ⊗ is produced by the induced current. So, it is clockwise.

5. By increasing the current in loop-1, magnetic field in ring-2 in downward direction will increase. Hence, induced current in ring-2 should produce upward magnetic field. Or current in ring should be in the same direction.



6. $2\pi R = 4L$

$$\therefore \quad L = \frac{\pi R}{2} = \frac{(\pi)(10)}{2}$$
$$= (5\pi)\text{ cm}$$
$$\Delta S = S_i - S_f = (\pi R^2) - L^2$$
$$= \pi\,(0.1)^2 - (5\pi)^2 \times 10^{-4}$$
$$= 0.0067\text{ m}^2$$
$$e = \frac{\Delta\phi}{\Delta t} = B\left(\frac{\Delta S}{\Delta t}\right)$$
$$= \frac{100 \times 0.0067}{0.1}$$
$$= 6.7\text{ V}$$

7. $\Delta\phi = 2\,(NBS)$

$$\Delta q = \frac{\Delta\phi}{R} = \frac{2NBS}{R}$$
$$= \frac{2 \times 500 \times 0.2 \times 4 \times 10^{-4}}{50}$$
$$= 1.6 \times 10^{-3}\text{C}$$

8. $\mathbf{S} = [(5 \times 10^{-4})\,\hat{\mathbf{k}}]\text{ m}^2$

$$\phi = |\mathbf{B} \cdot \mathbf{S}| = 9 \times 10^{-7}\text{ Wb}$$

INTRODUCTORY EXERCISE 27.2

1. $\because \quad e = Bvl = 1.1 \times 5 \times 0.8$
$$= 4.4\text{ V}$$

Apply right hand rule for polarity of this emf.

2. $\because \quad e = Bvl$
$$i = \frac{e}{R} = \frac{Bvl}{R}$$
$$F = ilB = \frac{B^2l^2v}{R}$$
$$= \frac{(0.15)^2\,(0.5)^2\,(2)}{3}$$
$$= 0.00375\text{ N}$$

3. $$V_A - V_C = \frac{B\omega l^2}{2}$$
$$V_D - V_C = \frac{B\omega(2l)^2}{2}$$

From these two equations, we find
$$V_A - V_D = -3B\omega l^2/2$$

4. Circuit is not closed. So, current is zero or magnetic force is zero.

INTRODUCTORY EXERCISE 27.3

1. $|e| = \left|L\frac{\Delta i}{\Delta t}\right|$ or $\left|L\frac{di}{dt}\right|$

Here, $\quad L = 1\text{H}$

and $\quad \frac{di}{dt} = 3\,[\sin t + t\cos t]$

$\therefore \quad |e| = 3\,(t\cos t + \sin t)$

2. $V_L = +L\frac{di}{dt} = (2)\frac{d}{dt}(10e^{-4t})$
$$= -80\,e^{-4t}$$

Further, $V_a - iR - L\frac{di}{dt} = V_b$

$$\therefore \quad V_a - V_b = iR + L\frac{di}{dt}$$

or $$V_{ab} = (10e^{-4t})(4) - 80e^{-4t}$$
$$= -40e^{-4t}$$

3. (a) $dI/dt = 16\,\text{A/s}$

$$\therefore \quad L = \left| \frac{e}{dI/dt} \right| = \frac{10 \times 10^{-3}}{16}$$

$$= 0.625 \times 10^{-3}\ \text{H} = 0.625\ \text{mH}$$

(b) At $t = 1\,\text{s}$, $I = 21$ A

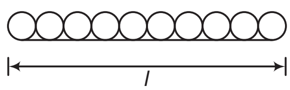
$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times (0.625 \times 10^{-3})(21)^2$$

$$= 0.137\ \text{J}$$

$$P = Ei = (10 \times 10^{-3})\,(21)$$

$$= 0.21\ \text{J/s}$$

4. (a)



$$L = \frac{\mu_0 N^2 S}{l}$$

$$N = \frac{l}{d}$$

$$\therefore \quad L = \frac{\mu_0 l S}{d^2} = \frac{(4\pi \times 10^{-7})\,(0.4)\,(0.9 \times 10^{-4})}{(0.1 \times 10^{-2})^2}$$

$$= 4.5 \times 10^{-5}\ \text{H}$$

(b) $e = \left| L \dfrac{\Delta i}{\Delta t} \right| = \dfrac{(4.5 \times 10^{-5})\,(10)}{0.1} = 4.5 \times 10^{-3}$ V

INTRODUCTORY EXERCISE 27.4

1. $\because \quad M = \dfrac{e_2}{di_1/dt} = \dfrac{(50 \times 10^{-3})}{(8/0.5)}$

$$= 3.125 \times 10^{-3}\ \text{H} = 3.125\ \text{mH}$$

$$e_1 = M\left(\frac{\Delta i_2}{\Delta t}\right) = \frac{(3.125 \times 10^{-3})\,(6)}{0.02}$$

$$= 0.9375\ \text{V}$$

2. (a) $M = \dfrac{N_2 \phi_2}{i_1} = \dfrac{(1000)\,(6 \times 10^{-23})}{3} = 2$ H

(b) $|e_2| = M\left(\dfrac{\Delta i_1}{\Delta t}\right) = \dfrac{(2)\,(3)}{0.2} = 30$ V

(c) $L_1 = \dfrac{N_1 \phi_1}{i_1} = \dfrac{(600)\,(5 \times 10^{-3})}{3} = 1\text{H}$

3. (a) $|e_2| = M\left(\dfrac{di_1}{dt}\right)$

$$= (3.24 \times 10^{-4})\,(830)$$

$$= 0.27\ \text{V}$$

(b) Result will remain same.

INTRODUCTORY EXERCISE 27.5

1. $\because \quad E = \dfrac{1}{2}Li^2 = i^2 Rt$

$\therefore \quad \dfrac{L}{R}$ has the units of time.

2. (a) $\tau_L = \dfrac{L}{R} = \dfrac{2}{10} = 0.2$ s

(b) $i_0 = \dfrac{E}{R} = \dfrac{100}{10} = 10$ A

(c) $i = i_0\,(1 - e^{-t/\tau_L})$

$$= 10\,(1 - e^{-1/0.2})$$

$$= 9.93\ \text{A}$$

3. $E = V_R + V_L$

INTRODUCTORY EXERCISE 27.6

1. $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}\dfrac{q^2}{C}$

$\therefore \quad \sqrt{LC} = \dfrac{q}{i} = \dfrac{it}{i} = t$

3. (a) $V_L = V_C$

$$L\frac{di}{dt} = \frac{q}{C}$$

$$q = (LC)\frac{di}{dt}$$

$$= (0.75 \times 18 \times 10^{-6})\,(3.4)$$

$$= 45.9 \times 10^{-6}\ \text{C}$$

$$= 45.9\ \mu\text{C}$$

(b) $\quad V_L = V_C = \dfrac{q}{C}$

$$= \frac{4.2 \times 10^{-4}}{18 \times 10^{-6}}$$

$$= 23.3\ \text{V}$$

4. $\dfrac{1}{2}Li_{\max}^2 = \dfrac{1}{2}CV_{\max}^2$

$\therefore \quad V_{\max} = \left(\sqrt{\dfrac{L}{C}}\right) i_{\max}$

$$= \left(\sqrt{\frac{20 \times 10^{-3}}{0.5 \times 10^{-6}}}\right)(0.1)$$

$$= 20\ \text{V}$$

INTRODUCTORY EXERCISE 27.7

1. In the theory we have already derived mutual inductance between solenoid and coil,

$$M = \frac{\mu_0 N_1 N_2 (\pi R_1^2)}{l_1}$$

$$= \frac{\mu_0 N_1 N_2 S_1}{l_1}$$

$$|e_2| = \left| M \frac{di}{dt} \right| = \left| \frac{\mu_0 N_1 N_2 S_1}{l_1} \frac{di_1}{dt} \right|$$

$$= \frac{(4\pi \times 10^{-7})(25)(10)(5 \times 10^{-4})(0.2)}{10^{-2}}$$

$$= 3.14 \times 10^{-6} \text{ V}$$

(b) $$El = \left| \frac{d\phi}{dt} \right| = e$$

$$\therefore \quad E = \frac{e}{l} = \frac{e}{2\pi R_2}$$

$$= \frac{3.14 \times 10^{-6}}{(2\pi)(0.25)}$$

$$= 2 \times 10^{-6} \text{ V/m}$$

2. (a) **At P_2**

$$El = \frac{d\phi}{dt}$$

$$E(2\pi r_2) = \pi R^2 \cdot \frac{dB}{dt}$$

$$\therefore \quad E = \frac{R^2}{2r_2}\left(\frac{dB}{dt}\right)$$

$$F = qE = \frac{eR^2}{2r_2}(6t^2 - 8t)$$

Substituting the values, we have

$$F = \frac{(1.6 \times 10^{-19})(2.5 \times 10^{-2})}{2 \times 5 \times 10^{-2}}[6(2)^2 - 8(2)]$$

$$= 8.0 \times 10^{-21} \text{N}$$

⊗ magnetic field at the given instant is increasing. Hence, induced current in an imaginary circular loop passing through P_2 should produced ⊙ magnetic field. Or, current in this should be anti-clockwise. Hence, electrons should move in clockwise direction. Or electron at P_2 should experience force in downward direction (perpendicular to r_2).

(b) **At P_1**

$$El = \frac{d\phi}{dt}$$

$$\therefore \quad E(2\pi r_1) = s\frac{dB}{dt} = (\pi r_1^2)\left(\frac{dB}{dt}\right)$$

$$\therefore \quad E = \frac{r_1}{2}\left(\frac{dB}{dt}\right)$$

$$= \frac{r_1}{2}[6t^2 - 8t]$$

$$= \frac{0.02}{2}[6(3)^3 - 8(3)]$$

$$= 0.3 \text{ V/m}$$

As discussed in the above part, direction of electric field is in the direction of induced current (anti-clockwise) in an imaginary circular conducting loop passing through P_1.

Exercises

LEVEL 1

Assertion and Reason

1. Due to non-uniform magnetic field (a function of x) magnetic flux passing through the loop obtained by integration. But that remains constant with time.

Hence, $$\frac{d\phi}{dt} = 0$$

or $$e = 0$$

Magnetic field is along $-\hat{\mathbf{k}}$ direction or in ⊗ magnetic is increasing. Hence, induced current should produce ⊙ magnetic field. Or induced current should be anti-clockwise.

2. At time t_0, magnetic field is negative or and increasing.

Hence, induced current will produce ⊗ magnetic field. Or induced current should be clockwise.

If $\frac{d\phi}{dt}$ = constant. Then, e = constant

$\therefore$ i or rate of flow of charge is constant.

3. It can exert a force on charged particle.

4. $\frac{di}{dt} = 2$ A/s

$$V_a - V_b = L\frac{di}{dt}$$

$$= (2)(2) = 4 \text{ V}$$

5. Comparing with spring-block system

$\left(\frac{dI}{dt}\right)$ is acceleration.

$$a_{max} = \omega^2 A = \omega\,(\omega A)$$
$$= \omega\,(v_{max})$$
$$\therefore \quad \left(\frac{dI}{dt}\right)_{max} = \omega\, I_{max}$$

6. Applying RHR, we can find that

$$V_a > V_b$$

8. Ferromagnetic substance will attack more number of magnetic lines through it. So, flux passing through it will increase. Hence, coefficient of self-inductance will increase.

L depends on number of turns in the coil's radius of coil etc. It does not depend on the current passing through it.

9. Current developed in the inductor wire will decrease exponentially through wire ab.

10. $V_{L_1} = V_{L_2}$

$$\therefore \quad L\frac{di_1}{dt} = L_2\frac{di_2}{dt}$$
$$\text{or} \quad L_1 di_1 = L_2 di_2$$
$$\text{or} \quad L_1 i_1 = L_2 i_2$$
$$\text{or} \quad i \propto \frac{1}{L}$$

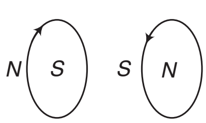
Objective Questions

1. $U = \frac{1}{2}Li^2$

$$\therefore \quad [L] = \left[\frac{U}{i^2}\right] = \left[\frac{ML^2T^{-2}}{A^2}\right]$$
$$= [ML^2T^{-2}A^{-2}]$$

2. $M \propto N_1 N_2$

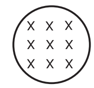
3.



When brought closer induced effects should produced repulsion. So, currents should increase, so that pole strength increases. Hence, repulsion increases.

4. Magnetic field of ring is also along its axis, or in the direction of velocity of charged particle. Hence, no magnetic force will act on charged particle. But, due to g velocity of charged particle will increase.

5.



If magnetic field in the shown cylindrical region is changing, then induced electric field exists even outside the cylindrical regions also where magnetic field does not exist.

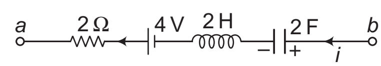
6. $i = \frac{dq}{dt} = (8t)\text{A}$

$$\frac{di}{dt} = 8 \text{ A/s}$$

At $\quad t = 1\text{ s},\ q = 4\text{ C},\ i = 8\text{ A}$

and $\quad \frac{di}{dt} = 8\text{ A/s}$

Charge on capacitor is increasing. So, charge on positive plate is also increasing. Hence, direction of current is towards left.



Now, $V_a + 2\times 8 - 4 + 2\times 8 + \frac{4}{2} = V_b$

$$\therefore \quad V_a - V_b = -30\text{ V}$$

7. $\frac{dI}{dt} = I_0\omega\cos\omega t$

$$e = M\frac{dI}{dt} = MI_0\omega\cos\omega t$$
$$\therefore \quad e_{max} = MI_0\omega$$
$$= 0.005\times 10\times 100\pi$$
$$= (5\pi)\text{ V}$$

8. $\frac{1}{2}Li^2 = \frac{1}{2}CV^2$

$$\therefore \quad V = \sqrt{\frac{L}{C}} \Rightarrow i = \sqrt{\frac{2}{4\times 10^{-6}}}\ (2)$$
$$= \sqrt{2}\times 10^3\text{ V}$$

9. $e = \frac{B\omega l^2}{2} = \text{constant}$

10. $\Delta q = \frac{\Delta\phi}{R}$

or $\quad i\Delta t = \frac{\Delta\phi}{R}$

$$\therefore \quad \Delta\phi = i(\Delta t)R = (10\times 10^{-3})\,(5)\,(0.5)$$
$$= 25\times 10^{-3}\text{ Wb}$$

11. $\otimes$ magnetic field is increasing. Therefore, induced electric lines are circular and anti-clockwise. Force on negative charge is opposite to electric field.

12. $e = \left|\frac{d\phi}{dt}\right| = (a\tau - 2at)$

$$i = \frac{e}{R} = \frac{a\tau - 2at}{R}$$

$$H = \int_0^{\tau} i^2 R dt$$

13. $e = L\left|\frac{di}{dt}\right| = L$ (Slope of i-t graph)

Initially, slope = 0 $\Rightarrow$ $e = 0$

Then in remaining two regions slopes are constants but of opposite signs. Hence, induced emfs are constants but of opposite signs.

14. $V_A - 1 \times 5 + 15 + (5 \times 10^{-3})(10^3) = V_B$

$\therefore$ $V_B - V_A = 15$ V

15. $I = (10t + 5)$ A

$$\frac{dI}{dt} = 10 \text{ A/s} = \text{constant}$$

At, $t = 0, I = 5$ A

Now, $V_A - 3 \times 5 - 1 \times 10 + 10 = V_B$

$\therefore$ $V_A - V_B = 15$ V

16. $$(V_C)_{\max} = (V_L)_{\max}$$

$$\frac{q_0}{C} = L\left(\frac{dI}{dt}\right)_{\max}$$

$\therefore$ $$\left(\frac{dI}{dt}\right)_{\max} = \frac{q_0}{LC}$$

17. $V_L = L\frac{di}{dt}$

18. $\phi_i = BS \cos 0° = 2$ Wb

$\phi_f = BS \cos 180° = -2$ Wb

$$|\Delta\phi| = 4 \text{ Wb}$$

$$|\Delta q| = \frac{|\Delta\phi|}{R}$$

$$= \frac{4}{10} = 0.4 \text{ C}$$

19. $\mathbf{S} = (ab)\,\hat{\mathbf{k}} \rightarrow$ perpendicular to xy-plane

$\phi = \mathbf{B} \cdot \mathbf{S} = (50)(ab) = \text{constant}$

$$\frac{d\phi}{dt} = 0$$

$\therefore$ $e = 0$

20. Back emf = Applied voltage potential drop across armature coil

$$= 200 - iR$$
$$= 200 - 1.5 \times 20$$
$$= 170 \text{ V}$$

21. $\frac{N_S}{N_P} = \frac{I_P}{I_S} = \frac{V_S}{V_P}$

$$V_P = V_i = \left(\frac{N_P}{N_S}\right) V_0$$

$$= \frac{2}{1} \times 20 = 40 \text{ V}$$

$$I_P = \left(\frac{N_S}{N_P}\right) I_S$$

$$= \left(\frac{1}{2}\right)(4) = 2 \text{ A}$$

22. Relative velocity = 0

$\therefore$ Charge in flux = 0

23. In case of free fall,

$$d = \frac{1}{2}gt^2$$

$$= \frac{1}{2}(10)(1)^2 = 5 \text{ m}$$

Here due to repulsion from induced effects

$$a < g$$

$\therefore$ $d < 5$ m

24. $V_A - V_B = L\frac{di}{dt}$

$$= L(-\alpha) = -\alpha L$$

25. $i_0 = \frac{E}{R} = \frac{12}{0.3} = 40$ A

$$U = \frac{1}{2}Li_0^2 = \frac{1}{2} \times 50 \times 10^{-3}(40)^2$$

$$= 40 \text{ J}$$

26. Value remains $\frac{1}{4}$th in 20 ms times. Hence, two half-lives are equal to 20 ms. So, one half-life is 10 ms.

$$t_{1/2} = (\ln 2)\,\tau_C = (\ln 2)\frac{L}{R}$$

$\therefore$ $$R = \frac{(\ln 2)\,L}{t_{1/2}}$$

$$= \frac{(\ln 2)(2)}{10 \times 10^{-3}} = (100 \ln 4)\ \Omega$$

27. $\because$ $i = \frac{e}{R}$

$$= \frac{N\,(\Delta\phi/\Delta t)}{R} = \frac{NS\,(\Delta B/\Delta t)}{R}$$

$$= \frac{10\,(10 \times 10^{-4})(10^4)}{20}$$

$$= 5 \text{ A}$$

28. In steady state, whole current passes through the inductor.

29. If current is passed through the straight wire, magnetic lines are circular and tangential to the loop. So, no flux is linked with the loop.

30. In second position, $\Delta\phi = 0$

$\therefore$ $$|Q_2| = \frac{\Delta\phi}{R} = 0$$

31. From Lenz's law, induced effects always oppose the cause due to which they are produced. So, when the first loop is moved towards the smaller loop, it will face repulsion.

32. $\tau_L = \dfrac{L}{R} = 2\text{ s}$

$$i_0 = \frac{E}{R} = 3\text{A}, \; t = 2\text{ s}$$

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Substituting the given values, we can find i.

33. In AB, **l** is parallel to its **v**. Hence, PD = 0

34. **v** is parallel to **l**.

35. For wire ab, velocity vector is parallel to **l**.

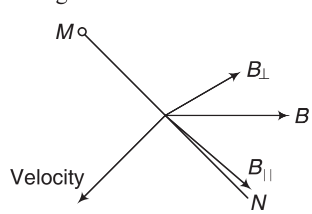
36. Current increases with time. So, flux passing through B will increase with time. From Lenz's law, it should have a tendency to move away from the coil to decrease flux.

37. For $E \neq 0$, ϕ must change

or $$\frac{d\phi}{dt} \neq 0$$

38. Even if radius is doubled, flux is not going to change.

39.



$B_{||}$ is parallel to MN (or **l**) and $B_\perp$ is parallel or antiparallel to velocity.

Subjective Questions

1. When switch is opened current suddenly decreasing from steady state value to zero. When switch is closed, it takes time to increase from 0 to steady state value.

$$e = \left| L\frac{\Delta i}{\Delta t} \right|$$

Δt in second case is large. Hence, induced emf is less.

2. $e = \dfrac{N\Delta\phi}{\Delta t} = \left(\dfrac{NS\Delta B}{\Delta t}\right)\cos 30°$

$$\therefore \quad S = \left(\frac{e\Delta t}{N\Delta B}\right)\sec 30°$$

$$= \left[\frac{(80 \times 10^{-3})\,(0.4)}{(50)\,(400 \times 10^{-6})}\right]\left[\frac{2}{\sqrt{3}}\right] = 1.85\text{ m}$$

$\therefore$ Side of square = 1.36 m

Total length of wire = 50 (4 × 1.36)

= 272 m

3. $\phi = BS = B_0 S\, e^{-at}$

$$\text{Induced emf} = \left|\frac{d\phi}{dt}\right| = aB_0 S e^{-at}$$

4. (a) At a distance x from the wire, magnetic field over the wire ab is

$$B = \frac{\mu_0}{2\pi}\frac{i}{x}$$

$$dV = Bvdx = \left(\frac{\mu_0 i}{2\pi x}\right) vdx$$

$$\therefore \quad \text{Total emf} = \int_{x=d}^{x=d+l} dV$$

(b) Magnetic field due to current i over the wire ab is inwards. Velocity of wire ab is towards right. Applying right hand rule, we can see that a point is at higher potential.

(c) Net change in flux through the loop $abcd$ is zero. Hence, induced emf is zero. So, induced current is zero.

5. At $t = 0$, inductor offers infinite resistance. Hence, current through inductor wire is zero. Whole current passes through two resistors of 4Ω each.

$$i_1 = \frac{10}{4+4} = 1.25\text{ A}$$

At $t = \infty$, inductor offers zero resistance.

$$R_{\text{net}} = 4 + \frac{8 \times 4}{8 + 4}$$

$$= 6.67\ \Omega$$

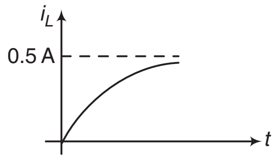
So, main current

$$i_2 = \frac{10}{R_{\text{net}}} = 1.5\text{A}$$

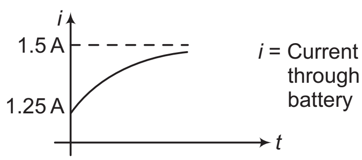
This distributes in 4Ω and 3Ω in inverse ratio of resistance. Hence, current through 4Ω is 1A and through 8Ω is 0.5A.

For equivalent τ_L of the circuit R_{net} across inductor after short-circuiting, the battery is 10 Ω.

$\therefore \qquad \tau_L = \frac{1}{R_{net}} = \frac{1}{10} = 0.1\ \text{s}$



$$i_L = 0.5\,(1 - e^{-\frac{t}{0.1}})$$
$$= 0.5\,(1 - e^{-10t})$$



$$i = 1.25 + 0.25\,(1 - e^{-t/0.1})$$
$$= 1.5 - 0.25\,e^{-10t}$$

6. Similar to above problem

$$\Delta q = \frac{2NBS}{R}$$

$$\therefore \qquad B = \frac{(\Delta q)R}{2NS}$$
$$= \frac{(4.5 \times 10^{-6})\,(40)}{(2)\,(60)\,(3 \times 10^{-6})}$$
$$= 0.5\ \text{T}$$

7. $e = \frac{d\phi}{dt} = s\left(\frac{dB}{dt}\right) = \pi R^2$ (Slope of B - t graph)

(a) $e = (\pi)\,(0.12)^2\left(\frac{0.5}{2}\right) = 0.011\ \text{V/m}$

(b) Slope of B-t graph is zero. Hence,

$$e = 0$$

(c) Slope is just opposite to the slope of part (a).

8. Induced emf ($e = BvL$) and therefore induced current is developed only during entering and during existing from the magnetic field.

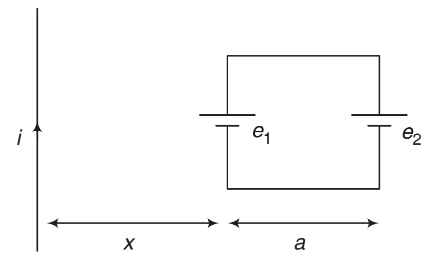
$$i = \frac{e}{R} = \frac{BvL}{R}$$
$$F = iLB = \frac{B^2L^2v}{R}$$

Further, magnetic force always opposes the change. Hence, external force is always positive.

During entering into the field, ⊗ magnetic field increases. Hence, induced current should produce ⊙ magnetic field. Or it should be anti-clockwise.

During existing from the magnetic field case is just opposite.

9.



$$e_{net} = e_1 - e_2$$
$$= B_1va - B_2va$$
$$= (B_1 - B_2)\,va$$
$$= \left[\frac{\mu_0}{2\pi}\frac{i}{x} - \frac{\mu_0}{2\pi}\frac{i}{x+a}\right] va$$
$$= \frac{\mu_0}{4\pi}\frac{2ia^2v}{x\,(x+a)}$$

10. $B = \frac{\mu_0}{2\pi}\frac{i}{r}$

$$dV = Bvdr = \frac{\mu_0 iv}{2\pi}\int \frac{dr}{r}$$
$$V = \int_{r_1}^{r_2} dV = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

11. $V_L = L\frac{di}{dt}$

$$\therefore \quad di = \frac{1}{L}\,(V_L dt)$$
$$\therefore \quad \int di = i = \frac{1}{L}\int V_L dt$$

or $\quad i = \frac{1}{L}$ (area under V_L *versus* t graph)

(a) **At $t = 2$ ms**

$$i = (150 \times 10^{-3})^{-1}\left(\frac{1}{2} \times 2 \times 10^{-3} \times 5\right)$$
$$= 3.33 \times 10^{-2}\ \text{A}$$

(b) **At $t = 4$ ms**

Area is just double. Hence, current is also double.

12. (a) $L = \left|\frac{e}{di/dt}\right| = \frac{0.016}{0.064} = 0.25\ \text{H}$

(b) $L = \frac{N\phi}{i}$

$$\therefore \qquad \phi = \frac{Li}{N} = \frac{(0.25)\,(0.72)}{400}$$
$$= 4.5 \times 10^{-4}\ \text{Wb}$$

13. (a) $M = \frac{N_2\phi_2}{i_1} = \frac{(400)(0.032)}{6.52} = 1.96\ \text{H}$

(b) $M = \frac{N_1\phi_1}{i_2}$

$$\phi_2 = \frac{Mi_2}{N_1} = \frac{(1.96)(2.54)}{700}$$

$$= 7.12 \times 10^{-3}\ \text{Wb}$$

14. $\tau_L = \frac{L}{R} = 0.1\ \text{s}$

The given time $t = 0.1$ s is one time constant.

The desired ratio is $\frac{iV_L}{iE}$ $\quad (\because\ P = Vi)$

$$= \frac{V_L}{E}$$

After on time constant $V_L = \frac{E}{e}$ as

$$V_L = Ee^{-t/\tau_L}$$

Hence, the desired ratio is $\frac{1}{e} \approx 0.37$.

15. $i_0 = \frac{V}{R} = \frac{3.24}{12.8} = 0.253\ \text{A}$

$\tau_L = \frac{L}{R} = \frac{3.56}{12.8} = 0.278\ \text{s}$

(a) After one time constant ($t = 0.278\ \text{s} = \tau_C$)

$$i = \left(1 - \frac{1}{e}\right) i_0$$

$$\approx 0.63\, i_0$$

$$= 0.16\ \text{A}$$

Power supplied by battery $= Ei$

$$P = (3.24)(0.16) = 0.518\ \text{W}$$

(b) $P_R = i^2 R$

$$= (0.16)^2 (12.8) = 0.328\ \text{W}$$

(c) $P_L = P - P_R = 0.191\ \text{W}$

16. (a) After one half-life,

$$t = t_{1/2} = (\ln 2)\,\tau_L$$

$$= 0.693\,\frac{L}{R}$$

$$= \frac{(0.693)(1.25 \times 10^{-3})}{50}$$

$$= 1.73 \times 10^{-5}\ \text{s}$$

(b) $\left(\frac{1}{2}Li^2\right) = \left(\frac{1}{2}Li_0^2\right)/2$

$$\therefore \quad i = \frac{i_0}{\sqrt{2}}$$

Now, apply

$$i = i_0\,(1 - e^{-t/\tau_L})$$

where, $\tau_L = \frac{L}{R}$

17. Steady state current developed in the inductor

$$= \frac{E}{r} = i_0\ (\text{say})$$

(a) Now this current decreases to zero exponentially through r and R.

$$\therefore \quad i = i_0 e^{-t/\tau_L}$$

where, $\tau_L = \frac{L}{R + r}$

Energy stored in inductor,

$$U_0 = \frac{1}{2}Li_0^2 = \left(\frac{1}{2}L\right)\left(\frac{E}{r}\right)^2$$

Now, this energy dissipates in r and R in direct ratio of resistances.

$$\therefore \quad H_r = \left(\frac{r}{R + r}\right)U_0 = \frac{E^2 L}{2r\,(R + r)}$$

18. In steady state, main current from the battery is

$$i_0 = \frac{E}{R} = \frac{20}{5} = 4\ \text{A}$$

Now, this current distributes in inverse ratio of inductor.

$$\therefore \quad i_5 = \left(\frac{10}{10 + 5}\right)(4\text{A}) = \frac{8}{3}\ \text{A}$$

19. (a) $\frac{1}{2}Li_0^2 = \frac{1}{2}CV_0^2$

$$\therefore \quad L = \frac{CV_0^2}{i_0^2} = \frac{(4 \times 10^{-6})(1.5)}{(50 \times 10^{-3})^2}$$

$$= 3.6 \times 10^{-3}\ \text{H}$$

(b) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(3.6 \times 10^{-3})(4 \times 10^{-6})}}$

$$= 0.133 \times 10^4\ \text{Hz}$$

$$= 1.33\ \text{kHz}$$

(c) $t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4 \times 1.33 \times 10^3}\ \text{s}$

$$= 0.188 \times 10^{-3}\ \text{s}$$

$$= 0.188\ \text{ms}$$

20. (a) $\omega = \frac{2\pi}{f}$, $T = \frac{1}{f}$

(b) At $t = 0$, $q = q_0 = CV_0 = (100\ \mu\text{C})$

Now, $\quad q = q_0 \cos \omega t$

(c) $\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C}$

(d) $|i| = \left|\frac{dq}{dt}\right| = q_0\omega \sin \omega t$

Average value of current in first quarter cycle

$$= \frac{\int_0^{T/4} dt}{T/4}$$

21. (a) $\frac{1}{2}\frac{q_i^2}{C} + \frac{1}{2}LI_i^2 = \frac{1}{2}CV_0^2$

$\therefore \quad V_0 = \frac{q_i}{C} \qquad$ (as $I_i = 0$)

(b) $\frac{1}{2}CV_0^2 = \frac{1}{2}LI_0^2$

$\therefore \quad I_0 = \sqrt{\frac{C}{L}}V_0$

(c) $U_{\max} = \frac{1}{2}LI_0^2$

(d) $U_L = \frac{1}{2}L\left(\frac{I_0}{2}\right)^2$

$$U_C = \frac{1}{2}\frac{q^2}{C} = U_{\max} - U_L$$

LEVEL 2

Single Correct Option

1. $\frac{1}{2}mv_0^2 = \frac{1}{2}Li_{\max}^2$

$\therefore \quad i_{\max} = \sqrt{\frac{m}{L}}\, v_0$

2. $V_C = Bvl$

$\therefore \quad q = CV_C = BvlC =$ constant

$\therefore \quad I_C = \frac{dq}{dt} = 0$

$$U_C = \frac{1}{2}CV^2 = \frac{1}{2}CB^2L^2v$$

3. From right hand rule, we can see that P and Q points are at higher potential than O.

4. At mean position, velocity is maximum. Hence, motional emf Bvl is also maximum. v oscillates simple harmonically. Hence, motional emf will also move simple harmonically. Further, polarity of induced emf will keep on changing.

5. At $t = t$ side of square,

$$l = (a + 2v_0 t)$$

Area, $\quad S = l^2 = (a + 2v_0 t)^2$

$$\phi = BS = B\,(a + 2v_0 t)^2$$

$$e = \frac{d\phi}{dt} = 4\,Bv_0\,(a + 2v_0 t)$$

$$R = \lambda\,[4l] = 4\lambda\,(a + 2v_0 t)$$

$\therefore \quad i = \frac{e}{R} = \frac{Bv_0}{\lambda}$

6. **At time *t***

Side of square $l = l_i - dt$

$$S = l^2 = (l_i - \alpha t)^2$$

At given time

$$l = l_i - \alpha t = a$$

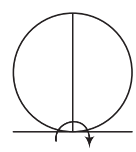
$$\phi = BS = B\,(l_i - \alpha t)^2$$

$$e = \left|\frac{d\phi}{dt}\right| = 2B\alpha\,(l_i - \alpha t)$$

But, $\quad (l_i - \alpha t) = a$

$\therefore \quad e = 2a\,\alpha\, B$

7.



$$\omega = \frac{v}{R} = \frac{v}{l/2} = \frac{2v}{l}$$

$$e = \frac{B\omega l^2}{2} = \frac{B\left(\frac{2v}{l}\right)l^2}{2} = Bvl$$

8. From right hand rule, we can see that

$$V_A > V_B$$

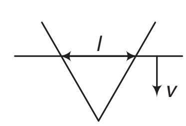
$\therefore \quad q_A$ is positive and q_B is negative.

$$q = CV = C\,(Bvl)$$

$$= (20 \times 10^{-6})\,(0.5)\,(0.2)\,(0.1)$$

$$= 0.2 \times 10^{-6}\text{C} = 0.2\,\mu\text{C}$$

9. $i = \frac{Bvl}{R}$



Let λ = resistance per unit length of conducting rod, then

$$i = \frac{Bvl}{\lambda l} = \frac{Bv}{\lambda} = \text{constant}$$

10. At time t, angle rotated by loop is $\theta = \omega t$. This is also the angle between **B** and **S**. Then,

$$\phi = BS \cos\theta$$
$$= Bb^2 \cos \omega t$$
$$e = \left|\frac{d\phi}{dt}\right| = b^2 B\omega \sin \omega t$$

11. $El = \frac{d\phi}{dt} = S\frac{dB}{dt}$

$\therefore \quad E(2\pi r) = (\pi r^2)\frac{dB}{dt}$

or $\quad E = \frac{r}{2}\frac{dB}{dt}$

$$F = qE = \frac{qr}{2}\frac{dB}{dt}$$
$$W = Fd = (2\pi r)$$
$$= \pi r^2 q\left(\frac{dB}{dt}\right)$$
$$= \left(\frac{22}{7}\right)(1)^2 (10^{-6})(2\times 10^{-3})$$
$$= 2\pi \times 10^{-9}\ \text{J}$$

12. Initial current $= \frac{10}{10} = 1$ A

$\therefore \quad \phi_i = L(I_i) = 500$ mWb $= 0.5$ Wb

Final current $= \frac{20}{5} = 4$ A

$\phi_f = L(I_f) = (0.5)\times 4 = 2$ Wb

$\therefore \quad \Delta\phi = 1.5$ Wb

13. $\frac{1}{2}Li^2 = \frac{1}{2}\left[\frac{1}{2}Li_0^2\right]$

$\therefore \quad i = \frac{i_0}{\sqrt{2}} = i_0(1 - e^{-t/\tau_L})$

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$\therefore \quad \frac{t}{\tau_L} = \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$

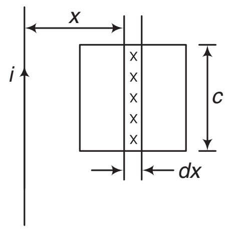
or $\quad t = \tau_L \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$

$$= \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

14. $B = \frac{\mu_0}{2\pi}\frac{i}{a}$

$$F = Bqv\sin 90° = \frac{\mu_0}{2\pi}\frac{i}{a}(qv)$$

15.



$$B = \frac{\mu_0}{2\pi}\frac{i}{x} \quad \Rightarrow \quad dS = c\,dx$$
$$d\phi = BdS = \frac{\mu_0}{2\pi}\frac{ic}{x}dx$$
$$\phi = \int_a^b d\phi = \frac{\mu_0 ic}{2\pi}\ln\left(\frac{b}{a}\right)$$
$$M = \frac{\phi}{i} = \frac{\mu_0 c}{2\pi}\ln\left(\frac{b}{a}\right)$$

16. $B_x = \frac{\mu_0}{2\pi}\frac{I}{x}$

$$de = B_x v dx = \frac{\mu_0}{2\pi}\frac{I}{x}v\,dx$$
$$e = \int_a^b de = \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)$$
$$i = \frac{e}{R} = \frac{\mu_0 Iv}{2\pi R}\ln\left(\frac{b}{a}\right) = \text{Induced current}$$
$$dF = (i)(dx)B_x$$
$$= \left[\frac{\mu_0 Iv}{2\pi R}\ln\left(\frac{b}{a}\right)\right]\left[\frac{\mu_0}{2\pi}\frac{I}{x}\right]dx$$
$$F = \int_a^b dF$$

17. $El = \frac{d\phi}{dt} = S\frac{dB}{dt}$

$\therefore \quad E(2\pi r) = \pi r^2\frac{dB}{dt}$

or $\quad E = \frac{r}{2}\frac{dB}{dt}$ or $E \propto r$

18. Magnetic field through Q (by I_2) is downwards. By decreasing I_1, downward magnetic field through Q will decrease. Hence, induced current in Q should produce magnetic field in same direction.

19. $i = 0(1 - e^{-t/\tau_L}) = \frac{E}{R}(1 - e^{-t/\tau_L})$

$$= \frac{E}{R} - \frac{Ee^{-t/\tau_L}}{R} = i_0 - \left(\frac{V_L}{R}\right) \quad (\text{as } V_L = Ee^{-t/\tau_L})$$

$\therefore \quad V_L = (i_0 R) - (R)\,i$

i.e. V_L *versus* i graph is a straight line with positive intercept and negative slope.

20. $e = Bvl = 0.5 \times 4 \times 0.25 = 0.5$ V

12 Ω and 4Ω are parallel. Hence, their net resistance $R = 3\ \Omega$.

$$i = \frac{e}{R+r} = \frac{0.5}{3+2} = 0.1 \text{ A}$$

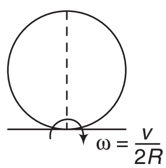
21. $El = \frac{d\phi}{dt} = S\,\frac{dB}{dt}$

$$E\,(2\pi R) = (\pi r^2)\,(\beta)$$

$\therefore \quad E = \frac{r^2}{2R}\beta \quad \Rightarrow \quad F = qE$

and $\quad \tau = FR = qER = \frac{1}{2}qr^2\beta$

22.



$$\text{PD} = \frac{B\omega l^2}{2} = \frac{(B)\,(v/2R)\,(4R)^2}{2} = 4\,BvR$$

23. $L_1 = \left(\frac{\eta}{\eta+1}\right)L,\ R_1 = \left(\frac{\eta}{\eta+1}\right)R$

$L_2 = \left(\frac{1}{\eta+1}\right)L,\ R_2 = \left(\frac{1}{\eta+1}\right)R$

$L_{\text{net}} = \frac{L_1L_2}{L_1 + L_2}$

Similarly, $\quad R_{\text{net}} = \frac{R_1R_2}{R_1 + R_2}$

$$\tau_L = \frac{L_{\text{net}}}{R_{\text{net}}} = \frac{L}{R}$$

24. $i = i_0e^{-t/\tau_L}$

$$\beta\, i_0 = i_0e^{-T/\tau_L}$$

$\therefore \quad \tau_L = \frac{T}{\ln\,(1/\beta)}$

25. $P = i_0^2R \quad \Rightarrow \quad i_0^2 = \frac{P}{R} \quad \Rightarrow \quad \tau = \frac{L}{R}$

$\therefore \quad L = \tau R$

Heat dissipated $= \frac{1}{2}Li_0^2 = \frac{1}{2}\,(\tau R)\left(\frac{P}{R}\right) = \frac{1}{2}P\tau$

26. In decay of current through *L-R* circuit, current can not remain constant.

27. By short-circuiting the battery, net resistance across inductor is $\frac{R}{2}$ (*R* and *R* in parallel).

$\therefore \quad \tau_{\text{net}} = \frac{L}{R_{\text{net}}} = \frac{2L}{R}$

28. At $t = 0$, $i = E/R$

Now, this current will decay in closed loop in anti-clockwise direction. So, $|i_2| = i_2 = E/R$ in upward or opposite direction.

Hence, $\quad i_2 = -\frac{E}{R}$

29. $\frac{1}{2}Li^2 = \frac{1}{4}\left[\frac{1}{4}Li_0^2\right]$

So, $\quad i = \frac{i_0}{2}$, half value

$\therefore \quad t = t_{1/2} = (\ln 2)\,\tau_L = (\ln 2)\left(\frac{L}{R}\right)$

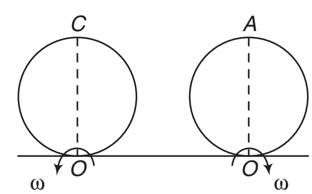
30. Steady state current through inductor in $\frac{E}{R}$.

So, at $t = 0$, current in closed loop (confiding of capacitor) will remain same.

31. At $t = 0$, $V_L = -E$

32. $V_A - V_0 = \frac{B\omega\,(2R)^2}{2} = 2B\omega R^2 \quad$ ...(i)

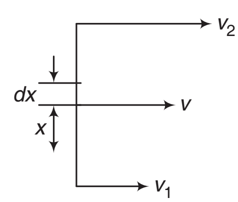
$V_0 - V_C = \frac{B\omega\,(2R)^2}{2} = 2B\omega R^2 \quad$ ...(ii)



Adding these two equations, we get

$$V_A - V_C = 4\,B\omega R^2$$

33.



$$v = v_1 + \left(\frac{v_2 - v_1}{l}\right)x$$

Small potential difference $= Bv\,(dx)$

$\therefore$ Total potential difference $= \int_0^l Bv\,dx$

$$= \frac{1}{2}B\,(v_1 + v_2)\,l$$

34. At time $t = 0$, resistance offered by a capacitor = 0 and resistance offered by an inductor = α.

$$R_{net} = \frac{R}{2} + \frac{R}{3} = \frac{5R}{6} = 5\ \Omega$$

$\therefore$ Current from the battery,

$$i = \frac{E}{R_{net}} = \frac{5}{5} = 1\ \text{A}$$

35. $\tau_L = \frac{L}{R} = \frac{0.01}{10} = 10^{-3}$ s

$\tau_C = CR = (0.1 \times 10^{-3})\ (10) = 10^{-3}$s

$(i_0)_L = \frac{20}{10} = 2$ A

$(i_0)_C = \frac{20}{10} = 2$ A

The given time is the half-life time of both the circuits.

$\therefore \qquad i_L = i_C = \frac{2}{2} = 1$ A

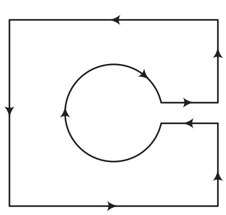
or total current is 2 A.

36. $|e| = \frac{d\phi}{dt} = S\ \frac{dB}{dt} = (4b^2 - \pi a^2)\ B_0$

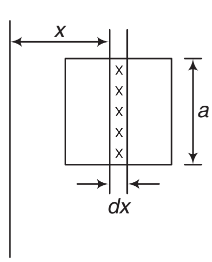
$$i = \frac{|e|}{R} = \frac{(4b^2 - \pi a^2)\ B_0}{R}$$

$\otimes$ magnetic field is increasing. So,

$\odot$ magnetic field is produced.



37. $\phi_i = \int_b^{b+a} \frac{\mu_0}{2\pi} \frac{i}{x} (adx) = \frac{\mu_0 ia}{2\pi} \ln \left(\frac{b+a}{b} \right)$



Similarly,

$$\phi_f = \frac{\mu_0 ia}{2\pi} \ln \left(\frac{b-a}{b} \right)$$

$$\Delta\phi = |Q_i - Q_f| = \frac{\mu_0 ia}{2\pi} \ln \left(\frac{b+a}{b-a} \right)$$

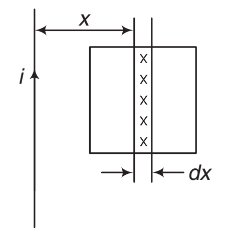
$$\Delta q = \frac{\Delta\phi}{R} = \frac{\mu_0 ia}{2\pi R} \ln \left(\frac{b+a}{b-a} \right)$$

More than One Correct Options

1. $e = Bvl, \quad$ where $\quad l = \frac{L}{2}$

For polarity of this motional emf, we can use right hand rule.

2. (a)



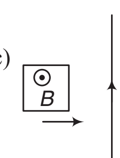
$$B_x = \frac{\mu_0}{2\pi} \frac{i}{x}$$

$$d\phi = (B_x)\ dS = \left(\frac{\mu_0}{2\pi} \frac{i}{x} \right) (adx)$$

$$\phi = \int_a^{2a} d\phi = \frac{\mu_0 ia}{2\pi} \ln 2$$

$$M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi} \ln 2$$

(c)



Wire produces $\odot$ magnetic field over the loop. If the loop is brought closer to the wire, $\odot$ magnetic field passing through the loop increases. Hence, induced current produces $\otimes$ magnetic field so, induced current is clockwise.

3. (a) $L = \frac{N\phi}{i} \quad \Rightarrow \quad \phi = \frac{Li}{N}$

So, SI unit of flux is Henry-ampere.

(c) $L = \frac{-e}{\Delta i / \Delta t} = -\frac{-e\Delta t}{\Delta i}$

Hence, SI unit of L is $\frac{\text{V - s}}{\text{ampere}}$.

4. $\tau_L = \frac{L}{R} = \frac{2}{2} = 1$ s

$t_{\frac{1}{2}} = (\ln 2)\ \tau_L = (\ln 2)$ s

Hence, the given time is half-life time.

$$\therefore \qquad i = \frac{i_0}{2} = \frac{8/2}{2} = 2\text{ A}$$

Rate of energy supplied by battery

$$= Ei = 8 \times 2 = 16\text{ J/s}$$

$$P_R = i^2R = (2)^2(2) = 8\text{ J/s}$$

$$V_a - V_b = E - iR = 8 - 2 \times 2 = 4\text{ V}$$

5. According to Lenz's law, induced effects always oppose the change i_1 and i_2 both are in same direction. Hence, magnetic lines from B due to both currents are from right to left. By bringing A closer to B or increasing i_1 right to left magnetic field from B will increase. So, i_2 should decrease.

6. $\phi_i = BS\cos 0° = (4)(2) = 8$ Wb

$\phi_f = BS\cos 90° = 0$

$$\Delta\phi = 8\text{ Wb}$$

$$|e| = \frac{\Delta\phi}{\Delta t} = \frac{8}{0.1} = 80\text{ V}$$

$$i = \frac{|e|}{R} = 20\text{ A}$$

$$\Delta q = \frac{\Delta\phi}{R} = 2\text{ C}$$

This current is not constant. So, we cannot find the heat generated unless current function with time is not known.

7. $i_{\max} = \omega q_0 = \left(\frac{1}{\sqrt{LC}}\right)q_0$

$$\left(\frac{di}{dt}\right)_{\max} = \omega^2 q_0 = \left(\frac{1}{LC}\right)q_0$$

8. If ⊗ magnetic field increases, then induced electric lines are anti-clockwise. If ⊗ magnetic field decreases, then induced electric lines are clockwise (both inside and outside the cylindrical region).

On positive charge, force is in the direction of **E**. On negative charge, force is in the opposite direction of **E**.

9. $q = 2t^2$

$$i = \frac{dq}{dt} = 4t$$

$$\frac{di}{dt} = 4\text{ A/s}$$

At $t = 1$ s, $q = 2$C, $i = 4$A

and $\qquad \dfrac{di}{dt} = 4\text{ A/s}$

$$V_a - V_b = L\frac{di}{dt} = 1 \times 4 = 4\text{ V}$$

$$V_b - V_c = \frac{q}{C} = \frac{2}{2} = 1\text{ V}$$

$$V_c - V_d = iR = 4 \times 4 = 16\text{ V}$$

$V_a - V_d$ is summation of above three, i.e. 21 V.

10. $V_a - V_c = 0$ as **l** is parallel to **v**.

$$V_a - V_b = V_c - V_b = \frac{B\omega l^2}{2}$$

Comprehension Based Questions

1. $\dfrac{dB}{dt} = (6t^2 + 24)$ T/s

At $t = 2$ s, $\dfrac{dB}{dt} = 48$ T/s

$$El = \frac{d\phi}{dt} = S\left(\frac{dB}{dt}\right)$$

or $\qquad E(2\pi r) = \pi r^2\left(\dfrac{dB}{dt}\right)$

$$\therefore \qquad E = \frac{r}{2}\cdot\frac{dB}{dt} \qquad \ldots\text{(i)}$$

$$F = qE = \frac{qr}{2}\frac{dB}{dt}$$

$$= \frac{(1.6 \times 10^{-19})(1.25 \times 10^{-2})}{2}(48)$$

$$= 48 \times 10^{-21}\text{N}$$

2. From Eq. (i) of above problem, we can see that

$$E \propto r$$

i.e. E - r graph is a straight line passing through origin.

3. ⊗ Magnetic field is increasing. Hence, ⊙ magnetic field is produced by a conducting circular loop placed there. For producing, magnetic field induced current should be anti-clockwise. Direction of induced circular electric lines are also anti-clockwise.

4. $|e| = \left|\dfrac{d\phi}{dt}\right| = S\dfrac{dB}{dt} = (\pi a^2)B_0$

5. $El = \dfrac{d\phi}{dt}$

$$\therefore \qquad E(2\pi a) = (\pi a^2)B_0$$

or $\qquad E = \dfrac{1}{2}aB_0$

6. $\qquad F = qE = \dfrac{1}{2}qaB_0$

$$\tau = Fa = \frac{1}{2} qa^2 B_0$$

$$\alpha = \frac{\tau}{I} = \frac{\left(\frac{1}{2} qa^2 B_0\right)}{ma^2} = \frac{qB_0}{2m}$$

7. $\omega = \alpha t = \left(\frac{qB_0}{2m}\right) t$

$$P = \tau\,\omega = \left(\frac{1}{2} qa^2 B_0\right)\left(\frac{qB_0}{2m}\right)$$

$$= \frac{q^2 B_0^2 a^2}{4m}$$

8. $|e| = \frac{d\phi}{dt} = S\frac{dB}{dt}$

$$= (0.2 \times 0.4)\,(2) = 0.16 \text{ V}$$

$$i = \frac{|e|}{R} = \frac{0.16}{(1)\,(40 + 40 + 20) \times 10^{-2}}$$

$$= 0.16 \text{ A}$$

$\odot$ Magnetic field passing through the loop is increasing. So, induced current should produce $\otimes$ magnetic field. Hence, induced current is clockwise.

9. At $t = 2$ s, rod will move 10 cm. Hence, 40 cm side will become 30 cm.

$$|e| = e_1 \text{ (say)} = S\left(\frac{dB}{dt}\right)$$

$(0.2 \times 0.3)\,(2) = 0.12$ V

At $t = 2$ s, $B = 4$ T

$$\therefore \quad e_2 = Bvl$$

$$= (4)\,(5 \times 10^{-2})\,(0.2)$$

$$= 0.04 \text{ V}$$

$$\therefore \quad e_{\text{net}} = e_1 - e_2 = 0.08 \text{ V}$$

10. $i = \frac{e_{\text{net}}}{R} = \frac{0.08}{(1)\,(30 + 30 + 20) \times 10^{-2}}$

$$= 0.1 \text{ A}$$

$$F = ilB$$

$$= (0.1)\,(0.2)\,(4) = 0.08 \text{ N}$$

11 to **13**

At terminal velocity,

$$iLB = mg$$

$$\therefore \quad i = \frac{mb}{LB} = \frac{0.2 \times 98}{1 \times 0.6}$$

$$i = 3.27 \text{ A} \qquad \ldots\text{(i)}$$

$$e = BvL \qquad (v = \text{terminal velocity})$$

$$= (0.6)\,(v)\,(1)$$

$$e = 0.6v$$

$$P_{R_1} = \frac{e^2}{R_1}$$

$$\therefore \quad 0.76 = \frac{0.36\,v^2}{R_1} \qquad \ldots\text{(ii)}$$

$$P_{R_2} = \frac{e^2}{R_2}$$

$$\therefore \quad 1.2 = \frac{0.36\,v^2}{R_2} \qquad \ldots\text{(iii)}$$

R_1 and R_2 are in parallel.

$$\therefore \quad R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2} \qquad \ldots\text{(iv)}$$

$$i = \frac{e}{R_{\text{net}}} \qquad \ldots\text{(v)}$$

Solving these five equations, we can get the results.

Match the Columns

1. (a) $B = \frac{F}{il}$

$$[B] = \left[\frac{\text{MLT}^{-2}}{\text{AL}}\right] = [\text{MT}^{-2}\text{A}^{-1}]$$

(b) $U = \frac{1}{2} Li^2$

$$\therefore \quad [L] = \left[\frac{U}{i^2}\right] = \left[\frac{\text{ML}^2\text{T}^{-2}}{\text{A}^2}\right] = [\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$$

(c) $\omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad [LC] = \left[\frac{L}{\omega^2}\right] = [\text{T}^2]$

(d) $[\phi] = [BS] = [\text{MT}^{-2}\text{A}^{-1}\text{L}^2]$

2. $V_L = Ee^{-t/\tau_L} = 10e^{-t/\tau_L}$

$$\tau_L = \frac{L}{R} = 1 \text{ s}$$

$$\therefore \quad V_L = 10e^{-t}$$

$$V_R = E - V_L = 10\,(1 - e^{-t})$$

Now, we can put $t = 0$ and $t = 1$ second.

3. $\omega = \frac{1}{\sqrt{LC}} = 2$ rad/s

(a) $i_{\max} = \omega q_0 = 8$ A

(b) $\left|\frac{di}{dt}\right|_{\max} = \omega^2 q_0 = 16$ A/s

(c) $V_L = V_C = \frac{q}{C} = \frac{2}{1/4} = 8$ V

(d) $V_C = V_L = L\frac{di}{dt} = (1)\left(\frac{16}{2}\right) = 8$ V

4. Steady state current through inductor,

$$i_0 = \frac{9}{3} = 3 \text{ A}$$

Now, this current decays exponentially across inductor and two resistors.

$$\tau_L = \frac{L}{R} = \frac{9}{6+3} = 1 \text{ s}$$

$$t_{1/2} = (\ln 2)\,\tau_L = (\ln 2) \text{ s}$$

Given time is half-life time. Hence, current will remain 1.5 A.

$$i = i_0 e^{-t/\tau_L} = 3e^{-t}$$

$$\left(-\frac{di}{dt}\right) = 3e^{-t}$$

In the beginning $\left(\frac{-di}{dt}\right) = 3 \text{ A/s}$

After one half-life time $\left(\frac{-di}{dt}\right) = 1.5 \text{ A/s}$

(a) $V_L = L\left(\frac{-di}{dt}\right) = 9 \times 1.5 = 13.5 \text{ V}$

(b) $V_{3\Omega} = iR = 1.5 \times 3 = 4.5 \text{ V}$

(c) $V_{6\Omega} = iR = 1.5 \times 6 = 9 \text{ V}$

(d) $V_{bc} = V_L - V_{3\Omega} = 9 \text{ V}$

5. $\phi = 2t$

(a) $e = \frac{d\phi}{dt} = 2 \text{ V}$

(b) $i = \frac{e}{R} = 1 \text{ A} = \text{constant}$

(c) $\Delta q = i\Delta t = 1 \times 2 = 2 \text{ C}$

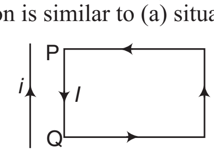
(d) $H = i^2 R\Delta t = (1)^2(2)(2) = 4\text{J}$

6. (a) If current is increased, ⊗ magnetic field passing through loop will increase. So, induced current will produce ⊙ magnetic field. Hence, induced current is anti-clockwise.

Now, i and I currents in PQ are in opposite directions. Hence, they will repel each other.

Same logic can be applied for (b) part.

(c) situation is similar to (b) situation and
(d) situation is similar to (a) situation.



Subjective Questions

1. $\tau_L = \frac{L}{R}$

and $\tau_C = CR$, $\frac{\tau_C}{\tau_L} = \frac{CR^2}{L} = \frac{C}{L} \cdot \frac{L}{C} = 1$

$\therefore \quad \tau_L = \tau_C$

$\therefore$ For the given condition $\tau_L = \tau_C = \tau$ (say)

Now, in L-R circuit

$$I_1 = \frac{V}{R}(1 - e^{-t/\tau})$$

In CR circuit, $I_2 = \frac{V}{R} e^{-t/\tau}$

$$\therefore \quad I = I_1 + I_2 = \frac{V}{R} = \text{constant}$$ **Ans.**

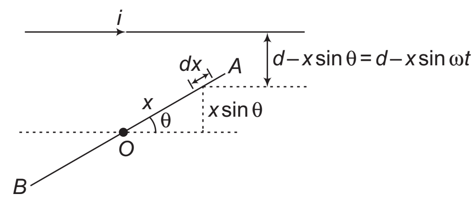
2. Motional emf, $V = Bvl$

Net resistance of the circuit $= R + \frac{R_1 R_2}{R_1 + R_2}$

$\therefore$ Current through the connector,

$$i = \frac{Bvl}{R + \frac{R_1 R_2}{R_1 + R_2}}$$ **Ans.**

3. $\theta = \omega t$



$$de = B\,(\omega x)\,dx$$

Here, $B = \frac{\mu_0}{2\pi} \frac{i}{d - x\sin\omega t}$

$$\therefore \quad de = \frac{\mu_0 i\omega}{2\pi} \frac{x}{d - x\sin\omega t} \cdot dx$$

$$V_{OA} = V_0 - V_A = \int_0^a de = \frac{\mu_0 i\omega}{2\pi} \int_0^a \frac{x}{d - x\sin\omega t}\, dx$$

$$= -\frac{\mu_0 i\omega}{2\pi \sin\omega t}\left[\frac{d}{\sin\omega t} \ln\left(\frac{d - a\sin\omega t}{d}\right) + a\right]$$

Similarly,

$$V_{OB} = V_O - V_B = \frac{\mu_0 i\omega}{2\pi} \int_0^a \frac{x}{d + x\sin\omega t}\, dx$$

$$= \frac{\mu_0 i\omega}{2\pi \sin\omega t}\left[a - \frac{d}{\sin\omega t} \ln\left(\frac{d + a\sin\omega t}{d}\right)\right]$$

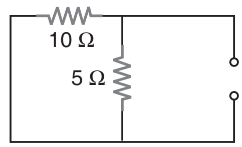
$$\therefore \quad V_{AB} = V_{OB} - V_{OA}$$

$$= \frac{\mu_0 i\omega}{2\pi \sin\omega t}\left[2a + \frac{d}{\sin\omega t} \ln\left(\frac{d - a\sin\omega t}{d + a\sin\omega t}\right)\right]$$

Ans.

Note *This function is discontinuous at $\omega t = n\pi$.*

4. At $t = 0$, equivalent resistance of an inductor in infinite and at $t = \infty$, equivalent resistance is zero.



$\therefore$ Initial current through inductor = 0 and

Final current through inductor $= \dfrac{36}{10} = 3.6$ A

To find equivalent time constant, we will have to short circuit the battery and find net resistance across inductor.

$$R_{net} = \frac{10 \times 5}{10 + 5} = \frac{10}{3}\ \Omega$$

$$\tau_L = \frac{L}{R_{net}} = \frac{3}{10}\ \text{ms}$$

Current through inductor will increase exponentially from 0 to 3.6 A.

$\therefore\ i = 3.6(1 - e^{-t/\tau_L})$, where $\tau_L = \dfrac{3}{10}$ ms = 300 μs

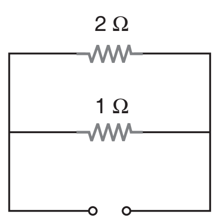
Current through 10 Ω will vary with time. **Ans.**

5. At $t = 0$, Current through inductor will be zero.

At $t = \infty$, net emf $= \dfrac{2/2 + 4/1}{1/2 + 1/1} = \dfrac{10}{3}$ V

Net resistance $= \dfrac{2 \times 1}{2 + 1} = \dfrac{2}{3}\ \Omega$

$$\therefore \quad i = \frac{10/3}{2/3} = 5 \text{ A}$$



To find equivalent time constant short circuit, both the batteries and find net resistance across inductor.

$$R_{net} = \frac{2 \times 1}{2 + 1} = \frac{2}{3}\ \Omega$$

$$\therefore \quad \tau_L = \frac{L}{R_{net}} = \frac{1 \times 10^{-3}}{2/3} = \frac{3}{2000}\ \text{s}$$

Current through inductor will increase exponentially from 0 to 5 A.

$$\therefore \quad i = 5\left(1 - e^{-\frac{2000t}{3}}\right)$$

6. These are two independent parallel circuits across the battery.

(a) $V_{ab} = E = 120$ volt (at all instants)

(b) a is at higher potential.

(c) V_{cd} will decrease exponentially from 120 V to zero.

$\therefore V_{cd} = 120$ volt, just after the switch is closed.

(d) c will be at higher potential.

(e) When switch is opened, current through R_1 will immediately become zero. While through R_2, will decrease to zero from the value $\dfrac{E}{R_2} = 2.4$ A $= i_0$ (say), exponentially. Path of this decay of current will be *cdbac*.

$\therefore$ Just after the switch is opened,

$V_{ab} = -i_0 R_1 = -2.4 \times 30 = -72$ volt

(f) Point b is at higher potential.

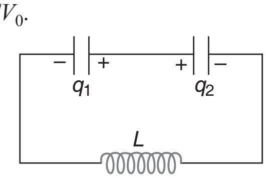
(g) $V_{cd} = -i_0(R_1 + R_2) = -2.4(80) = -192$ volt

(h) This time point d will be at higher potential.

7. $q_1 = 8\,CV_0 \quad : \quad q_2 = CV_0$

$$q_1 + q_2 = 9CV_0$$

In the absence of inductor, this $9C_0V$ will distribute as $6\,CV_0$ in $2C$ and $3CV_0$ in C. Thus, mean position of q_1 is $6\,CV_0$ and mean position of q_2 is $3CV_0$.



At $t = 0$, q_1 is $2CV_0$ more than its mean position and q_2 is $2CV_0$ less.

Thus, $\quad q_0 = 2CV_0$

$$C_{net} = \frac{2C}{3}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC_{net}}} = \sqrt{\frac{3}{2LC}}$$

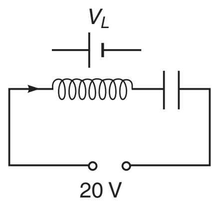
(a) $I_{max} = q_0\omega$

(b) $V_1 = \dfrac{6CV_0}{2C} = 3V_0$

and $V_2 = \dfrac{3CV_0}{C}$

$= 3V_0$

(c) $i = q_0 \sin \omega t$

8. 

(a) $V_L = L\dfrac{di}{dt} = (1 \times 10^{-3})\dfrac{d}{dt}(20t) = 0.02$ V

$= 20$ mV

(b) $q = \int_0^t i\,dt = \int_0^t (20t)\,dt = 10t^2$

$V = \dfrac{q}{C} = \dfrac{10t^2}{10^{-6}} = (10^{+7}t^2)$ V

(c) $\dfrac{q^2}{2C} > \dfrac{1}{2}Li^2$

or $\dfrac{(10t^2)^2}{2 \times 10^{-6}} > \dfrac{1}{2} \times 10^{-3} \times (20t)^2$

or $t > 63.2 \times 10^{-6}$ s

or $t > 63.2\ \mu$s **Ans.**

9. In steady state when switch was closed,

$$i_0 = E/R = (1/5)\text{A} = 0.2 \text{ A}$$

After switch is opened, it becomes L-C circuit in which peak value current is 0.2 A.

$\therefore$ $\dfrac{1}{2}Li_0^2 = \dfrac{1}{2}CV_0^2$

or $L = \dfrac{V_0^2}{i_0^2}.C$

$= \dfrac{(150)^2}{(0.2)^2} \times 0.5 \times 10^{-6}$

$= 0.28$ H **Ans.**

10. (a) $e = Bvl = 0.8 \times 7.5 \times 0.5 = 3$ V

(b) Current will flow in anti-clockwise direction, as magnetic field in $\otimes$ direction passing through the closed loop is increasing. Therefore, induced current will produce magnetic field in $\odot$ direction.

(c) $F = F_m = ilB = \dfrac{e}{R}lB = \left(\dfrac{Bvl}{R}\right)Bl = \dfrac{B^2l^2}{R}v$

$= \dfrac{(0.8)^2(0.5)^2}{1.5} \times 7.5 = 0.8$ N

(d) $Fv = 0.8 \times 7.5 = 6$ W **Ans.**

$i^2R = \left(\dfrac{Bvl}{R}\right)^2 R = \dfrac{B^2l^2}{R}.v^2$

$= \dfrac{(0.8)^2(0.5)^2}{1.5} \times (7.5)^2 = 6$ W **Ans.**

So, we can see that both rates are equal.

11. (a) Magnitude of induced electric field due to change in magnetic flux is given by

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d\phi}{dt} = S.\frac{dB}{dt}$$

or $El = \pi R^2(2B_0t)$ $\left(\dfrac{dB}{dt} = 2B_0t\right)$

Here, E = induced electric field due to change in magnetic flux

or $E(2\pi R) = 2\pi R^2 B_0 t$

or $E = B_0Rt$

Hence, $F = QE = B_0QRt$

This force is tangential to ring. Ring starts rotating when torque of this force is greater than the torque due to maximum friction ($f_{\max} = \mu mg$) or when

$$\tau_F \geq \tau_{f_{\max}}$$

Taking the limiting case

$\tau_F = \tau_{f_{\max}}$ or $FR = (\mu mg)R$

or $F = \mu mg$

or $B_0QRt = \mu mg$

It is given that ring starts rotating after 2. So, putting $t = 2$, we get

$\mu = \dfrac{2B_0RQ}{mg}$ **Ans.**

(b) After 2

$$\tau_F > \tau_{f_{\max}}$$

Therefore, net torque is

$\tau = \tau_F - \tan_{f_{\max}} = B_0QR^2t - \mu mgR$

Substituting $\mu = \dfrac{2B_0QR}{mg}$, we get

$$\tau = B_0QR^2(t-2)$$

or $I\left(\dfrac{d\omega}{dt}\right) = B_0QR^2(t-2)$

or $mR^2\left(\dfrac{d\omega}{dt}\right) = B_0QR^2(t-2)$

or $\int_0^\omega d\omega = \dfrac{B_0Q}{m}\int_2^4 (t-2)\,dt$

or $\omega = \dfrac{2B_0Q}{m}$ ...(i)

Now, magnetic field is switched off, i.e only retarding torque is present due to friction. So, angular retardation will be

$$\alpha = \frac{\tau_{f_{\max}}}{I} = \frac{\mu mgR}{mR^2} = \frac{\mu g}{R}$$

Therefore, applying

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

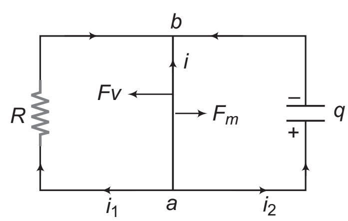
or $$0 = \left(\frac{2B_0Q}{m}\right)^2 - 2\left(\frac{\mu g}{R}\right)\theta$$

or $$\theta = \frac{2B_0^2Q^2R}{\mu m^2 g}$$

Substituting $$\mu = \frac{2B_0RQ}{mg}$$

We get $$\theta = \frac{B_0Q}{m}$$ **Ans.**

12. Let v be the velocity of connector at some instant of time. Then,



$$V_{ab} = Bvl,\ i_1 = \frac{Bvl}{R},\ q = C(Bvl)$$

$\therefore$ $$i_2 = \frac{dq}{dt} = CBl\frac{dv}{dt}$$

Now, $$i = i_1 + i_2 = \frac{Bvl}{R} + CBl\frac{dv}{dt}$$

Magnetic force, $$F_m = ilB = \frac{B^2l^2}{R}\cdot v + B^2l^2C\cdot\frac{dv}{dt}$$

Further, $$F_{\text{net}} = F - F_m$$

or $$m\frac{dv}{dt} = F - \frac{B^2l^2}{R}v - B^2l^2C\frac{dv}{dt}$$

$\therefore$ $$\int_0^v \frac{dv}{F - \frac{B^2l^2}{R}v} = \int_0^t \frac{dt}{m + B^2l^2C}$$

Integrating we get,

$$v = \frac{FR}{B^2l^2}\left[1 - e^{-\left(\frac{B^2l^2}{mR + RB^2l^2C}\right)t}\right]$$

Terminal velocity in this case is : $v_T = \dfrac{FR}{B^2l^2}$ **Ans.**

13. With key K_1 closed, C_1 and C_2 are in series with the battery in steady state.

$\therefore$ $C_{\text{net}} = 1\,\mu\text{F}$ or $q_0 = C_{\text{net}}\ V = 20\,\mu\text{C}$

(a) With K_1 opened and K_2 closed, charge on C_2 will remain as it is, while charge on C_1 will oscillate in L-C_1 circuit.

$$\omega = \frac{1}{\sqrt{LC_1}}$$

$$= \frac{1}{\sqrt{0.2\times10^{-3}\times2\times10^{-6}}}$$

$$= 5\times10^4 \text{ rad/s}$$ **Ans.**

(b) Since, at $t = 0$, charge is maximum $(= q_0)$. Therefore, current will be zero.

$$\frac{1}{2}Li^2 = \frac{1}{3}\left(\frac{1}{2}\frac{q^2}{C}\right)$$

or $$i = \frac{q}{\sqrt{3LC}} = \frac{q\omega}{\sqrt{3}}$$

From the expression,

$$i = \omega\sqrt{q_0^2 - q^2}$$

We have, $$\frac{q\omega}{\sqrt{3}} = \omega\sqrt{q_0^2 - q^2}$$

or $$q = \frac{\sqrt{3}}{2}q_0$$

Since at $t = 0$, charge is maximum or q_0, so we can write

$$q = q_0\cos\omega t \text{ or } \frac{\sqrt{3}q_0}{2} = q_0\cos\omega t$$

or $$\omega t = \frac{\pi}{6} \text{ or } t = \frac{\pi}{6\omega} = \frac{\pi}{6\times5\times10^4}$$

$$= 1.05\times10^{-5}\text{ s}$$

(c) $q = \dfrac{\sqrt{3}}{2}q_0 = \dfrac{\sqrt{3}}{2}\times20 = 10\sqrt{3}\,\mu\text{C}$ **Ans.**

14. In the capacitor,

$$q_i = CV_i = (10\times10^{-3})(5) = 0.05\text{ C}$$

$$q_f = CV_f = (10\times10^{-3})(10) = 0.1\text{ C}$$

$\therefore$ Charge in capacitor will increase from 0.05 C to 0.1 C exponentially.

Time constant for this increase would be

$$\tau_C = CR = 1\text{ s.}$$

$\therefore$ Charge at time t will be

$$q = 0.05 + (0.1 - 0.05)(1 - e^{-t/\tau_C})$$

$$= 0.1 - 0.05\,e^{-t/\tau_C}$$

(a) At $t = 1$ s, $q = 0.1 - 0.05\, e^{-1} = 0.0816$ C

$$V = \frac{q}{C} = \frac{0.0816}{10 \times 10^{-3}} = 8.16 \text{ V} \qquad \textbf{Ans.}$$

(b) This charge 0.0816 C is also the maximum charge q_0 of L-C oscillations.

From energy conservation equation,

$\frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}Li_0^2$ we have,

$$i_0 = \frac{q_0}{\sqrt{LC}} = \frac{0.0816}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-3}}}$$

$$= 5.16 \text{ A} \qquad \textbf{Ans.}$$

Further, $\omega = \frac{1}{\sqrt{LC}}$ or $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{25 \times 10^{-3} \times 10 \times 10^{-3}}}$$

$$= 10 \text{ Hz} \qquad \textbf{Ans.}$$

15. (a) Let at time t velocity of rod be v (towards right) and current in the circuit is i (from a to b). The magnetic force on it is ilB (towards right). Writing the equation of motion of the rod,

$$m \cdot \frac{dv}{dt} = i\,l\,B = \left(\frac{E_0 - Blv}{R}\right) lB$$

$$\int_0^v \frac{dv}{\frac{E_0 Bl}{mR} - \frac{B^2 l^2}{mR} v} = \int_0^t dt$$

$$\therefore \quad v = \frac{E_0}{Bl}\left(1 - e^{-\frac{B^2 l^2}{mR} t}\right) \qquad \textbf{Ans.}$$

(b) $i = \frac{E_0 - Blv}{R}$

16. Let v be the velocity at some instant. Then, motional emf, $V = Bvl$

Charge stored in capacitor $q = CV = (CBl)v$

Current in the wire $= \frac{dq}{dt} = (CBl)\frac{dv}{dt}$

Magnetic force, $F_m = ilB = (CB^2l^2)\frac{dv}{dt}$ (upwards)

$\therefore$ Net force, $F_{\text{net}} = mg - F_m$

or $\quad m\frac{dv}{dt} = mg - (CB^2l^2)\frac{dv}{dt}$

$\therefore \quad \frac{dv}{dt}$ = acceleration, $a = \frac{mg}{m + CB^2l^2}$

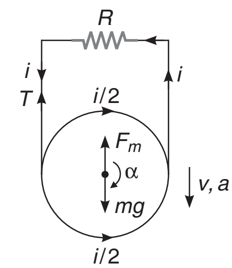
Since, a = constant

$$\therefore \quad x = \frac{1}{2}at^2 = \frac{mgt^2}{2(m + CB^2l^2)}$$

17. Let at time t velocity of ring be v (downwards)

$$e = Bv\,(2r) = 2Bvr$$

(Two batteries of emf $2Bvr$ are connected in parallel)



$$\therefore \quad i = \frac{e}{R} = \frac{2Bvr}{R}$$

Now, $\quad a = \frac{mg - F_m - T}{m}$

Here, $F_m = 2\left[\left(\frac{i}{2}\right)(2r)B\right] = 2irB = \frac{4B^2r^2v}{R}$

$$\therefore \quad a = g - \frac{4B^2r^2v}{mR} - \frac{T}{m} \qquad \text{...(i)}$$

$$\alpha = \frac{Tr}{mr^2} = \frac{T}{mr} \qquad \text{...(ii)}$$

$$a = r\alpha = \frac{T}{m} \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$a = \frac{g}{2} - \frac{2B^2r^2v}{mR}$$

or $$\int_0^v \frac{dv}{\frac{g}{2} - \frac{2B^2r^2v}{mR}} = \int_0^t dt$$

or $$v = \frac{mgR}{4B^2r^2}\left(1 - e^{-\frac{2B^2r^2}{mR}t}\right) \qquad \textbf{Ans.}$$

$$i = \frac{2Bvr}{R} = \frac{mg}{2Br}\left(1 - e^{-\frac{2B^2r^2}{mR}t}\right)$$

and $$v_T = \frac{mgR}{4B^2r^2} \qquad \textbf{Ans.}$$

18. (a) Suppose v be the velocity of rod ef when it has fallen a distance, x. Then,

$V_{fe} = V_{cb}$ or $Bvl = L(di/dt)$

or $B(dx/dt)l = L(di/dt)$ or $Bl(dx) = L(di)$

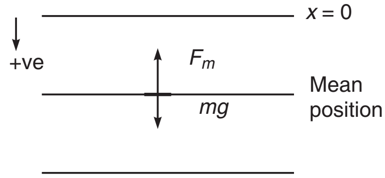
Integrating, we get $Li = Blx$

or $$i = \left(\frac{Bl}{L}\right)x \qquad \text{...(i)}$$

Now, magnetic force opposite to displacement x will be

$$F = F_m = ilB = \left(\frac{B^2l^2}{L}\right)x$$

A constant downward force is mg.



So, this is similar situation like spring-block system in vertical position. In which a force $F = kx$ acts upwards and a constant force mg acts downwards.

Hence, the wire will execute SHM, where

$$k = \frac{B^2l^2}{L}$$

Amplitude will be at $F_m = mg$

or $$\left(\frac{B^2l^2}{L}\right)A = mg \Rightarrow A = \frac{mgL}{B^2l^2}$$

At $t = 0$, rod is in its extreme position. Therefore, if we write the equation from mean position we will write,

$$X = -A\cos\omega t$$

But, $x = X + A = A - A\cos\omega t = A(1 - \cos\omega t)$

where, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{B^2l^2}{mL}}$ **Ans.**

(b) From Eq. (i),

$$i_{\max} = \left(\frac{Bl}{L}\right)x_{\max}$$

Here, $x_{\max} = 2A = \frac{2mgL}{B^2l^2}$

$\therefore$ $i_{\max} = \left(\frac{Bl}{L}\right)\left(\frac{2mgL}{B^2l^2}\right) = \frac{2mg}{Bl}$ **Ans.**

(c) Maximum velocity,

$$v_0 = \omega A = \left(\sqrt{\frac{B^2l^2}{mL}}\right)\left(\frac{mgL}{B^2l^2}\right) = g\sqrt{\frac{mL}{B^2l^2}} = \frac{g\sqrt{mL}}{Bl}$$

Ans.

19. (a) At time t, $v = a_0t$

Motional emf, $V = Bvl = Ba_0lt$

Total resistance $= \frac{R_1R_2}{R_1 + R_2}$

$\therefore$ $i = \frac{(Ba_0lt)(R_1 + R_2)}{R_1R_2}$ **Ans.**

(b) From right hand rule, we can see that points a and b will be at higher potential and c and d at lower potentials.

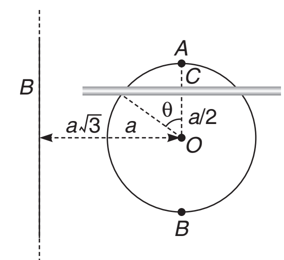
$$F_m = ilB = \frac{B^2l^2a_0t}{R_1R_2}(R_1 + R_2)$$

Let F be the external force applied, then,

$$F - F_m = ma_0$$

$$\therefore \quad F = F_m + ma_0 = \frac{B^2l^2a_0t}{R_1R_2}(R_1 + R_2) + ma_0$$

20. (a) At the given instant,



$$AC = \frac{a}{2}, \quad OC = \frac{a}{2}$$

and $\cos\theta = \frac{a/2}{a} = \frac{1}{2}, \quad \theta = \frac{\pi}{3}$

$\therefore$ Velocity of rod

$= \left(+\frac{v_0}{2}\right)$ along the direction of current.

Emf induced across the ends M and N

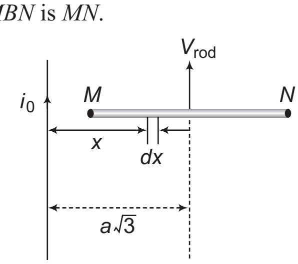
$$E_{\text{rod}} = \int_{a\sqrt{3} - \frac{a\sqrt{3}}{2}}^{a\sqrt{3} + \frac{a\sqrt{3}}{2}} v_{\text{rod}}B\,dx$$

$$= v_{\text{rod}}\int_{a\frac{\sqrt{3}}{2}}^{3a\frac{\sqrt{3}}{2}} \frac{\mu_0 i_0}{2\pi x}\,dx$$

$$= \left(\frac{v_0}{2}\right)\frac{\mu_0 i_0}{2\pi}\ln\left(\frac{3}{1}\right)$$

with end M at higher potential.

Since, the effective length of both the arcs MAN and MBN is MN.



$$\therefore \quad E_{MAN} = E_{MBN} = v_{\text{loop}} \frac{\mu_0 i_0}{2\pi} \ln 3$$

$$= v_0 \frac{\mu_0 i_0}{2\pi} \ln 3$$

with point M at higher potential.

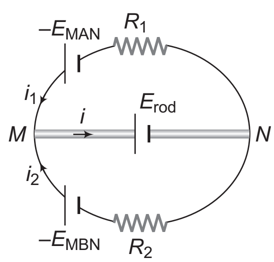
Resistance of arc MAN

$$\Rightarrow \quad R_1 = (R)2(a\theta) = 2aR\,\frac{\pi}{3}$$

$\Rightarrow$ Resistance of arc MBN

$$\Rightarrow \quad R_2 = (R)a(2\pi - 2\theta) = 4aR\,\frac{\pi}{3}$$

Equivalent circuit at the given instant is shown in the figure.



Current through the rod MN,

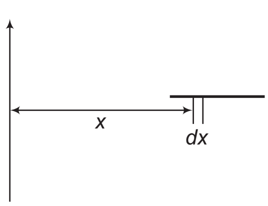
$$i = (i_1 + i_2) = \left(\frac{E_{MAN} - E_{\text{rod}}}{R_1}\right) + \left(\frac{E_{MBN} - R_{\text{rod}}}{R_2}\right)$$

$$i = (E_{MAN} - E_{\text{rod}})\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$= \frac{v_0 \mu_0 i_0 (\ln 3)}{4\pi}\left[\left(\frac{1}{2} + \frac{1}{4}\right)\frac{3}{aR\pi}\right]$$

$$= \frac{9 v_0 i_0 \mu_0}{16 aR\pi^2} \ln (3)$$ **Ans.**

(b) Force on the rod



$$F_{\text{rod}} = \int_{\frac{a\sqrt{3}}{2}}^{\frac{3a\sqrt{3}}{2}} i\, dxB$$

$$= \frac{i\mu_0 i_0}{2\pi} \ln 3 = \frac{9\,\mu_0^2 i_0^2 v_0}{32 aR\pi^3} (\ln 3)^2$$

21. Since, PQ and DC both cut the lines of field.

$\therefore$ Motional emf will be induced across both of them.

Integrating, potential difference across

$$dx \Rightarrow \int de = \int_a^{2a} v \left(\frac{\mu_0 i_0}{2\pi x}\right) dx$$

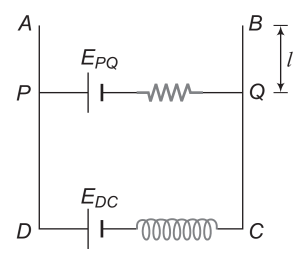
$$e_{DC} = \frac{v\mu_0 i_0}{2\pi} \ln 2 \text{ with } D \text{ at higher potential}$$

$$e_{PQ} = \frac{2v\mu_0 i_0}{2\pi} \ln 2 \text{ with } P \text{ at higher potential}$$

The relative velocity of the rod PQ w.r.t. U frame

$$v_{\text{rel}} = 2v - v = v$$

Now, time taken by it to loose the contact $t = \dfrac{l}{v}$



From equivalent electrical network

Net emf in the closed loop $QPDC$.

$$e = e_{PQ} - e_{DC} = \frac{v\mu_0 i_0}{2\pi} \ln 2$$

Growth of current in the L-R circuit is given by

$$i = i_0(1 - e^{-tR/L}) = \left(\frac{e}{R}\right)(1 - e^{-tR/L})$$

At time $t = \dfrac{l}{v}$

$$\Rightarrow \quad i = \left(\frac{e}{R}\right)(1 - e^{-\frac{Rl}{vL}})$$

28. Alternating Current

INTRODUCTORY EXERCISE 28.1

1. (a) $X_L = 2\pi fL$

(b) $L = \dfrac{X_L}{2\pi f}$

(c) $X_C = \dfrac{1}{2\pi fC}$

(d) $C = \dfrac{1}{2\pi fX_C}$

2.
$$V_L = \sqrt{V^2 - V_R^2}$$
$$= \sqrt{(150)^2 - (100)^2}$$
$$= 111.8 \text{ V}$$
$$V_L = IX_L = I\,(2\pi fL)$$
$$\therefore \quad L = \frac{V_L}{2\pi fI} = \frac{111.8}{2\pi \times 50 \times 10}$$
$$= 0.036 \text{ H}$$

3. $\phi = 0$, if
$$X_L = X_C$$
or
$$2\pi f\,L = \frac{1}{2\pi\, fC}$$
$$\therefore \quad L = \frac{1}{(2\pi f)^2\, C}$$
$$= \frac{1}{(360)^2 \times 10^{-6}}$$
$$= 7.7 \text{ H}$$
$$Z = R \qquad \text{(When } X_L = X_C\text{)}$$
$$\therefore \quad I = \frac{V}{Z} = \frac{V}{R} = \frac{120}{20}$$
$$= 6 \text{ A}$$

INTRODUCTORY EXERCISE 28.2

1.
$$f = \frac{1}{2\pi\sqrt{2C}} = \text{resonance frequency}$$
$$= \frac{1}{2\pi\sqrt{0.03 \times 2 \times 10^{-6}}}$$
$$= 650 \text{ Hz}$$

At resonance, $X_L = X_C$ and $Z = R$

$$\therefore \quad \cos\phi = \frac{R}{Z} = 1 \quad \text{or} \quad \phi = 0°$$

2. $R = \dfrac{V}{I} = \dfrac{40}{10} = 4\Omega$

$Z = \dfrac{V}{I} = \dfrac{200}{10} = 20\ \Omega$

Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{4}{20}$

$= 0.2$

Exercises

LEVEL 1

Assertion and Reason

1. $Z = \sqrt{R^2 + (X_C - X_L)^2}$

From this expression, we can see that X_C may be greater than Z also.

2. At resonance frequency f_r,
$$X_C = X_L$$
Now, $X_L = 2\pi fL$ or $X_L \propto f$

For $f > f_r$, $X_L > X_C$

At resonance, $X_L = X_C \Rightarrow Z = R$
$$\therefore \quad \cos\phi = \frac{R}{Z} = 1$$
or
$$\phi = 0°$$

3. $\omega = \dfrac{1}{\sqrt{LC}}$

By inserting a slab, C will increase. So, ω will decrease.

4. Average value $= \dfrac{\text{total area under } i\text{-}t \text{ graph}}{\text{total time interval}}$
$$= \frac{8 + 2 + 2 + 4 + 2}{6}$$
$$= \frac{18}{6} = 3 \text{ A}$$

5. $Z = \sqrt{R^2 + (X_C \sim X_L)^2}$

X_L will increase. So, Z may increase or decrease, depending on the value of X_C.Therefore, current may decrease or increase.

6. $V_L = V_C \Rightarrow X_L = X_C$

So this resonance condition.

8. $$P = I_{rms}^2 R = \left(\frac{2}{\sqrt{2}}\right)^2 (10) = 20 \text{ W}$$

9. $I_{DC} = \frac{V_{DC}}{r}$ (r = internal resistance of inductor)

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{V_{AC}}{\sqrt{r^2 + X_L^2}}$$

If $V_{DC} = V_{AC}$, then $I_{DC} > I_{AC}$

10. $$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + (2\pi fL)^2}}$$

with increase in frequency, I will decrease.

$$\tan\phi = \frac{X_L}{R} = \frac{2\pi fL}{R}$$

with increase in frequency $\tan\phi$ and therefore ϕ will increase.

11. At resonance, $X_L = X_C$

$\Rightarrow$ $Z = R$

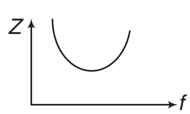
Hence, $I = \frac{V}{Z} = \frac{V}{R}$

So, current at resonance depends on R.

Objective Questions

2. Average value in AC comes out to be zero.

3.



Impedance first decreases, then increases. At resonance frequency Z is minimum.

4. In case of only capacitor and inductor phase difference between current and voltage should be 90°.

5. $I_{rms} = \frac{I_0}{\sqrt{2}} \approx 0.707\, I_0$

6. $\phi = 90°$ between V and I functions.

$\therefore$ $P = V_{rms} I_{rms} \cos\phi = 0$

7. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

8. $\because$ $$P = V_{rms} I_{rms} \cos\theta = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta = \frac{V_0 I_0}{2}\cos\theta$$

9. $V_0 = 240$ V

$\therefore$ $$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{240}{\sqrt{2}} = 170 \text{ V}$$

$\omega = 120$ rad/s

$$f = \frac{\omega}{2\pi} = \frac{120}{2 \times 3.14} = 19 \text{ Hz}$$

10. $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}} = 500 \text{ rad/s}$$

11. $$P = V_{rms} I_{rms} \cos\phi = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{100 \times 10^{-3}}{\sqrt{2}}\right) \cdot \cos\left(\frac{\pi}{3}\right) = 2.5 \text{ W}$$

12. $X_C = \frac{1}{\omega C}$

$\omega = 0$ for DC

$\therefore$ $X_C = \infty$

or it becomes a perfect insulator.

13. Substituting $t = \frac{1}{600}$ s in the given equation, we have

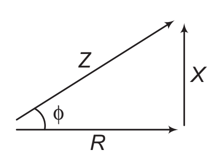
$$V = 10\cos(100\,\pi)\left(\frac{1}{600}\right) = 10\cos\frac{\pi}{6} = 5\sqrt{3} \text{ V}$$

15. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$ or $X_C \propto \frac{1}{f}$

i.e. X_C *versus* f graph is a rectangular hyperbola.

16. $\sin\phi = \frac{X}{Z} = \frac{1}{\sqrt{3}}$

$\therefore$ $\phi = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$



17. $i = \sqrt{2}\sin\left[\omega t + \frac{\pi}{4} + \frac{\pi}{2}\right]$

Hence, phase difference between V and i is $\frac{\pi}{2}$. So, power consumed = 0.

18. $I_{DC} = \dfrac{V_{DC}}{R}$

$$I = \frac{100}{R} \Rightarrow R = 100\ \Omega$$

$$I_{AC} = \frac{V_{AC}}{\sqrt{R^2 + X_L^2}}$$

$$0.5 = \frac{100}{\sqrt{(100)^2 + X_L^2}}$$

or $X_L = 100\sqrt{3}\ \Omega = (2\pi fL)$

$\therefore$ $L = \dfrac{100\sqrt{3}}{2\pi f} = \dfrac{100\sqrt{3}}{2\pi\ (50)}$

$$= \left(\frac{\sqrt{3}}{\pi}\right)\text{H}$$

19. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{100 \times 10^{-6}} = 10^4\ \Omega$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{(200\sqrt{2})/\sqrt{2}}{10^4}$$

$$= 0.02\text{ A } = 20\text{ mA}$$

20. $V = \sqrt{V_R^2 + V_L^2}$

$$= \sqrt{(20)^2 + (15)^2} = 25\text{ V}$$

But this is the rms value.

$\therefore$ Peak value $= \sqrt{2}\ V_{rms} = 25\sqrt{2}$ V

22. Resistance does not depend on the frequency of AC.

23. An ideal choke coil should have almost zero internal resistance. Otherwise, it will consume some power.

24. 45° phase angle means,

$$X_L = R$$

$\therefore$ $(2\pi fL) = R$

$\therefore$ $L = \dfrac{R}{2\pi f}$

$$= \frac{100}{(2\pi)(10^3)}$$

$$= 0.0159\text{ H}$$

$$\approx 16\text{ mH}$$

25. $T = \dfrac{1}{f} = \dfrac{1}{50}$ s

$$t = \frac{T}{4} = \frac{1}{200}\text{ s}$$

$$= 5 \times 10^{-3}\text{ s}$$

$$= 5\text{ ms}$$

26. $P = V_{rms}\, I_{rms} \cos\phi$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos 60°$$

$$= \left(\frac{220}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= 220\text{ W}$$

27. I_C is 90° ahead of the applied voltage and I_L lags behind the applied voltage by 90°. So, there is a phase difference of 180° between I_L and I_C.

$\therefore$ $I = I_C - I_L = 0.2$ A

28. V_L function is cos function, which is 90° ahead of the current function. Hence, current function should be sin function.

29. $H_{DC} = I^2Rt$

$$H_{AC} = I_{rms}^2\, Rt = \left(\frac{I}{\sqrt{2}}\right)^2 Rt = \frac{I^2\, Rt}{2}$$

$\therefore$ $\dfrac{H_{DC}}{H_{AC}} = \dfrac{2}{1}$

30. $V_C = \sqrt{V^2 - V_R^2} = \sqrt{(20)^2 - (12)^2}$

$$= 16\text{ V}$$

Subjective Questions

1. $X_L = \omega L = 100\ \Omega$

$$X_C = \frac{1}{\omega C} = 312.5\ \Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(300)^2 + (312.5 - 100)^2}$$

$$= 368\ \Omega$$

(a) $I_0 = \dfrac{V_0}{Z} = \dfrac{120}{368} = 0.326$ A

(b) Since, $X_C > X_L$, voltage lags the current by an angle given by

$$\phi = \cos^{-1}\left(\frac{R}{Z}\right) = \cos^{-1}\left(\frac{300}{368}\right) = 35.3°$$

(c) $(V_0)_R = I_0R = (0.326)\ 300 = 97.8$ V

$(V_0)_L = I_0X_L = (0.326)\ (100) = 32.6$ V

$(V_0)_C = I_0X_C = (0.326)\ (312.5) = 102$ V

2. (a) Voltage lags

$\therefore$ $X_C > X_L$

Power factor, $\cos\phi = \dfrac{R}{Z}$

$$= \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$$

To increase the power factor denominator should decrease. Hence, X_L should increase. Therefore, an inductor is required to be connected.

(b) $\cos\phi = \dfrac{R}{Z} = 0.72$

$$\therefore \quad R = 0.72\,Z = 0.72 \times 60 = 43.2\ \Omega$$

$$(X_C - X_L) = \sqrt{(60)^2 - (43.2)^2} = 41.64\ \Omega$$

New inductor of inductance 41.64 Ω should be added in the circuit.

$$L = \frac{X_L}{2\pi f} = \frac{41.64}{2\pi\,(50)} = 0.133\ \text{H}$$

3. (b) $f = \dfrac{\omega}{2\pi} = \dfrac{6280}{2 \times 3.14} = 1000$ Hz

(c) $\phi = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$ or $30°$

Power factor $= \cos\phi = \cos 30° = \dfrac{\sqrt{3}}{2}$

From the given functions of V and i, we can see that current function leads the voltage function.

(d) $Z = \dfrac{170}{8.5} = 20\ \Omega$ ...(i)

$$\cos\phi = \frac{\sqrt{3}}{2} = \frac{R}{Z} = \frac{R}{20}$$

$$\therefore \quad R = 10\sqrt{3}\ \Omega = 17.32\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = 10\ \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{6280 \times 10}\ \text{F} = 15.92 \times 10^{-6}\ \text{F}$$

4. $I_0 = \dfrac{V_0}{X_L} = \dfrac{V_0}{\omega L}$

5. (a) $I_0 = \dfrac{(V_0)_R}{R} = \dfrac{2.5}{300}$

$$= 8.33 \times 10^{-3}\ \text{A} = 8.33\ \text{mA}$$

Current function and V_R function are in phase. Hence,

$$I = (8.33\ \text{mA}) \cos[(950\ \text{rad/s})\,t]$$

(b) $X_L = \omega L = 950 \times 0.8 = 760\ \Omega$

(c) $(V_0)_L = I_0 X_L$

$$= (8.33 \times 10^{-3})\,(760) = 6.33\ \text{V}$$

Now, V_L function leads the current (or V_R) function by 90°.

$$\therefore \quad V_L = 6.33 \cos(950\,t + 90°) = -6.33 \sin(950\,t)$$

6. $X_L = 2\pi f L = 301\ \Omega$

$$X_C = \frac{1}{2\pi f C} = 55\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240)^2 + (301 - 55)^2} = 343\ \Omega$$

(a) $\phi = \cos^{-1}\left(\dfrac{R}{Z}\right)$

$$= \cos^{-1}\left(\frac{240}{343}\right) = 45.8°$$

$$\cos\phi = \frac{R}{Z} = \frac{240}{343} = 0.697$$

Since, $X_L > X_C$, voltage leads the current.

(b) Impedance $= Z = 343\ \Omega$

(c) $V_{\text{rms}} = I_{\text{rms}}\,Z$

$$= 0.45 \times 343 = 155\ \text{V}$$

(d) $P = I_{\text{rms}}^2 R$

$$= (0.45)^2\,(240) = 48.6\ \text{W}$$

(e) $P = P_R = 48.6$ W

(f) $P_C = 0$

(g) $P_L = 0$

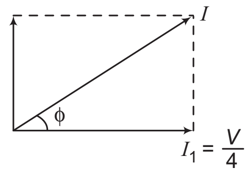
LEVEL 2

Single Correct Option

1. $I_2 = \dfrac{V}{X_C} = \dfrac{V}{3}$ (here V = rms value)

$$I_1 = \frac{V}{R} = \frac{V}{4}$$

I_2 is 90° ahead of applied voltage function and I_1 is in phase with it.



$$\tan\phi = \frac{V/3}{V/4} = \frac{4}{3}$$

$$\therefore \qquad \phi = 53°$$

2. I_R and I_L are in same phase and phase difference between them and applied voltage lies between 0° and 90°.

3. $X_L = \omega L = (5 \times 10^{-3})\,(2000) = 10\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)\,(50 \times 10^{-6})} = 10\ \Omega$$

Since, $X_L = X_C$ circuit is in resonance.

$$Z = R = (6 + 4) = 10\ \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{(20/\sqrt{2})}{10} = 1.414\ \text{A}$$

This is also the reading of ammeter.

$$V = 4I_{\text{rms}}$$
$$\approx 5.6\ \text{volt}$$

4.
$$I_R = \frac{V_{\text{rms}}}{R} = \frac{200}{100} = 0.2\ \text{A}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{(2\pi)\,(5 \times 10^3)\left(\frac{1}{\pi} \times 10^{-6}\right)}$$

$$= 100\ \Omega$$

$$\therefore \quad I_C = \frac{V_{\text{rms}}}{X_C} = \frac{200}{100} = 2\ \text{A}$$

I_C is 90° ahead of the applied voltage and I_R is in phase with the applied voltage. Hence, there is a phase difference of 90° between I_R and I_C too.

$$\therefore \qquad I = \sqrt{I_R^2 + I_C^2}$$
$$= \sqrt{(2)^2 + (2)^2}$$
$$= 283\ \text{A}$$

5. Average value of 5 sin 100 ωt is zero. But average value of 5 A (= constant current) is 5 A. Hence, average value of total given function is 5 A.

6. V function is sin function. I function is ahead of V function. Hence, the circuit should be capacitive in nature.

Further, $\qquad \phi = 45°$

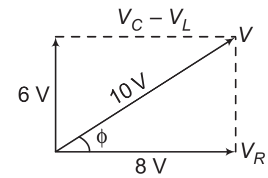
$$\therefore \qquad X_C = R \quad \text{or} \quad \omega C = R$$

or
$$C = \frac{R}{\omega} = \frac{R}{100} = 0.01\,R$$

In option (b), this condition is satisfied.

7. $V = \sqrt{V_R^2 + (V_C - V_L)^2} = 10\ \text{V}$

$V_C > V_L$, hence current leads the voltage.



$$\text{Power factor} = \cos\phi = \frac{8}{10} = 0.8$$

8. See the hint of miscellaneous example numbers 6 and 7 of solved examples.

9.
$$V_S = \sqrt{V_R^2 + V_L^2}$$
$$= \sqrt{(70)^2 + (20)^2} = 72.8\ \text{V}$$

$$\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{20}{70} = \frac{2}{7}$$

10. In first case, $X_C = \dfrac{V}{I} = \dfrac{220}{0.25} = 880\ \Omega$

In the second case, $R = \dfrac{V}{I} = \dfrac{220}{0.25} = 880\ \Omega$

In the combination of P and Q,

$$\tan\phi = \frac{X_C}{R} = 1$$

$$\therefore \qquad \phi = 45°$$

Since the circuit is capacitive, current leads the voltage. Further,

$$Z = \sqrt{R^2 + X_C^2} = 880\sqrt{2}\ \Omega$$

$$I = \frac{V}{Z} = \frac{220}{880\sqrt{2}} = \frac{1}{4\sqrt{2}}\ \text{A}$$

11. See the hints of miscellaneous example numbers 6 and 7 of solved examples.

12. $i = \dfrac{V}{R}$, i.e. circuit is in resonance. Hence,

$$V_C = V_L = 200\ \text{V}$$

13.
$$P = I_{\text{rms}}^2\,R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R$$

$$= \left[\frac{(V_0/\sqrt{2})^2}{R^2 + \omega^2 L^2}\right] R$$

$$= \frac{V_0^2 R}{2\,(R^2 + \omega^2 L^2)}$$

14. $X_L = \omega L$

If ω is very low, then $X_L \approx 0$

$\therefore \quad V_L \approx 0$

or $\quad V = V_C = V_0$

15. $I_{\max} = \dfrac{V}{R}$ (at resonance)

$$6 = \frac{24}{R}$$

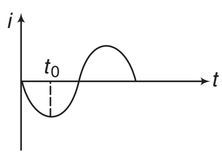
$$\therefore \quad R = 4\ \Omega$$

$$I_{DC} = \frac{V}{R + r} = \frac{12}{4 + 4} = 1.5\ \text{A}$$

16. $V_R = \sqrt{V^2 - V_C^2} = \sqrt{(10)^2 - (8)^2} = 6\ \text{V}$

$$\tan\phi = \frac{X_C}{X_R} = \frac{V_C}{V_R} = \frac{8}{6} = \frac{4}{3}$$

17. Current will lead the voltage function by 90° voltage function is cos function. Therefore, current function will be – sin function.



$$t_0 = \frac{T}{4} = \frac{(2\pi/\omega)}{4} = \frac{\pi}{2\omega}$$

$$= \frac{\pi}{2\,(\pi/2)} = 1\ \text{s}$$

18. $R = \dfrac{1}{\omega C} = X_C$

$$\therefore \quad Z = \sqrt{R^2 + X_C^2} = \sqrt{2}\,R \qquad (\text{as } X_C = R)$$

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{2}\,R} \qquad \text{...(i)}$$

When ω becomes $\dfrac{1}{\sqrt{3}}$ times, X_C will become $\sqrt{3}$ times or $\sqrt{3}\,R$.

$$Z' = \sqrt{(R^2) + (\sqrt{3}R)^2} = 2R$$

$$I_0' = \frac{V_0}{Z'} = \frac{V_0}{2R} = \frac{I_0}{\sqrt{2}}$$

More than One Correct Options

1. $\sqrt{V_R^2 + V_L^2} = 100 \qquad \text{...(i)}$

$V_L \sim V_C = 120 \qquad \text{...(ii)}$

$\sqrt{V_R^2 + (V_L \sim V_C)^2} = 130 \qquad \text{...(iii)}$

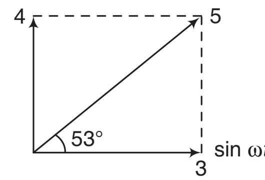
Solving these three equations, we get

$$V_R = 50\ \text{V},\ V_L = 86.6\ \text{V and}$$
$$V_C = 206.6\ \text{V}$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$$

Since $V_C > V_L$, circuit is capacitive in nature.

2. $i = 5\sin(\omega t + 53°)$



$$i_0 = 5\ \text{A}$$

$$\therefore \quad i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}\ \text{A}$$

Mean value of current in positive half cycle is

$$\frac{2}{\pi}\,i_0 = \left(\frac{2}{\pi}\right)(5) = \left(\frac{10}{\pi}\right)\ \text{A}$$

In $V = V_m \sin\omega t$, current $i = 5\sin(\omega t + 53°)$ leads the voltage function. Hence, circuit is capacitive in nature. Same is the case with part (*d*).

3. $P_R = V_R\, i$

$$\therefore \quad i = \frac{P_R}{V_R} = \frac{60}{60} = 1\ \text{A}$$

Now, $\quad V_L = \sqrt{V^2 - V_R^2}$

$$= \sqrt{(100)^2 - (60)^2}$$

$$= 80\ \text{V} = i\,X_L = i\,(2\pi f L)$$

$$L = \frac{80}{2\pi f i}$$

$$= \frac{80}{(2\pi)(50)(1)} = \frac{4}{5\pi}\ \text{H}$$

If we connect another resistance R in series, then it should consume 40 V, so that remaining 60 V is used by the tube light.

$$R = \frac{V}{i} = \frac{40}{1} = 40\ \Omega$$

4. Power factor, $\cos\phi = \dfrac{R}{Z}$

When circuit contains only resistance, then

$$Z = R \quad \Rightarrow \quad \cos\phi = 1$$

When circuit contains only inductance, then

$$R = 0$$

$$\therefore \quad \cos\phi = 0$$

5. (a) $X_L > X_C$, hence voltage function will lead the current function.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(10)^2 + (20 - 10)^2}$

$= 10\sqrt{2}\ \Omega$

(c) $\cos\phi = \dfrac{R}{Z} = \dfrac{1}{\sqrt{2}}$

Hence, $\phi = 45°$

(d) Power factor $= \cos\phi = \dfrac{R}{Z} = \dfrac{1}{\sqrt{2}}$

6. (b) At resonance frequency (ω_r)

$$X_L > X_C$$

In the given values, $X_L > X_C$. Hence,

$$\omega > \omega_r$$

As, $X_L = \omega L \Rightarrow X_L \propto \omega$

and $X_C = \dfrac{1}{\omega C} \Rightarrow X_C \propto \dfrac{1}{\omega}$

(c) If frequency is increased from the given value, X_L will further increase. So, $X_L - X_C$ will increase. Hence, net impedance will increase.

(d) If frequency is decreased from the given value, then X_C will increase and X_L will decrease. So, $X_L - X_C$ may be less than the previous value or $X_C - X_L$ may be greater than the previous. So, Z may either increase or decrease. Hence, current may decrease or increase.

7. (a) $V_R = IR = 80$ V

(b) $X_C = \dfrac{V_C}{I} = \dfrac{100}{2} = 50\ \Omega$

(c) $V_L = IX_L = 40$ V

(d) $V = V_{rms} = \sqrt{V_R^2 + (V_C - V_L)^2}$

$= \sqrt{(80)^2 + (100 - 40)^2}$

$= 60$ V

$\therefore\ V_0 = \sqrt{2}\,V_{rms} = 60\sqrt{2}$ V

8. $I = \dfrac{V}{Z}$

$$= \frac{V}{\sqrt{R^2 + \left(\omega L \sim \dfrac{1}{\omega C}\right)^2}}$$

By increasing R, current will definitely decrease by change in L or C, current may increase or decrease.

Comprehension Based Questions

1 to **3.**

$$V_{DC} = I_{DC} R$$

$\therefore\quad R = \dfrac{V_{DC}}{I_{DC}} = \dfrac{12}{4} = 3\ \Omega$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{V_{AC}}{\sqrt{R^2 + X_L^2}}$$

$$2.4 = \frac{12}{\sqrt{(3)^2 + X_L^2}}$$

Solving this equation, we get

$X_L = 4\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{50 \times 2500 \times 10^{-6}}$

$= 8\ \Omega$

$Z = \sqrt{R^2 + (X_C - X_L)^2} = 5\Omega$

$\therefore\quad I = \dfrac{V_{DC}}{Z} = \dfrac{12}{5} = 2.4\ \text{A} = I_{rms}$

$P = I_{rms}^2 R = (2.4)^2 (3)$

$= 17.28$ W

At given frequency, $X_C > X_L$. If ω is further decreased, X_C will increase $\left(\text{as } X_C \propto \dfrac{1}{\omega}\right)$ and X_L will increase (as $X_L \propto \omega$).

Therefore, $X_C - X_L$ and hence Z will increase. So, current will decrease.

4. $\omega = \dfrac{1}{\sqrt{LC}}$

$= \dfrac{1}{\sqrt{4.9 \times 10^{-3} \times 10^{-6}}}$

$= \dfrac{10^5}{7}$ rad/s

5. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{\left(\dfrac{10^5}{7}\right)(10^{-6})} = 70\ \Omega$

$Z_P = \sqrt{R_P^2 + X_C^2}$

$= \sqrt{(32)^2 + (70)^2}$

$\approx 77\ \Omega$

6. At maximum current means at resonance,

$$X_L = X_C,\ Z = R$$

$\therefore\quad$ Power factor $= \cos\phi = \dfrac{R}{Z} = 1$

Match the Columns

2. (a) $\phi = 0°$ between voltage function and current function.

(b) $I = I_0 \sin(\omega t - 90°)$

i.e. $\phi = 90°$ and voltage function leads the current function.

(c) Current function leads the voltage function. So,

$$X_C > X_L$$

(d) Voltage function leads the current function. So,

$$X_L > X_C$$

3. See the hint of Q.No. 6 and 8 of section more than one correct options. Then,

$$P = I_{\text{rms}}^2 R$$

By increasing R, current i_{rms} will decrease but the power, $P = I_{\text{rms}}^2 R$ may increase or decrease.

4. (a) $R = \dfrac{V_R}{I} = \dfrac{40}{2} = 20\ \Omega$

(b) $V_C = IX_C = 2 \times 30 = 60$ V

(c) $V_L = I\ X_L = 2 \times 15 = 30$ V

(d) $V = \sqrt{V_R^2 + (V_C - V_L)^2} = 50$ V

5. (a) Resistance does not depend on the value of ω.

(b) $X_C = \dfrac{1}{\omega C}$ or $X_C \propto \dfrac{1}{\omega}$

(c) $X_L = \omega L$ or $X_L \propto \omega$

(d) Z is minimum at $\omega = \omega_r$ and $Z_{\min} = R$

Below or above ω_r

$$Z = \sqrt{R^2 + (X_L \sim X_C)^2}$$

or $\qquad Z > R$

Subjective Questions

1. $P = V_{\text{rms}} i_{\text{rms}} \cos\phi$

or $\qquad 200 = 230 \times 8 \times \cos\phi$

$\therefore \qquad \cos\phi = 0.108$

or $\qquad \phi = 83.8°$

Further, $\qquad P = i_{\text{rms}}^2 R$

$\therefore \qquad R = \dfrac{P}{i_{\text{rms}}^2} = \dfrac{200}{(8)^2} = 3.125\ \Omega$

(a) $\tan\phi = \dfrac{X_C - X_L}{R}$

$\therefore \qquad \dfrac{1}{2\pi fC} - (2\pi fL) = R\tan\phi$

$\therefore \qquad L = \dfrac{1}{(2\pi f)^2 C} - \dfrac{R\tan\phi}{2\pi f}$

$$= \frac{1}{(2\pi \times 50)^2 \times 20 \times 10^{-6}} - \frac{3.125 \tan 83.8°}{2\pi \times 50}$$

$$= 0.416 \text{ H}$$

(b) $\tan\phi = \dfrac{X_L - X_C}{R}$

or $\qquad 2\pi fL - \dfrac{1}{(2\pi fC)} = R\tan\phi$ **Ans.**

$$L = \frac{1}{(2\pi f)^2 C} + \frac{R\tan\phi}{(2\pi f)}$$

$$= \frac{1}{(2 \times \pi \times 50)^2 \times 20 \times 10^{-6}} + \frac{3.125 \tan 83.8°}{(2\pi \times 50)}$$

$= 0.597$ H **Ans.**

2. Average current will be zero as positive and negative half cycles are symmetrical RMS current can also be obtained from 0 to $\tau/2$.

$$I = \left(\frac{I_0}{\tau/2}\right)t = \left(\frac{2I_0}{\tau}\right)t$$

$$\Rightarrow \qquad I^2 = \left(\frac{4I_0^2}{\tau^2}\right)t^2$$

$$\Rightarrow \qquad \langle I^2 \rangle_{0-\tau/2} = \frac{\int_0^{\tau/2} \frac{4I_0^2}{\tau^2} t^2 dt}{\tau/2}$$

$$\text{or} \qquad \langle I^2 \rangle_{0-\tau/2} = \frac{I_0^2}{3}$$

$$\Rightarrow \qquad I_{\text{rms}} = \sqrt{\frac{I_0^2}{3}} = \frac{I_0}{\sqrt{3}}$$

3. (a) $0.5 = \dfrac{R_1}{Z_1}$

Further, $\quad P = V_{\text{rms}} i_{\text{rms}} \cos\phi$

or $\qquad 100 = 230 \times \dfrac{230}{Z_1} \times 0.5$

$\therefore \qquad Z_1 = 264.5\ \Omega$

and $\qquad R_1 = 132.25\ \Omega$

Further, $X_L = \sqrt{Z_1^2 - R_1^2} = \dfrac{\sqrt{3}}{2} Z_1$

$= 229\ \Omega$

In second case, $0.6 = \dfrac{R_2}{Z_2}$

and $\qquad 60 = \dfrac{230 \times 230}{Z_2} \times 0.6$

$\therefore \quad Z_2 = 529\ \Omega$

and $\quad R_2 = 317.4\ \Omega$

Further, $\quad X_C = \sqrt{Z_2^2 - R_2^2}$

$= 423.2\ \Omega$

When connected in series,

$R = R_1 + R_2 = 449.65\ \Omega$

$X_C - X_L = 194.2$

$\therefore \quad Z = \sqrt{(449.65)^2 + (194.2)^2}$

$= 489.79\ \Omega$

Power factor, $\cos\phi = \dfrac{R}{Z} = 0.92$ (leading)

$$P = V_{\text{rms}} i_{\text{rms}} \cos\phi$$

$$= (230)\left(\frac{230}{489.79}\right)(0.92)$$

$$= 99\ \text{W}$$ **Ans.**

(b) Since, $X_C - X_L = 194.2\ \Omega$

Therefore, if $194.2\ \Omega$ inductive reactance is to be added in series, then it will become only R circuit and power factor will become unity.

4. (a) $i_{\text{rms}} = \dfrac{V_1}{R} = \dfrac{40}{4} = 10$ A

$$i_0 = \sqrt{2} i_{\text{rms}} = 10\sqrt{2}\ \text{A}$$ **Ans.**

(b) $\dfrac{E_0}{\sqrt{2}} = \sqrt{V_1^2 + (V_2 - V_1)^2} = 50$ V

$\therefore \quad E_0 = 50\sqrt{2}$ V

(c) $\quad X_L = (\omega L) = \dfrac{V_2}{i_{\text{rms}}} = \dfrac{40}{10} = 4\ \Omega$

$\therefore \quad L = \dfrac{4}{\omega} = \dfrac{4}{100\pi}\ \text{H} = \dfrac{1}{25\pi}$ H **Ans.**

$X_C = \dfrac{1}{\omega C} = \dfrac{V_1}{i_{\text{rms}}} = \dfrac{10}{10} = 1\ \Omega$

$\therefore \quad C = \dfrac{1}{\omega} = \dfrac{1}{100\pi}$ F **Ans.**

5. $\quad \cos\phi_1 = 0.5$

$\therefore \quad \phi_1 = 60°$

$\cos\phi_2 = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \phi_2 = 30°$

Let R be the effective resistance of the box. Then,

$$\tan\phi_1 = \frac{X_C}{R} \quad \text{or} \quad \sqrt{3} = \frac{X_C}{R} \qquad \ldots\text{(i)}$$

$$\tan\phi_2 = \frac{X_C}{R + 10} \quad \text{or} \quad \frac{1}{\sqrt{3}} = \frac{X_C}{R + 10} \qquad \ldots\text{(ii)}$$

From these two equations, we get $R = 5\ \Omega$

6. (a) $V_R = IR = 80$ V, $V_C = 100$ V

and $\quad V_L = IX_L = 160$ V

$\therefore \quad V = \sqrt{V_R^2 + (V_L - V_C)^2} = 100$ V **Ans.**

Note *Value of X_L have been taken from part (b).*

(b) Since the current is lagging behind, there should be an inductor in the box.

$$X_C = \frac{V_C}{I} = 100\ \Omega$$

Now, $\quad 0.8 = \dfrac{R}{Z} = \dfrac{80}{\sqrt{(80)^2 + (X_L - 100)^2}}$

Solving, we get

$X_L = 160\ \Omega$

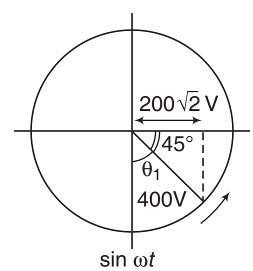
or $\quad \omega L = 160$

$\therefore \quad (2\pi f L) = 160$

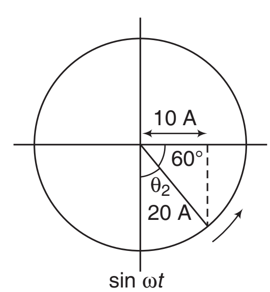
$\therefore \quad L = \dfrac{160}{2\pi f} = \dfrac{160}{(2\pi) \times 50}$

$= \dfrac{1.6}{\pi}$ H

7. (a)



Reference circle for voltage



Reference circle for current

$$\omega = 2\pi f = (100\pi)\ \text{rad/s}$$

From the above two figures, we can write

$V = 400 \sin(\omega t + \theta_1) = 400 \sin\left[100\pi t + \dfrac{\pi}{4}\right]$ **Ans.**

$i = 20 \sin(\omega t + \theta_2) = 20 \sin\left[100\pi t + \dfrac{\pi}{6}\right]$ **Ans.**

(b) Phase difference between V and i

$$\phi = (\pi/4 - \pi/6) = \frac{\pi}{12} \text{ or } 15°$$

$$P = V_{rms} i_{rms} \cos\phi = \left(\frac{400}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right) \cos 15°$$

$$= 3864 \text{ W}$$ **Ans.**

8. $\omega = \dfrac{2}{\sqrt{LC}} = \dfrac{2}{\sqrt{5 \times 10^{-3} \times 20 \times 10^{-6}}}$

$= 6324.5$ rad/s

$X_L = \omega L = (6324.5)(5 \times 10^{-3}) = 31.62\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{6324.5 \times 20 \times 10^{-6}} = 7.9\ \Omega$

$\therefore \quad Z = X_L - X_C = 23.72\ \Omega$

(a) Maximum voltage across capacitor

$= i_0 X_C = (0.211)(7.9) = 1.67$ mV

$\therefore$ Maximum charge

$q_0 = (20 \times 10^{-6})(1.67 \times 10^{-3}) = 33.4$ nC

(b) $i_0 = \dfrac{V_0}{Z} = \dfrac{5}{23.72}$ mA $= 0.211$ mA

(c) Since $X_L > X_C$, current in the circuit will lag behind the applied voltage by $\pi/2$.

Further voltage across the inductor will lead this current by $\pi/2$.

Therefore, applied voltage and voltage across inductor are in phase.

Voltage across the capacitor will lag the circuit current by $\pi/2$.

Therefore, phase difference between V_L and V_C will be 180°.

9. $X_{L_1} = \omega L_1 = (2\pi \times 50)(0.02) = 6.28\ \Omega$

$\therefore \quad Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$

$= \sqrt{(5)^2 + (6.28)^2} = 8.0\ \Omega$

$P_1 = (I_{rms})_1 V_{rms} \cos\phi_1 = \left(\dfrac{100}{8}\right)(100)\left(\dfrac{5}{8}\right)$

$= 781.25$ W

$X_{L_2} = \omega L_2 = (2\pi \times 50)(0.08)$

$= 25.13\ \Omega$

$\therefore \quad Z_2 = \sqrt{R_2^2 + X_{L_2}^2}$

$= 25.15\ \Omega$

$\therefore \quad P_2 = (i_{rms})_2 V_{rms} \cos\phi_2$

$= \left(\dfrac{100}{25.15}\right)(100)\left(\dfrac{1}{25.15}\right)$

$= 15.8$ W

$\therefore \quad P_{Total} = P_1 + P_2 = 797$ W

10. $Z_1 = \dfrac{115}{3} = 38.33\ \Omega$

$\cos\phi_1 = \dfrac{R_1}{Z_1}$

$\Rightarrow \quad R_1 = Z_1 \cos\phi_1$

$= (38.33)(0.6) = 23\ \Omega$

$X_L = \sqrt{Z_1^2 - R_1^2} = 30.67\ \Omega$

$Z_2 = \dfrac{115}{5} = 23\ \Omega$

$R_2 = Z_2 \cos\phi_2 = (23)(0.707)$

$= 16.26$

$X_C = \sqrt{Z_2^2 - R_2^2}$

$= \sqrt{(23)^2 - (16.26)^2}$

$= 16.26\ \Omega$

When connected in series,

$R = R_1 + R_2 = 39.26\ \Omega$

$X_L - X_C = 14.41\ \Omega$

$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= 41.82\ \Omega$

(a) $i = \dfrac{V}{Z} = \dfrac{230}{41.82} = 5.5$ A

(b) $P = i^2 R = (5.5)^2 (39.26)$

$= 1187.6 \text{ W} \approx 1.188$ kW

(c) Power factor $= \cos\phi = \dfrac{R}{Z} = \dfrac{39.26}{41.92} = 0.939$

Since $X_L > X_C$, this power factor is lagging.

JEE Main and Advanced

Previous Years' Questions (2018-13)

JEE Main

1. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is **(2018)**

(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

2. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be **(2018)**

(a) 1.2 nC (b) 0.3 nC (c) 2.4 nC (d) 0.9 nC

3. In an AC circuit, the instantaneous emf and current are given by

$e = 100 \sin 30\, t,\ i = 20 \sin\left(30\, t - \frac{\pi}{4}\right)$

In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively

(a) 50 , 10 (b) $\frac{1000}{\sqrt{2}}$, 10

(c) $\frac{50}{\sqrt{2}}$, 0 (d) 50 , 0

4. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between **(2018)**

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V

(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

5. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field B. The relation between r_e, r_p, r_α is **(2018)**

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$

(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

6. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is **(2018)**

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

7. For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by **(2018)**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$

(c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

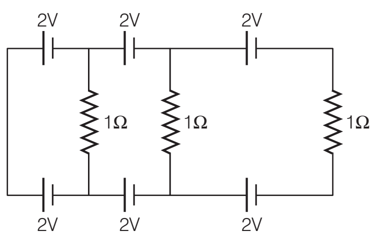
8. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **(2018)**

(a) 1 Ω (b) 1.5 Ω (c) 2 Ω (d) 2.5 Ω

9. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances? **(2018)**

(a) 990 Ω (b) 505 Ω (c) 550 Ω (d) 910 Ω

10. In the below circuit, the current in each resistance is **(2017)**



(a) 0.25 A (b) 0.5 A (c) 0 A (d) 1 A

11. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is **(2017)**

(a) 2.045×10^3 Ω (b) 2.535×10^3 Ω
(c) 4.005×10^3 Ω (d) 1.985×10^3 Ω

12. Which of the following statements is false? **(2017)**

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed
(b) A rheostat can be used as a potential divider
(c) Kirchhoff's second law represents energy conservation
(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

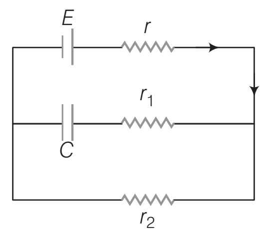
13. An electric dipole has a fixed dipole moment **p**, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **(2017)**

(a) 45° (b) 60° (c) 90° (d) 30°

14. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **(2017)**

(a) 16 (b) 24 (c) 32 (d) 2

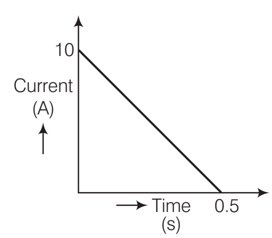
15. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be **(2017)**



(a) $CE \dfrac{r_1}{(r_2 + r)}$ (b) $CE \dfrac{r_2}{(r + r_2)}$

(c) $CE \dfrac{r_1}{(r_1 + r)}$ (d) CE

16. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **(2017)**

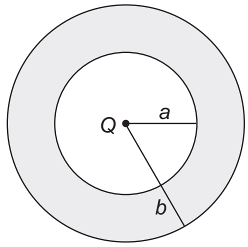


(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

17. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is **(2016)**

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

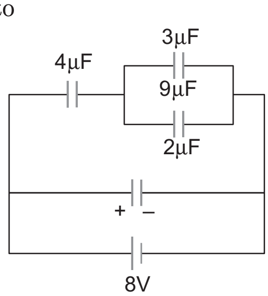
18. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$,



where, A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A, such that the electric field in the region between the spheres will be constant, is **(2016)**

(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$
(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

19. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\mu F$ and $9\mu F$ capacitors), at a point distant 30 m from it, would equal to **(2016)**

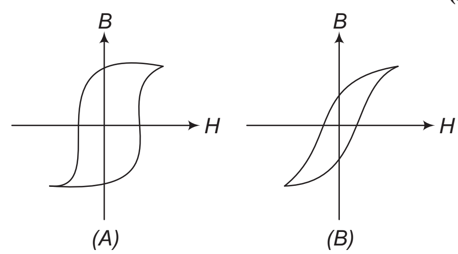


(a) 240 N/C (b) 360 N/C
(c) 420 N/C (d) 480 N/C

20. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is **(2016)**

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

21. Hysteresis loops for two magnetic materials A and B are as given below: **(2016)**



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

22. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to **(2016)**

(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

23. Arrange the following electromagnetic radiations in the order of increasing energy. **(2016)**

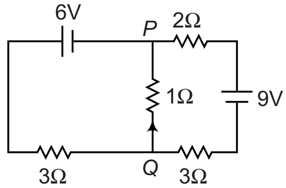
A. Blue light B. Yellow light
C. X-ray D. Radio wave

(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

24. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5 \times 10^{-4}\, ms^{-1}$. If the electron density in the wire is $8 \times 10^{28}\, m^{-3}$ the resistivity of the material is close to **(2015)**

(a) $1.6 \times 10^{-8}\ \Omega$-m
(b) $1.6 \times 10^{-7}\ \Omega$-m
(c) $1.6 \times 10^{-5}\ \Omega$-m
(d) $1.6 \times 10^{-6}\ \Omega$-m

25. In the circuit shown below, the current in the 1Ω resistor is



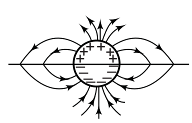
(2015)

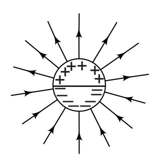
(a) 1.3 A, from P to Q
(b) 0.13 A, from Q to P
(c) 0 A
(d) 0.13 A, from P to Q

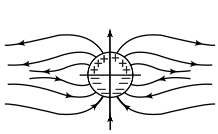
26. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then, **(2015)**

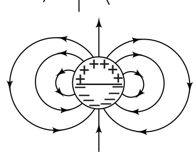
(a) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(b) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(c) $2R < R_4$
(d) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

27. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge – σ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **(2015)**

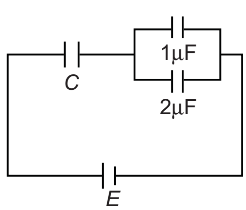
(a) 

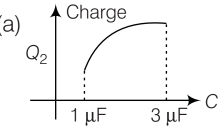
(b) 

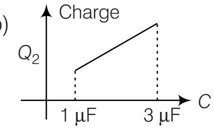
(c) 

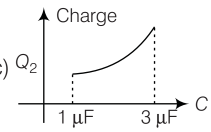
(d) 

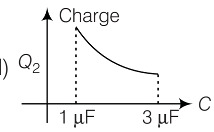
28. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale) **(2015)**



(a) 

(b) 

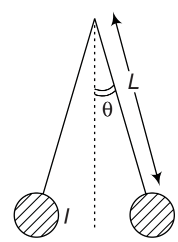
(c) 

(d) 

29. Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then, **(2015)**

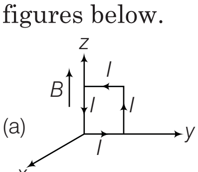
(a) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$
(b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards
(c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$
(d) $\mathbf{F}_{21} = \mathbf{F}_2 = 0$

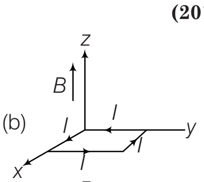
30. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is (g = gravitational acceleration) **(2015)**

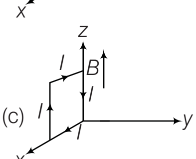


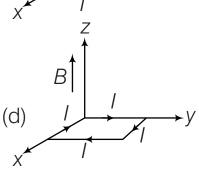
(a) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$
(b) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$
(c) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$
(d) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

31. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below. **(2015)**

(a) 

(b) 

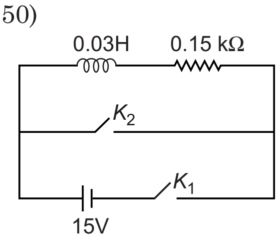
(c) 

(d) 

If there is a uniform magnetic field of 0.3 T in the positive z-direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b) respectively
(b) (b) and (d) respectively
(c) (a) and (c) respectively
(d) (b) and (c) respectively

32. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be ($e^5 \cong 150$) **(2015)**



(a) 100 mA (b) 67 mA (c) 0.67mA (d) 6.7 mA

33. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be **(2014)**

(a) 8 A (b) 10 A (c) 12 A (d) 14 A

34. Assume that an electric field $\mathbf{E} = 30x^2\,\hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is **(2014)**

(a) 120 J (b) −120 J (c) −80 J (d) 80 J

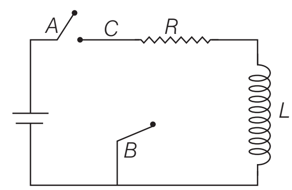
35. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **(2014)**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^4 C/m^2 (d) 6×10^4 C/m^2

36. The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is **(2014)**

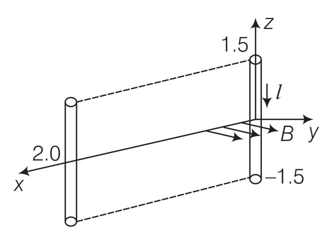
(a) 30 mA (b) 60 mA
(c) 3 A (d) 6 A

37. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **(2014)**



(a) $\frac{e}{1-e}$ (b) 1 (c) −1 (d) $\frac{1-e}{e}$

38. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure).
For a field $\mathbf{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3}s.
Assume parallel motion along the x-axis. **(2014)**



(a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

39. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **(2013)**

Statement I Higher the range, greater is the resistance of ammeter.

Statement II To increase the range of ammeter, additional shunt needs to be used across it.

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

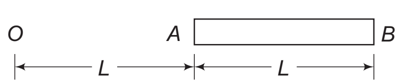
40. The supply voltage in a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **(2013)**

(a) zero (b) 2.9 V (c) 13.3 V (d) 10.4V

41. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $y\,(y << a)$ along the y-axis, the net force acting on the particle is proportional to **(2013)**

(a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

42. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **(2013)**



(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$
(c) $\frac{Q}{4\pi\varepsilon_0 L \ln(2)}$ (d) $\frac{Q\ln(2)}{4\pi\varepsilon_0 L}$

43. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then **(2013)**

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

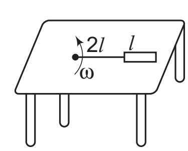
44. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **(2013)**

(a) 3.6×10^{-5} Wb /m^2 (b) 2.56×10^{-4} Wb /m^2
(c) 3.50×10^{-4} Wb /m^2 (d) 5.80×10^{-4} Wb /m^2

45. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the smaller loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the bigger loop, then the flux linked with smaller loop is **(2013)**

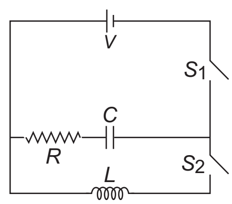
(a) 9.1×10^{-11} Wb (b) 6×10^{-11} Wb
(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

46. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is **(2013)**



(a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$
(c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

47. In a L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 and S_2, are closed. (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **(2013)**



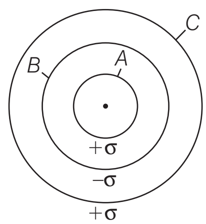
(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau$, $q = CV/2$
(c) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(d) At $t = \tau/2$, $q = CV(1 - e^{-1})$

48. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s, it will decrease to α times its original magnitude, where α equals **(2013)**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

Answer *with* Explanations

1. **(b)** Potential of B = Potential due to charge on A + Potential due to charge on B + Potential due to charge on C.



$$\therefore \quad V_B = \frac{k(Q_A + Q_B)}{b} + \frac{kQ_C}{c}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$= \frac{\sigma\varepsilon}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + \frac{c^2}{c}\right) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

2. **(a)** Magnitude of induced charge is given by

$$Q' = (K - 1)CV_0$$

$$= \left(\frac{5}{3} - 1\right) 90 \times 10^{-12} \times 20 = 1.2 \times 10^{-9}\text{C}$$

$$\Rightarrow \quad Q' = 1.2 \text{ nC}$$

3. **(b)** Given, $e = 100 \sin 30t$

and $\quad i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

$\therefore$ Average power,

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4} = \frac{1000}{\sqrt{2}} \text{ watt}$$

Wattless current is,

$$I = I_{\text{rms}} \sin\phi$$

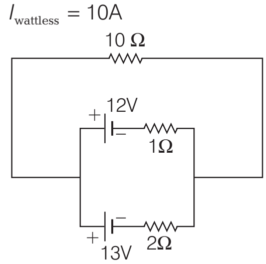
$$= \frac{20}{\sqrt{2}} \times \sin\frac{\pi}{4}$$

$$= \frac{20}{2} = 10\text{A}$$

$$\therefore \quad P_{\text{av}} = \frac{1000}{\sqrt{2}} \text{ watt}$$

and $\quad I_{\text{wattless}} = 10\text{A}$

4. **(b)**



For parallel combination of cells,

$$E_{\text{eq}} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

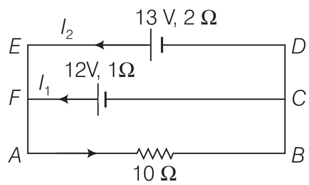
$$\therefore \quad E_{\text{eq}} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3} \text{ V}$$

Potential drop across 10 Ω resistance,

$$V = \left(\frac{E}{R_{\text{total}}}\right) \times 10 = \frac{\frac{37}{3}}{\left(10 + \frac{2}{3}\right)} \times 10 = 11.56\text{V}$$

$$\therefore \quad V = 11.56 \text{ V}$$

Alternative Method



Applying KVL, in loop $ABCFA$,

$$-12 + 10(I_1 + I_2) + 1 \times I_1 = 0$$

$$\Rightarrow \qquad 12 = 11I_1 + 10I_2 \qquad \ldots(\text{i})$$

Similarly,

in loop $ABDEA$,

$$-13 + 10(I_1 + I_2) + 2 \times I_2 = 0$$

$$\Rightarrow \qquad 13 = 10I_1 + 12I_2 \qquad \ldots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$I_1 = \frac{7}{16}\text{A}, \; I_2 = \frac{23}{32}\text{A}$$

$\therefore$ Voltage drop across 10 Ω resistance is,

$$V = 10\left(\frac{7}{16} + \frac{23}{32}\right) = 11.56\,\text{V}$$

5. **(b)** From $Bqv = \frac{mv^2}{r}$, we have

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

where, K is the kinetic energy.

As, kinetic energies of particles are same;

$$r \propto \frac{\sqrt{m}}{q} \quad \Rightarrow r_e : r_p : r_\alpha = \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{4m_p}}{2e}$$

Clearly, $r_p = r_\alpha$ and r_e is least $\qquad [\because m_e < m_p]$

So, $r_p = r_\alpha > r_e$

6. **(c)** As $m = IA$, so to change dipole moment (current is kept constant), we have to change radius of loop.

Initially, $\quad m = I\pi R^2$ and $B_1 = \frac{\mu_0 I}{2R_1}$

Finally, $\quad m' = 2m = I\pi R_2^2$

$\Rightarrow \quad 2I\pi R_1^2 = I\pi R_2^2$

or $\quad R_2 = \sqrt{2}R_1$

So, $\quad B_2 = \frac{\mu_0 I}{2(R_2)} = \frac{\mu_0 I}{2\sqrt{2}R_1}$

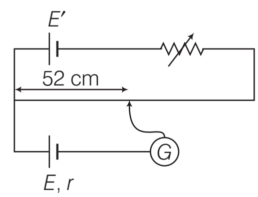
Hence, $\quad$ ratio $\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{2R_1}\right)}{\left(\frac{\mu_0 I}{2\sqrt{2}R_1}\right)} = \sqrt{2}$

$\therefore$ Ratio $\frac{B_1}{B_2} = \sqrt{2}$

7. **(a)** Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the selectivity of circuit. This ratio is also called ''Quality Factor'' or Q-factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

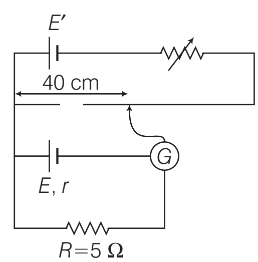
8. **(b)** With only the cell,



On balancing, $\quad E = 52 \times x \qquad \ldots(\text{i})$

where, x is the potential gradient of the wire.

When the cell is shunted,



Similarly, on balancing,

$$V = E - \frac{Er}{(R + r)} = 40 \times x \qquad \ldots(\text{ii})$$

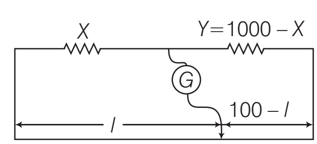
Solving Eqs. (i) and (ii), we get

$$\frac{E}{V} = \frac{1}{1 - \frac{r}{R + r}} = \frac{52}{40}$$

$$\Rightarrow \quad \frac{E}{V} = \frac{R + r}{R} = \frac{52}{40} \quad \Rightarrow \quad \frac{5 + r}{5} = \frac{52}{40}$$

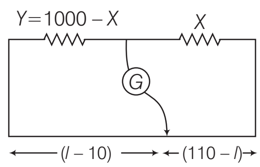
$$\Rightarrow \quad r = \frac{3}{2}\,\Omega \quad \Rightarrow \quad r = 1.5\,\Omega$$

9. **(c)** We have, $\quad X + Y = 1000\,\Omega$



Initially, $\qquad \frac{X}{l} = \frac{1000 - X}{100 - l} \qquad \ldots(\text{i})$

When X and Y are interchanged, then



$$\frac{1000-X}{l-10}=\frac{X}{100-(l-10)}$$

or $$\frac{1000-X}{l-10}=\frac{X}{110-l} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{100-l}{l}=\frac{l-10}{110-l}$$

$$(100-l)(110-l)=(l-10)l$$

$$11000-100l-110l+l^2=l^2-10l$$

$\Rightarrow \quad 11000=200l$

$\therefore \quad l=55\text{ cm}$

Substituting the value of l in Eq. (i), we get

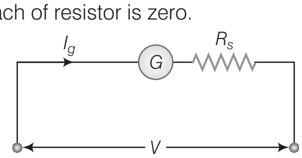
$$\frac{X}{55}=\frac{1000-55}{100-55}$$

$\Rightarrow \quad 20X=11000$

$\therefore \quad X=550\ \Omega$

10. **(c)** A potential drop across each resistor is zero, so the current through each of resistor is zero.

11. **(d)** For a voltmeter,



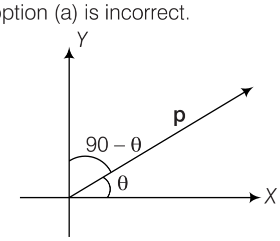
$I_g(G+R_s)=V$

$\Rightarrow \quad R=\frac{V}{I_g}-G$

$\Rightarrow R=1985=1.985\text{ k}\Omega$ or $R=1.985\times10^3\ \Omega$

12. **(a)** In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

13. **(b)** Torque applied on a dipole $\tau=pE\sin\theta$



where θ = angle between axis of dipole and electric field.

For electric field $E_1=E\hat{\mathbf{i}}$ it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1=pE_1\sin\theta$$

In electric field $E_2=\sqrt{3}\,E\hat{\mathbf{j}}$, it means field is directed along positive Y-axis, so angle between dipole and field will be $90-\theta$

Torque in this direction $T_2=pE\sin(90-\theta)$.

$$=p\sqrt{3}\,E_1\cos\theta$$

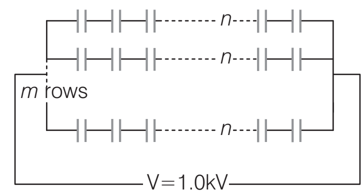
According to question $\tau_2=-\tau_1 \Rightarrow |\tau_2|=|\tau_1|$

$\therefore \quad pE_1\sin\theta=p\sqrt{3}\,E_1\cos\theta$

$\tan\theta=\sqrt{3}\Rightarrow\tan\theta=\tan60°$

$\therefore \quad \theta=60°$

14. **(c)** Let there are n capacitors in a row with m such rows in parallel.



As voltage not to exceed 300 V

$\therefore \quad n\times300>1000$

[a voltage greater than 1 kV to be withstand]

$\Rightarrow \quad n>\frac{10}{3}\Rightarrow n=4$ (or 3.33)

Also, $\quad C_{Eq}=\frac{mC}{n}=2\mu\text{F}$

$\Rightarrow \quad \frac{m}{n}=2\Rightarrow m=8 \quad [\because C=1\mu\text{F}]$

So, total number of capacitors required

$$=m\times n=8\times4=32$$

15. **(b)** In steady state no current flows through the capacitor.

So, the current in circuit $I=\frac{E}{r+r_2}$

$\because$ Potential drop across capacitor = Potential drop across r_2 $=Ir_2=\frac{Er_2}{r+r_2}$

$\therefore$ Stored charge of capacitor, $Q=CV=\frac{CEr_2}{r+r_2}$

16. **(b)** Induced constant, $I=\frac{e}{R}$

Here, e = induced emf $=\frac{d\phi}{dt}$

$$I=\frac{1}{R}=\left(\frac{d\phi}{dt}\right)\cdot\frac{1}{R}$$

$\Rightarrow \quad d\phi=IRdt$

$\Rightarrow \quad \phi=\int IRdt$

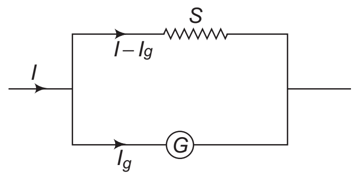
$\therefore$ Here, R is constant $\Rightarrow \phi=R\int Idt$

$\int I\cdot dt$ = Area under $I-t$ graph

$$=\frac{1}{2}\times10\times0.5=2.5$$

$\therefore \quad \phi=R\times2.5=100\times2.5=250\text{ Wb.}$

17. **(a)**



In parallel, current distributes in inverse ratio of resistance. Hence,

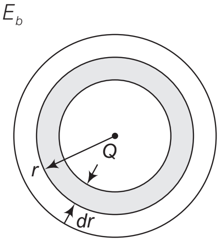
$$\frac{I - I_g}{I_g} = \frac{G}{S} \quad \Rightarrow \quad S = \frac{GI_g}{I - I_g}$$

As I_g is very small, hence

$$S = \frac{GI_g}{I} \quad \Rightarrow b = \frac{(100)\,(1 \times 10^{-3})}{10} = 0.01\ \Omega$$

18. **(a)** As E is constant,

Hence, $E_a = E_b$



As per Guass theorem, only Q_{in} contributes in electric field.

$$\therefore \qquad \frac{kQ}{a^2} = \frac{k\left[Q + \int_a^b 4\pi r^2 dr \cdot \dfrac{A}{r}\right]}{b^2}$$

Here, $$k = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow \quad Q\frac{b^2}{a^2} = Q + 4\pi A\left[\frac{r^2}{2}\Bigg|_a^b\right] = Q + 4\pi A \cdot \left(\frac{b^2 - a^2}{2}\right)$$

$$\Rightarrow \quad Q\left(\frac{b^2}{a^2}\right) = Q + 2\pi A\,(b^2 - a^2)$$

$$\Rightarrow \quad Q\left(\frac{b^2 - a^2}{a^2}\right) = 2\pi A\,(b^2 - a^2) \;\Rightarrow\; A = \frac{Q}{2\pi a^2}$$

19. **(c)** $3\,\mu\text{F}$ and $9\mu\text{F} = 12\mu\text{F}$

$$4\mu\text{F and } 12\mu\text{F} = \frac{4 \times 12}{4 + 12} = 3\mu\text{F}$$

$$Q = CV = 3 \times 8 = 24\mu\text{C (on } 4\mu\text{F and } 3\mu\text{F)}$$

Now, this $24\mu\text{C}$ distributes in direct ratio of capacity between

$3\mu\text{F}$ and $9\mu\text{F}$. Therefore,

$$Q_{9\,\mu\text{F}} = 18\mu\text{C}$$

$$\therefore Q_{4\mu\text{F}} + Q_{9\,\mu\text{F}} = 24 + 18 = 42\mu\text{C} = Q \quad \text{(say)}$$

$$E = \frac{kQ}{R^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30^2} = 420\ \text{N/C}$$

20. **(d)** B at centre of a circle $= \dfrac{\mu_0 I}{2R}$

B at centre of a square

$$= 4 \times \frac{\mu I}{4\pi \cdot \dfrac{l}{2}}[\sin 45° + \sin 45°] = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi l}$$

Now, $\quad R = \dfrac{L}{2\pi}$ and $l = \dfrac{L}{4} \qquad$ (as $L = 2\pi R = 4l$)

where, $\quad L$ = length of wire.

$$\therefore \qquad B_A = \frac{\mu_0 I}{2 \cdot \dfrac{L}{2\pi}} = \frac{\pi\mu_0 I}{L} = \pi\left[\frac{\mu_0 I}{L}\right]$$

$$B_B = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi\left(\dfrac{L}{4}\right)} = \frac{8\sqrt{2}\mu_0 I}{\pi L} = \frac{8\sqrt{2}}{\pi}\left[\frac{\mu_0 I}{L}\right]$$

$$\therefore \qquad \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

21. **(d)** We need high retentivity and high coercivity for electromagnets and small area of hysteresis loop for transformers.

22. **(d)** $V^2 = V_R^2 + V_L^2 \Rightarrow 220^2 = 80^2 + V_L^2$

Solving, we get

$$V_L \approx 205\ \text{V}$$

$$X_L = \frac{V_L}{I} = \frac{205}{10} = 20.5\ \Omega = \omega L$$

$$\therefore \qquad L = \frac{20.5}{2\pi \times 50} = 0.065\ \text{H}$$

23. **(a)** Theoretical question. Therefore, no solution is required.

24. **(c)** $i = neAv_d$ or $\dfrac{V}{R} = neAv_d \Rightarrow \dfrac{V}{\left(\dfrac{\rho l}{A}\right)} = neAv_d$

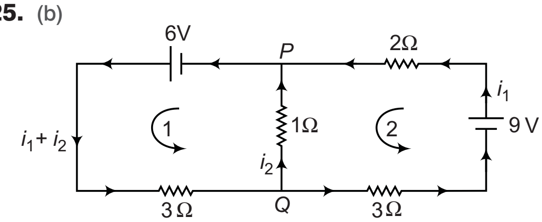
$$\therefore \qquad \rho = \frac{V}{nelv_d} = \text{resistivity of wire}$$

Substituting the given values we have

$$\rho = \frac{5}{(8 \times 10^{28})(1.6 \times 10^{-19})(0.1)(2.5 \times 10^{-4})}$$

$$\approx 1.6 \times 10^{-5}\ \Omega\text{-m}$$

25. **(b)**



Applying Kirchhoff's loop law in loops 1 and 2 in the directions shown in figure we have

$$6 - 3(i_1 + i_2) - i_2 = 0 \quad \text{...(i)}$$
$$9 - 2i_1 + i_2 - 3i_1 = 0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii) we get,

$$i_2 = 0.13\text{ A}$$

Hence, the current in 1 Ω resister is 0.13 A from Q to P.

26. **(b, c)** V_0 = potential on the surface $= \frac{Kq}{R}$

where, $K = \frac{1}{4\pi\varepsilon_0}$ and q is total charge on sphere.

Potential at centre $= \frac{3}{2}\frac{Kq}{R} = \frac{3}{2}V_0$

Hence, $R_1 = 0$

From centre to surface potential varies between $\frac{3}{2}V_0$ and V_0 From surface to infinity, it varies between V_0 and 0, $\frac{5V_0}{4}$ will be potential at a point between centre and surface. At any point, at a distance $r (r \le R)$ from centre potential is given by

$$V = \frac{Kq}{R^3}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right)$$
$$= \frac{V_0}{R^2}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right) \quad \left(\text{as } V_0 = \frac{Kq}{R}\right)$$

Putting $V = \frac{5}{4}V_0$ and $r = R_2$ in this equation, we get

$$R_2 = \frac{R}{\sqrt{2}}$$

$\frac{3V_0}{4}$ and $\frac{V_0}{4}$ are the potentials lying between V_0 and zero hence these potentials lie outside the sphere. At a distance $r (\ge R)$ from centre potential is given by

$$V = \frac{Kq}{r} = \frac{V_0 R}{r}$$

Putting $V = \frac{3}{4}V_0$ and $r = R_3$ in this equation we get,

$R_3 = \frac{4}{3}R$

Further putting $V = \frac{V_0}{4}$ and $r = R_4$ in above equation,

we get $\qquad R_4 = 4R$

Thus, $R_1 = 0$, $R_2 = \frac{R}{\sqrt{2}}$, $R_3 = \frac{4R}{3}$ and $R_4 = 4R$ with these values, option (b) and (c) are correct.

27. **(d)** Electric field lines originate from position charge and termination negative charge. They cannot form closed loops and they are smooth curves. Hence the most appropriate answer is (d).

28. **(a)** Resultant of 1 μF and 2 μF is 3 μF. Now in series, potential difference distributes in inverse ratio of capacity.

$$\therefore \quad \frac{V_{3\mu\text{F}}}{V_c} = \frac{C}{3} \quad \text{or} \quad V_{3\mu\text{F}} = \left(\frac{C}{C+3}\right)E$$

This is also the potential difference across 2 μF.

$$\therefore \quad Q_2 = (2\ \mu\text{F})(V_{2\mu\text{F}})$$

$$\text{or} \quad Q_2 = \left(\frac{2CE}{C+3}\right) = \left(\frac{2}{1+\frac{3}{C}}\right)E$$

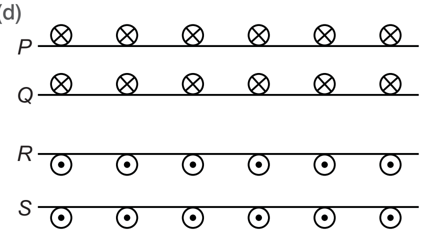
From this expression of Q_2, we can see that Q_2 will increase with increase in the value of c (but not linearly). Therefore, only options (a) and (b) may be correct.

Further, $\frac{d}{dc}(Q_2) = 2E\left[\frac{(C+3) - C}{(C+3)^2}\right] = \frac{6E}{(C+3)^2}$

= Slope of Q_2 *versus* C graph.

i.e. slope of Q_2 *versus* C graph decreases with increase in the value of C. Hence, the correct graph is (a).

29. **(d)**

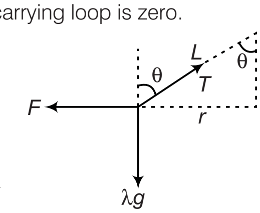


If we calculate the force on inner solenoid. Force on Q due to P is outwards (attraction between currents in same direction. Similarly, force on R due to S is also outwards. Hence, net force $\mathbf{F}_1$ is zero)

Force on P due to Q and force on S due to R is inwards. Hence, net force $\mathbf{F}_2$ is also zero.

Alternate Thought Field of one solenoid is uniform and other solenoid may be assumed a combination of circular closed loops. In uniform magnetic field, net force on a closed current carrying loop is zero.

30. **(a)**



$$r = L\sin\theta$$

F = Magnetic force (repulsion) per unit length

$$= \frac{\mu_0}{2\pi}\frac{I^2}{2r} = \frac{\mu_0}{4\pi}\frac{I^2}{L\sin\theta}$$

λg = weight per unit length

Each wire is in equilibrium under three concurrent forces as shown in figure. Therefore, applying Lami's theorem.

$$\frac{F}{\sin(180-\theta)} = \frac{\lambda g}{\sin(90+\theta)} \quad \text{or} \quad \frac{\frac{\mu_0}{4\pi}\frac{I^2}{L\sin\theta}}{\sin\theta} = \frac{\lambda g}{\cos\theta}$$

$$\therefore \quad I = 2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$$

31. **(b)** Direction of magnetic dipole moment $\mathbf{M}$ is given by screw law and this is perpendicular to plane of loop.

In stable equilibrium position, angle between **M** and **B** is 0° and in unstable equilibrium this angle is 180°.

32. (c) Steady state current i_0 was already flowing in the L-R circuit when K_1 was closed for a long time. Here,

$$i_0 = \frac{V}{R} = \frac{15\text{ V}}{150\ \Omega} = 0.1\text{ A}$$

Now, K_1 is opened and K_2 is closed. Therefore, this i_0 will decrease exponentially in the L-R circuit. Current i at time t will be given by $i = i_0 e^{\frac{-t}{\tau_L}}$

where, $\tau_L = \frac{L}{R} \Rightarrow \therefore \quad i = i_0 e^{\frac{-Rt}{L}}$

Substituting the values, we have

$$i = (0.1)\, e^{\frac{-(0.15\times10^3)(10^{-3})}{(0.03)}} = (0.1)(e^{-5})$$

$$= \frac{0.1}{150} = 6.67 \times 10^{-4}\text{ A}$$

$$= 0.67\text{ mA}$$

33. (c) Total power (P) consumed

$$= (15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000)$$

$$= 2500\text{ W}$$

As we know,

Power i.e. $P = VI \Rightarrow I = \frac{2500}{220}\text{ A} \quad = \frac{125}{11} = 11.3\text{ A}$

Minimum capacity should be 12 A.

34. (c) As we know, potential difference $V_A - V_O$ is

$$dV = -\,Edx$$

$$\Rightarrow \int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx \Rightarrow V_A - V_O = -\,30 \times \left[\frac{x^3}{3}\right]_0^2$$

$$= -\,10 \times [2^3 - (0)^3] = -\,10 \times 8 = -\,80\ \text{ J}$$

35. (a) When free space between parallel plates of capacitor, $E = \frac{\sigma}{\varepsilon_0}$

When dielectric is introduced between parallel plates of capacitor, $E' = \frac{\sigma}{K\varepsilon_0}$

Electric field inside dielectric, $\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

where, K = dielectric constant of medium = 2.2

ε_0 = permitivity of free space = 8.85×10^{-12}

$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

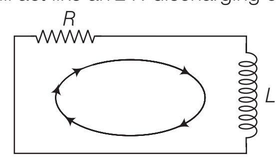
$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7} = 6 \times 10^{-7}\text{ C/m}^2$

36. For solenoid, the magnetic field needed to be magnetised the magnet. $\quad B = \mu_0 nI$

where, $n = 100$, $l = 10\text{cm} = \frac{10}{100}\text{m} = 0.1\text{ m}$

$$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I \Rightarrow I = 3\text{ A}$$

37. (c) After connecting C to B hanging the switch, the circuit will act like an L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \quad \Rightarrow \quad V_R = -\,V_L \quad \Rightarrow \quad \frac{V_R}{V_L} = -\,1$$

38. (b) When force exerted on a current carrying conductor $F_{\text{ext}} = BIL$

Average power = $\frac{\text{Work done}}{\text{Time taken}}$

$$P = \frac{1}{t}\int_0^2 F_{\text{ext.}}\,.\,dx = \frac{1}{t}\int_0^2 B(x)IL\,dx$$

$$= \frac{1}{5 \times 10^{-3}}\int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3\,dx$$

$$= 9\,[1 - e^{-0.4}] = 9\left[1 - \frac{1}{e^{0.4}}\right] = 2.967 \approx 2.97\text{ W}$$

39. (d) Statement I is false and Statement II is true.

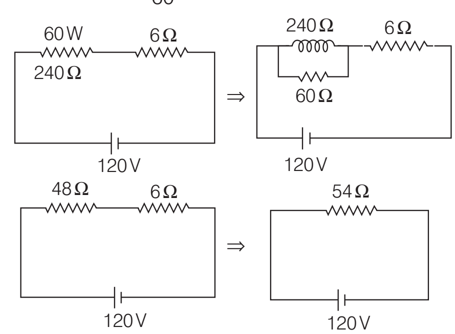
40. (d) As, $P = \frac{V^2}{R}$

where, P = power dissipates in the circuit,

V = applied voltage,

R = net resistance of the circuit

$$R = \frac{120 \times 120}{60} = 240\ \Omega \quad \text{[resistance of bulb]}$$



$$R_{eq} = 240 + 6 = 246\ \Omega \quad \Rightarrow \quad i_1 = \frac{V}{R_{eq}} = \frac{120}{246}$$

[before connecting heater]

$$R = \frac{V^2}{R} = \frac{120 \times 120}{240}$$

$\Rightarrow \quad R = 60\ \Omega$ [resistance of heater]

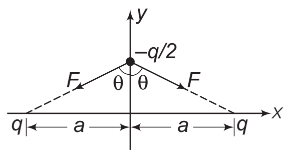
So, from figure,

$$V_1 = \frac{240}{246} \times 120 = 117.073\text{ V} \qquad [\because V = IR]$$

$\Rightarrow \quad i_2 = \dfrac{V}{R_2} = \dfrac{120}{54} \quad \Rightarrow V_2 = \dfrac{48}{54} \times 120 = 106.66 \text{ V}$

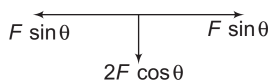
$V_1 - V_2 = 10.04 \text{ V}$

41. (a) $F_{\text{net}} = 2F\cos\theta$



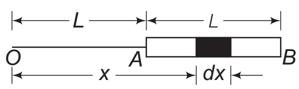
$$F_{\text{net}} = \frac{2kq\,(q/2)}{(\sqrt{y^2+a^2})^2} \cdot \frac{y}{\sqrt{y^2+a^2}}$$

$$F_{\text{net}} = \frac{2kq\,(q/2)\,y}{(y^2+a^2)^{3/2}}$$



$$\Rightarrow \frac{kq^2 y}{a^3} \propto y$$

42. (d)



$$V = \int_L^{2L} \frac{kdQ}{x} = \int_L^{2L} \frac{k\left(\frac{Q}{L}\right)dx}{x} = \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L} = \frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L]$$

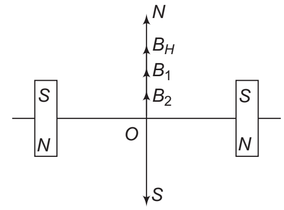
$$= \frac{Q}{4\pi\varepsilon_0 L}\ln(2)$$

43. (b, c) Polarity should be mentioned in the question.
Potential on each of them can be zero if, $q_{\text{net}} = 0$

or $\quad q_1 \pm q_2 = 0$

or $\quad 120C_1 \pm 200C_2 = 0$ or $3C_1 \pm 5C_2 = 0$

44. (b) $B_{\text{net}} = B_1 + B_2 + B_H$



$$B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{(M_1+M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}\,(1.2+1)}{(0.1)^3} + 3.6\times10^{-5} = 2.56\times10^{-4}\ \text{Wb/m}^2$$

45. (a) Magnetic field at the centre of smaller loop

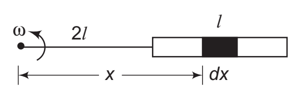
$$B = \frac{\mu_0 i R_2^2}{2(R_2^2 + x^2)^{3/2}}$$

Area of smaller loop $S = \pi R_1^2$

$\therefore$ Flux through smaller loop $\phi = BS$

Substituting the values, we get, $\phi \approx 9.1\times10^{-11}$ Wb

46. (d) $e = \displaystyle\int_{2l}^{3l}(\omega x)\,B dx = B\omega\frac{[(3l)^2 - (2l)^2]}{2} = \frac{5B\omega l^2}{2}$



47. (c) For charging of capacitor $q = CV(1 - e^{t/\tau})$

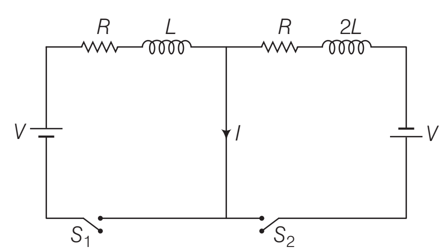
At $\ t = 2\tau;\ \ q = CV(1 - e^{-2})$

48. (c) Amplitude decreases exponentially. In 5 s, it remains 0.9 times. Therefore, in total 15 s it will remains (0.9) (0.9) (0.9) = 0.729 times its original value.

JEE Advanced

1. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude $I_{\max}$ at time $t = \tau$. Which of the following statements is (are) true?

(More than One Correct Option, 2018)



(a) $I_{\max} = \dfrac{V}{2R}$ (b) $I_{\max} = \dfrac{V}{4R}$

(c) $\tau = \dfrac{L}{R}\ln 2$ (d) $\tau = \dfrac{2L}{R}\ln 2$

2. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive, if it is in the $+\hat{\mathbf{j}}$-direction. Which of the following statements regarding the magnetic field $\mathbf{B}$ is (are) true?

(More than One Correct Option, 2018)

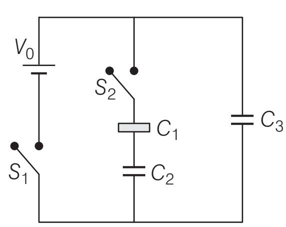
(a) If $I_1 = I_2$, then $\mathbf{B}$ cannot be equal to zero at the origin (0,0,0)

(b) If $I_1 > 0$ and $I_2 < 0$, then $\mathbf{B}$ can be equal to zero at the origin (0,0,0)

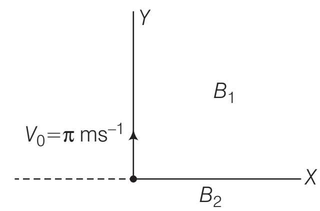
(c) If $I_1 < 0$ and $I_2 > 0$, then $\mathbf{B}$ can be equal to zero at the origin (0,0,0)

(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

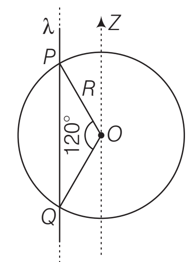
3. Three identical capacitors C_1, C_2 and C_3 have a capacitance of $1.0\mu F$ each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be $5\mu C$. The value of ε_r =........... **(Numerical Value, 2018)**



4. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{\mathbf{k}}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{\mathbf{k}}$. A positively charged particle is projected from the origin along the positive Y-axis with speed $v_0 = \pi \text{ ms}^{-1}$ at $t = 0$, as shown in figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the X-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the X-axis in the time interval T is........... . **(Numerical Value, 2018)**



5. An infinitely long thin non-conducting wire is parallel to the Z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **(More than One Correct Option, 2018)**



(a) The electric flux through the shell is $\sqrt{3}\, R\lambda / \varepsilon_0$.

(b) The z-component of the electric field is zero. at all the points on the surface of the shell.

(c) The electric flux through the shell is $\sqrt{2}\, R\lambda / \varepsilon_0$.

(d) The electric field is normal to the surface of the shell at all points.

6. A particle of mass 10^{-3}kg and charge 1.0 C is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\mathbf{E}(t) = E_0 \sin \omega t\, \hat{\mathbf{i}}$, where $E_0 = 1.0 \text{ NC}^{-1}$ and $\omega = 10^3 \text{ rad s}^{-1}$. Consider the effect of only the electrical force on the particle. Then, the maximum speed in m s^{-1}, attained by the particle at subsequent times is ....

(Numerical Value, 2018)

7. A moving coil galvanometer has 50 turns and each turn has an area $2 \times 10^{-4} \text{m}^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is $10^{-4} \text{N - m rad}^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs, if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance in ohms, is ....

(Numerical Value, 2018)

8. The electric field E is measured at a point $P\,(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

(Matching Type, 2018)

	List-I		List-II
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at (0, 0, l) and – Q at (0, 0, – 1). (Take, $2l << d$)
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the X-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying a uniform linear charge density parallel to the X- axis. The one along (y = 0, z = l) has a charge density + λ and the one along (y = 0, z = – l) has a charge density – λ. (Take, $2l << d$).
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(a) P → 5; Q → 3,4; R → 1; S → 2
(b) P → 5; Q → 3; R → 1, 4; S → 2
(c) P → 5; Q → 3; R → 1, 2; S → 4
(d) P → 4; Q → 2, 3; R → 1; S → 5

Passage (Q. Nos. 9-10)

Consider a simple RC circuit as shown in Figure 1.

Process 1 In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T >> RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_c.

Process 2 In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then, the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1. These two processes are depicted in Figure 2.

(Passage Type, 2017)

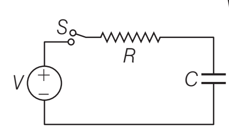


Figure 1

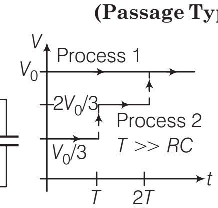


Figure 2

9. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

(a) $E_C = E_D \ln 2$
(b) $E_C = E_D$
(c) $E_C = 2E_D$
(d) $E_C = \frac{1}{2} E_D$

10. In Process 2, total energy dissipated across the resistance E_D is

(a) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$
(b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$
(c) $E_D = 3CV_0^2$
(d) $E_D = \frac{1}{2}CV_0^2$

Directions (Q.Nos. 11 to 13) Matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x=0$, $y=0$, $z=0$) with a given initial velocity **v**. A uniform electric field **E** and a uniform magnetic field **B** exist everywhere. The velocity **v**, electric field **E** and magnetic field **B** are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

(Matching Type, 2017)

Column 1	Column 2	Column 3
(I) Electron with $\mathbf{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\mathbf{E} = E_0\hat{z}$	(P) $\mathbf{B} = -B_0\hat{x}$
(II) Election with $\mathbf{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\mathbf{E} = -E_0\hat{y}$	(Q) $\mathbf{B} = B_0\hat{x}$
(III) Proton with $\mathbf{v} = 0$	(iii) $\mathbf{E} = -E_0\hat{x}$	(R) $\mathbf{B} = B_0\hat{y}$
(IV) Proton with $\mathbf{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\mathbf{E} = E_0\hat{x}$	(S) $\mathbf{B} = B_0\hat{z}$

11. In which case would the particle move in a straight line along the negative direction of Y-axis?

(a) (IV) (ii) (S)
(b) (II) (iii) (Q)
(c) (III), (ii) (R)
(d) (III) (ii) (P)

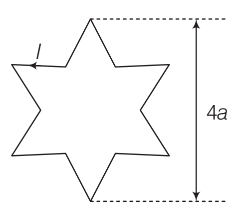
12. In which case will the particle move in a straight line with constant velocity?

(a) (II) (iii) (S)
(b) (III) (iii) (P)
(c) (IV) (i) (S)
(d) (III) (ii) (R)

13. In which case will the particle describe a helical path with axis along the positive z-direction?

(a) (II) (ii) (R)
(b) (III) (iii) (P)
(c) (IV) (i) (S)
(d) (IV) (ii) (R)

14. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the center of the loop is

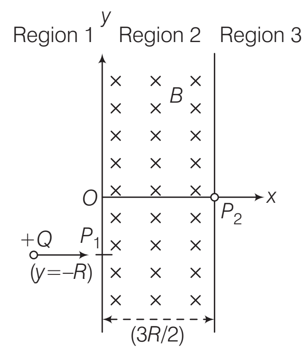


(Single Correct Option, 2017)

(a) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$
(b) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$
(c) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$
(d) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

15. A uniform magnetic field B exists in the region between $x=0$ and $x=\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along X-axis enters region 2 from region 1 at point P_1 ($y=-R$).

Which of the following option(s) is/are correct? **(More than One Correct Option, 2017)**



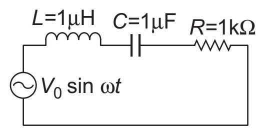
(a) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from Y-axis is $\frac{p}{\sqrt{2}}$

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on X-axis

(c) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

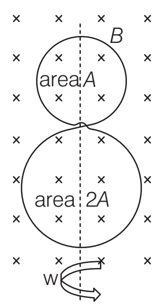
(d) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

16. In the circuit shown, $L = 1\,\mu\text{H}$, $C = 1\,\mu\text{F}$ and $R = 1\,\text{k}\Omega$. They are connected in series with an AC source $V = V_0 \sin\omega t$ as shown. Which of the following options is/are correct? **(More than One Correct Option, 2017)**



(a) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero
(b) The frequency at which the current will be in phase with the voltage is independent of R
(c) The current will be in phase with the voltage if $\omega = 10^4\ \text{rads}^{-1}$
(d) At $\omega >> 10^6\ \text{rads}^{-1}$, the circuit behaves like a capacitor

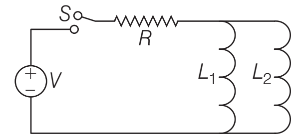
17. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\mathbf{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(More than One Correct Option, 2017)

(a) the emf induced in the loop is proportional to the sum of the areas of the two loops
(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
(c) The net emf induced due to both the loops is proportional to $\cos\omega t$
(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

18. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct? **(More than One Correct Option, 2017)**



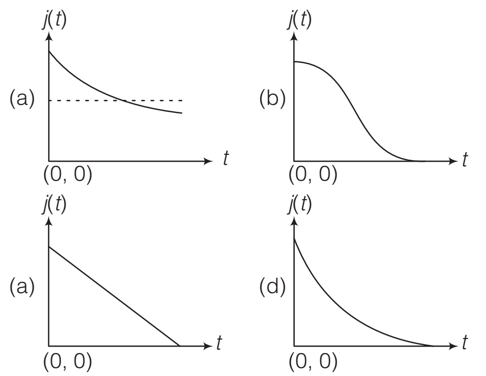
(a) After a long time, the current through L_1 will be $\dfrac{V}{R}\left(\dfrac{L_2}{L_1 + L_2}\right)$
(b) After a long time, the current through L_2 will be $\dfrac{V}{R}\left(\dfrac{L_1}{L_1 + L_2}\right)$
(c) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)
(d) At $t = 0$, the current through the resistance R is $\dfrac{V}{R}$

19. The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin\omega t, V_Y = V_0 \sin\left(\omega t + \dfrac{2\pi}{3}\right)$ and $V_Z = V_0 \sin\left(\omega t + \dfrac{4\pi}{3}\right)$.

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be **(More than One Correct Option, 2017)**

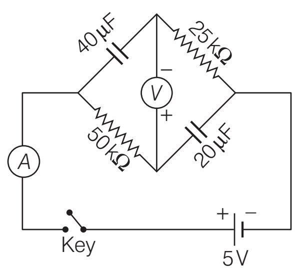
(a) $V_{YZ}^{\text{rms}} = V_0\sqrt{\dfrac{1}{2}}$
(b) $V_{XY}^{\text{rms}} = V_0\sqrt{\dfrac{3}{2}}$
(c) independent of the choice of the two terminals
(d) $V_{XY}^{\text{rms}} = V_0$

20. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material? **(Single Correct Option, 2016)**



21. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true?

(More than One Correct Option, 2016)



(a) The voltmeter display −5 V as soon as the key is pressed and displays +5 V after a long time

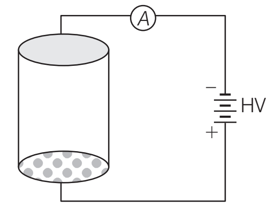
(b) The voltmeter will display 0 V at time $t = \ln 2$ seconds

(c) The current in the ammeter becomes $1/e$ of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time

Passage (Q. Nos. 22-23)

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now, a high voltage source (HV) connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it.



The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to te soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity) **(Passage Type, 2016)**

22. Which one of the following statements is correct?

(a) The balls will execute simple harmonic motion between the two plates

(b) The balls will bounce back to the bottom plate carrying the same charge they went up with

(c) The balls will stick to the top plate and remain there

(d) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

23. The average current in the steady state registered by the ammeter in the circuit will be

(a) proportional to V_0^2

(b) proportional to the potential V_0

(c) zero

(d) proportions to $V_0^{1/2}$

24. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true? **(More than One Correct Option, 2016)**

(a) The temperature distribution over the filament is uniform

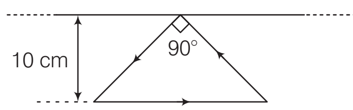
(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of requencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

25. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counter- clockwise direction and increased at a constant rate of $10\,\text{As}^{-1}$. Which of the following statement(s) is (are) true?

(More than One Correct Option, 2016)



(a) There is a repulsive force between the wire and the loop

(b) If the loop is rotated at a constant angular speed about the wire, an additional emf of (μ_0 / π) volt is induced in the wire

(c) The magnitude of induced emf in the wire is $\left(\dfrac{\mu_0}{\pi}\right)$ volt

(d) The induced current in the wire is in opposite direction to the current along the hypotenuse

26. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_c < R/2$, which of the following statement(s) about anyone of the galvanometers is (are) true?

(More than One Correct Option, 2016)

(a) The maximum voltage range is obtained when all the components are connected in series

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(c) The maximum current range is obtained when all the components are connected in parallel

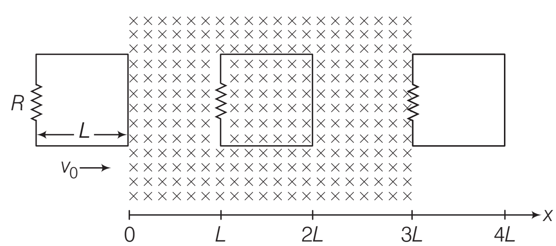
(d) The maximum current range is obtained when the two galvanometers are connected in series, and the combination is connected in parallel with both the resistors

27. Two inductors L_1 (inductance 1mH, internal resistance $3\,\Omega$) and L_2 (inductance 2 mH, internal resistance $4\,\Omega$), and a resistor R (resistance $12\,\Omega$) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is **(Single Integer Type, 2016)**

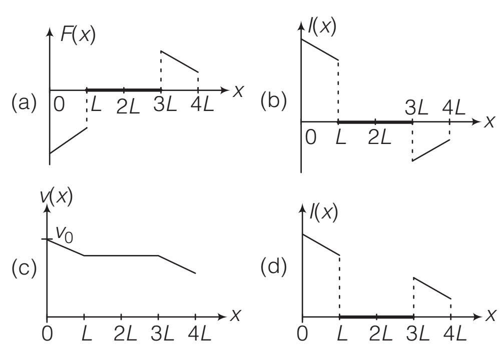
28. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper .

At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.

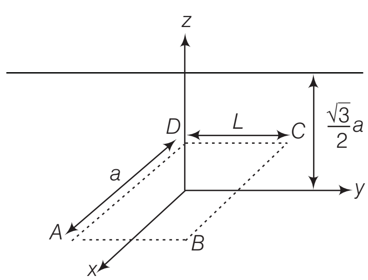
(More than One Correct Option, 2016)



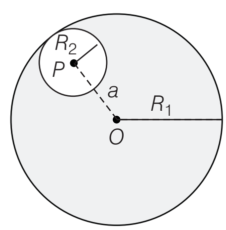
Which of the following schematic plot(s) is (are) correct? (Ignore gravity)



29. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is = 6) **(Single Integer Type, 2015)**



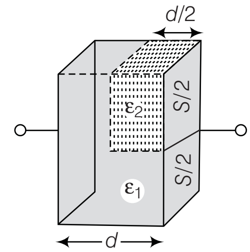
30. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\mathbf{r}$ is $\mathbf{E}(\mathbf{r})$, then the correct statement(s) is/are



(Single Correct Option, 2015)

(a) $\mathbf{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\mathbf{r}$
(b) $\mathbf{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\mathbf{r}$
(c) $\mathbf{E}$ is uniform, its magnitude is independent of 'a' but its direction depends on $\mathbf{a}$
(d) $\mathbf{E}$is uniform and both its magnitude and direction depend on $\mathbf{a}$

31. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is

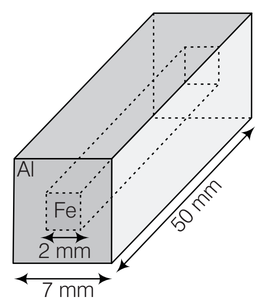


(a) $\frac{6}{5}$ (b) $\frac{5}{3}$
(c) $\frac{7}{5}$ (d) $\frac{7}{3}$

(Single Correct Option, 2015)

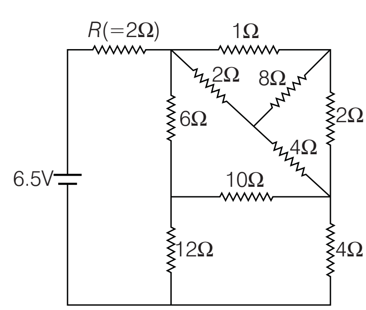
32. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$m and $1.0 \times 10^{-7}\ \Omega$m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

(Single Correct Option, 2015)

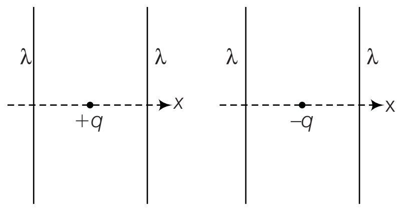


(a) $\frac{2475}{64}\mu\,\Omega$ (b) $\frac{1875}{64}\mu\,\Omega$
(c) $\frac{1875}{49}\mu\,\Omega$ (d) $\frac{2475}{132}\mu\,\Omega$

33. In the following circuit, the current through the resistor $R(= 2\,\Omega)$ is I amperes. The value of I is **(Single Integer Type, 2015)**



34. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other.



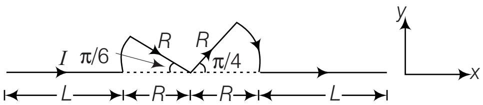
In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements is/are

(Single Correct Option, 2015)

(a) both charges execute simple harmonic motion
(b) both charges will continue moving in the direction of their displacement
(c) charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement
(d) charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement

35. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\mathbf{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are

(More than One Correct Option, 2015)

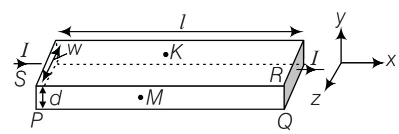


(a) if $\mathbf{B}$ is along $\hat{\mathbf{z}}$, $F \propto (L + R)$
(b) if $\mathbf{B}$ is along $\hat{\mathbf{x}}$, $F = 0$
(c) if $\mathbf{B}$ is along $\hat{\mathbf{y}}$, $F \propto (L + R)$
(d) if $\mathbf{B}$ is along $\hat{\mathbf{z}}$, $F = 0$

Passage (Q. Nos. 36-37)

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field $\mathbf{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction.
This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.

(Passage Type, 2015)



36. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statements is/are

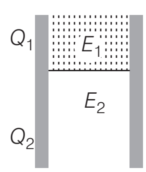
(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

37. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct options is/are

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

38. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.

(More than One Correct Option, 2014)



(a) $\dfrac{E_1}{E_2} = 1$ (b) $\dfrac{E_1}{E_2} = \dfrac{1}{K}$

(c) $\dfrac{Q_1}{Q_2} = \dfrac{3}{K}$ (d) $\dfrac{C}{C_1} = \dfrac{2+K}{K}$

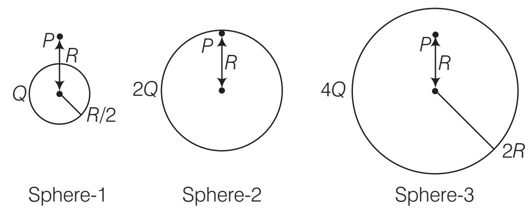
39. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(More than One Correct Option, 2014)

(a) $Q = 4\sigma\pi r_0^2$ (b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(c) $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$ (d) $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

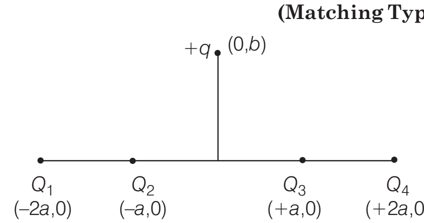
40. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **(Single Correct Option, 2014)**



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$
(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

41. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a, -a, +a$ and $+2a$ respectively. A positive charge q is placed on the positive y-axis at a distance $b > 0$. Four options of the signs of these charges are given in Column I. The direction of the forces on the charge q is given in Column II. Match Column I with Column II and select the correct answer using the code given below the lists.

(Matching Type, 2014)



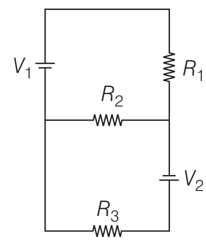
Column I		Column II	
P.	Q_1, Q_2, Q_3, Q_4 all positive	1.	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	2.	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	3.	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	4.	$-y$

Codes

	P	Q	R	S
(a)	3,	1,	4,	2
(b)	4,	2,	3,	1
(c)	3,	1,	2,	4
(d)	4,	2,	1,	3

42. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if

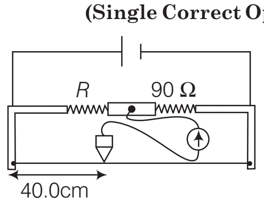
(More than One Correct Option, 2014)



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(b) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(c) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(d) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

43. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is

(Single Correct Option, 2014)



(a) $60 \pm 0.15\ \Omega$ (b) $135 \pm 0.56\ \Omega$
(c) $60 \pm 0.25\ \Omega$ (d) $135 \pm 0.23\ \Omega$

44. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, and value of $\frac{R_1}{R_2}$ is

(Single Integer Type, 2014)

45. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0-30 V. If connected to a $\frac{2n}{249}$ Ω resistance, it becomes an ammeter of range 0-1.5 A. The value of n is **(Single Integer Type, 2014)**

Passage (Q. Nos. 46-47)

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counter-clockwise direction if seen from above. **(Passage Type, 2014)**

46. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(a) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$
(b) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$
(c) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$
(d) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

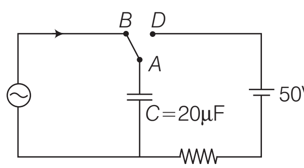
47. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\frac{\mu_0 I^2 a^2}{d}$ (b) $\frac{\mu_0 I^2 a^2}{2d}$
(c) $\frac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$ (d) $\frac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

48. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and

$\omega = 500\ \mathrm{rad\ s^{-1}}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu\mathrm{F}$, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).

(More than One Correct Option, 2014)



(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(c) Immediately after A is connected to D, the current in R is 10 A

(d) $Q = 2\times 10^{-3}$ C

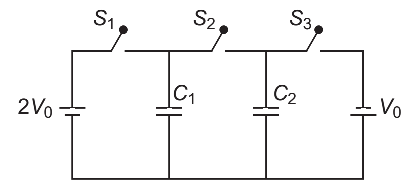
49. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centre of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be **(More than One Correct Option, 2013)**

(a) -4 (b) $-\frac{32}{25}$

(c) $\frac{32}{25}$ (d) 4

50. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time

(More than One Correct Option, 2013)



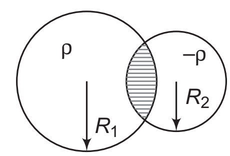
(a) the charge on the upper plate of C_1 is $2CV_0$

(b) the charge on the upper plate of C_1 is CV_0

(c) the charge on the upper plate of C_2 is 0

(d) the charge on the upper plate of C_2 is $-CV_0$

51. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region **(More than One Correct Option, 2013)**



(a) the electrostatic field is zero

(b) the electrostatic potential is constant

(c) the electrostatic field is constant in magnitude

(d) the electrostatic field has same direction

52. A particle of mass M and positive charge Q, moving with a constant velocity $\mathbf{u}_1 = 4\mathbf{i}\,\mathrm{ms^{-1}}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\mathbf{u}_2 = 2(\sqrt{3}\mathbf{i} + \mathbf{j})\ \mathrm{ms^{-1}}$. The correct statement(s) is (are)

(More than One Correct Option, 2013)

(a) the direction of the magnetic field is $-z$ direction.

(b) the direction of the magnetic field is $+z$ direction

(c) the magnitude of the magnetic field is $\frac{50\pi M}{3Q}$ units.

(d) the magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

53. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)

(More than One Correct Option, 2013)

(a) In the region $0 < r < R$, the magnetic field is non-zero

(b) In the region $R < r < 2R$, the magnetic field is along the common axis

(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis

(d) In the region $r > 2R$, the magnetic field is non-zero

Passage (Q. Nos. 54-55)

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The applications of the magnetic field induces an emf in the orbit.

The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

(Passage Type, 2013)

54. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(a) γBQR^2 (b) $-\gamma \frac{BQR^2}{2}$

(c) $\gamma \frac{BQR^2}{2}$ (d) γBQR^2

55. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\frac{BR}{4}$ (b) $\frac{-BR}{2}$ (c) BR (d) $2BR$

Passage (Q. Nos. 56-57)

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the current and voltages mentioned are rms values.

(Passage Type, 2013)

56. If the direct transmission method with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is

(a) 20 (b) 30 (c) 40 (d) 50

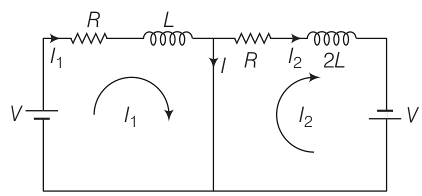
57. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1

(c) 100 : 1 (d) 50 : 1

Answer *with* Explanations

1. **(b, d)** $I_1 = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$



$$I_2 = \frac{V}{R}\left(1 - e^{-\frac{tR}{2L}}\right)$$

From principle of superposition,

$$I = I_1 - I_2$$

$$\Rightarrow \quad I = \frac{V}{R} e^{-\frac{tR}{2L}}\left(1 - e^{-\frac{tR}{2L}}\right) \qquad \ldots\text{(i)}$$

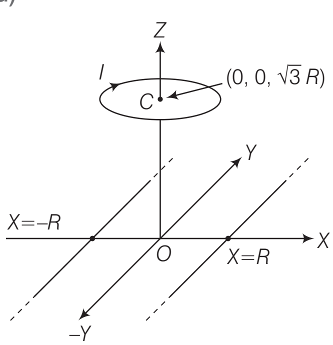
I is maximum when $\frac{dI}{dt} = 0$, which gives $e^{-\frac{tR}{2L}} = \frac{1}{2}$ or

$t = \frac{2L}{R} \ln 2$

Substituting this time in Eq. (i), we get

$$I_{max} = \frac{V}{4R}$$

2. **(a,b,d)**

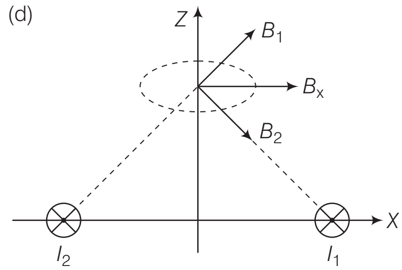


(a) At origin, $\mathbf{B} = 0$ due to two wires if $I_1 = I_2$, hence $(\mathbf{B}_{net})$ at origin is equal to $\mathbf{B}$ due to ring. which is non-zero.

(b) If $I_1 > 0$ and $I_2 < 0$, $\mathbf{B}$ at origin due to wires will be along $+\hat{\mathbf{k}}$. Direction of $\mathbf{B}$ due to ring is along $-\hat{\mathbf{k}}$ direction and hence $\mathbf{B}$ can be zero at origin.

(c) If $I_1 < 0$ and $I_2 > 0$, $\mathbf{B}$ at origin due to wires is along $-\hat{\mathbf{k}}$ and also along $-\hat{\mathbf{k}}$ due to ring, hence $\mathbf{B}$ cannot be zero.

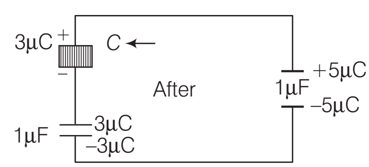
(d)



At centre of ring, $\mathbf{B}$ due to wires is along X-axis.

Hence Z-component is only because of ring which $\mathbf{B} = \frac{\mu_0 i}{2R}(-\hat{\mathbf{k}})$.

3. **(1.50)**

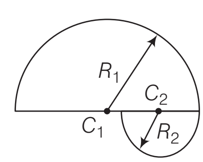


Applying loop rule,

$$\frac{5}{1} - \frac{3}{\varepsilon_r} - \frac{3}{1} = 0 \quad \Rightarrow \quad \frac{3}{\varepsilon_r} = 2 \quad \Rightarrow \varepsilon_r = 1.50$$

4. (2 m/s) If average speed is considered along X-axis,

$$R_1 = \frac{mv_0}{qB_1}, R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1} \quad \Rightarrow \quad R_1 > R_2$$



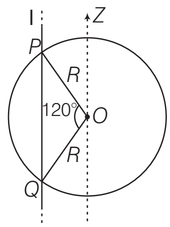
Distance travelled along x-axis, $\Delta x = 2(R_1 + R_2) = \frac{5mv_0}{2qB_1}$

$$\text{Total time} = \frac{T_1}{2} + \frac{T_2}{2} = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}$$

$$= \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$$

$$\text{Magnitude of average speed} = \frac{\frac{5mv_0}{2qB_1}}{\frac{5\pi m}{4qB_1}} = 2 \text{ m/s}$$

5. **(a, b)** $PQ = (2)\,R \sin 60°$



$$= (2R)\frac{\sqrt{3}}{2} = (\sqrt{3}R)$$

$$q_{\text{enclosed}} = \lambda\,(\sqrt{3}R)$$

We have, $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0} \Rightarrow \phi = \left(\dfrac{\sqrt{3}\,\lambda R}{\varepsilon_0}\right)$

Also, electric field is perpendicular to wire, so Z-component will be zero.

6. **(2)** $a = \dfrac{F}{m} = \dfrac{qE}{m} = 10^3 \sin(10^3 t)$

$$\frac{dv}{dt} = 10^3 \sin(10^3 t) \Rightarrow \int_0^v dv = \int_0^t 10^3 \sin(10^3 t)\,dt$$

$$\therefore \quad v = \frac{10^3}{10^3}[1 - \cos(10^3 t)]$$

Velocity will be maximum when $\cos(10^3 t) = -1$

$$v_{\max} = 2 \text{ m/s}$$

7. **(5.55)** Given, $N = 50$,

$$A = 2 \times 10^{-4}\text{ m}^2, C = 10^{-4}, R = 50\,\Omega,$$

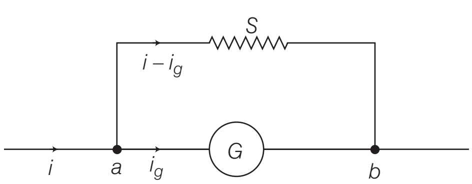
$$B = 0.02\text{ T}, \theta = 0.2\text{ rad}$$

$$\therefore \quad N i_g AB = C\theta$$

$$\Rightarrow \quad i_g = \frac{C\,\theta}{N\,AB} = \frac{10^{-4} \times 0.2}{50 \times 2 \times 10^{-4} \times 0.02} = 0.1\text{ A}$$

$$\therefore \quad V_{ab} = i_g \times G = (i - i_g)S$$

$$0.1 \times 50 = (1 - 0.1) \times S$$



$$5 = 0.9 \times S$$

$$\therefore \quad S = \frac{50}{9}\,\Omega = 5.55\,\Omega$$

8. **(b)** **List-II**

(1) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{d^2} \quad \Rightarrow \quad E \propto \dfrac{1}{d^2}$

(2) $E_{\text{axis}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q(2l)}{d^3} \quad \Rightarrow \quad E \propto \dfrac{1}{d^3}$

(3) $E = \dfrac{\lambda}{2\pi\varepsilon_0 d} \quad \Rightarrow \quad E \propto \dfrac{1}{d}$

(4) $E = \dfrac{\lambda}{2\pi\varepsilon_0(d-l)} - \dfrac{\lambda}{2\pi\varepsilon_0(d+l)} = \dfrac{\lambda(2l)}{2\pi\varepsilon_0 d^2}$

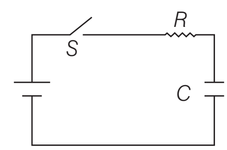
$$\Rightarrow \quad E \propto \frac{1}{d^2}$$

(5) $E = \dfrac{\sigma}{2\varepsilon_0} \quad \Rightarrow \quad E$ is independent of d

9. **(b)** When switch is closed for a very long time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor,

$$E_C = \frac{1}{2}CV^2 \qquad \ldots\text{(i)}$$



Work done by a battery,

$$W = Vq = VCV = CV^2$$

Energy dissipated across resistance

E_D = (work done by eq. battery) – (energy stored)

$$E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$E_D = E_C$$

10. **(a)** **For process (1)**

Charge on capacitor $= \dfrac{CV_0}{3}$

Energy stored in capacitor $= \dfrac{1}{2}C\dfrac{V_0^2}{9} = \dfrac{CV_0^2}{18}$

Work done by battery $= \dfrac{CV_0}{3} \times \dfrac{V}{3} = \dfrac{CV_0^2}{9}$

Heat loss $= \dfrac{CV_0^2}{9} - \dfrac{CV_0^2}{18} = \dfrac{CV_0^2}{18}$

For process (2)

Charge on capacitor $= \dfrac{2CV_0}{3}$

Extra charge flow through battery $= \dfrac{CV_0}{3}$

Work done by battery $= \dfrac{CV_0}{3} \cdot \dfrac{2V_0}{3} = \dfrac{2CV_0^2}{9}$

Final energy stored in capacitor $= \dfrac{1}{2}C\left(\dfrac{2V_0}{3}\right)^2 = \dfrac{4CV_0^2}{18}$

Energy stored in process 2 $= \dfrac{4CV_0^2}{18} - \dfrac{CV_0^2}{18} = \dfrac{3CV_0^2}{18}$

Heat loss in process (2) = work done by battery in process (2) – energy store in capacitor process (2)

$$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$$

For process (3)

Charge on capacitor $= CV_0$

Extra charge flown through battery

$$= CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$$

Work done by battery in this process

$$= \left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$$

Final energy stored in capacitor $= \frac{1}{2}CV_0^2$

Energy stored in this process

$$= \frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$$

Heat loss in process (3)

$$= \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$$

Now, total heat loss (E_D)

$$= \frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$$

Final energy stored in capacitor $= \frac{1}{2}CV_0^2$

So we can say that $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

11. **(c)** For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial velocity $\neq 0$) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity $= 0$)

12. **(a)** $\mathbf{F}_{\text{net}} = \mathbf{F}_e + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

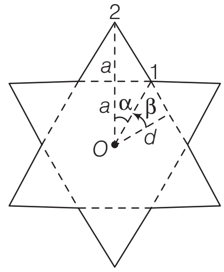
For particle to move in straight line with constant velocity, $\mathbf{F}_{\text{net}} = 0$

$\therefore \quad q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$

13. **(c)** For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.

For the given set of options only option (c) satisfy the condition. Path is helical with increasing pitch.

14. **(a)**



$$B_{12} = \frac{\mu_0 I}{4\pi d}[\sin\alpha + \sin\beta]$$

$\alpha = 60°$ and $\beta = -30°$

$$= \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

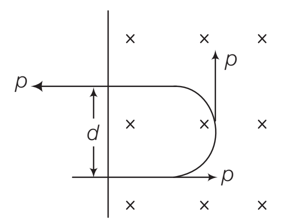
$$B_{12} = \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}-1}{2}\right]$$

$d = a$

$B_0 = 12\ B_{12}$

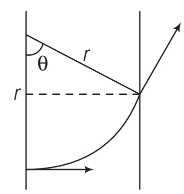
$$= 12 \times \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}-1}{2}\right] = \frac{\mu_0 I}{4\pi a} 6\,[\sqrt{3}-1]$$

15. **(b,c)** (a) $|\Delta\mathbf{P}| = \sqrt{2}p$



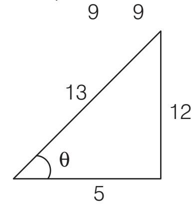
(b) $r(1-\cos\theta) = R \quad \Rightarrow \quad r\sin\theta = \frac{3R}{2}$

$$\frac{\sin\theta}{1-\cos\theta} = \frac{3}{2}\ \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{3}{2}$$



$$\cot\frac{\theta}{2} = \frac{3}{2} \quad \Rightarrow \quad \tan\frac{\theta}{2} = \frac{2}{3}$$

$$\Rightarrow \qquad \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{12}{5}$$



$\sin\theta = \frac{12}{13}\ r\left(\frac{12}{13}\right) = \frac{3R}{2}$; $r = \frac{13R}{8} = \frac{P}{QB}$; $B = \frac{8P}{13QR}$

(c) $\frac{P}{QB} < \frac{3R}{2}, B > \frac{2P}{3QR}$

(d) $r = \frac{mv}{QB}, d = 2r = \frac{2mv}{QB} \quad \Rightarrow \quad d \propto m$

16. **(a,b)** At $\omega \approx 0$, $X_C = \dfrac{1}{\omega C} = \infty$. Therefore, current is nearly zero.

Further at resonance frequency, current and voltage are in phase. This resonance frequency is given by,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rad/s}$$

We can see that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$

At, $\omega = \omega_r = 10^6$ rad/s, $X_L = X_C$.

For $\omega > \omega_r$, $X_L > X_C$. So, circuit is inductive.

17. **(b,d)** The net magnetic flux through the loops at time t is

$$\phi = B(2A - A)\cos \omega t = BA \cos \omega t$$

so, $\left|\dfrac{d\phi}{dt}\right| = B\omega A \sin\omega t$

$\therefore \left|\dfrac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \pi/2$

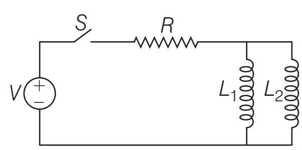
The emf induced in the smaller loop,

$\varepsilon_{\text{smaller}} = -\dfrac{d}{dt}(BA \cos \omega t) = B\omega A \sin\omega t$

$\therefore$ Amplitude of maximum net emf induced in both the loops

= Amplitude of maximum emf induced in the smaller loop alone.

18. **(a,b,c)**



Since inductors are connected in parallel

$$V_{L_1} = V_{L_2}; \quad L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$$

$$L_1 I_1 = L_2 I_2 ; \quad \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

Current through resistor at any time t is given by

$$I = \frac{V}{R}(1 - e^{-\frac{RT}{L}}), \quad \text{where} \quad L = \frac{L_1 L_2}{L_1 + L_2}$$

After long time $I = \dfrac{V}{R}$

$$I_1 + I_2 = I \qquad \ldots\text{(i)}$$

$$L_1 I_1 = L_2 I_2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I_1 = \frac{V}{R}\frac{L_2}{L_1 + L_2} \Rightarrow I_2 = \frac{V}{R}\frac{L_1}{L_1 + L_2}$$

(d) Value of current is zero at $t = 0$

Value of current is V/R at $t = \infty$

Hence option (d) is incorrect.

19. **(b,c)** $V_{XY} = V_0 \sin\left(\omega t + \dfrac{2\pi}{3}\right) - V_0 \sin \omega t$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin(\omega t + \pi)$$

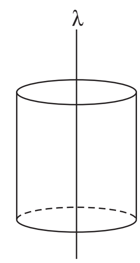
$$\Rightarrow \quad \phi = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\Rightarrow \quad V_0' = 2V_0 \cos\left(\frac{\pi}{6}\right) = \sqrt{3}\, V_0$$

$$\Rightarrow \quad V_{XY} = \sqrt{3} V_0 \sin(\omega t + \phi)$$

$$\Rightarrow \quad (V_{XY})_{\text{rms}} = (V_{YZ})_{\text{rms}} = \sqrt{3}\frac{V_0}{\sqrt{2}}$$

20. **(d)** Suppose charger per unit length at any instant is λ.



Initial value of λ is suppose λ_0.

Electric field s at a distance r at any instant is

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$J = \sigma E = \sigma \frac{\lambda}{2\pi\varepsilon r}$$

$$i = \frac{dq}{dt} = J(A) = -J\sigma 2\pi r l$$

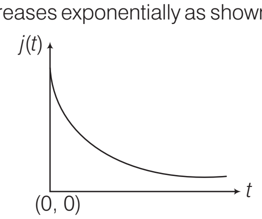
$$\frac{d\lambda l}{dt} = -\frac{\lambda}{2\pi\varepsilon r} \times \sigma 2\pi r l \qquad (q = \lambda l)$$

$$\int_{\lambda_0}^{\lambda} \frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\int_0^t dt \quad \Rightarrow \lambda = \lambda_0 e^{-\frac{\sigma}{\varepsilon}t}$$

$$J = \frac{\sigma}{2\pi\varepsilon r}\lambda = \frac{\sigma\lambda_0}{2\pi\varepsilon r} e^{-\frac{\sigma}{\varepsilon}t} = J_0 e^{-\frac{\sigma}{\varepsilon}t}$$

Here, $J_0 = \dfrac{\sigma\lambda_0}{2\pi\varepsilon r}$

$\therefore J(t)$ decreases exponentially as shown in figure below.



21. **(a,b,c,d)** Just after pressing key,

$5 - 25000 i_1 = 0$

$5 - 50000 i_2 = 0$ (As charge in both capacitors = 0)

$\Rightarrow i_1 = 0.2$ mA $\Rightarrow i_2 = 0.1$ mA

and $V_B + 25000 i_1 = V_A \Rightarrow V_B - V_A = -5$ V

After a long time, i_1 and $i_2 = 0$ (steady state)

$$\Rightarrow \quad 5 - \frac{q_1}{40} = 0 \Rightarrow q_1 = 200\mu\text{C}$$

and $\quad 5 - \dfrac{q_2}{20} = 0 \Rightarrow q_2 = 100\mu\text{C}$

$$V_B - \frac{q_2}{20} = V_A \Rightarrow V_B - V_A = +5 \text{ V}$$

$\Rightarrow$ (a) is correct.

For capacitor 1, $q_1 = 200[1 - e^{-t/1}]\,\mu C$

$$i_1 = \frac{1}{5}e^{-t/1} \text{ mA}$$

For capacitor 2, $q_2 = 100[1 - e^{-t/1}]\,\mu C$

$$i_2 = \frac{1}{10}e^{-t/1} \text{ mA}$$

$$\Rightarrow \quad V_B - \frac{q_2}{20} + i_1 \times 25 = V_A$$

$$\Rightarrow \quad V_B - V_A = 5[1 - e^{-t}] - 5e^{-t}$$

$$= -5[1 - 2e^{-t}]$$

At $t = \ln 2$, $V_B - V_A = 5[1 - 1] = 0$

$\Rightarrow$ (b) is correct.

At $t = 1$, $i = i_1 + i_2 = \frac{1}{5}e^{-1} + \frac{1}{10}e^{-1} = \frac{3}{10}\cdot\frac{1}{e}$

At $t = 0$, $i = i_1 + i_2 = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

$\Rightarrow$ (c) is correct.

After a long time, $i_1 = i_2 = 0 \Rightarrow$ (d) is correct.

22. **(d)** Balls will gain positive charge and hence move towards negative plate.

On reaching negative plate, balls will attain negative charge and come back to positive plate.

and so on, balls will keep oscillating.

But oscillation is not S.H.M.,

As force on balls is not $\propto x$.

$\Rightarrow$ option (d) is correct.

23. **(a)** As the balls keep on carrying charge form one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q,

$$\frac{kq}{r} = V_0 \left(k = \frac{1}{4\pi\varepsilon_0}\right) \Rightarrow q = \frac{V_0 r}{k}$$

Inside cylinder, electric field

$$E = [V_0 - (-V_0)]h = 2V_0 h.$$

$\Rightarrow$ Acceleration of each ball,

$$a = \frac{qE}{m} = \frac{2hr}{km}\cdot V_0^2$$

$\Rightarrow$ Time taken by balls to reach other plate,

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h.km}{2hrV_0^2}} = \frac{1}{V_0}\sqrt{\frac{km}{r}}$$

If there are n balls, then

Average current,

$$i_{av} = \frac{nq}{t} = n \times \frac{V_0 r}{k} \times V_0\sqrt{\frac{r}{km}} \quad \Rightarrow i_{av} \propto V_0^2$$

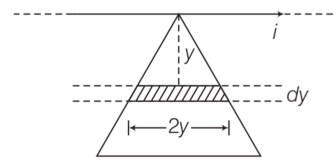
24. **(c,d)** Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as whole increases with time.

Overall resistance increases hence power decreases. $\left(P = \frac{V^2}{R} \text{ or } P \propto \frac{1}{R} \text{ as } V \text{ is constant}\right)$. At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

25. **(a,c)** By reciprocity theorem of mutual induction, it can be assumed that current in infinite wire is varying at 10A/s and EMF is induced in triangular loop.



Flux of magnetic field through triangle loop, if current in infinite wire is ϕ, can be calculated as follows:

$$d\phi = \frac{\mu_0 i}{2\pi y}\cdot 2y\,dy \quad \Rightarrow \quad d\phi = \frac{\mu_0 i}{\pi}dy$$

$$\Rightarrow \quad \phi = \frac{\mu_0 i}{\pi}\left(\frac{l}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \text{EMF} = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0}{\pi}\left(\frac{l}{\sqrt{2}}\right)\cdot\frac{di}{dt}$$

$$= \frac{\mu_0}{\pi}(10 \text{ cm})\left(10\frac{\text{A}}{\text{s}}\right) = \frac{\mu_0}{\pi} \text{ volt}$$

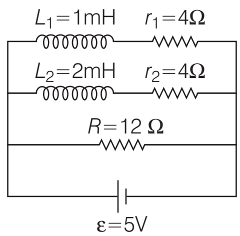
If we assume the current in the wire towards right then as the flux in the loop increases we know that the induced current in the wire is counter clockwise. Hence, the current in the wire is towards right.

Field due to triangular loop at the location of infinite wire is into the paper. Hence, force on infinite wire is away from the loop.

By cylindrical symmetry about infinite wire, rotation of triangular loop will not cause any additional EMF.

26. **(a,c)** For maximum range of voltage resistance should be maximum. So, all four should be connected in series. For maximum range of current, net resistance should be least. Therefore, all four should be connected in parallel.

27. **(8)**



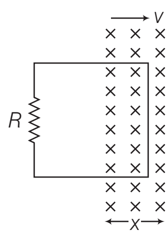
$$I_{max} = \frac{\varepsilon}{R} = \frac{5}{12}\text{A} \qquad \text{(Initially at } t = 0)$$

$$I_{min} = \frac{\varepsilon}{R_{eq}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right) \qquad \text{(finally in steady state)}$$

$$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3}\text{A}$$

$$\frac{I_{max}}{I_{min}} = 8$$

28. **(b,c)**



When loop was entering ($x < L$)

$$\phi = BLx$$

$$e = -\frac{d\phi}{dt} = -BL\frac{dx}{dt}$$

$$|e| = BLv$$

$$i = \frac{e}{R} = \frac{BLv}{R} \qquad \text{(anticlockwise)}$$

$$F = ilB \text{ (Left direction)} = \frac{B^2L^2v}{R} \qquad \text{(in left direction)}$$

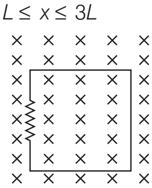
$$\Rightarrow \quad a = \frac{F}{m} = -\frac{B^2L^2v}{mR} \quad \Rightarrow \quad a = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = -\frac{B^2L^2v}{mR} \quad \Rightarrow \quad \int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx$$

$$\Rightarrow v = v_0 - \frac{B^2L^2v}{mR}x$$

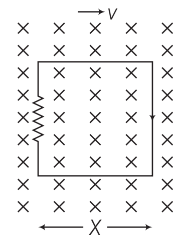
(straight line of negative slope for $x < L$)

$I = \frac{BL}{R}v \Rightarrow$ (I vs x will also be straight line of negative slope for $x < L$) $L \le x \le 3L$



$$\frac{d\phi}{dt} = 0 \quad e = 0, i = 0$$

$$F = 0 \Rightarrow x > 4L \Rightarrow e = Blv$$



Force also will be in left direction.

$$i = \frac{BLv}{R} \text{ (clockwise)}$$

$$a = -\frac{B^2L^2v}{mR} = v\frac{dv}{dx}$$

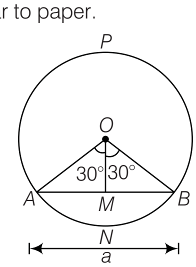
$$F = \frac{B^2L^2v}{R}\int_L^x -\frac{B^2L^2}{mR}dx = \int_{v_i}^{v_f} dv$$

$$\Rightarrow \quad -\frac{B^2L^2}{mR}(x - L) = v_f - v_i$$

$$v_f = v_i - \frac{B^2L^2}{mR}(x - L) \qquad \text{(straight line of negative slope)}$$

$$I = \frac{BLv}{R} \rightarrow \text{(Clockwise)} \qquad \text{(straight line of negative slope)}$$

29. **(6)** *ANBP* is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$$AM = \frac{a}{2}, MO = \frac{\sqrt{3}a}{2}$$

$$\therefore \quad \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ Electric flux passing through *ABCD* plane surface (shown only *AB*) = Electric flux passing through cylindrical surface *ANB*

$$= \left(\frac{60°}{360°}\right)(\phi_1) = \frac{\lambda L}{6\varepsilon_0}$$

$$\therefore \qquad n = 6$$

30. **(d)** The sphere with cavity can be assumed as a complete sphere with positive charge of radius R_1 + another complete sphere with negative charge and radius R_2.

$\mathbf{E}_+ \rightarrow \mathbf{E}$ due to total positive charge

$\mathbf{E}_- \rightarrow \mathbf{E}$ due to total negative charge.

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$

If we calculate it at P, then $\mathbf{E}_-$ comes out to be zero.

$\therefore \quad \mathbf{E} = \mathbf{E}_+$

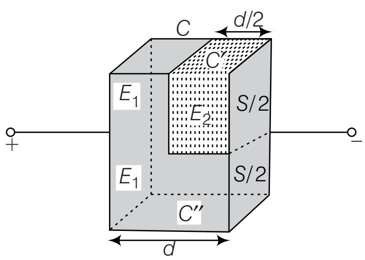
and $\mathbf{E}_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R_1^3}(OP)$, in the direction of OP.

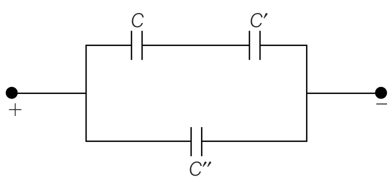
Here, q is total positive charge on whole sphere.

It is in the direction of OP or $\mathbf{a}$.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

31. **(d)**





$$C_1 = \frac{\varepsilon_0 S}{d}, C = \frac{2\varepsilon_0 \frac{S}{2}}{d/2} = \frac{2\varepsilon_0 S}{d}$$

$$C' = \frac{4\varepsilon_0 \frac{S}{2}}{d/2} = \frac{4\varepsilon_0 S}{d}$$

and $$C'' = \frac{2\varepsilon_0 \frac{S}{2}}{d} = \frac{\varepsilon_0 S}{d}$$

$$C_2 = \frac{CC'}{C + C'} + C'' = \frac{4}{3}\frac{\varepsilon_0 S}{d} + \frac{\varepsilon_0 S}{d}$$

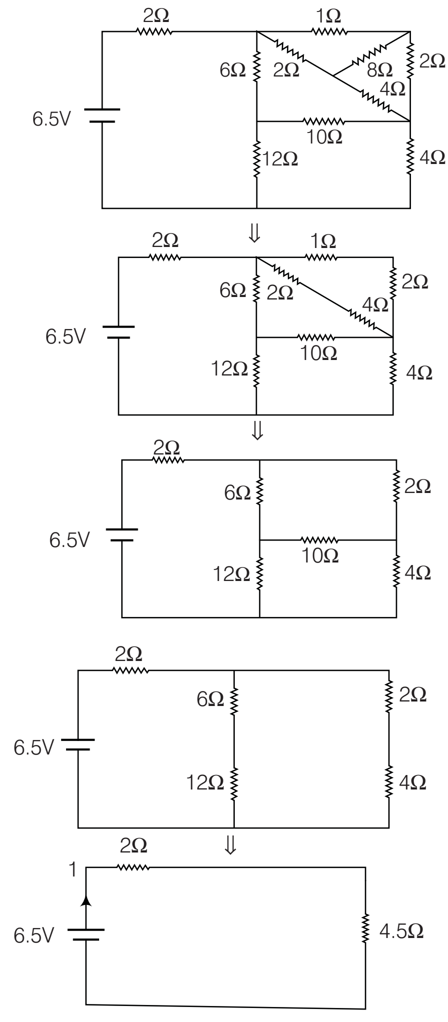
$$= \frac{7}{3}\frac{\varepsilon_0 S}{d} \quad \frac{C_2}{C_1} = \frac{7}{3}$$

32. **(b)** $\dfrac{1}{R} = \dfrac{1}{R_{\text{Al}}} + \dfrac{1}{R_{\text{Fe}}} = \left(\dfrac{A_{\text{Al}}}{\rho_{\text{Al}}} + \dfrac{A_{\text{Fe}}}{\rho_{\text{Fe}}}\right)\dfrac{1}{\ell}$

$$= \left[\frac{(7^2 - 2^2)}{2.7} + \frac{2^2}{10}\right]\frac{10^{-6}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

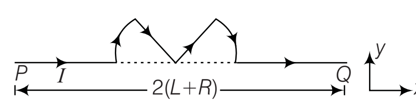
Solving we get, $R = \dfrac{1875}{64} \times 10^{-6}\ \Omega = \dfrac{1875}{64}\ \mu\Omega$

33. **(1)**



34. **(c)** At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

35. **(a,b,c)**



Force on the complete wire = force on straight wire PQ carrying a current I.

$$\mathbf{F} = I(\mathbf{PQ} \times \mathbf{B})$$

$$= I[\{2(L + R)\hat{\mathbf{i}}\} \times \mathbf{B}]$$

This force is zero if $\mathbf{B}$ is along $\hat{\mathbf{i}}$ direction or x-direction.
If magnetic field is along $\hat{\mathbf{j}}$ direction or $\hat{\mathbf{k}}$ direction,

$$|\mathbf{F}| = F = (I)(2)(L + R)B\sin 90°$$

or $\quad F = 2I(L + R)B$

or $\quad F \propto (L + R)$

$\therefore$ Options (a), (b) and (c) are correct.

36. **(a,d)** $F_B = Bev = Be\dfrac{I}{nAe} = \dfrac{BI}{nA}$

$$F_e = eE \Rightarrow F_e = F_B$$

$$eE = \frac{BI}{nA} \Rightarrow E = \frac{B}{nAe}$$

$$V = Ed = \frac{BI}{nAe}\cdot w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$$

$$\frac{V_1}{V_2} = \frac{d_2}{d_1}$$

$\Rightarrow$ if $w_1 = 2w_2$ and $d_1 = d_2$

$$V_1 = V_2$$

$\therefore$ Correct answers are (a) and (d).

37. **(a,c)** $V = \dfrac{BI}{ned} \Rightarrow \dfrac{V_1}{V_2} = \dfrac{B_1}{B_2}\times\dfrac{n_2}{n_1}$

If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
$\therefore$ Correct answers are (a) and (c).

38. **(a,d)** $C = C_1 + C_2$

$$C_1 = \frac{K\varepsilon_0 A/3}{d}, C_2 = \frac{\varepsilon_0 2A/3}{d}$$

$$\Rightarrow \quad C = \frac{(K+2)\varepsilon_0 A}{3d} \Rightarrow \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \dfrac{V}{d}$, where V is potential difference between the plates.

39. **(c, d)** $\dfrac{Q}{4\pi\varepsilon_0 r_0^2} = \dfrac{\lambda}{2\pi\varepsilon_0 r_0} = \dfrac{\sigma}{2\varepsilon_0}$

$$Q = 2\pi\sigma r_0^2$$

(a) is incorrect, $r_0 = \dfrac{\lambda}{\pi\sigma}$

(b) is incorrect, $E_1\left(\dfrac{r_0}{2}\right) = 4E_1(r_0)$

As $\quad E_1 \propto \dfrac{1}{r^2}$

$$E_2\left(\frac{r_0}{2}\right) = 2E_2(r_0) \text{ as } E_2 \propto \frac{1}{r}$$

$\Rightarrow$ (c) is correct

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$$

as $\quad E_3 \propto r^0$

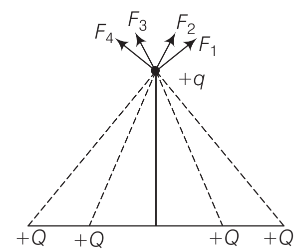
$\Rightarrow$ (d) is incorrect

40. **(c)** $E_1 = \dfrac{kQ}{R^2}$, where $k = \dfrac{1}{4\pi\varepsilon_0}$

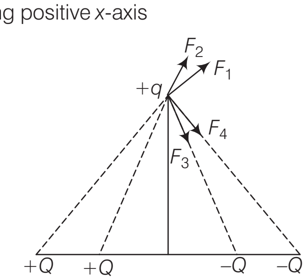
$$E_2 = \frac{k(2Q)}{R^2} \Rightarrow E_2 = \frac{2kQ}{R^2}$$

$$E_3 = \frac{k(4Q)R}{(2R)^3} \Rightarrow E_3 = \frac{kQ}{2R^2} \quad E_3 < E_1 < E_2$$

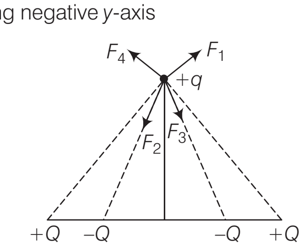
41. **(c)** (P) Component of forces along x-axis will vanish. Net force along positive y-axis



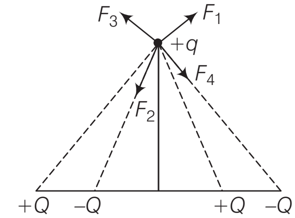
(Q) Component of forces along y-axis will vanish. Net force along positive x-axis



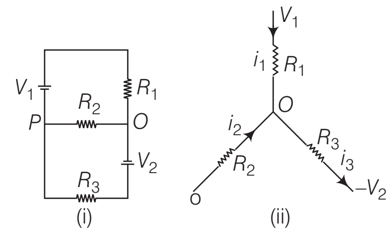
(R) Component of forces along x-axis will vanish. Net force along negative y-axis



(S) Component of forces along y-axis will vanish. Net force along negative x-axis



42. **(a,b,d)** Let us take $V_P = 0$. Then potentials across R_1, R_2 and R_3 are as shown in figure (ii)
In the same figure



$$i_1 + i_2 = i_3$$

$$\therefore \quad \frac{V_1 - V_0}{R_1} + \frac{0 - V_0}{R_2} = \frac{V_0 - (-V_2)}{R_3}$$

Solving this equation we get

$$V_0 = \frac{\frac{V_1}{R_1} + 0 - \frac{V_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Current through R_2 will be zero if

$$V_0 = 0 \quad \Rightarrow \quad \frac{V_1}{V_2} = \frac{R_1}{R_3}$$

In options (a), (b) and (d) this relation is satisfied.

43. **(c)** For balanced meter bridge

$$\frac{X}{R} = \frac{l}{(100 - l)} \qquad \text{(where, } R = 90\ \Omega)$$

$$\therefore \quad \frac{X}{90} = \frac{40}{100 - 40}$$

$$\therefore \quad X = 60\ \Omega$$

$$X = R\,\frac{l}{(100 - l)}$$

$$\frac{\Delta X}{X} = \frac{\Delta l}{l} + \frac{\Delta l}{100 - l} = \frac{0.1}{40} + \frac{0.1}{60}$$

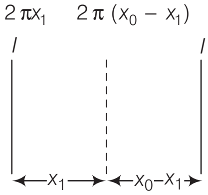
$$\Delta X = 0.25$$

So, $X = (60 \pm 0.25)\ \Omega$

44. **(3)** $B_2 = \dfrac{\mu_0 I}{2\pi x_1} + \dfrac{\mu_0 I}{2\pi (x_0 - x_1)}$

(when currents are in opposite directions)

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi (x_0 - x_1)}$$



(when currents are in same direction)

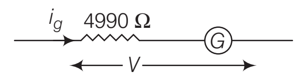
Substituting $x_1 = \dfrac{x_0}{3}$ $\quad$ (as $\dfrac{x_0}{x_1} = 3$)

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$R_1 = \frac{mv}{qB_1} \quad \text{and} \quad B_2 = \frac{9\mu_0 I}{4\pi x_0}$$

$$R_2 = \frac{mv}{qB_2} \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

45. **(5)**



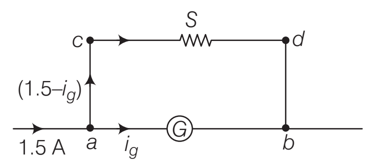
$$i_g (G + 4990) = V$$

$$\Rightarrow \quad \frac{6}{1000}(G + 4990) = 30$$

$$\Rightarrow \quad G + 4990 = \frac{30{,}000}{6} = 5000$$

$$\Rightarrow \quad G = 10\ \Omega$$

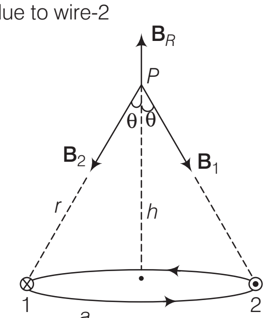
$$V_{ab} = V_{cd} \quad \Rightarrow \iota_\gamma \Gamma = (1.5 - \iota_\gamma)\,\Sigma$$



$$\Rightarrow \quad \frac{6}{1000} \times 10 = \left(1.5 - \frac{6}{1000}\right) S$$

$$\Rightarrow \quad S = \frac{60}{1494} = \frac{2n}{249} \quad \Rightarrow n = \frac{249 \times 30}{1494} = \frac{2490}{498} = 5$$

46. **(c)** $\mathbf{B}_R = \mathbf{B}$ due to ring
$\mathbf{B}_1 = \mathbf{B}$ due to wire-1
$\mathbf{B}_2 = \mathbf{B}$ due to wire-2



In magnitudes $\mathbf{B}_1 = \mathbf{B}_2 = \dfrac{\mu_0 I}{2\pi r}$

Resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$

$$= 2\mathbf{B}_1 \cos\theta = 2\left(\frac{\mu_0 I}{2\pi r}\right)\left(\frac{h}{r}\right) = \frac{\mu_0 I h}{\pi r^2}$$

$$B_R = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} = \frac{2\mu_0 I \pi a^2}{4\pi r^3}$$

As, $R = a,\ x = h$ and $a^2 + h^2 = r^2$

For zero magnetic field at P,

$$\frac{\mu_0 I h}{\pi r^2} = \frac{2\mu_0 I \pi a^2}{4\pi r^3} \Rightarrow \pi a^2 = 2rh \quad \Rightarrow \eta \approx 1.2\alpha$$

47. **(b)** Magnetic field at mid-point of two wires

$= 2$ (magnetic field due to one wire) $= 2\left[\frac{\mu_0}{2\pi}\frac{I}{d}\right]$

$= \frac{\mu_0 I}{\pi d} \otimes$

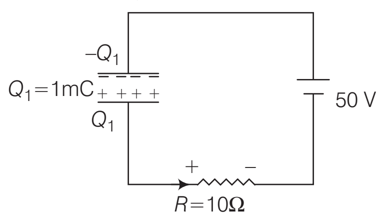
Magnetic moment of loop $M = IA = I\pi a^2$

Torque on loop $= MB \sin 150° = \frac{\mu_0 I^2 a^2}{2d}$

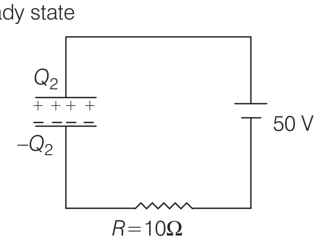
48. **(c,d)** $\frac{dQ}{dt} = I \quad \Rightarrow \quad Q = \int I\,dt = \int (I_0 \cos \omega t)\,dt$

$\therefore \quad Q_{max} = \frac{I_0}{\omega} = \frac{1}{500} = 2 \times 10^{-3}$ C

Just after switching



In steady state



At $\quad t = \frac{7\pi}{6\omega} \quad$ or $\quad \omega t = \frac{7\pi}{6}$

Current comes out to be negative from the given expression. So, current is anti-clockwise.

Charge supplied by source from $t = 0$ to

$$t = \frac{7\pi}{6\omega} \quad \Rightarrow \quad Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500t)\,dt$$

$$= \left[\frac{\sin 500t}{500}\right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin \frac{7\pi}{6}}{500} = -1\,\text{mC}$$

Apply Kirchhoff's loop law just after changing the switch to position D

$$50 + \frac{Q_1}{C} - IR = 0$$

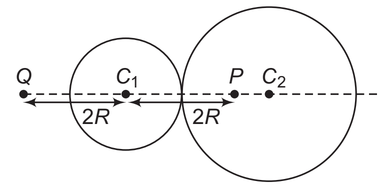
Substituting the values of Q_1, C and R we get

$$I = 10 \text{ A}$$

In steady state $Q_2 = CV = 1\,\text{mC}$

$\therefore$ Net charge flown from battery $= 2$ mC

49. **(b,d)** **At point P**



If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3}R \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$$

At point Q

If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25} \quad (\rho_1 \text{ must be negative})$$

50. **(b, d)** After pressing S_1 charge on upper plate of C_1 is $+2CV_0$.

After pressing S_2 this charge equally distributes in two capacitors. Therefore charge an upper plates of both capacitors will be $+CV_0$.

When S_2 is released and S_3 is pressed, charge on upper plate of C_1 remains unchanged $(= +CV_0)$ but charge on upper plate of C_2 is according to new battery $(= -CV_0)$.

51. **(c,d)** For electrostatic field,

Field at P

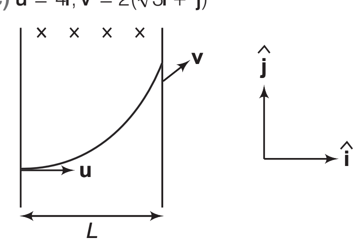
$$\mathbf{E}_P = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{P} + \frac{(-\rho)}{3\varepsilon_0}\mathbf{C}_2\mathbf{P}$$

$$= \frac{\rho}{3\varepsilon_0}(\mathbf{C}_1\mathbf{P} + \mathbf{PC}_2)$$

$$\mathbf{E}_P = \frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{C}_2$$

For electrostatic. Since, electric field is non-zero so it is not equipotential.

52. **(a,c)** $\mathbf{u} = 4\mathbf{i}$; $\mathbf{v} = 2(\sqrt{3}\mathbf{i} + \mathbf{j})$



According to the figure, magnetic field should be in ⊗ direction, or along $-z$ direction.

Further, $\tan\theta = \frac{v_y}{v_x} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30° \quad \text{or} \quad \frac{\pi}{6}$

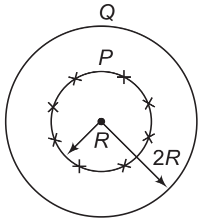
$= \text{angle of } \mathbf{v} \text{ with } x\text{-axis}$

$= \text{angle rotated by the particle}$

$= Wt = \left(\frac{BQ}{M}\right)t$

$\therefore \quad B = \frac{\pi M}{6Qt} = \frac{50\pi M}{3Q} \text{ units} \qquad (\text{as } t = 10^{-3} \text{ second})$

53. (a,d)



In the region, $0 < r < R$

$B_P = 0,$

$B_Q \neq 0$, along the axis

$\therefore \quad B_{\text{net}} \neq 0$

In the region, $R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

$\therefore B_{\text{net}} \neq 0$ neither in the directions mentioned in options (b) or (c).

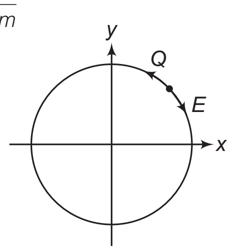
In region, $r > 2R$

$B_P \neq 0$

$B_Q \neq 0$

$\therefore \quad B_{\text{net}} \neq 0$

54. (b) $\frac{M}{L} = \frac{Q}{2m}$



$\therefore \quad M = \left(\frac{Q}{2m}\right)L \quad \Rightarrow \quad M \propto L,$

where $\gamma = \frac{Q}{2m} = \left(\frac{Q}{2m}\right)(I\omega)$

$= \left(\frac{Q}{2m}\right)(mR^2\omega) = \frac{Q\omega R^2}{2}$

Induced electric field is opposite. Therefore,

$\omega' = \omega - \alpha t$

$$\alpha = \frac{\tau}{I} = \frac{(QE)R}{mR^2} = \frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2} = \frac{QB}{2m}$$

$$\therefore \quad \omega' = \omega - \frac{QB}{2m}\cdot 1 = \omega - \frac{QB}{2m}$$

$$M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\therefore \quad \Delta M = M_f - M_i = -\frac{Q^2BR^2}{4m}$$

$$M = -\gamma\frac{QBR^2}{2} \qquad \left(\text{as } \gamma = \frac{Q}{2m}\right)$$

55. (b) The induced electric field is given by,

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\phi}{dt} \quad \text{or} \quad El = -s\left(\frac{dB}{dt}\right)$$

$\therefore \quad E(2\pi R) = -(\pi R^2)(B)$

or $\quad E = -\frac{BR}{2}$

56. (b) $\quad P = Vi$

$\therefore \quad i = \frac{P}{V} = \frac{600 \times 10^3}{4000} = 150 \text{ A}$

Total resistance of cables,

$R = 0.4 \times 20 = 8\Omega$

$\therefore$ Power loss in cables

$= i^2R = (150)^2(8)$

$= 180000 \text{ W} = 180 \text{ kW}$

This loss is 30% of 600 kW.

57. (a) During step-up,

$\frac{N_p}{N_s} = \frac{V_p}{V_s}$

or $\quad \frac{1}{10} = \frac{4000}{V_s}$

or $\quad V_s = 40{,}000 \text{ V}$

In step, down transformer,

$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = \frac{200}{1}$